# 量子Zeno 效应与量子测量问题

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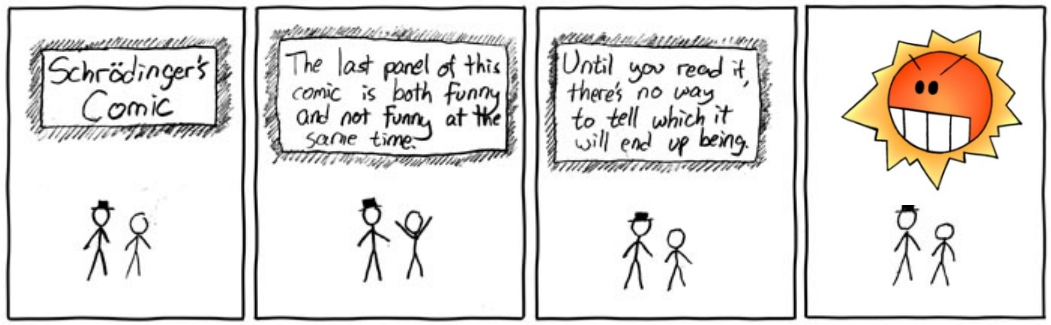


# Content

1. Quantum Zeno effect (QZE)

   "proves" projective measurement?

2. No Projection in reality for QZE:

   Revisiting existing experiments

3. Anti-QZE for open quantum system

# 参考文献


**Quantum Zeno Effect:**

1. L. A. Khalhin, JETP Lett. 8, 65 (1968).
2. B. Misra, E. C. G. Sudarshan, J. Math. Phys. 18, 756 (1977).

**Quantum Anti-Zeno Effect:**

3. A. M. Lane, Phys. Lett. A 99, 359 (1983).
4. A. G. Kofman & G. Kurizki, Nature 405, 546 (2000).
5. P. Facchi, H. Nakazato, and S. Pascazio, Phys. Rev. Lett. 86, 2699 (2001).

**Quantum (Anti-)Zeno and Rotating-Wave Approximation:**

6. H. Zheng, S. Y. Zhu, M. S. Zubairy, Phys. Rev. Lett. 101, 200404 (2008).
7. Q. Ai, Y. Li, H. Zheng, C. P. Sun, Phys. Rev. A 81, 042116 (2010).
8. Q. Ai, J. Q. Liao, C. P. Sun, arXiv:1003.4587 (2010).

**Dynamic Approach for Quantum Zeno Effect:**

9. W. M. Itano, D. J. Heinzen, J. J. Bollinger, D. J. Wineland, Phys. Rev. A 41, 2295 (1990).
10. L. E. Ballentine, Phys. Rev. A 43, 5165 (1991).
11. C. P. Sun, in Quantum Classical Correspondence: The 4th Drexel Symposium on Quantum Nonintegrability, 1994, edited by D. H. Feng and B. L. Hu (International Press, Cambridge, MA), p. 99. For a review, see C. P. Sun, X. X. Yi and X. J. Liu, Fortschr. Phys. 43, 585 (1995).

**Dynamic Approach for Quantum Anti-Zeno Effect:**

12. Q. Ai, D. Z. Xu, A. G. Kofman, C. P. Sun, F. Nori, arXiv:1007.4859 (2010).

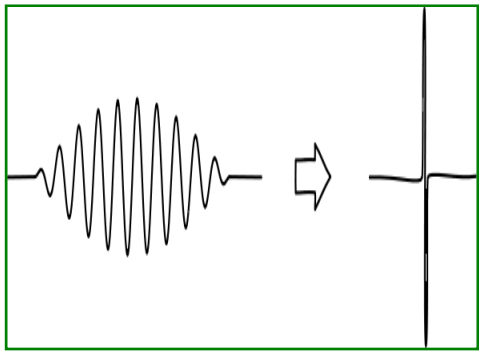
**Review on Quantum Zeno Effect:**

13. K. Koshinoa and A. Shimizu, Phys. Rep. 412, 191 (2005).

# Motivation for our recent studies

Quantum Zeno Effects： state evolution is inhibited by repetitive projective measurements

**How to predict/explain quantum Zeno/anti-Zeno phenomena ( including the recent experimental data) , only based on the dynamic model of quantum measurement, without the use of postulate of wave packet collapse**



1. D. Z. Xu, Q. Ai, C. P. Sun, arXiv:1007.4634 :
   Decoherence -based quantum Zeno effect in a cavity-QED System
2. P.Zhang, Q.Ai, C.P.Sun in preparation
   Geaneral approach to quatum Zeno/anti-Zeno efect without projective measurement

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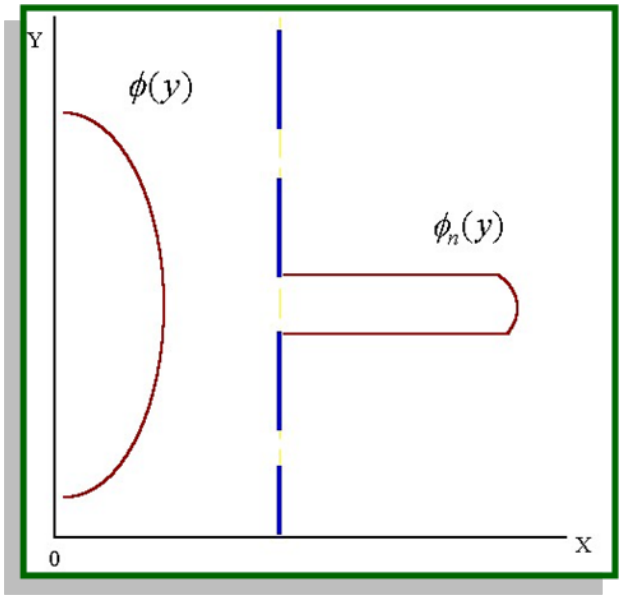
1. Quantum Zeno effect (QZE)

"proves" projective measurement?

# Von Neumann's projective postulate in quantum mechanics

**von Neumann 1929**




**Born rule**

$$|\psi\rangle = \sum_a c_a |a\rangle$$

$$(A|a\rangle = a|a\rangle)$$

**Probability of getting value of observable *A***

$$P_a = |c_a|^2$$

**Wave Packet Collapse (WPC),**
**Projection Measurement ,**
**Wave Function Reduction,**
**Strong measurement**

**State selection after projection:**

$$|\Psi\rangle \Rightarrow |a\rangle$$

# WPC results in quantum Zeno effect

**Misra, Sudarshan, J. Math. Phys. (N.Y.) 18, 756 (1977)**

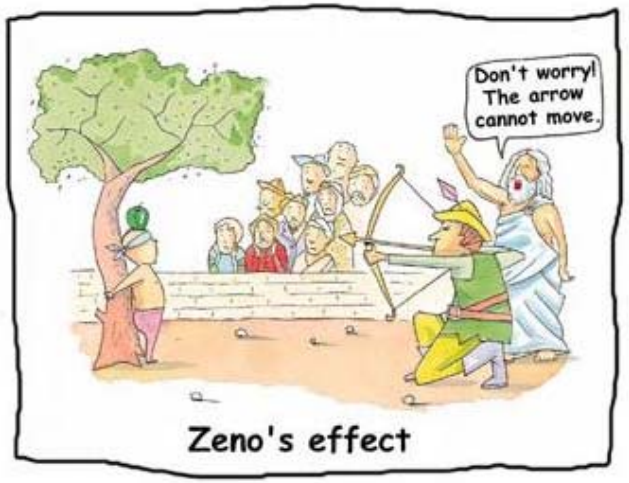
WFC postulate predicted the survival probability in initial state after *n* projection measurements

$$P(\tau=n\tau)=\left|\langle\psi_0|e^{-iH\tau}|\psi_0\rangle\right|^{2n}$$

$$\approx\left|\langle\psi_0|1-iH\tau-\frac{1}{2}H^2\tau^2|\psi_0\rangle\right|^{2n}$$

$$=\left[1-(\Delta H)^2\tau^2\right]^n\approx e^{-(\Delta H)^2\frac{t^2}{n}}$$

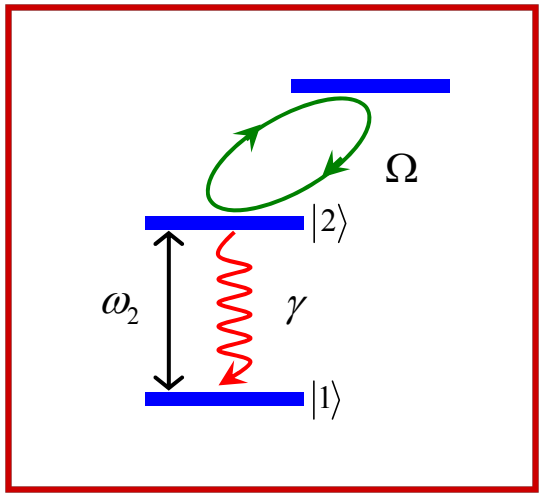
$$\xrightarrow{n\to\infty}1\quad,\ t=n\tau\ \text{fixed}$$




QZE: state evolution is inhibited by repetitive projective measurements

# A Peres' conjecture

A. Peres, Am. J. Phys 48, 931 (1980).




The coupling of an unstable quantum system with a measuring apparatus alters the dynamical properties of the former, in particular, its decay law.

The decay is usually slowed down and can be completely halted by a very tight monitoring, namely $\Omega >> \gamma$.

Projective measurements can be approximately by strong couplings to external agents

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# Question about QZE

Ballentine,1991 et al

Is the von Neumann's projection postulate necessary for explaining these existing experiments of QZE ?

We even made theoretical analysis for the trapped ion experiment for QZE by Wineland' group [Phys. Rev. A 41, 2295 (1990) ],concluding its un-necessarity

C. P. Sun, in Quantum Classical Correspondence: The 4th Drexel Symposium on Quantum Nonintegrability, 1994, edited by D. H. Feng and B. L. Hu (International Press, Cambridge, MA), p. 99.

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# Experiments testing QZE

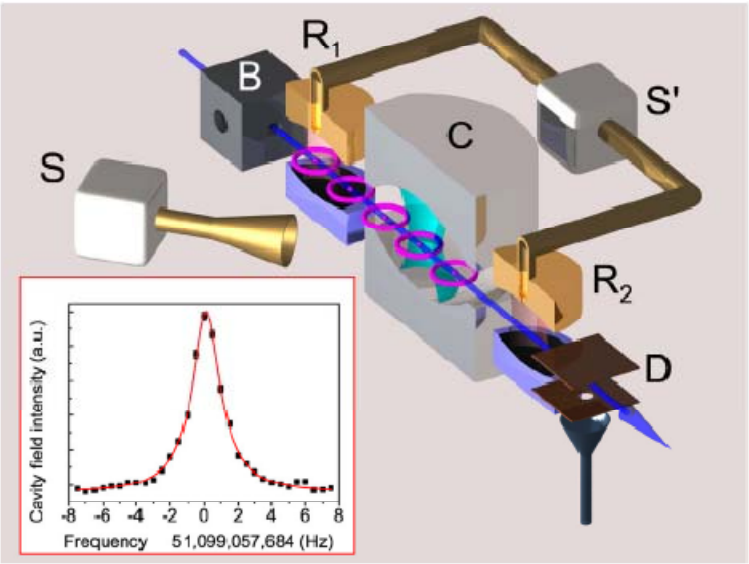
**Seem to well support von Neumann's projection postulate !**

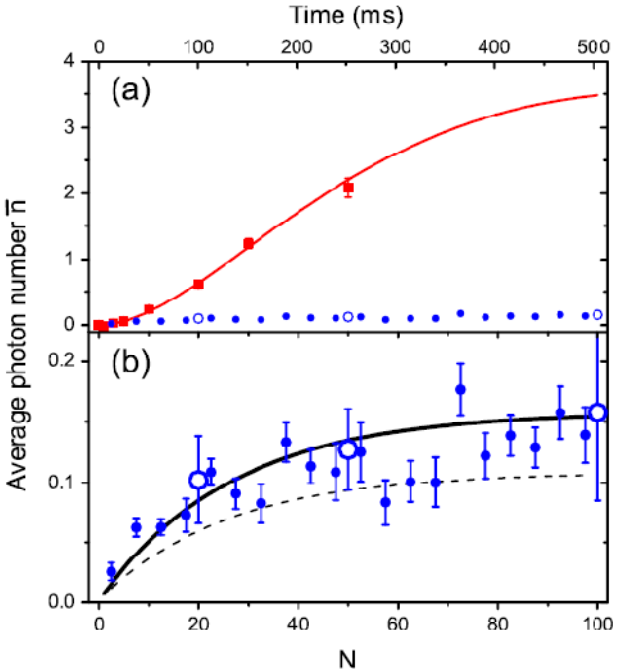
Cold atoms: Raizen's Group, Phys. Rev. Lett. 87, 040402 (2001)
BEC: Pritchard' group, Phys. Rev. Lett. 97, 260402 (2006)

Cavity QED: Haroche's group, Phys. Rev. Lett. 101, 180402 (2008)

# Rivist the QZE experiment

Cavity QED:.Haroche's Phys. Rev. Lett. **101**, 180402 (2008)




Freezing the photon number in the cavity

# Foundamental problems in QM

Can the results from projective measurements be generally realized by special couplings to external agents – measuring apparatus?

QZE (or QAZE) depends on quantum mechanics interpretation (QMI), or is Universal ?

# Our dynamic approach to quantum measurement

1. **Quantum dynamical model for wave-function reduction in classical and macroscopic limits**
   **C.P. Sun, Phys. Rev. A 48, 898 (1993)**

And a series of 17 following papers by my collaborators and me on Schroedinger cat paradox, decoherence and localization of macroscopic objects and open quantum systems

**e.g.,**
**Consistent approach for quantum measurement**
**P. Zhang, X. F. Liu, C. P. Sun,Phys. Rev. A 66, 042104 (2002)**

For a review of our related works before "quantum information time", pleases see,
**C. P. Sun, X. X. Yi, X. J. Liu, Fortschr. Phys. 43, 585 (1995).**

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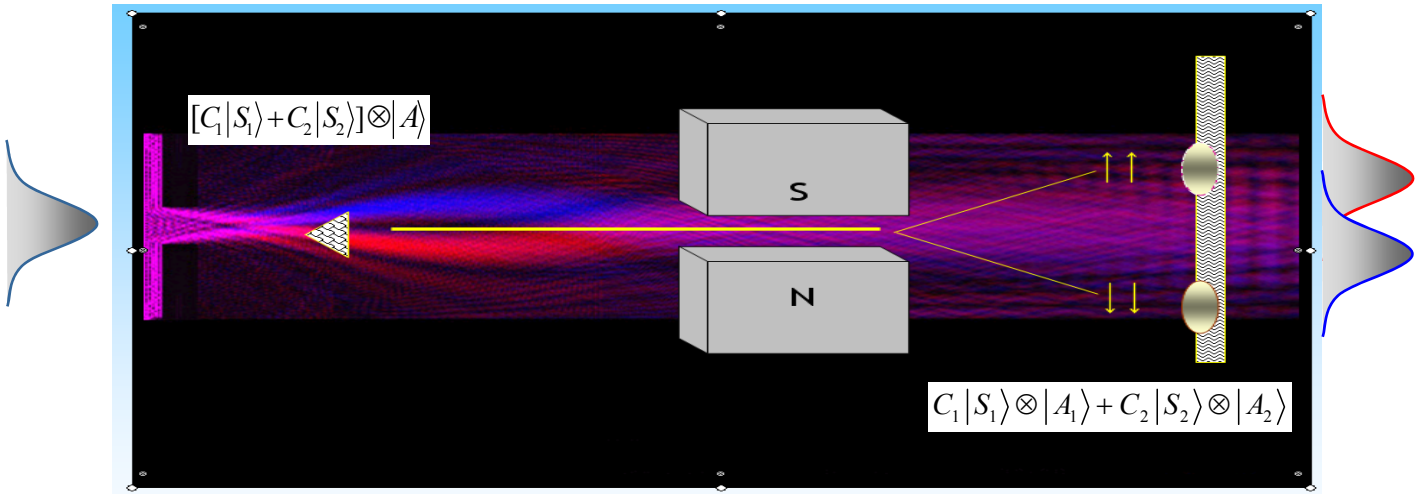
## 2 . No need projective measurement in reality for QZE: Revisiting existing experiments

# Decoherence-based quantum measurement




$$M(\tau_m) = \text{Measurememt Operator}$$

$$|\psi(0)\rangle = \left(\sum_j C_j |S_j\rangle\right)\otimes|A\rangle$$

system S

apparatus A

$$M(\tau_m)|\psi(0)\rangle = |\psi(\tau_m)\rangle = \sum_j C_j |S_j\rangle\otimes|A_j(\tau_m)\rangle$$

# "Unitary measurement"--pre-measurement

## Creating entanglement between system and apparatus

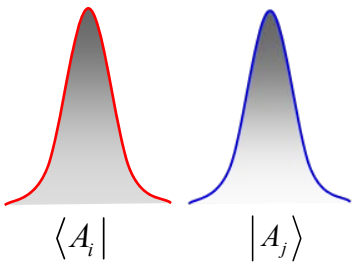
$$M(\tau_m)[\sum_j C_j \left|S_j\right\rangle \otimes \left|A\right\rangle] = \sum_j C_j \left|S_j\right\rangle \otimes \left|A_j(\tau_m)\right\rangle$$

**On system basis :** Unitary and Diagonal!

$$M(\tau_m) = \begin{pmatrix} \exp[-i\hat{h}_1(\tau_m)] & 0 & \dots & 0 \\ 0 & \exp[-i\hat{h}_2(\tau_m)] & 0 & 0 \\ \dots & \dots & \ddots & \dots \\ 0 & 0 & \dots & \exp[-i\hat{h}_M(\tau_m)] \end{pmatrix}$$

$\hat{h}_i(\tau_m)$ operators valued in the variables of apparatus

# "Ideal measurement"




$$\boxed{\langle A_i | A_j \rangle = \delta_{ij}}$$

$$|A_j(\tau_m)\rangle = e^{-i\hat{h}_j(\tau_m)}|A\rangle$$

$$\rho(S) = Tr_A(|\psi(\tau_m)\rangle\langle\psi(\tau_m)|$$
$$= \sum_j |C_j|^2 |S_j\rangle\langle S_j|$$

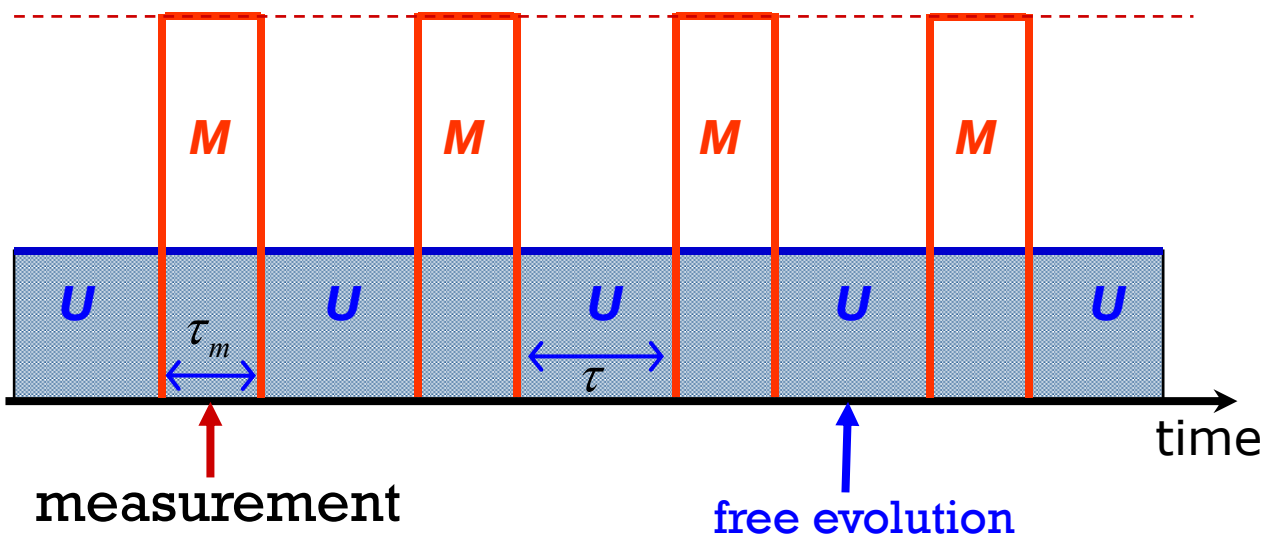
Apparatus induces decoherence
No further details , e.g., Zurek's en-selection

No more philosophy issue discussed

# QZE by unitary kicked evolution

$$U_c(\tau,\tau_m) = [M(\tau_m)U(\tau)]^N$$

$N\tau = t$ fixed $\qquad N \to \infty$



# A Core theorem

*For a series of measurements performs N times within a given free evolution time,* $N\tau = t$ *the total evolution operator will approach an unitary diagonal one:*

$$\lim_{\tau \to 0} [M(\tau_m)U(\tau)]^N \to U_c(\tau, \tau_m) \equiv e^{-iH_d t} M(\tau_m)^N$$

*$H_d$= Ddiagonal part of the free evolution Hamiltonian H*

**which could not results in any transition with respect the measurement basis**

# Our proof-1

$$\left[M(\tau_m)U(\tau)\right]^N = \left[\prod_{n=1}^{N} M(\tau_m)^n U(\tau) M(\tau_m)^{-n}\right] M(\tau_m)^N$$

To the first order of $\tau$:

Diagonal part of H

$$U(\tau) = e^{-iH\tau} \approx 1 - iH\tau = 1 - iH_d\tau - iH_{off}\tau$$

Off-diagonal part of H

$$M(\tau_m)^n U(\tau) M(\tau_m)^{-n}$$

$$\approx M(\tau_m)^n \left(1 - iH_d\tau - iH_{off}\tau\right) M(\tau_m)^{-n}$$

$$= 1 - iH_d\tau - iM(\tau_m)^n H_{off} M(\tau_m)^{-n}\tau$$

# Our proof -2

$$\prod_{n=1}^{N} M(\tau_m)^n U(\tau) M(\tau_m)^{-n} = 1 - i\sum_{n=1}^{N} H_d \tau - i\tau \sum_{n=1}^{N} M(\tau_m)^n H_{off} M(\tau_m)^{-n}$$

$$= 1 - iH_d t - \frac{it}{N}\sum_{n=1}^{N} M(\tau_m)^n H_{off} M(\tau_m)^{-n}$$

The off-diagonal part:

$$\lim_{N\to\infty} \frac{t}{N}\sum_{n=1}^{N} M(\tau_m)^n H_{off} M(\tau_m)^{-n}$$

$$\approx \lim_{N\to\infty} \frac{t}{N}\sum_{n=1}^{N}\sum_{ij} e^{-in\left(\hat{h}_i - \hat{h}_j\right)} H_{ij} \left|s_i\right\rangle\left\langle s_j\right|$$

$$= \lim_{N\to\infty} \frac{t}{N}\sum_{ij} \frac{\left[1 - e^{-iN\left(\hat{h}_i - \hat{h}_j\right)}\right]}{e^{i\left(\hat{h}_i - \hat{h}_j\right)} - 1} H_{ij} \left|s_i\right\rangle\left\langle s_j\right|$$

$$= 0$$

$N\tau = t$ fixed

$$H_{off} = \sum_{ij} H_{ij} \left|s_i\right\rangle\left\langle s_j\right|$$

$$\left(\hat{h}_i \neq \hat{h}_j, \forall i, j \in \mathbb{N}\right)$$

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# QZE in large N limit

$$\prod_{n=1}^{N} M(\tau_m)^n U(\tau) M(\tau_m)^{-n} = 1 - i\sum_{n=1}^{N} H_d \tau - i\frac{t}{N}\sum_{n=1}^{N} M(\tau_m)^n H_{off} M(\tau_m)^{-n}$$

$$\lim_{\tau\to 0}[M(\tau_m)U(\tau)]^N = e^{-iN\tau H_d} M(\tau_m)^N$$

We have proved that von Neumann's projection postulate is not necessary for QZE

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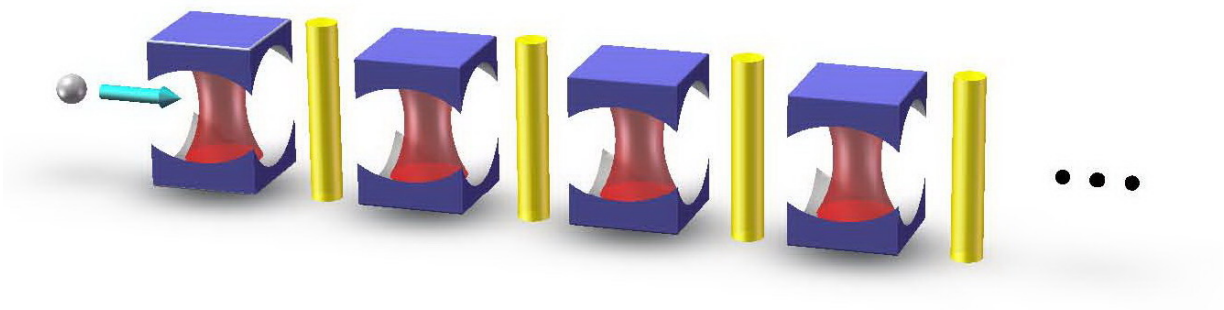
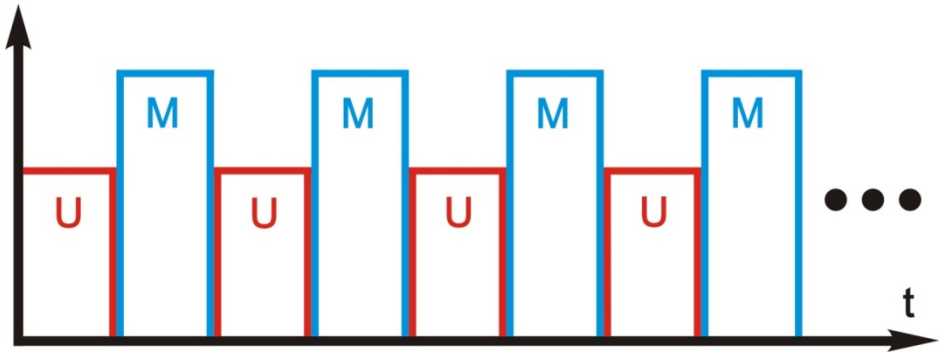
# Experimental test

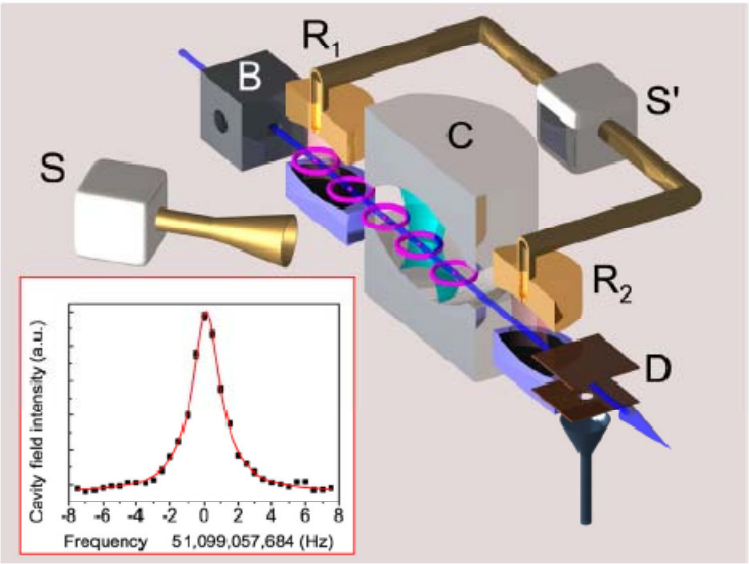
$$[M(\tau_m)U(\tau)]^N$$

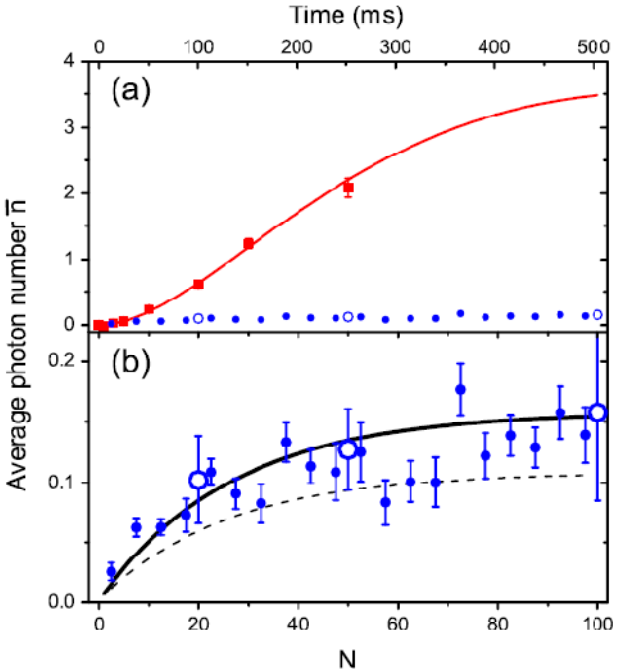




# Rivist the QZE experiment

Cavity QED:.Haroche's Phys. Rev. Lett. **101**, 180402 (2008)




Freezing the photon number in the cavity

# Projective measurement approach

## 1. Without measurement

$$|\phi(t)\rangle = D(\lambda t)|0\rangle = |\lambda t\rangle$$

$$D(\alpha) = e^{(\alpha a^{\dagger} - \alpha^{*} a)}$$

$$\bar{n} = |\lambda| t^2$$

**nonlinear to time**

## 2. Continuous projective measurements

$$t = n\tau$$

Fixed!

Single measurement $|\lambda\tau\rangle = D(\lambda\tau)|0\rangle = |0\rangle + \lambda\tau|1\rangle$

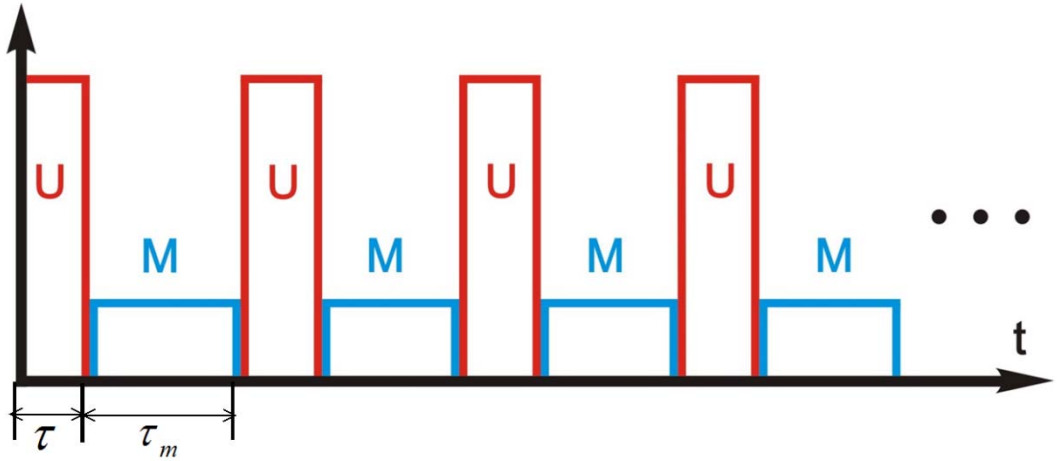
Probability of one photon after N measurements

$$p_0^N \approx [1-(|\lambda|\tau)^2]^N \approx 1-|\lambda|^2 t\tau$$

$$\bar{n} = |\lambda|^2 \tau t$$

**linear to time**

# Our approach : free evolution + unitary measurement




**U:** $$H_U(\tau) = \omega a^\dagger a + f e^{-i\omega_F \tau} a^\dagger + f e^{i\omega_F \tau} a$$

**M:** $$H_M = \frac{g^2}{\Delta}(|e\rangle\langle e| - |g\rangle\langle g|) a^\dagger a$$

# Time evolution U-operator

## In the time dependent frame of reference

$$\left|\psi'(t)\right\rangle = e^{i\omega a^{\dagger} a t}\left|\psi(t)\right\rangle \qquad (\delta = \omega_F - \omega)$$

$$H_{eff} = e^{i\omega a^{\dagger} a t} H_U(t) e^{-i\omega a^{\dagger} a t} - i e^{i\omega a^{\dagger} a t} \partial_t e^{i\omega a^{\dagger} a t}$$
$$= f e^{-i\delta t} a^{\dagger} + f e^{i\delta t} a$$

**Use Wei-Norman algebra method**

$$U(t) = e^{A(t)a^{\dagger}} e^{B(t)a} e^{C(t)}$$

$$\mathrm{i}[\partial_t U(\mathrm{t})] U^{-1}(\mathrm{t}) = H_{eff}$$

$$\Rightarrow \begin{cases} i\dot{A}(t) = f e^{-i\delta t} \\ i\dot{B}(t) = f e^{i\delta t} \\ \dot{C}(t) = \dot{B}(t) A(t) \end{cases}$$

# Time evolution U-operator

$$A(t) = \frac{f}{\delta}(e^{-i\delta t} - 1)$$

$$B(t) = -\frac{f}{\delta}(e^{i\delta t} - 1)$$

$$C(t) = -\frac{if^2 t}{\delta} + \frac{f^2}{\delta^2}(e^{i\delta t} - 1)$$

## The time evolution U operator

$$\boxed{U(\tau) = e^{i\phi(\tau)} D[\alpha(\tau)]}$$

$$D[\alpha(\tau)] = e^{\alpha(\tau)a^\dagger - \alpha^*(\tau)a} \qquad \alpha(\tau) = \frac{f}{\delta}[e^{-i\delta\tau} - 1]$$

# Time evolution U-process

## Initial State

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\underbrace{(|e\rangle + |g\rangle)}_{\text{ATOM}} \otimes \underbrace{|0\rangle}_{\text{CAVITY}}$$

## After the first U process

$$|\psi(\tau)\rangle = T^{\dagger}(\tau)U(\tau)T(0)|\psi(0)\rangle = \frac{e^{i\phi(\tau)}}{\sqrt{2}}(|e\rangle + |g\rangle) \otimes |\alpha(\tau)\, e^{-i\omega\tau}\rangle$$

$$T(t) = e^{i\omega a^{\dagger} a t}$$

# Measurement process

## Time evolution operator due to measurement

$$M(\tau_m) = \exp[-i\frac{g^2}{\Delta}\tau_m(|e><e| - |g><g|)a^\dagger a e]$$

## After the first measurement:

$$M(\tau_m)|\psi(\tau)\rangle = \frac{1}{\sqrt{2}}e^{i\phi(\tau)}\left(|e\rangle\otimes|\alpha_+\rangle + |g\rangle\otimes|\alpha_-\rangle\right)$$

$$\alpha_\pm = \alpha(\tau)e^{-i\omega\tau \mp i\frac{g^2}{\Delta}\tau_m}$$

# Time evolution for N steps

$$\left|\psi[N(\tau+\tau_m)]\right\rangle$$

$$= T^{\dagger}(N\tau)[M(\tau_m)U(\tau)]^N T(0)\left|\psi(0)\right\rangle$$

$$= e^{iN\phi(\tau)} T^{\square}(N\tau)[M(t)D[\alpha(\tau)]]^N \left|\psi(0)\right\rangle$$

$$= \frac{e^{iN\phi(\tau)}}{\sqrt{2}} T^{\dagger}(N\tau)\left(\left[\underline{e^{-i\frac{g^2}{\Delta}ta^{\dagger}a} D[\alpha(\tau)]}\right]^N \left|e\right\rangle\otimes\left|0\right\rangle + \left[\underline{e^{i\frac{g^2}{\Delta}ta^{\dagger}a} D[\alpha(\tau)]}\right]^N \left|g\right\rangle\otimes\left|0\right\rangle\right)$$

$$\left[e^{\mp i\frac{g^2 t}{\Delta}a^{\dagger}a} D\left[\alpha(\tau)\right]\right]^N$$

$$= \prod_{n=1}^{N}\left[e^{\mp in\frac{g^2 t}{\Delta}a^{\dagger}a} D\left[\alpha(\tau)\right] e^{\pm in\frac{g^2 t}{\Delta}a^{\dagger}a}\right] e^{\mp iN\frac{g^2 t}{\Delta}a^{\dagger}a} = \prod_{n=1}^{N} D\left[\alpha(\tau) e^{\mp in\frac{g^2 t}{\Delta}}\right] e^{\mp iN\frac{g^2 t}{\Delta}a^{\dagger}a}$$

$$= D\left[\alpha(\tau)\sum_{n=1}^{N} e^{\mp in\frac{g^2 t}{\Delta}}\right] e^{i\theta_{\pm}(N)} \equiv D\left[\alpha_{\pm N}(\tau)\right] e^{i\theta_{\pm}(N)}$$

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# Time evolution

## After N free evolutions and measurements:

$$\left|\psi[N(\tau+\tau_m)]\right\rangle = T^{\dagger}(N\tau)[M(\tau_m)U(\tau)]^N T(0)\left|\psi(0)\right\rangle$$
$$= \frac{e^{i\phi_+}}{\sqrt{2}}\left|e\right\rangle \otimes \left|\alpha_{+N} e^{-i\omega \mathrm{N}\tau}\right\rangle + \frac{e^{i\phi_-}}{\sqrt{2}}\left|g\right\rangle \otimes \left|\alpha_{-N} e^{-i\omega \mathrm{N}\tau}\right\rangle$$

$$\begin{cases} \phi_{\pm} = N\phi(\tau) + \theta_{\pm}(N) \\ \alpha_{\pm N} = \alpha(\tau) e^{\mp i(N+1)\frac{g^2\tau_m}{\Delta}} \dfrac{sin(Ng^2\tau_m / 2\Delta)}{sin(g^2\tau_m / 2\Delta)} \end{cases}$$

$$\theta_{\pm}(N) = \pm \frac{|\alpha(\tau)|^2}{2} \frac{N\sin\xi_m - \sin N\xi_m}{1-\cos\xi_m}$$

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# Average photon number

$$\bar{n} = |\alpha(\tau)|^2 \frac{\sin^2(Ng^2\tau_m / 2\Delta)}{\sin^2(g^2\tau_m / 2\Delta)}$$

**Return to the projective measurement**

$\tau \to 0:$

$\alpha(\tau) \longrightarrow if\ \tau$

$$\bar{n} \approx f^2 N^2 \tau^2$$

$$\bar{n} \longrightarrow 0$$

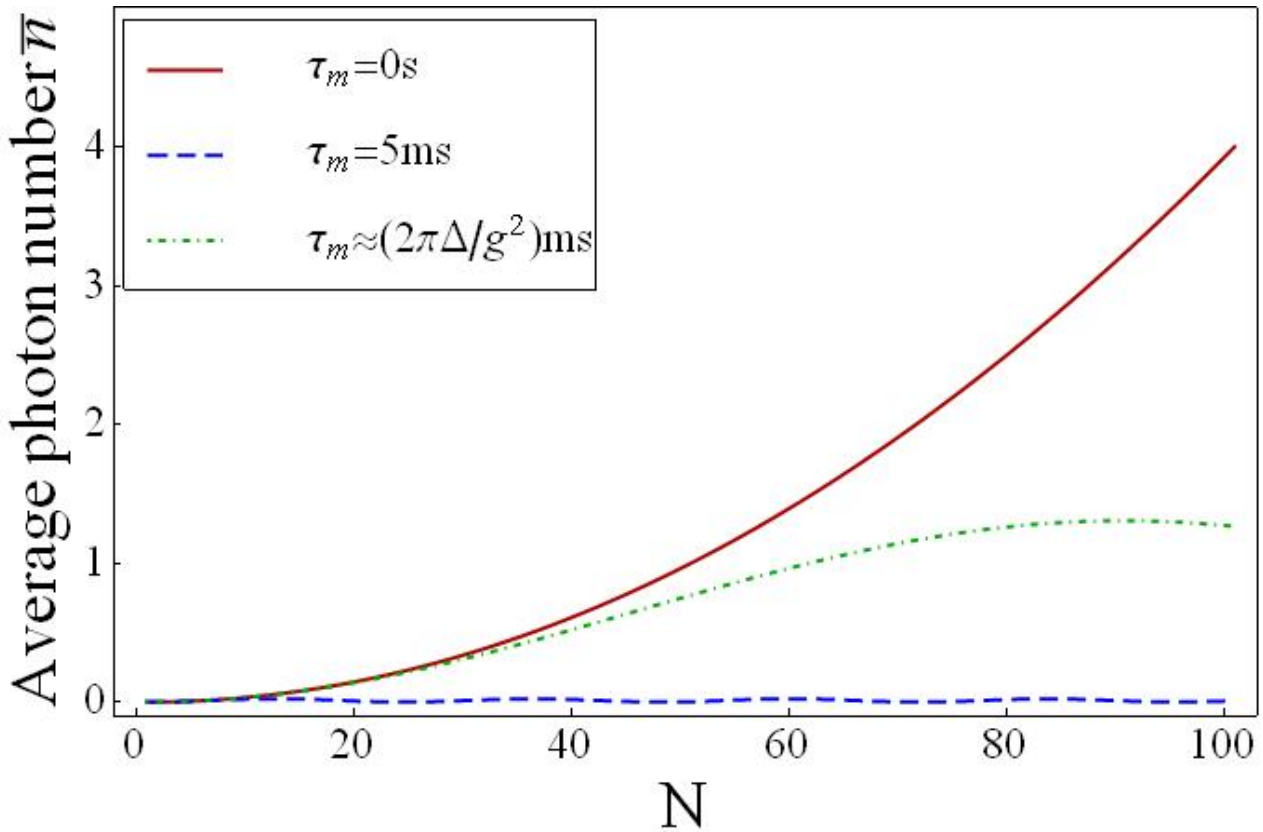
**Except for**

$$\tau_m = \frac{2\pi k \Delta}{g^2}$$

$k = 0, 1, 2, \ldots$

# Average photon number

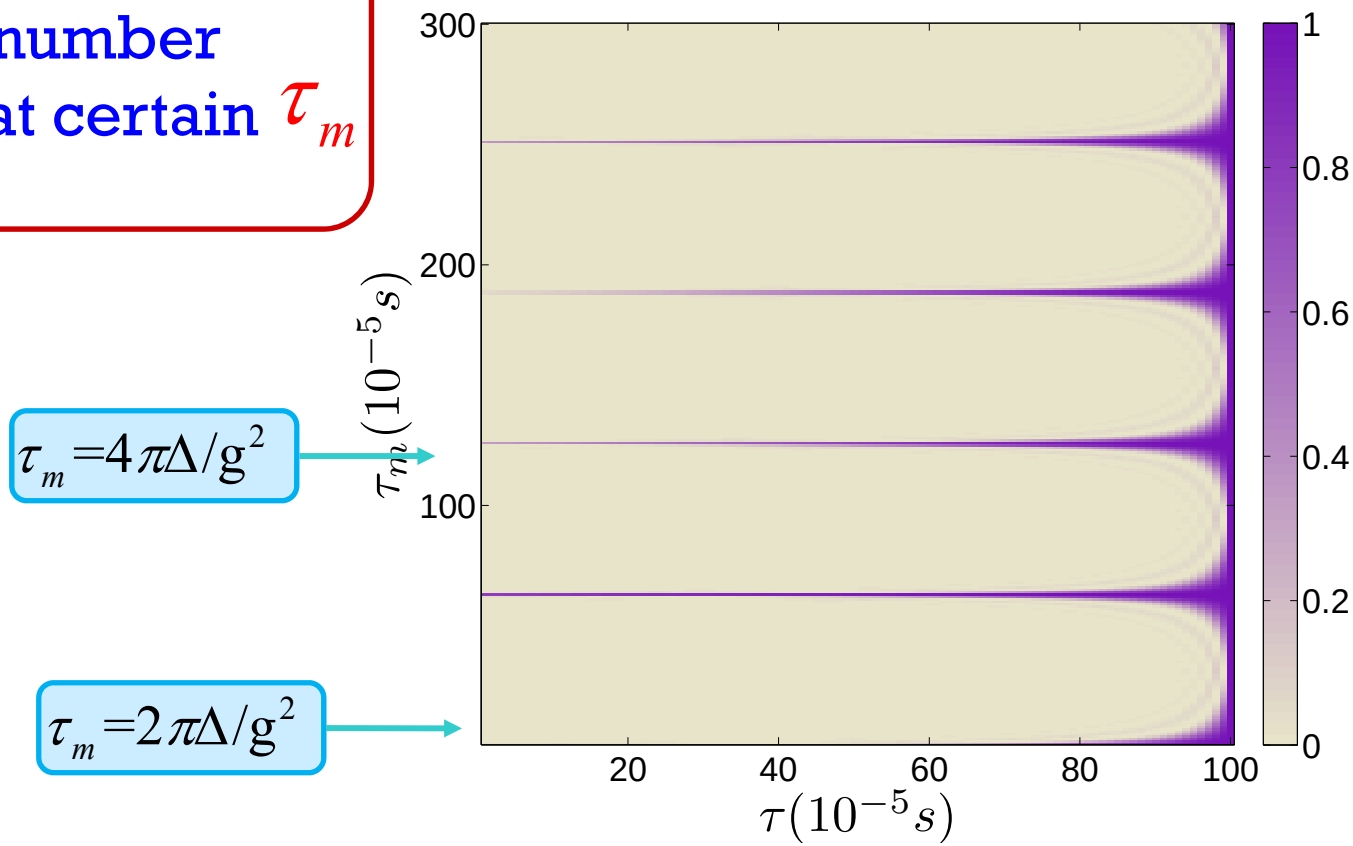
Different from Projection Measurement!




$N\tau = t$ Fixed

# Average photon number vs. $\tau$ and $\tau_m$

Photon number revival at certain $\tau_m$



# Two papers was post almost at the same time

1. Decoherence-Based Quantum Zeno Effect in a Cavity-QED System
arXiv:1007.4634, D. Z. Xu, Q. Ai, C. P. Sun (ITP,CAS)

2. Phase space tweezers for tailoring cavity fields by quantum Zeno dynamics

arXiv:1007.4942, J.Raimond ,C. Sayrin,
S. Gleyzes ,I. Dotsenko M. Brune (LKB - Lhomond),
S. Haroche (LKB - Lhomond, CDF),
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**Experimentalists made a theoretical paper , put in arXiv at once after our paper**

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# Temporary summary

1. We generally prove that the quantum Zeno effect can also appear for a close quantum system when the measurements is dynamically treated as some couplings of the system to the apparatus, rather than the use of the von Neumann's projection.

2. Our theory not only recovers the existing experimental data ( J. Bernu, et al., Phys. Rev. Lett, 101, 180402 (2008)]0), but also predict the observable effect that distinguishes this dynamic QZE from the usual one based on projective measurements.

# 3. Anti-QZE for open quantum system

with dynamic approach to measurement

# QZE vs. QAZE for open system

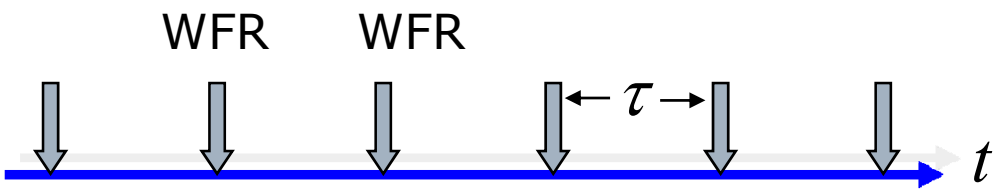
Kofman & Kurizki, Nature 405, 546 (2000).

heat bath

measured system

$$H = \sum_k \omega_k b_k^+ b_k + \frac{\Omega}{2}\sigma_z + \sum_k g_k(\sigma^+ b_k + \sigma^- b_k^+)$$

**Sequence of frequent Measurements**




Quantum Anti-Zeno Effect (QAZE)
happens for certain open cases !!!

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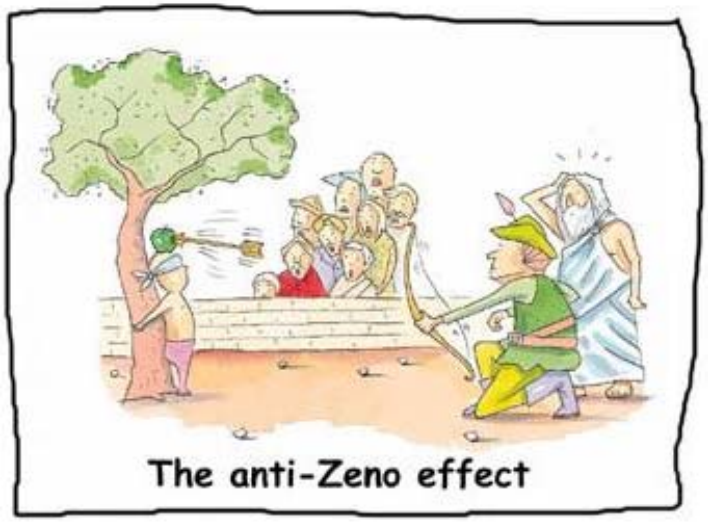
# Quantum Anti-Zeno Effect

**Kofman & Kurizki, (2000);**
**Fachi & Pascazio, (2001)**

Decay of unstable state can be accelerated by measurements due to its couplings to reservoir




Zheng et.al , PRL 101, 200404 (2008).

no QAZE if RWA is not made for some cases

Ai, Li, Zheng, Sun, PRA 81, 042116 (2010).

QAZE happens without RWA is for some cases :For hydrogen spontaneous decay

2p-1s; 2p-1s

# QZE with projection measurements

Wave function:

$$\left|\psi(t)\right\rangle = \alpha\left|e,0\right\rangle + \sum_k \beta_k \left|g,1_k\right\rangle$$

Schrödinger equation:

$$d\alpha / dt = -i\sum_k g_k \beta_k e^{i(\Omega-\omega_k)t},$$

$$d\beta_k / dt = -ig_k \alpha e^{-i(\Omega-\omega_k)t}.$$

Survival probability amplitude after one projection:

$$\alpha(\tau) = \alpha(0) - \sum_k g_k^2 \int_0^\tau dt' \int_0^{t'} \alpha(t'') e^{i(\Omega-\omega_k)(t'-t'')} dt''$$

$$\approx \alpha(0) - \alpha(0)\sum_k g_k^2 \int_0^\tau dt' \int_0^{t'} e^{i(\Omega-\omega_k)(t'-t'')} dt''$$

$$= \alpha(0) + \alpha(0)\sum_k g_k^2 \left[\frac{\tau}{i(\Omega-\omega_k)} + \frac{e^{i(\Omega-\omega_k)\tau}-1}{(\Omega-\omega_k)^2}\right]$$

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# QZE with projection measurements

Survival probability after *n* projections:

$$P(t = n\tau) = \left|\alpha(\tau)\right|^{2n} \approx \exp\left(-Rt\right)$$

Projection measurements

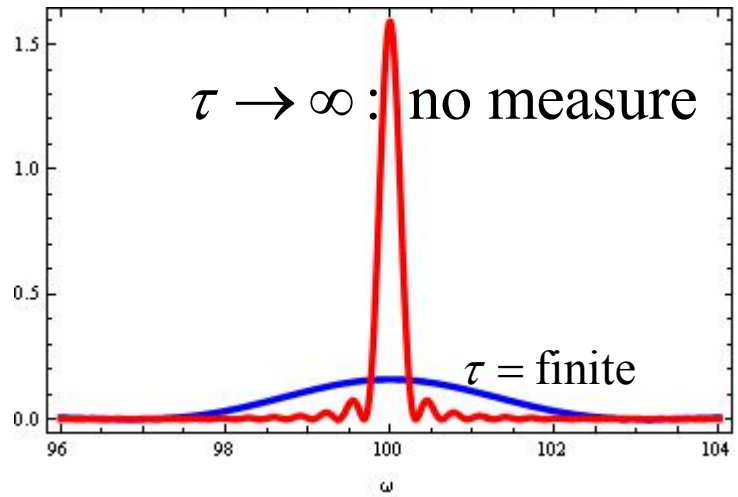
Decay rate:

$$R = 2\pi \int_{-\infty}^{\infty} d\omega F(\omega) G(\omega)$$

# Measurement induces spectrum broadening

Effective level broadening induced by measurements

$$F(\omega) = \frac{\tau}{2\pi} \mathrm{sinc}^2 \frac{(\omega - \Omega)\tau}{2}$$




*F*(ω) for different $\tau$

Interacting spectrum

$$G(\omega) = \sum_k g_k^2 \delta(\omega - \omega_k)$$

*F*(ω) is a delta function in the absence of measurement

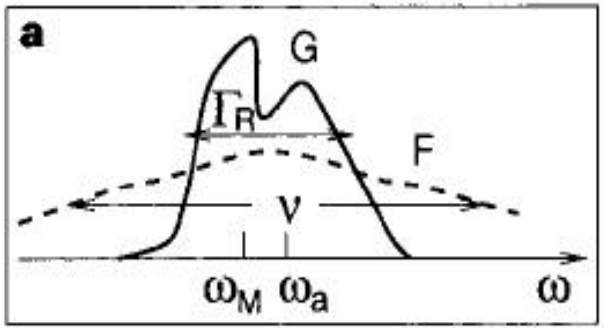
$$\tau \to \infty$$

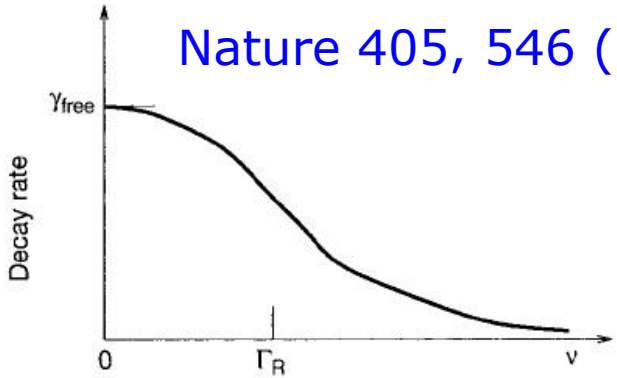
Nature 405, 546 (2000).

# QZE by spectrums overlap

**Zeno Effect**

$R < \gamma$

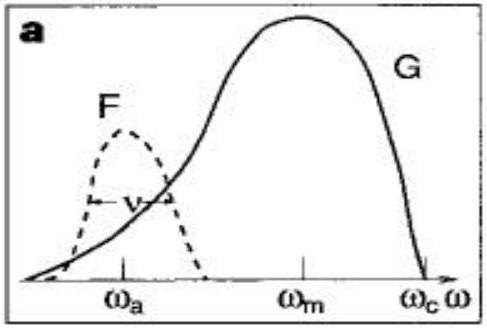


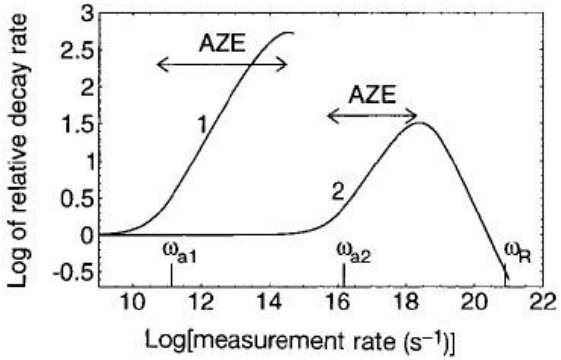




Nature 405, 546 (2000).

**Anti-Zeno Effect !!!**

$R > \gamma$



# Some results

1. In general, QAZE happens even without RWA.
2. With respect to bare excited state, QAZE for transitions of hydrogen atom is recovered by both ALZS' and ZZZ's approaches.

| Excited State → | Bare with RWA | Bare without RWA | Renormalized with RWA | Renormalized without RWA |
|---|---|---|---|---|
| ZZZ's Approach | QAZE | QAZE | Not studied | No QAZE |
| ALZS's Approach | QAZE | QAZE | Not studied | Not studied |

# Our present motivation

In the conventional approach, the QZE and QAZE were studied with projection measurements. It seems that the QZE and QAZE are based on the wave function reduction postulate.

However, an experiment demonstrating the QZE was explained in a dynamical fashion:
L. E. Ballentine, Phys. Rev. A 43, 5165 (1991).
W. M. Itano, *et. al.*, Phys. Rev. A 41, 2295 (1990).

Our Motivations

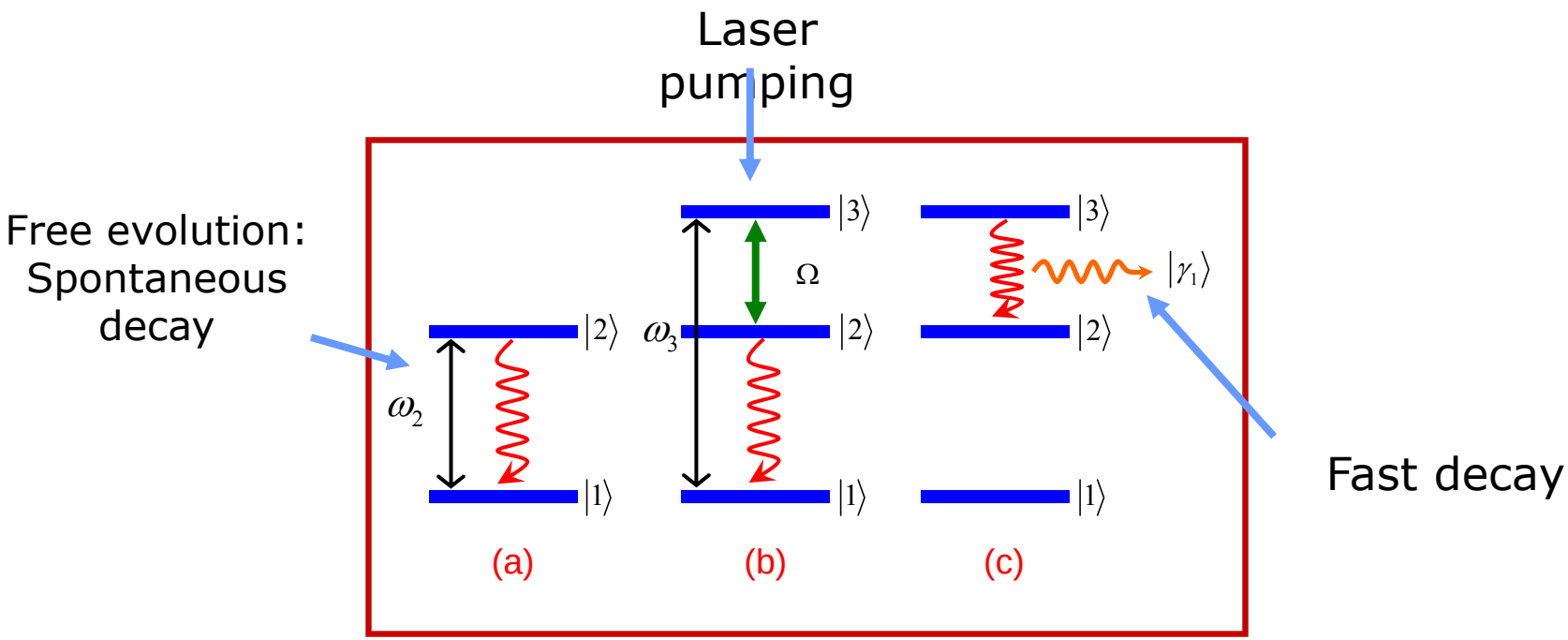
To check universality of QZE and QAZE

They result from short-interval interruption

Or they are only caused by collapse postulate

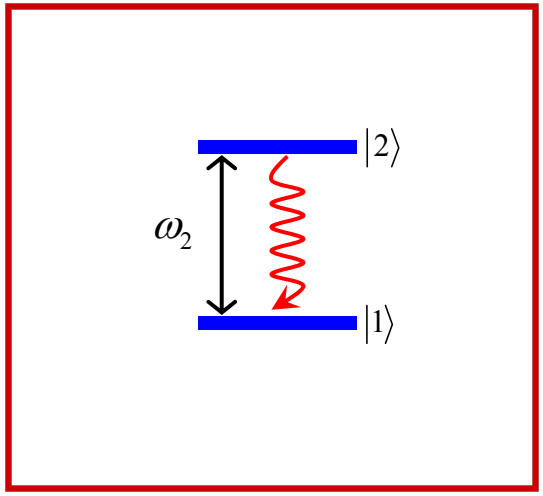
# Three processes in dynamic QZE model

"Dynamic" = Non WPC !!



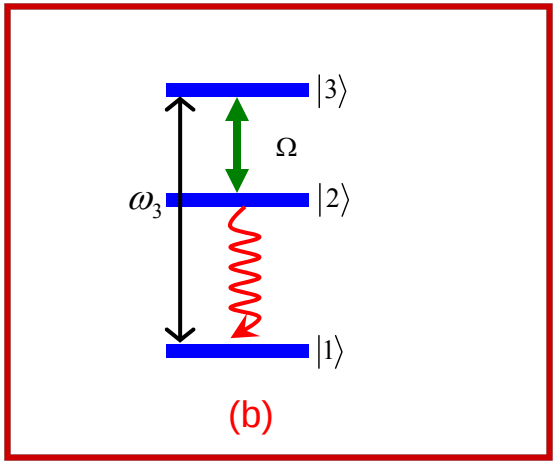

Ai, Xu, Yi, Kofman, Sun, Nori, arXiv:1007.4859.

# Free evolution: spontaneous decay




$$H = \sum_k \omega_k a_k^\dagger a_k + \omega_2 |2\rangle\langle 2| + \sum_k g_k (a_k^\dagger |1\rangle\langle 2| + a_k |2\rangle\langle 1|).$$

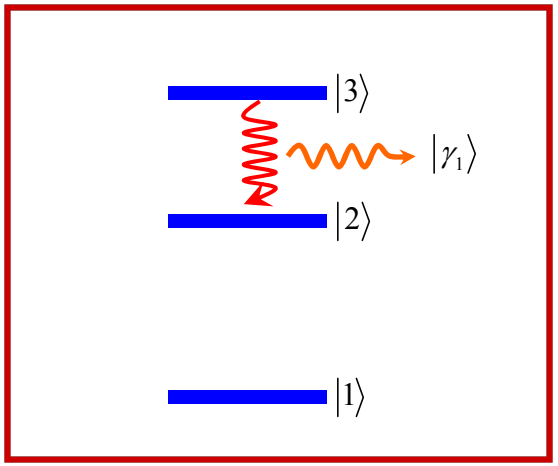
# Coupling-based measurement




**Laser pumping**

$$H = \omega_2 |2\rangle\langle 2| + \sum_k \omega_k a_k^\dagger a_k + \sum_k g_k (|1\rangle\langle 2| a_k^\dagger + \text{H.c.})$$
$$+ \omega_3 |3\rangle\langle 3| + \Omega \cos \Delta t (|3\rangle\langle 2| + \text{H.c.})$$

**Fast Spontaneous decay**

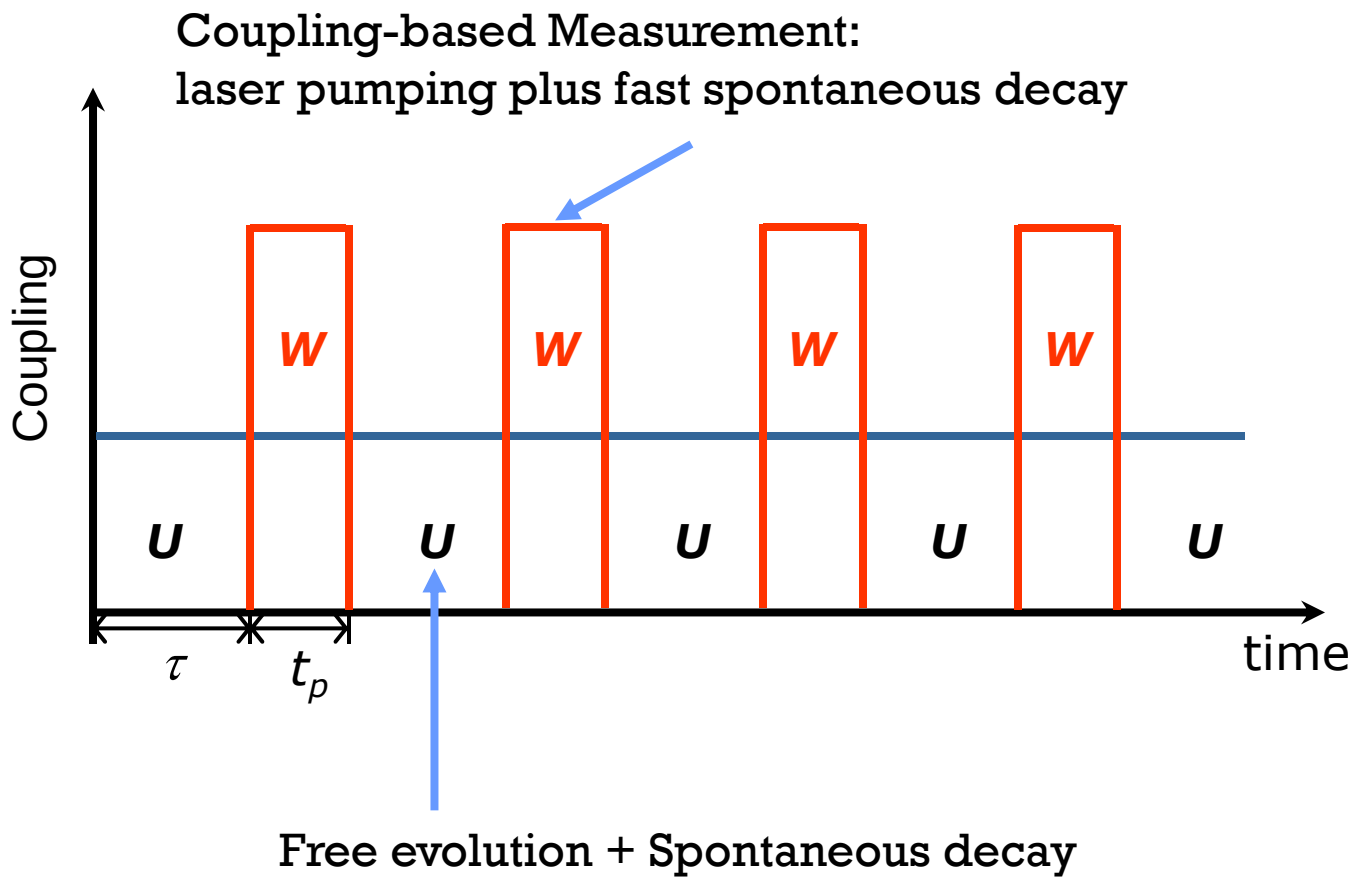



single photon in $\gamma$ mode

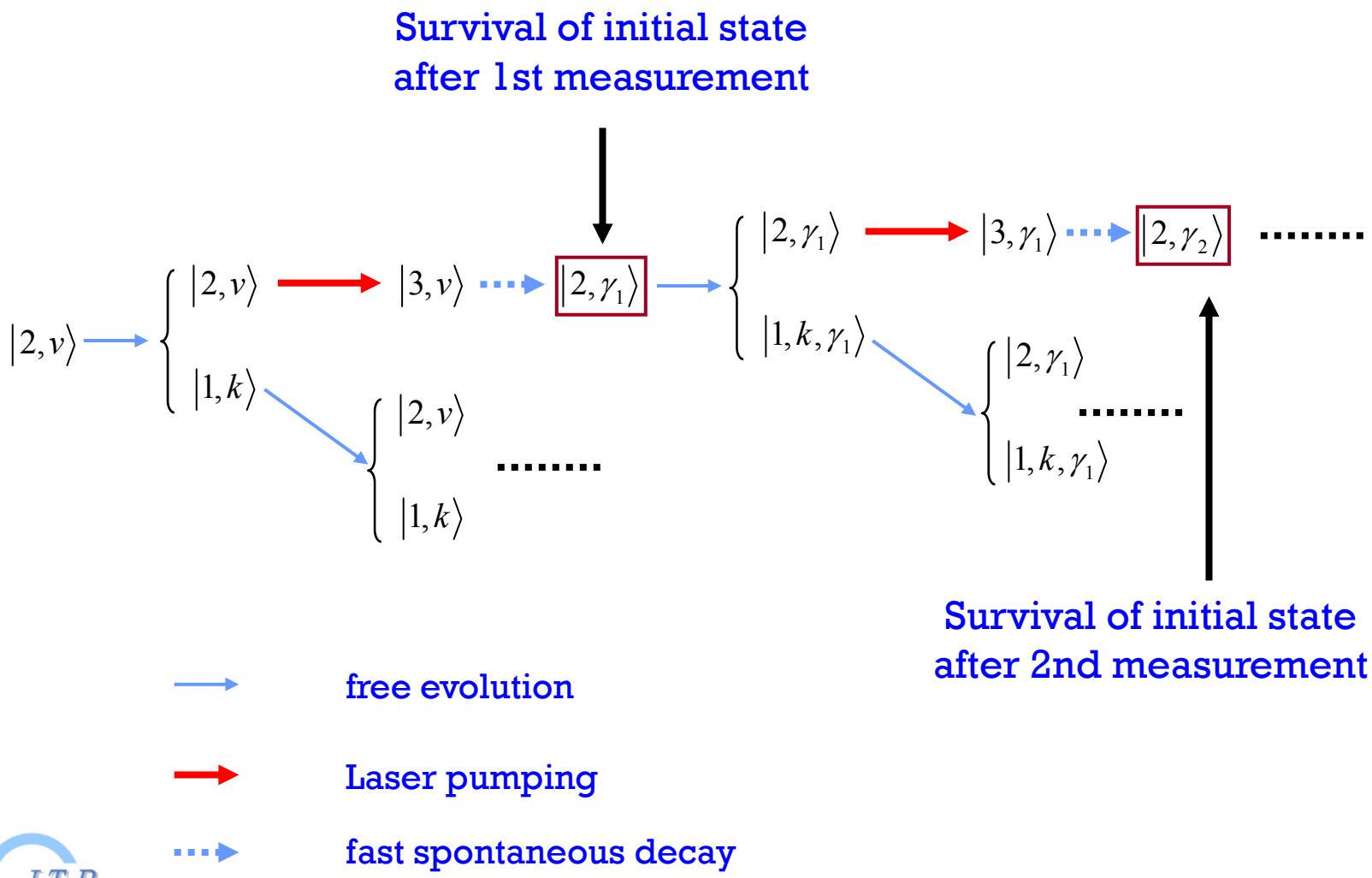
$$|3\rangle|v\rangle \rightarrow |2\rangle|\gamma_1\rangle$$

vacuum of reservoir

# Pulse sequence for dynamical QAZE



# State evolution




free evolution

Laser pumping

fast spontaneous decay

# Dynamic process in free evolution

Hamiltonian

$$H = \sum_k \omega_k a_k^\dagger a_k + \omega_2 |2\rangle\langle 2| + \sum_k g_k (a_k^\dagger |1\rangle\langle 2| + a_k |2\rangle\langle 1|).$$

Wave function

$$|\Psi(t)\rangle = \alpha|2\rangle + \sum_k \beta_k |1, k\rangle,$$

where coefficients fulfill Schrodinger equation

$$i\dot{\alpha}' = \sum_k g_k \beta_k' e^{i(\omega_2 - \omega_k)t},$$

$$i\dot{\beta}_k' = g_k \alpha' e^{-i(\omega_2 - \omega_k)t}.$$

with slowly-varying varibles

$$\alpha'(t) = \alpha e^{i\omega_2 t}, \ \beta_k'(t) = \beta_k e^{i\omega_k t}$$

# Dynamical evolution after n cycles

Survival probability amplitude $\left[C'(t_p)\exp(-i\omega_3 t_p)\right]^n$

Survival probability

$$\boxed{P = \left|\left[C'(t_p)\exp(-i\omega_3 t_p)\right]^n\right|^2 = \left|a_1(t_p)\right|^{2n} = e^{-Rt}}$$

In the case with a strong laser

$$\begin{aligned} h_{t_p}(\Omega) &= \frac{e^{i\pi} - 1}{\Omega} = \frac{-2}{\Omega}, \\ h_{t_p}(\omega_\pm) &= \frac{e^{i\omega_\pm t_p} - 1}{\omega_\pm} \approx 2\frac{i \mp 1}{\Omega}, \\ \omega_\pm &\approx \pm\frac{\Omega}{2}, \end{aligned}$$

$$a_1(t_p) \approx \left[1 + \sum_k g_k^2 \frac{\cos(\omega_k - \omega_2)\tau - 1}{(\omega_k - \omega_2)^2} - \sum_k g_k^2 \frac{2\sin(\omega_2 - \omega_k)\tau}{\Omega(\omega_2 - \omega_k)} + \mathrm{Im}\right] e^{-i\omega_2\tau}$$

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# Effective decay rate

Effective decay rate:

$$R(t) = 2\pi \int_{-\infty}^{\infty} F(\omega, \tau, \Omega) G(\omega) d\omega.$$

Measurement-induced level-broadening function:

$$F(\omega, \tau, \Omega) = \frac{4\,\mathrm{sinc}(\omega_2 - \omega)\tau}{2\pi\Omega} + \frac{\tau}{2\pi}\mathrm{sinc}^2\frac{(\omega_2 - \omega)\tau}{2}.$$

$F_1(\omega,\tau,\Omega)$ due to finite Rabi frequency

$F_2(\omega,\tau)$ same as one with projection measurements

Interacting spectral distribution:

$$G(\omega) = g_k^2 \rho(\omega_k)\big|_{\omega_k = \omega}$$

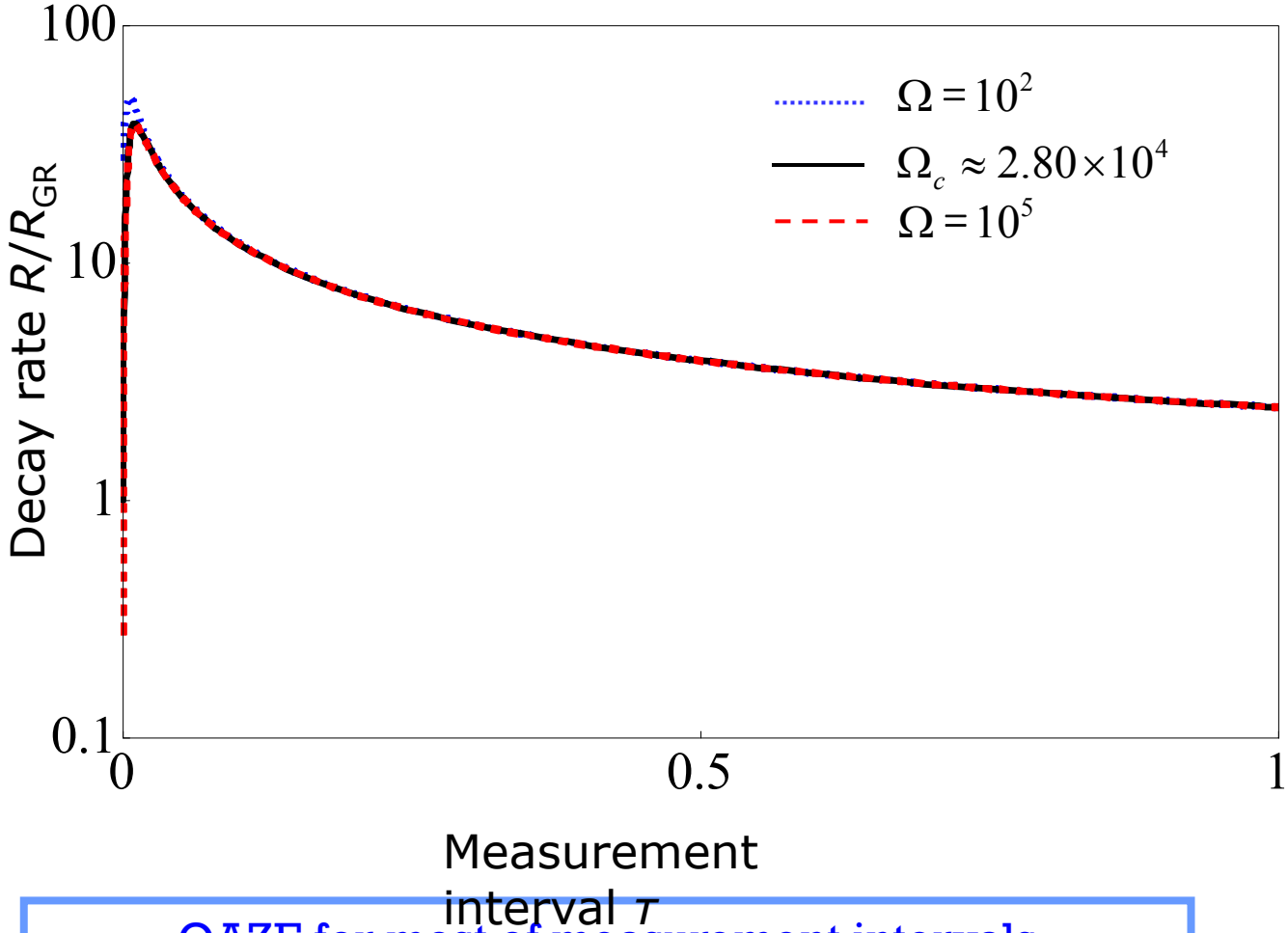
# Very strong coupling $\longrightarrow$ von Neumann projection

$$F(\omega,\tau,\Omega) = \frac{\tau}{2\pi}\mathrm{sinc}^2\frac{(\omega_2-\omega)\tau}{2} + \cancel{\frac{4\mathrm{sinc}(\omega_2-\omega)\tau}{2\pi\Omega}}$$

$$\Omega \quad \Rightarrow \quad \infty$$

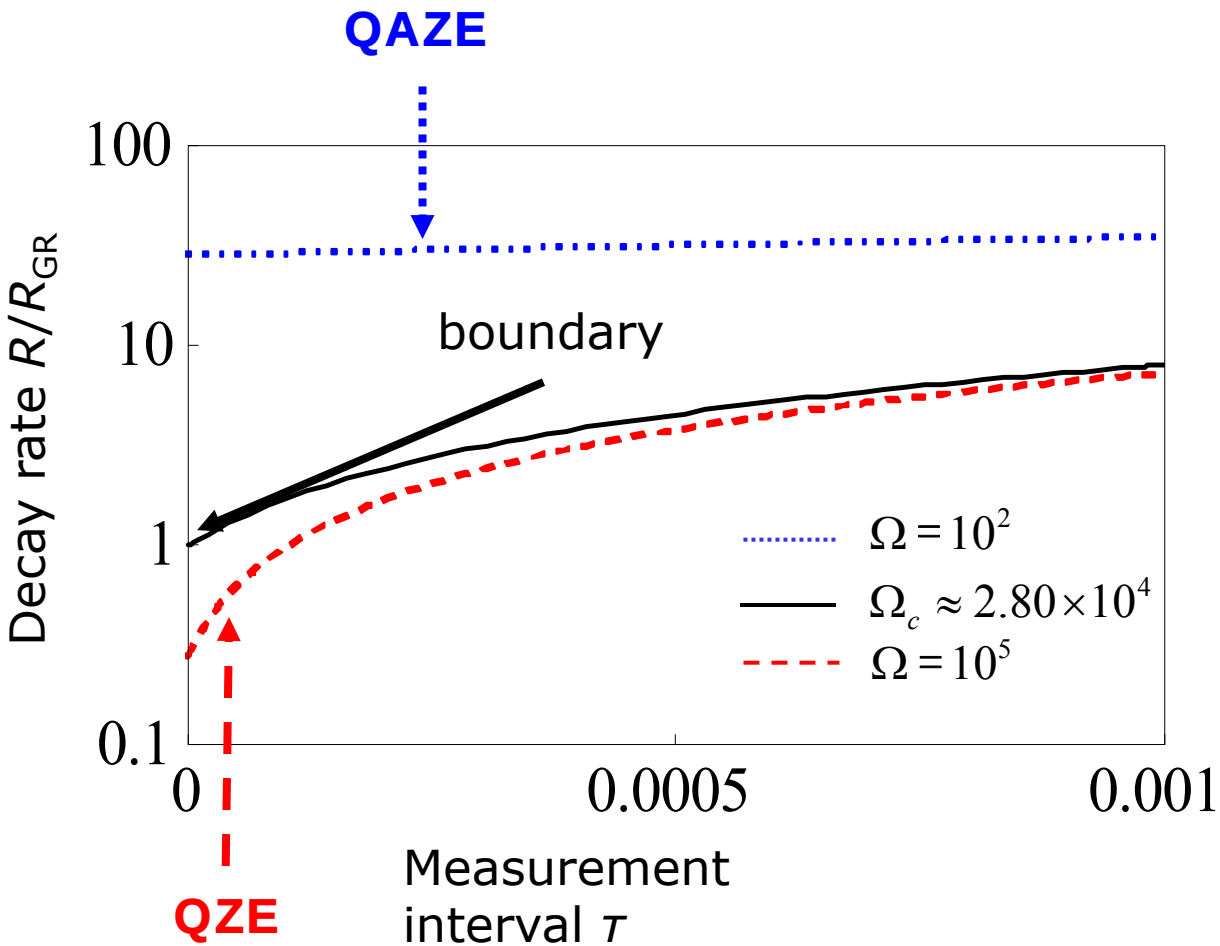
$$F(\omega,\tau,\Omega) = \frac{\tau}{2\pi}\mathrm{sinc}^2\frac{(\omega_2-\omega)\tau}{2}$$

# Decay rate for 2p-1s transition in hydrogen

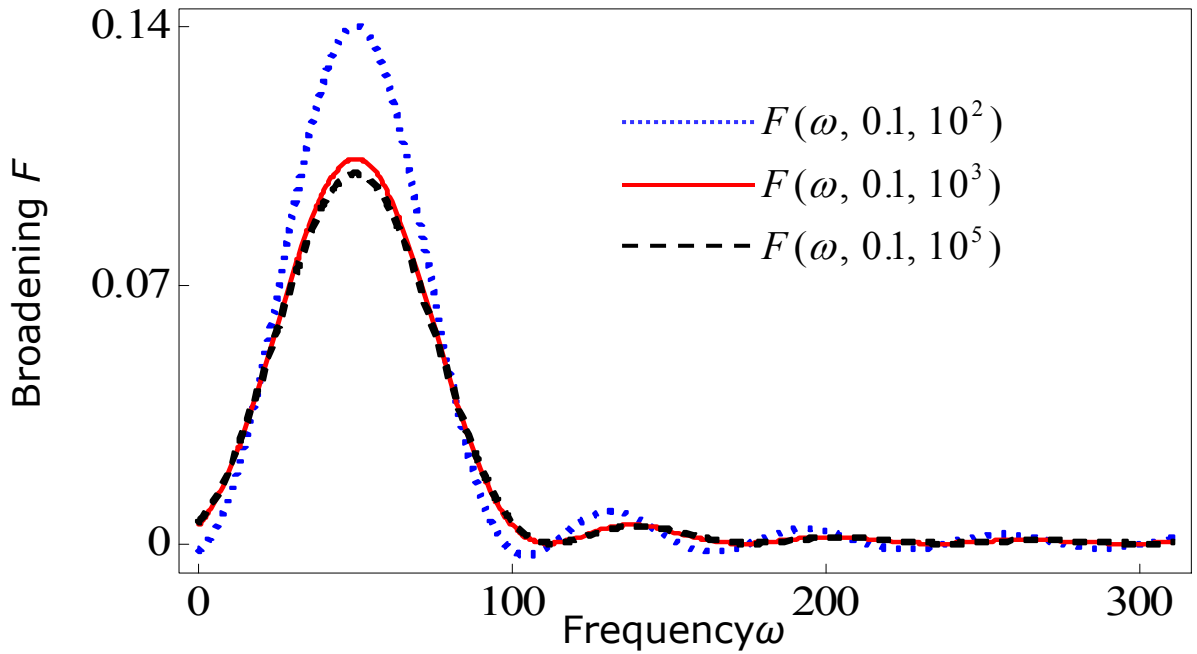



QAZE for most of measurement intervals

# QZE to QAZE transition in short-time regime



# Level broadening in short-time regime




**$F_1(\omega,\tau,\Omega)$ dominates**
**$F(\omega,\tau,\Omega)$ sensitive to variance of Rabi frequency $\Omega$**

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# Second summary

We study quantum anti-Zeno effect for a two-level subsystem, where measurements are treated as couplings between the excited state and an auxiliary state rather than the von Neumann's wave function reduction.

We also have present the proof that the quantum Zeno/anti-Zeno effect could happen without projective measurement

We studied in very details for the role of rotating wave approximation investigating the quantum Zeno/anti-Zeno effect

# Thank You !

# Thank You !