Lecture 4

Decoherence by Quantum Phase Transition

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Outlines

1. What is quantum decoherence ?

2. Quantum Measurement: Model of Decoherence

3. Environment induced Decoherence and Quantum Chaos

4. Quantum Phase Transition

5. Emergence of Decoherence as a Phenomenon in

Quan, Song , Liu, Zanardi, Sun, PRL, 2006

Quantum Mechanics

Wave Mechanics	Matrix Mechanics

Wave Mechanics

Schrödinger Equation

$$i\hbar \frac{d}{dt}\psi(x,t) = \mathrm{H}\psi(x,t)$$

Wawe function

$$|\psi(\mathrm{x},\mathrm{t})|^2$$

Probability finding Particle

Matrix Mechanics

Observable spectrum relates two "orbits"

$$P_{mn},\quad Q_{mn}\quad \rightarrow \text{Matrix}\begin{bmatrix} x & x & .. \\ x & x & .. \\ .. & .. & .. \end{bmatrix}$$

$$[Q,\ P] \neq 0$$

$$\Delta Q\ \Delta P \sim \hbar$$

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Quantum Measurement Postulate

$$|\psi\rangle = \sum_n c_n |n\rangle \qquad (A|n\rangle = a_n |n\rangle)$$

Measuring A, once you obtain a_n , then

$$|\psi\rangle = \sum_n c_n |n\rangle \rightarrow |n\rangle$$

Wave Function Collapse (WFC) !!!

5'th Solvay conference (1927)

"Electrons and photons"

Quantum Theory – "Quantum Wave Mechanics" - takes flight

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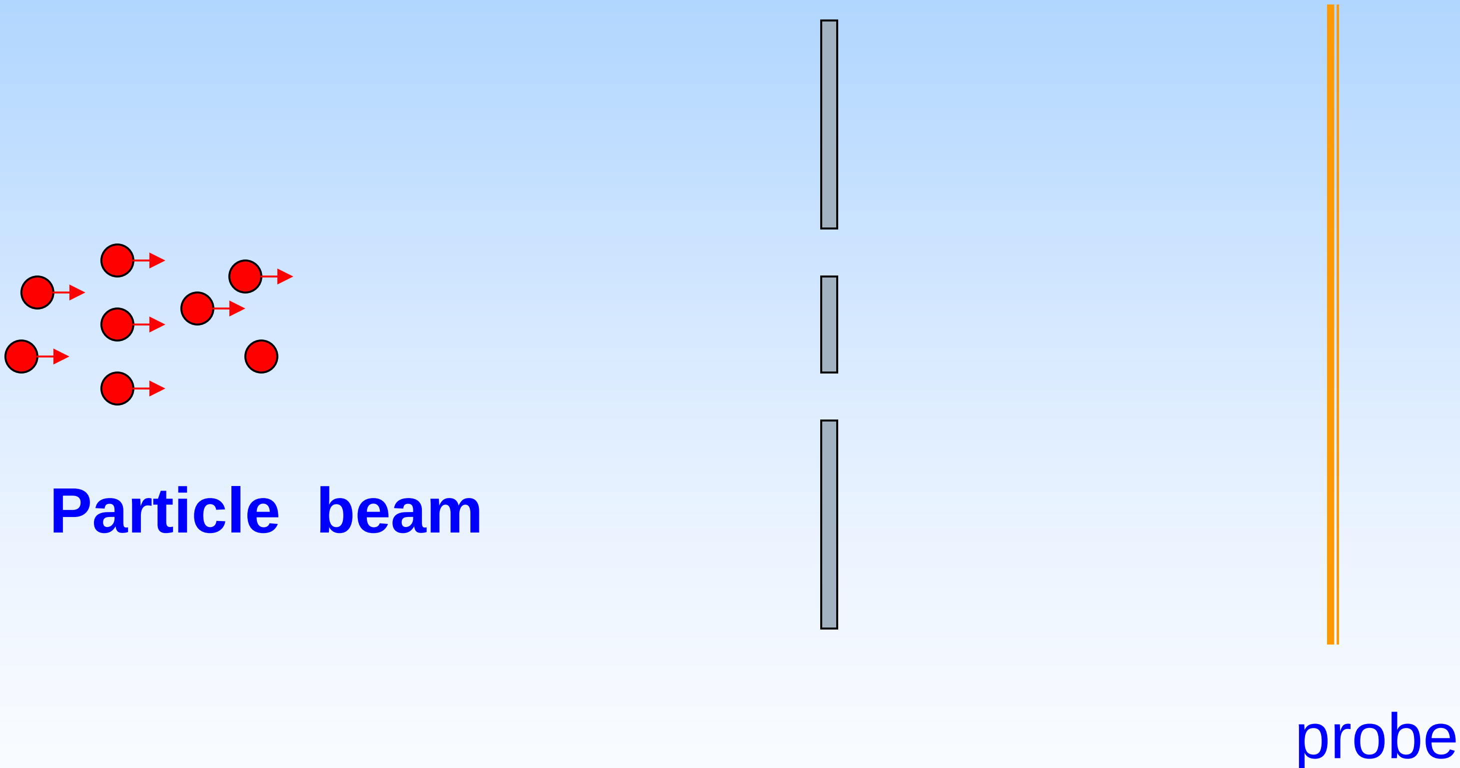
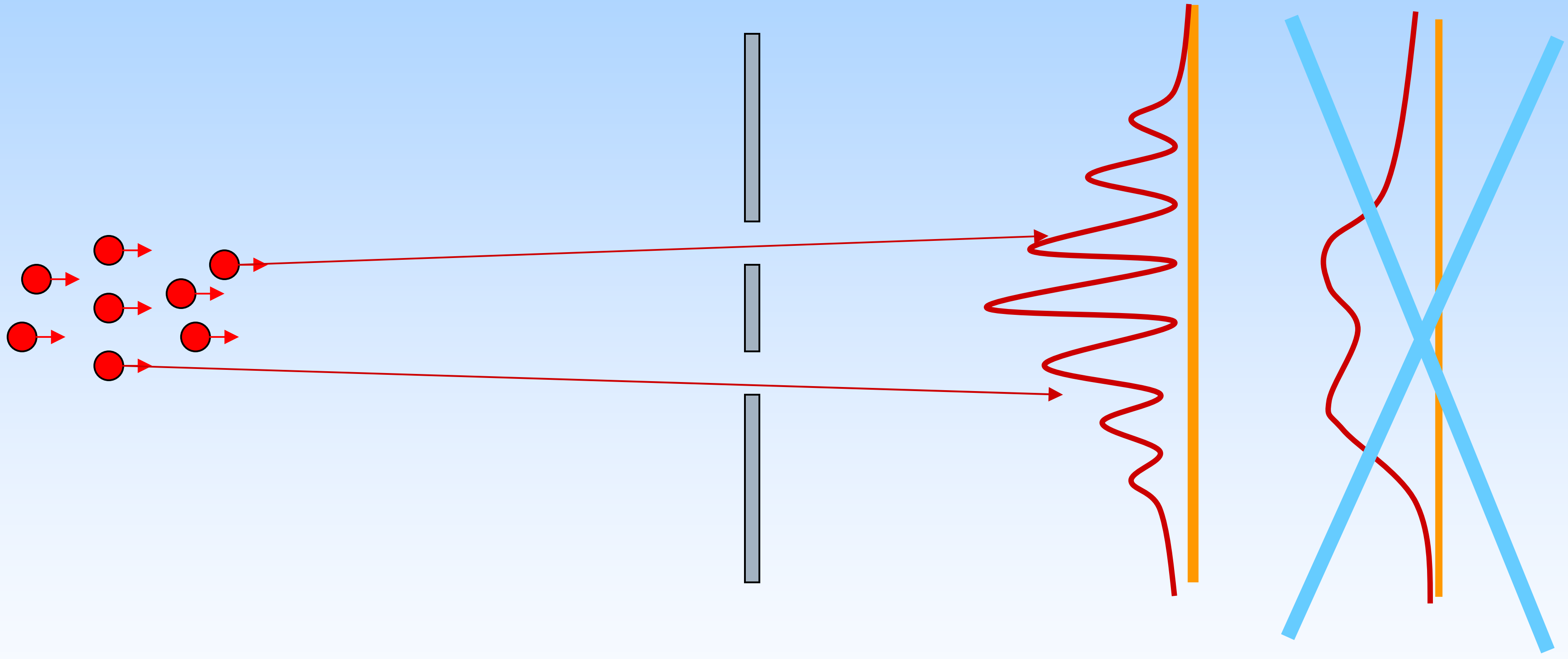
Wave-Particle Duality

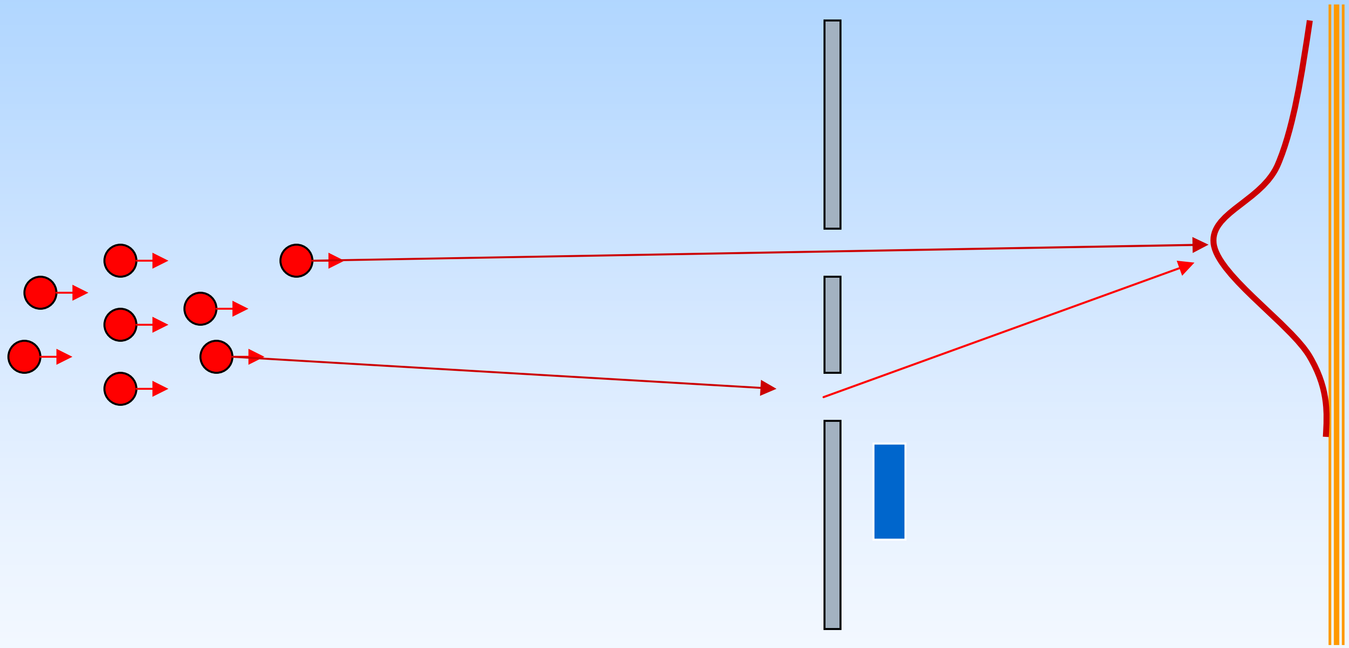


"two slit" experiment

$$|\psi\rangle = C_0|0\rangle + C_1|1\rangle$$



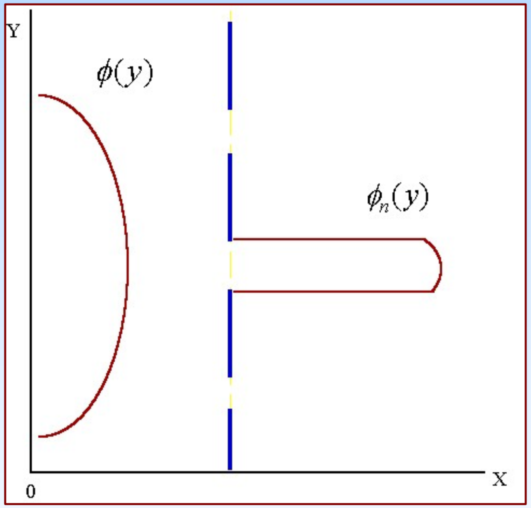
Quantum Decoherence



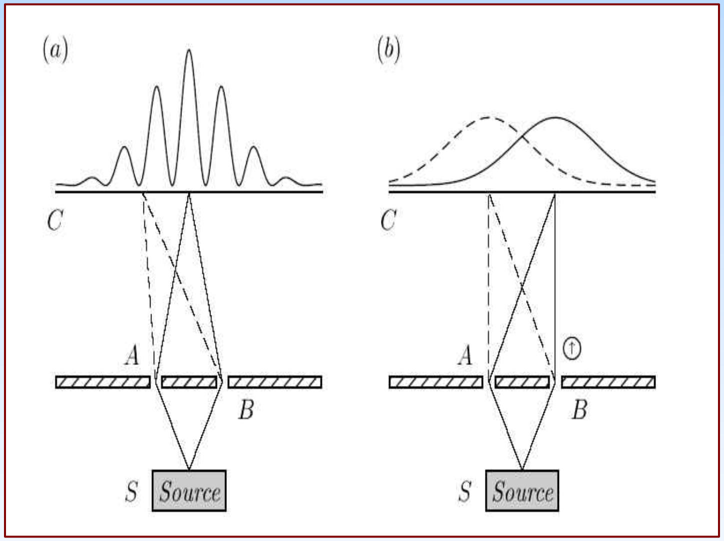
Wave Packet Collapse (WPC)

Decoherence in Quantum Measurement

Single Particle Picture



Ensemble Picture



$$|\psi\rangle = \sum_n c_n |n\rangle \rightarrow |n\rangle$$

$$|\psi\rangle\langle\psi| \rightarrow \sum_n |c_n|^2 |n\rangle\langle n|$$

Ensemble Explanation For Quantum Measurement

Quantum Probability to Classical Probability

$$\rho = |\psi\rangle\langle\psi| = \sum_n c^*_m c_n |n\rangle\langle m| \rightarrow \rho_M = \sum_n |c_n|^2 |n\rangle\langle n|$$

Vanishing of Diagonal Elements due to QM

$$\langle x|\psi\rangle = \sum_n c_n \phi_n(x) = c_1\phi_1(x) + c_2\phi_2(x)$$

$$\rho(x,x) = |c_1|^2 |\phi_1(x)|^2 + |c_2|^2 |\phi_2(x)|^2 + \underbrace{c^*_1 c_2 \phi^*_1(x)\phi_2(x) + c.c.}_{\text{interference:}}$$

interference:

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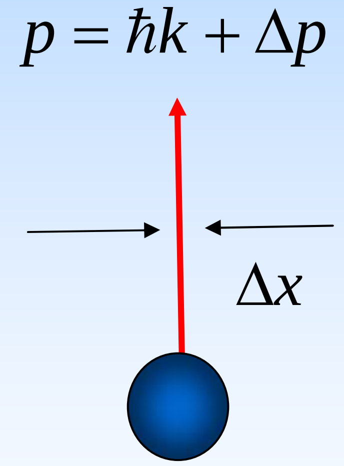
Bohr's Complementarity

Matter possesses particle-wave duality, but the particle and wave natures are repulsive with each other in a same experiment

Uncertainty Principle

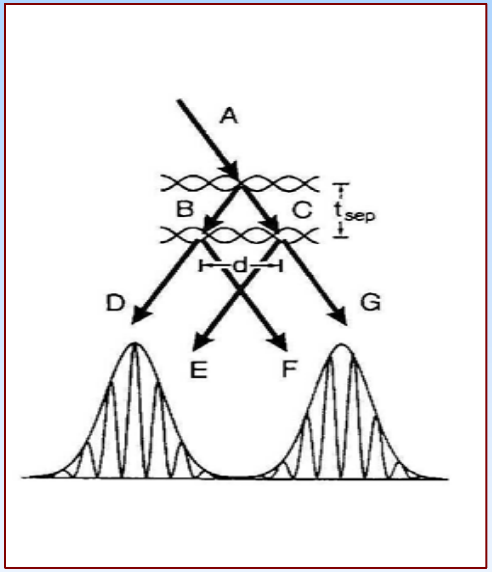
$$\Delta x \bullet \Delta p \sim \hbar$$

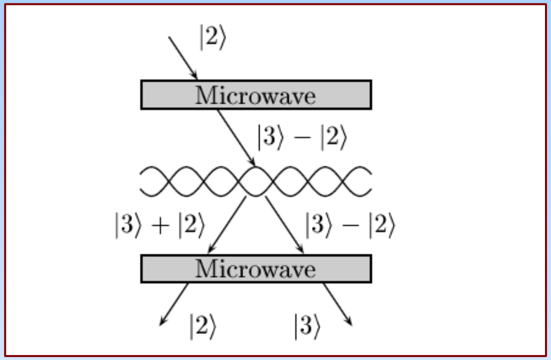
Particle (Δx) — **wave** (Δp)

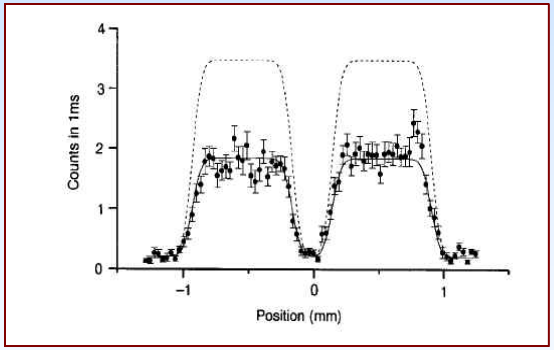


Vanishing interference due to Perturbation of Momentum (PM) ?

Nature, 395,33(1998)







PM is Not Unique,

Quantum Entanglement

From Conventional Copenhagen to Modern Point of View for Quantum Measurement

Copenhagen

1. Need not time to complete QM with WFC for the System (S)
2. Measuring Apparatus (D) must be Classical

van Neuman

Entanglement by interaction between S and D

Modern Approach : Zurek, Zeh, Joos, Sun, et al

Decoherence :
Classical Correlation from Entanglement By Environment

Von-Neumann Model for QM

D – S interaction

$$H_I = SP$$

$$[X, P] = i$$

$$[X, S] = [P, S] = 0$$

$S \leftrightarrow$ measured system

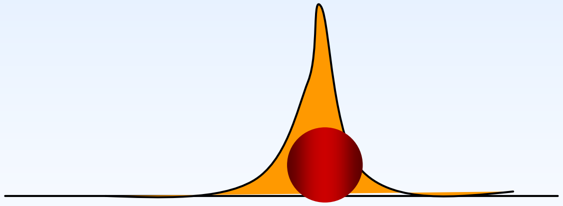
$X, P \leftrightarrow$ detector

$$S|s\rangle = s|s\rangle$$

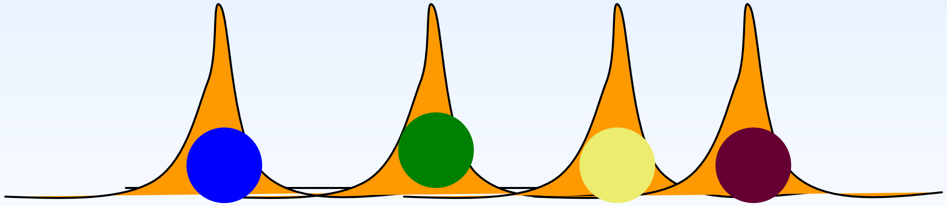
$$X|x\rangle = x|x\rangle$$

$$\psi(0) = \phi(x) \sum c_s |s\rangle$$

$$\psi(x,t) = \langle x|\psi(t)\rangle = \sum c_s \phi(x+st)|s\rangle$$



$$\phi(x) = \langle x|\phi\rangle$$



Simple Mathematics

$$e^{isPt}\left|x\right\rangle = \left|x+st\right\rangle$$

$$\left|\psi(0)\right\rangle = (\sum c_s \left|s\right\rangle) \otimes \left|\phi\right\rangle$$

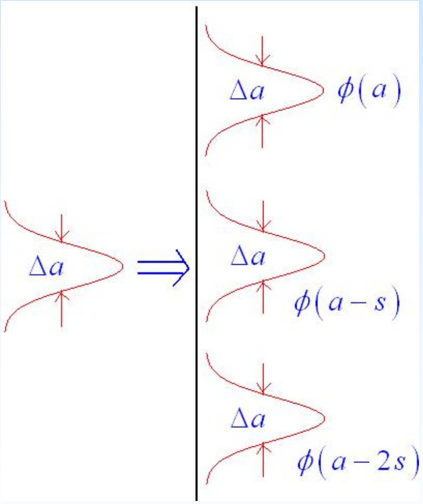
$$\psi(x,t) = \left\langle x \middle| \psi(t)\right\rangle$$

$$= \sum c_s \left\langle x \right| e^{-isPt} \left|\phi\right\rangle \left|s\right\rangle$$

$$= \sum c_s \phi(x+st) \left|s\right\rangle$$

$$\left|\psi(t)\right\rangle = e^{-iSPt}(\sum c_s \left|s\right\rangle) \otimes \left|\phi\right\rangle$$

$$= (\sum c_s \left|s\right\rangle) \otimes e^{-isP} \left|\phi\right\rangle$$



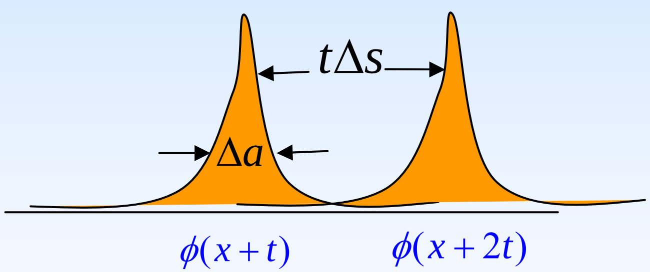
Ideal Measurement =Schmidt decomposation

Schmidt decomposition

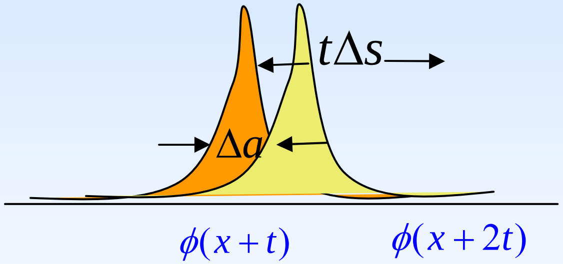
$$|\psi\rangle = \sum c_{sk} |s\rangle \otimes |k\rangle = \sum b_n |\chi_n\rangle \otimes |\phi_n\rangle$$

$$\langle \chi_m | \chi_n \rangle = \delta_{mn} = \langle \phi_m | \phi_n \rangle$$

Overlaps :



$$\Delta a \ll t\Delta s$$



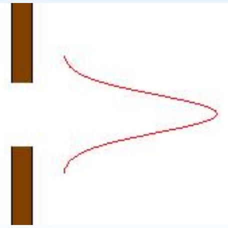
$$\Delta a \sim t\Delta s$$

Stern-Gerlach Experiment

$$H = \frac{p^2}{2M} + g\sigma_z B(z)$$

$$|\psi(0)\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |D\rangle$$

$$|D\rangle =$$

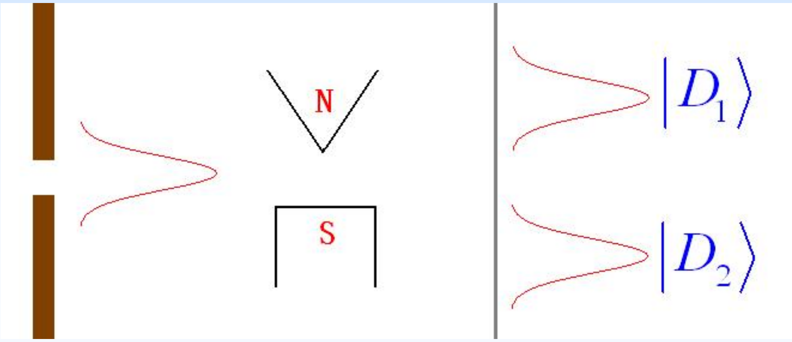


$$\langle x|D\rangle \sim \exp\left[-\frac{x^2}{4\sigma^2}\right]$$

Dynamic Schmidt Decomposation

$$|\psi(0)\rangle \to |\psi(t)\rangle = \alpha|1\rangle \otimes |D_1\rangle + \beta|0\rangle \otimes |D_0\rangle$$

$$\langle x|D_{1(\text{or }0)}\rangle = \langle x|\exp\left\{\pm iB(z) - i\frac{p^2}{2M}\right\}|D\rangle$$



Reduced Density Matrix

$$\rho_s = Tr_D(\left|\psi(t)\right\rangle\left\langle\psi(t)\right|)$$

$$\left|\alpha\right|^2 \left|1\right\rangle\left\langle 1\right| + \left|\beta\right|^2 \left|0\right\rangle\left\langle 0\right| + \alpha\beta^* F \left|1\right\rangle\left\langle 0\right| + h.c$$

$$B(z) \sim kz$$

Deacying Decoherence Factor

$$F = \left\langle D_1 \middle| D_0 \right\rangle = \exp\left\{-2a^2 f^2 t^2 - \xi t^4\right\} \xrightarrow{t\to\infty} 0$$

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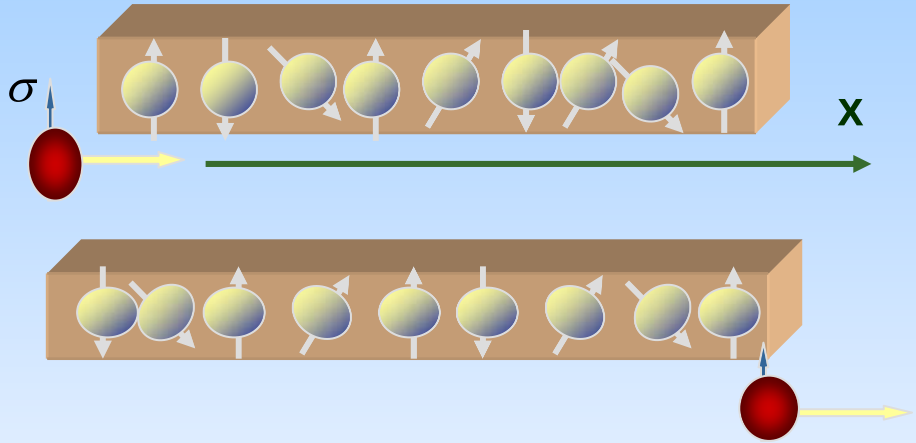
HEPP-Coleman Model

(Hepp,1975, Sun ,1993)



$$H_S = C P$$

Ultra-Relativistic particle + N Spin Array

Model Hamiltonian

$$H_I = \sum_{j=1}^{N} \frac{1}{2} V(x - j + ct)[1 + \sigma_3(0)]\sigma_2(j),$$

Exact Solution

$$U_I(t) = \prod_{j=1}^{N} e^{-iF(x-j+ct)\sigma_2(j)\frac{1}{2}[1+\sigma_3(0)]}$$

$$F(x-j+ct) = \int_{-\infty}^{t} V(x-j+ct')dt' = \frac{1}{c}\int_{-\infty}^{x-j+ct} V(y)dy$$

$$U_j(t)|\downarrow\rangle = |u_j\rangle = \begin{pmatrix} -\sin F(x-j) \\ \cos F(x-j) \end{pmatrix}$$

$$U(t)|\uparrow(0)\rangle \otimes |\downarrow\rangle \otimes \cdots \otimes |\downarrow\rangle \sim |\uparrow(0)\rangle|\uparrow\rangle \otimes \ldots \otimes|\uparrow\rangle,$$
$$U(t)|\downarrow(0)\rangle \otimes |\downarrow\rangle \otimes \cdots \otimes |\downarrow\rangle \sim |\downarrow(0)\rangle \otimes |\downarrow\rangle \otimes \cdots \otimes |\downarrow\rangle$$

$$\int_{-\infty}^{\infty} V(x)dx = \frac{\pi}{2}$$

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Decoherence due to the macroscopicness

$$\Delta S_z = \frac{1}{2}\sum_{j=1}^{N}\langle\Psi(t)|\sigma_3(j)|\Psi(t)\rangle + \frac{1}{2}N = |\alpha|^2\sum_{j=1}^{N}\sin^2 F(x-j)$$

$$\sin^2 F(x-j) \simeq F^2(x-j) \qquad \Delta S_z = 2NV^2t^2$$

$$N \to \infty \qquad V\sqrt{N} \to g$$

$$F(N,t) = \prod_{j=1}^{N}\langle\downarrow|u_j\rangle = \prod_{j=1}^{N}\cos F(x-j)$$

$$|\cos F(x-j)| \leq 1$$

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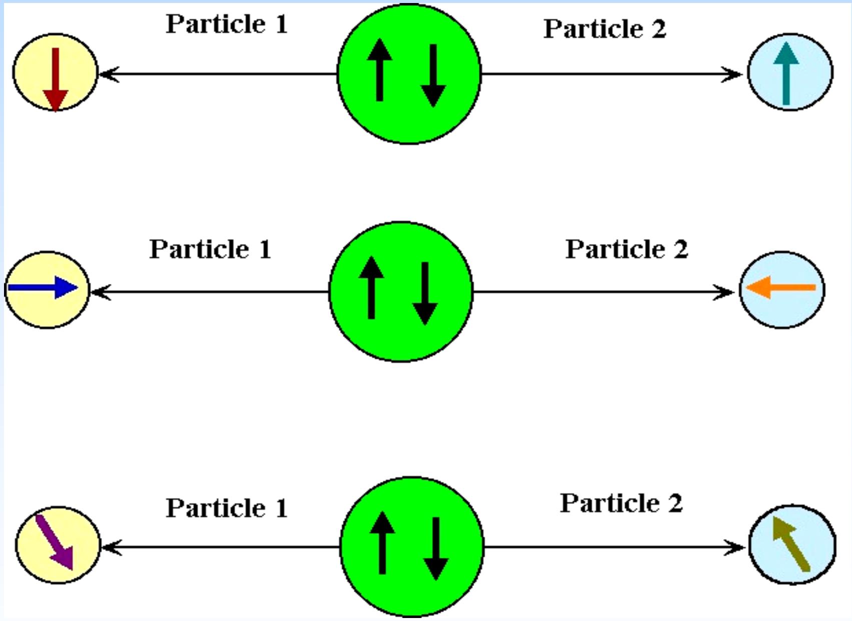
Principle Difficulty :

Entanglement $\neq$ Quantum Mesurement

Uncertainty in Entanglement

$$|\eta\rangle = \tfrac{1}{\sqrt{2}}\left[|1, D_1\rangle - |0, D_0\rangle\right]$$
$$= \tfrac{1}{\sqrt{2}}\left[|+, D_+\rangle - |-, D_-\rangle\right]$$

$$|\pm\rangle = \tfrac{1}{\sqrt{2}}\left[|1\rangle \pm |0\rangle\right],$$
$$|D_\pm\rangle = \tfrac{1}{\sqrt{2}}\left[|D_1\rangle \mp |D_-\rangle\right]$$



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QM Needs Classical Correlation

$$|\alpha|^2 |1, D_1\rangle\langle 1, D_1| + |\beta|^2 |0, D_0\rangle\langle 0, D_0|$$

$$\sum_n C_n |n\rangle \otimes |D_n\rangle = \sum_n B_n |S_n\rangle \otimes |A_n\rangle$$

$|n\rangle \otimes |D_n\rangle$ or $|S_n\rangle \otimes |A_n\rangle$

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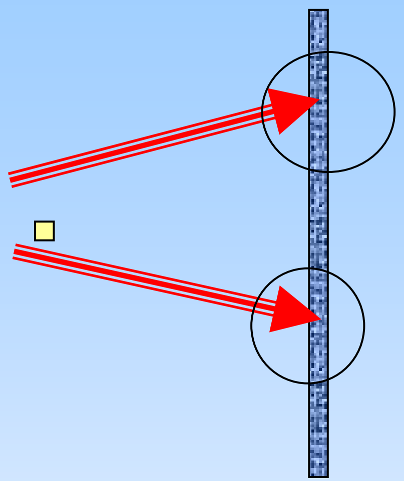
Zurek Triple Model

$$H = \underbrace{\frac{p^2}{2M} + g\sigma_z B(z)}_{H_1} + H_{\mathrm{ES}}$$

Interaction with Screen



Zurek: Environment Induced Decoherence

$$|\psi(0)\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |D\rangle \otimes |E\rangle$$

Step 1: $(\alpha|1\rangle + \beta|0\rangle) \otimes |D\rangle \otimes |E\rangle \to (\alpha|1\rangle \otimes |D_1\rangle + \beta|0\rangle \otimes |D_0\rangle) \otimes |E\rangle$

Step 2: $(\alpha|1\rangle \otimes |D_1\rangle + \beta|0\rangle \otimes |D_0\rangle) \otimes |E\rangle \to \alpha|1\rangle \otimes |D_1\rangle \otimes |E_1\rangle + \beta|0\rangle \otimes |D_0\rangle \otimes |E_0\rangle$

$$\langle E_0 | E_1 \rangle = 0$$

$$\rho_{\mathrm{SD}} = Tr_E(|\psi\rangle\langle\psi|) = |\alpha|^2 |1, D_1\rangle\langle 1, D_1| + |\beta|^2 |0, D_0\rangle\langle 0, D_0|$$

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More About Environment induced decoherence

C.P. Sun, Phys. Rev. A (1993).

$$H = H_E + H_S + H_{ES}$$

Initial state: $\left|\psi(0)\right\rangle = \left|\phi_{S,D}(0)\right\rangle \otimes \left|\varphi_E(0)\right\rangle; \quad \left|\phi_{SD}(0)\right\rangle = C_g\left|g\right\rangle + C_e\left|e\right\rangle$

State evolution: $\left|\psi(t)\right\rangle = C_g\left|g\right\rangle \otimes \left|\varphi_g(t)\right\rangle + C_e\left|e\right\rangle \otimes \left|\varphi_e(t)\right\rangle$

$$\left|\varphi_\alpha(t)\right\rangle = \exp\left(-iH_\alpha t\right)\left|\varphi_E(0)\right\rangle, \qquad \left(\alpha = g, e\right)$$

K. Hepp, Hev. Phys. Acta, 45, 237 (1972).
J.S. Bell, Hev. Phys. Acta, 48, 93 (1975).

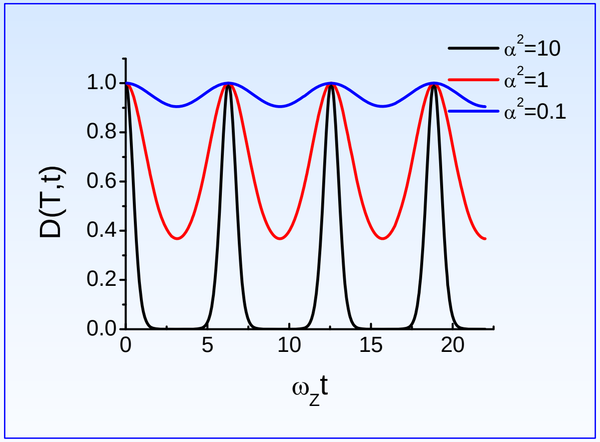
Reduced density matrix of the system:

$$\rho_S(t) = Tr_E\left(|\psi(t)\rangle\langle\psi(t)|\right) = |C_g|^2 |g\rangle\langle g| + |C_e|^2 |e\rangle\langle e|$$

$$+\langle\varphi_g(t)|\varphi_e(t)\rangle C_e C_g^* |e\rangle\langle g| + \langle\varphi_e(t)|\varphi_g(t)\rangle C_g C_e^* |g\rangle\langle e|$$

Decoherence due to Factorization :

$$\langle\varphi_e(t)|\varphi_g(t)\rangle = \prod_{j=1}^{N}\langle e_j|g_j\rangle$$

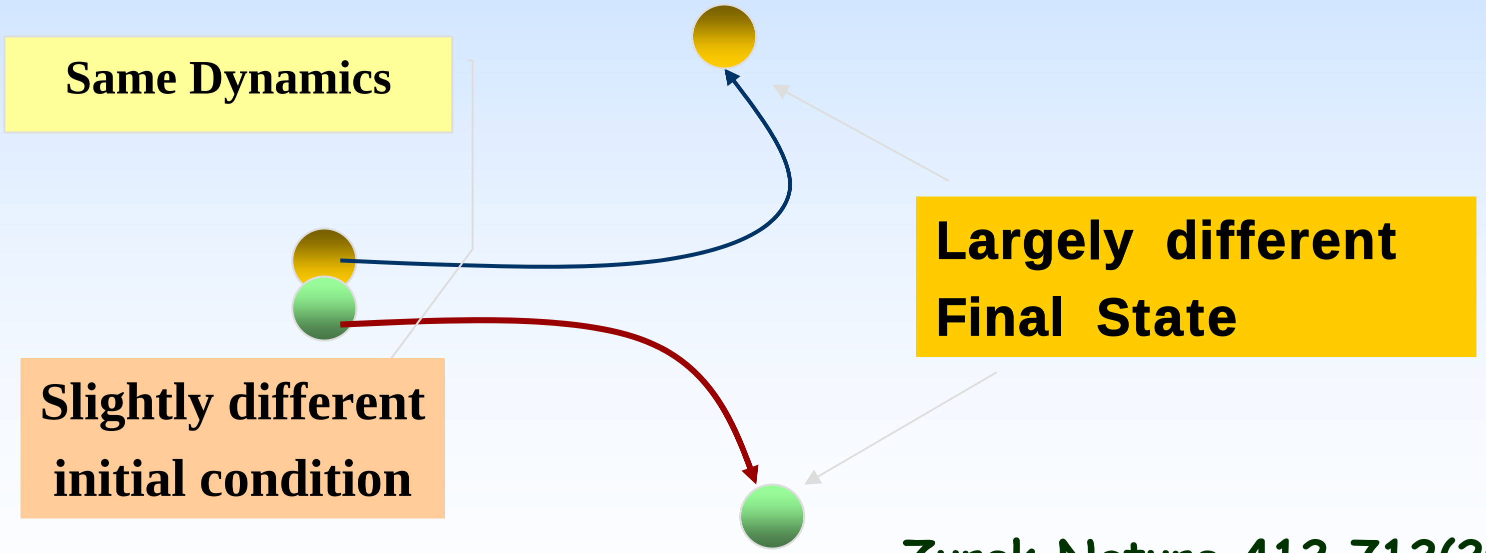


C.P. Sun , PRA, 1993

Decoherence due to Quantum Chaos

Classical Chaos: Butterfly Effect:

Slightly different initial condition leads to exponential divergence of trajectories

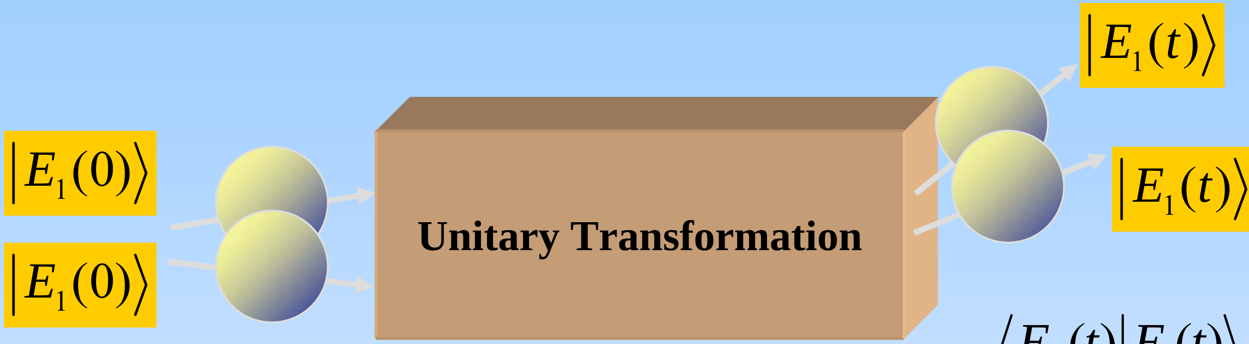


Zurek, Nature, 412, 712(2001);

PRL, 89, 170405 (2002)

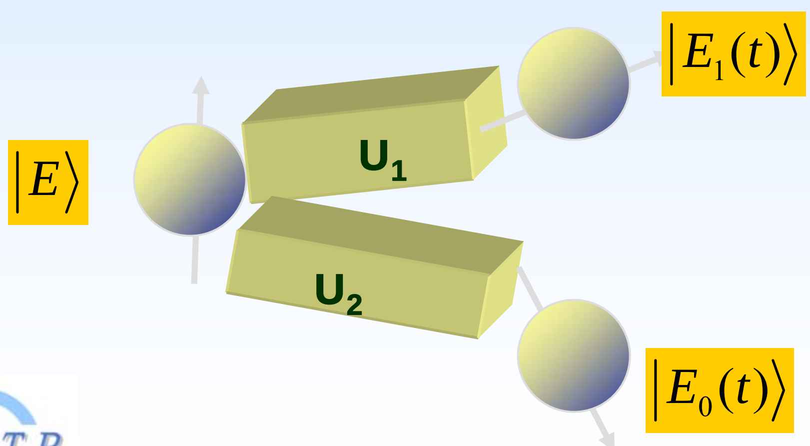
No Butterfly Effect in Quantum Mechanics?

Quantum Chaotic Environment :



$$\langle E_0(t)|E_1(t)\rangle = \langle E_0(0)|U^\dagger(t)U(t)|E_1(0)\rangle$$
$$= \langle E_0(0)|E_1(0)\rangle$$

Same initial state evolve according to two slightly different Hamiltonian



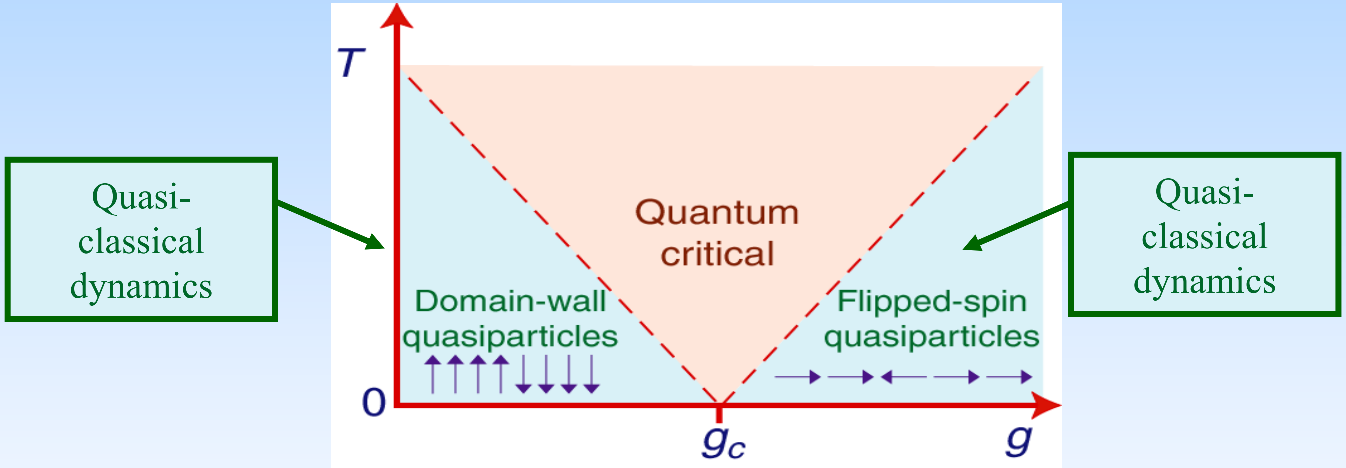
Peres, Conception of Quantum Chaos ,1995

$$\langle E_0(t)|E_1(t)\rangle = \langle E|U^\dagger(t)U(t)|E\rangle = 0$$

Quantum phase transition (QPT)

Ising model in a transverse field

$$H = H_0 + H_1 = -J\sum_j \left(g\sigma_j^x + \sigma_j^z\sigma_{j+1}^z\right)$$



$$g \ll 1: \quad |G\uparrow\rangle = |\cdots\uparrow\uparrow\uparrow\ \uparrow\uparrow\uparrow\uparrow\cdots\rangle - \frac{g}{2}|\cdots\uparrow\ \uparrow\uparrow\downarrow\uparrow\ \uparrow\uparrow\cdots\rangle - \cdots$$

$$g \gg 1: \quad |G\rangle = |\cdots\rightarrow\rightarrow\rightarrow\rightarrow\ \rightarrow\rightarrow\cdots\rangle - \frac{1}{2g}|\cdots\rightarrow\ \rightarrow\leftarrow\leftarrow\rightarrow\ \rightarrow\rightarrow\cdots\rangle - \cdots$$

Motivation (uncertainty relation)

In decoherence process, e.g., vanishing of interference paptern, the randomness of relative phase has its source in uncertainty relation

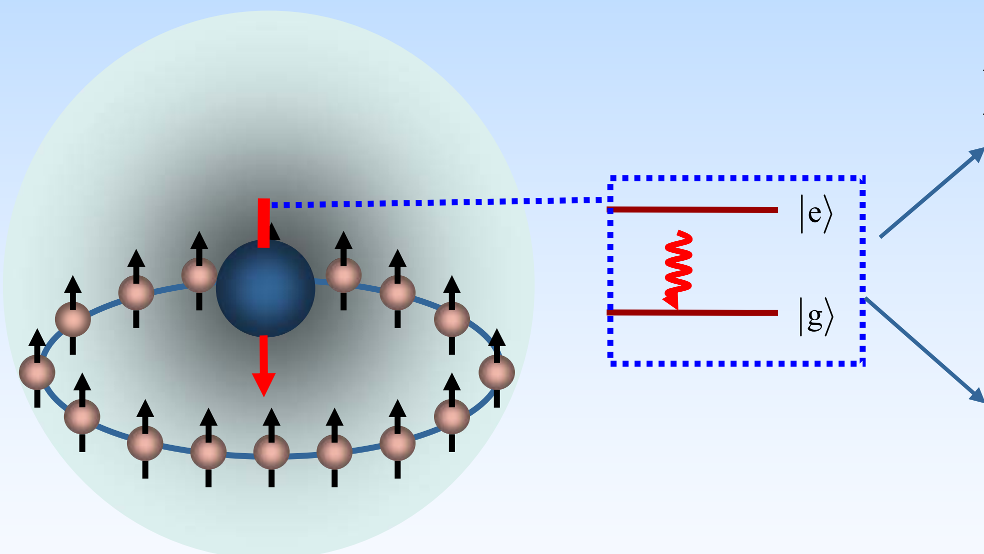
In QPT, the quantum fluctuation is also due to the uncertainty relation

Is there any intriguing relation between them???

Generalized Hepp-Coleman model:

Ising model in a transverse field

$$H = -J\sum_j \left(g|e\rangle\langle e|\sigma_j^x + \sigma_j^z \sigma_{j+1}^z \right)$$



$$H_e = H_e(\lambda) = H_g + V_e$$

$$H_g = -J\sum_j \sigma_j^z \sigma_{j+1}^z$$

$$V_e = -\lambda J \sum_j \sigma_j^x.$$

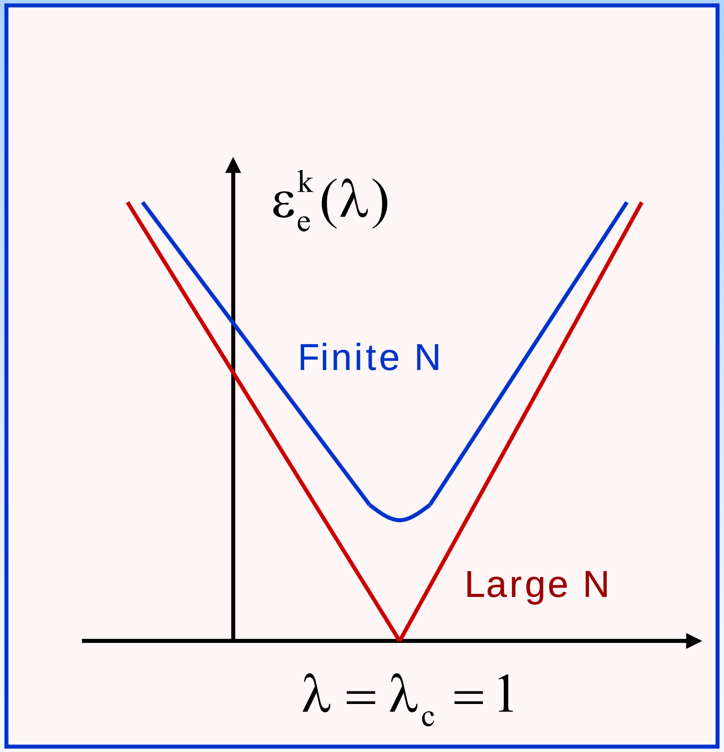
Exact solution of Loschmidt echo

$$H_e(\lambda) = \sum_k \varepsilon_e^k \left(A_k^+ A_k - 1/2 \right)$$

$$A_k = \sum_l \frac{e^{-ikal}}{\sqrt{N}} \prod_{s<l} \sigma_s^{[x]} \left(u_e^k \sigma_l^{[+]} - i v_e^k \sigma_l^{[-]} \right),$$

$$\varepsilon_e^k = \varepsilon_e^k(\lambda) = 2J\sqrt{\left(1 + \lambda^2 - 2\lambda\cos(ka)\right)}$$

K=2n/Na



Ground states

$$H_e = -J\sum_j \sigma_j^z \sigma_{j+1}^z - \lambda J \sum_j \sigma_j^x$$

$$H_g = -J\sum_j \sigma_j^z \sigma_{j+1}^z$$

$$A_k |G\rangle_e = 0$$

$$|G\rangle_g = |\downarrow\downarrow \ldots\ldots \downarrow\rangle_g$$

$$B_k |G\rangle_g = 0$$

$$B_{\pm k} = \cos(\alpha_k) A_{\pm k} - i\sin(\alpha_k)(A_{\mp k})^+,$$

$$|G\rangle_g = \prod_{k>0} \left[i\cos(\alpha_k) + \sin(\alpha_k) A_k^+ A_{-k}^+ \right] |G\rangle_e$$

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Decoherence & Loschmidt echo

Reduced Density matrix

$$[\rho_s(t)]_{eg} = c_g c_e^* D(t)$$

$$D(t) = \langle \varphi_g(t) | \varphi_e(t) \rangle \qquad |\varphi_\alpha(t)\rangle = \exp[-iH_\alpha t]|G\rangle_g$$

Loschmidt echo

$$L(\lambda, t) = | D(t) |^2 = | \langle \varphi_g(t) | \varphi_e(t) \rangle |^2$$

$$L(\lambda, t) = \prod_{k>0} [1 - \sin^2(2\alpha_k)\sin^2(\varepsilon_e^k t)].$$

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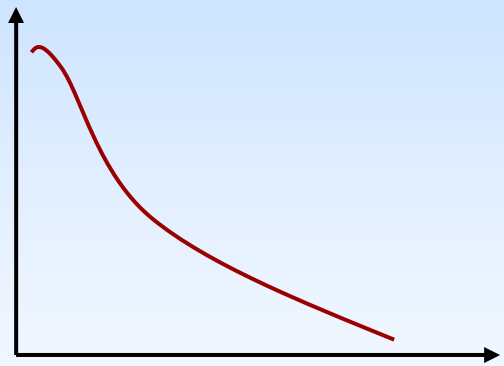
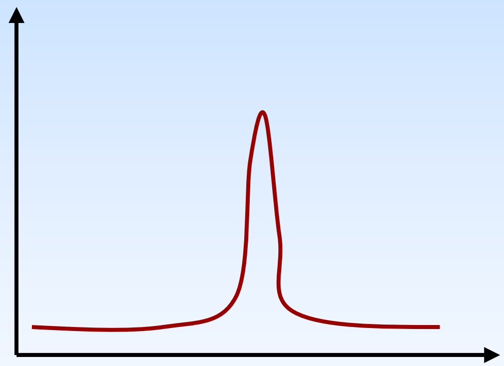
Loschmidt echo via Local density of state

$$L(\omega) = \int L(t)e^{-i\omega t}dt = \sum_s |\langle G | E_s \rangle|^2 \delta(\omega - E_s),$$





$$L(t) \sim e^{-\gamma t}$$

$$L(\omega) \sim \frac{1}{(\omega - \Omega)^2 + \gamma^2}$$

A heuristic analysis

$$L_c(\lambda,t) \equiv \prod_{k>0}^{K_c} F_k > L(\lambda,t),$$

$$S(\lambda,t) = \ln L_c \equiv -\sum_{k>0}^{K_c} |\ln F_k|$$

$$\sin^2[2\alpha_k] \approx \left(k\bar{\lambda}a\right)^2/(1-\lambda)^2$$

$$\varepsilon_e^k \approx 2J\,|1-\lambda|$$

$$S(\lambda,t) \approx -\frac{\bar{\lambda}^2 E(K_c)}{(1-\lambda)^2}\sin^2\left(2J[1-\lambda]t\right)$$

$$E(K_c) = 4\pi^2 N_c(N_c+1)(2N_c+1)/(6N^2)$$

$$L_c(\lambda,t) \approx \exp\left(-\gamma t^2\right)$$

$$\gamma = 4J^2 E(K_c)$$

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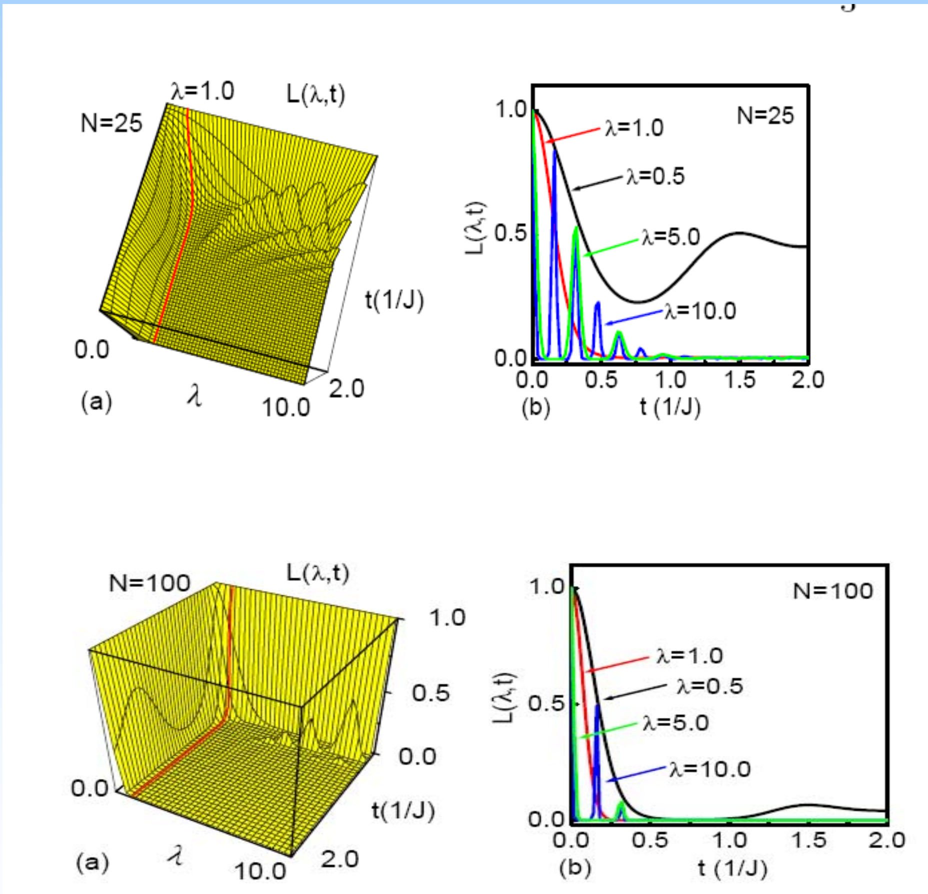
Numerical results: far from the critical point



Our result: Unpublished ,

but even posted in Arxive before the PRL paper

Short time behavior

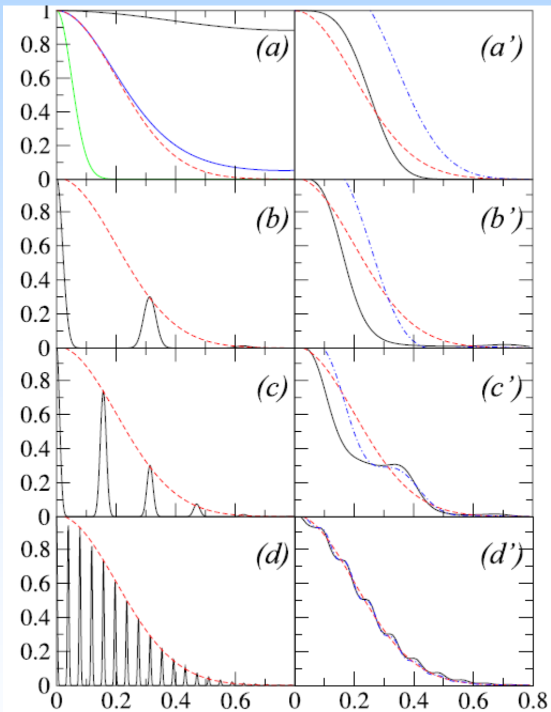
$$L_c(\lambda, t) \approx \exp\left(-\gamma t^2\right)$$

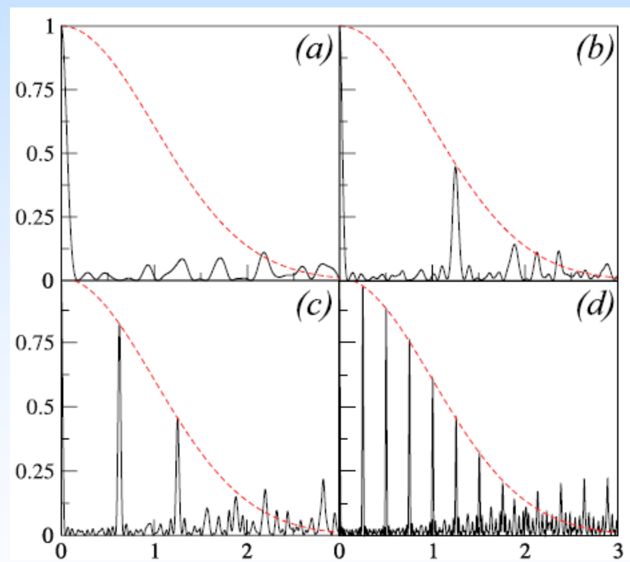
Universality of Loschmidt Echo

Cucchietti, Fernandez-Vidal, Paz, quantph/0604136

$$L_c(\lambda, t) \approx \exp\left(-ut^2\right)F(t)$$

Coupling independent u



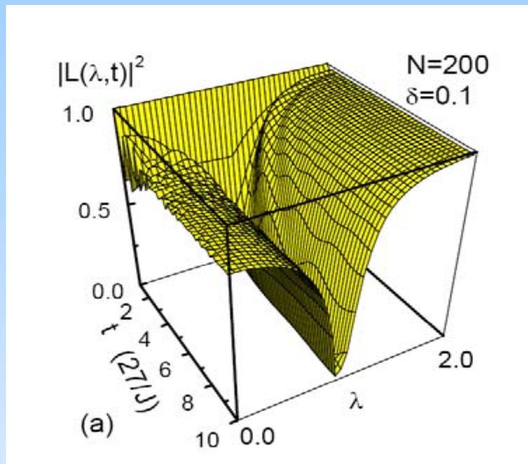


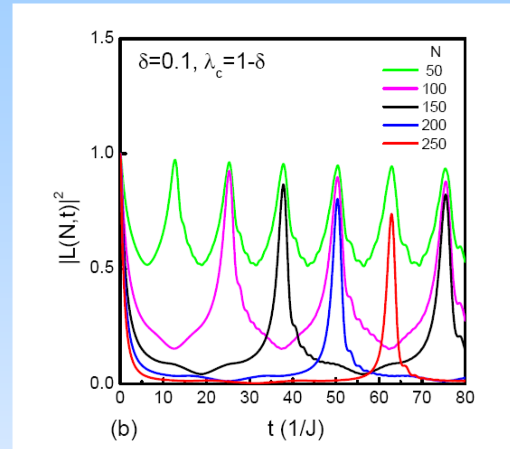
transverse field Ising model

Boson Habburd model

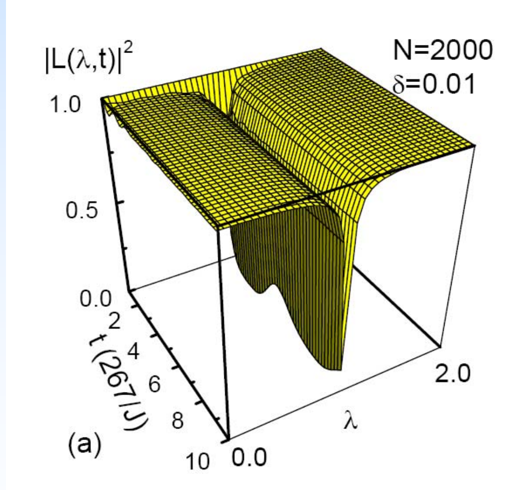
Numerical results : near the critical point

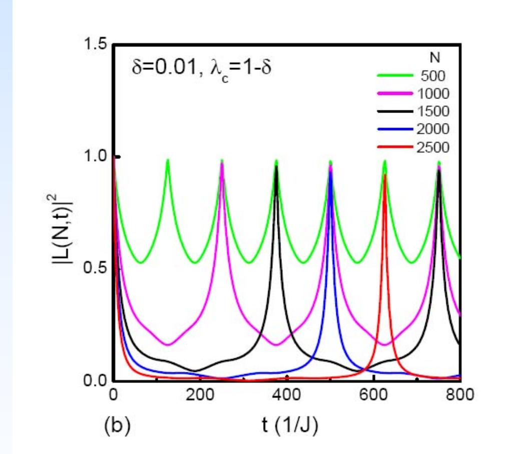
Small N



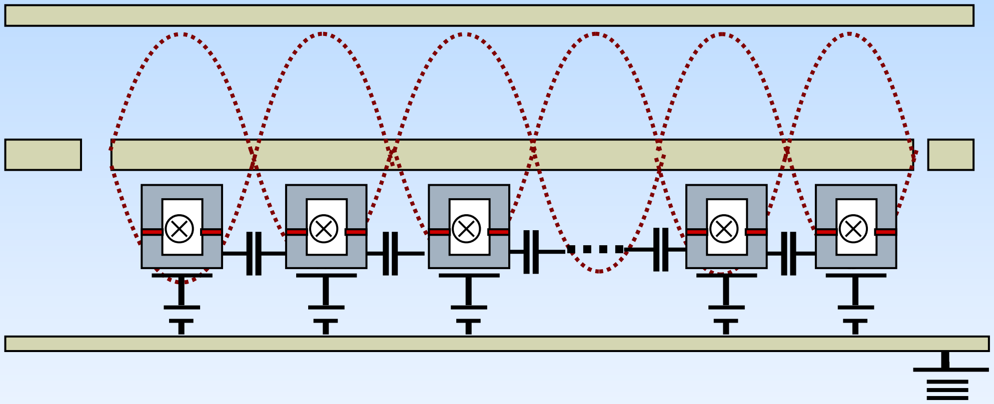


Large N





Implementation :
Superconducting Quantum Network



Circuit QED for Charge Qubit array

Model Hamiltonian

$$H_0 = h[\lambda] \equiv B(\lambda \sum_{\alpha} \sigma_x^{(\alpha)} + \sum_{\alpha} \sigma_z^{(\alpha)} \sigma_z^{(\alpha+1)})$$

$$NEC : C_m / C_\Sigma \approx 0.05$$

$$\phi_x = \eta\left(a + a^\dagger\right) \qquad \eta = (S/d)\left(\hbar l \omega / L\right)^{1/2}$$

$$H_F = \hbar \omega a^\dagger a - g \sum_{\alpha} \left(a^\dagger a + a a^\dagger\right) \sigma_x^{(\alpha)}$$

Pseudo-Spin Representation:

$$S_{zk} = \gamma_k^\dagger \gamma_k + \gamma_{-k}^\dagger \gamma_{-k} - 1,$$
$$S_{xk} = i\gamma_{-k}\gamma_k + \gamma_{-k}^\dagger \gamma_k^\dagger,$$
$$S_{yk} = \gamma_{-k}^\dagger \gamma_k^\dagger - \gamma_{-k}\gamma_k.$$

$$|G\rangle = \prod_{k>0} |-\rangle_k \equiv \prod_{k>0} |O_k, O_{-k}\rangle$$

$$H_n = \sum_{k>0} H_n^{(k)}$$

$$H_n^{(k)} = \varepsilon_{nk}(S_{zk}\cos 2\alpha_{nk} + S_{xk}\sin 2\alpha_{nk})$$

$$D_{mn} = \prod_{k>0} \langle -| e^{iH_m^k t} e^{-iH_n^k t} |-\rangle_k$$

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Summary and conclusion

1. We establish the connection between the QPT and the decoherence for the first time.

2. We obtain the decohence factor (Loschmidt echo) quantitatively through the exact calculation

3. Both the analysis and the numerical result confirm our assumption that the quantum critical behavior of the environment strongly enhances its ability of inducing decoherence.

4. A possible physical implementation is proposed based on the superconducting Circuit QED

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Recent Objectives and Their Status

Sources of Decoherence in Multi-Qubit System (esp. in Solid State System, 1/f) ?V ↔ Decoherence Models for Many Body System with Inner or External Environment

Non-Desolation Readout of Quantum Information ? ↔ Theory of Quantum Measurement in Practice

Protocols for Quantum Storage and Memory V ↔ Exotic Light Propagation in Coherence Medium

Quantum Information Transfer for Multi-Qubits V ↔ Correlation in Engineered FM (AF) Quantum Spin Chain

Fast Quantum Logic Gate with Noise Suppression ? ↔ Quantum State Control for Quantum Open System

Quantum Information | Quantum Physics

We have worked More

We have initialed some

We are ready to work

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Thank you!

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