

Cavity QED Construct:

Quantum Information Processing and Quantum Optics for Solid State Artificial Atoms

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Quantum Physics and Quantum Information Processing

QPOQIP in ITP CAS

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Outline

Lecture1: (Introductions)

Cavity QED Architecture: From Optical to Solid State Systems
---Superconducting Quantum Circuits and Nano-Mechanical Systems
Apr 4, Tue 16:30-17:15 & 17:30-18:15 2.

Lecture2:

Superconducting Quantum Circuit QED for Quantum Computing
Apr 8, Sat 11:30-12:15 (PHY5580)

Lecture 3.

Quantum State Transfer and Storage with Quantum Spin System
Apr 18, Tue 16:30-17:15 & 17:30-18:15

Lecture 4:

Decoherence induced by Quantum Phase Transition In Solid Systems
Apr 25, Tue 16:30-17:15 & 17:30-18:15

Related Publications and References therein

Lecture1 & 2:

1. P. Zhang, Y. D. Wang, and C. P. Sun, *Phys. Rev. Lett.* 95, 097204 (2005)
2. Y.-x. Liu, J. Q. You, L. F. Wei, C. P. Sun, and F. Nori, *Phys. Rev. Lett.* 95, 087001 (2005)
3. C. P. Sun, L. F. Wei, Y.-x. Liu, and F. Nori, *Phys. Rev. A* **73**, 022318 (2006)
4. Y. B. Gao, Y. D. Wang, and C. P. Sun, *Phys. Rev. A* 71, 032302 (2005)
5. Y. D. Wang, Z. D. Wang, and C. P. Sun, *Phys. Rev. B* 72, 172507 (2005)
6. Y. D. Wang, P. Zhang, D. L. Zhou, and C. P. Sun, *Phys. Rev. B* **70**, 224515 (2004)
7. Y. D. Wang, Y. B. Gao and C. P. *Eur. Phys. Jour. B* **40**, 321-326 (2004).
8. P. Zhang, Z. D. Wang, J. D. Sun, and C. P. Sun, *Phys. Rev. A* **71**, 042301 (2005)

Lecture 3:

1. H.T. Quan, Z. Song , X.F. Liu, P. Zanardi , C.P.Sun, *Phys. Rev. Lett.* (2006) in press
2. X.-F. Qian, T. Shi, Y. Li, Z. Song, and C. P. Sun, *Phys. Rev. A* 72, 012333 (2005)
4. H. T. Quan, P. Zhang, and C. P. Sun, *Phys. Rev. E* 72, 056110 (2005)
5. H. T. Quan, P. Zhang, and C. P. Sun, *Phys. Rev. E* **73**, 036122 (2006)
6. L. Zheng, C. Li, Y. Li, and C. P. Sun, *Phys. Rev. A* 71, 062101 (2005)
7. F. Xue, S. X. Yu, and C. P. Sun, *Phys. Rev. A* **73**, 013403 (2006)

Related Publications and References therein

Lecture 4:

1. C. P. Sun, Y. Li, and X. F. Liu, *Phys. Rev. Lett.* 91, 147903 (2003)
2. S. Yang, Z. Song, and C. P. Sun, *Phys. Rev. A* 73, 022317 (2006)
3. X.-F. Qian, Y. Li, Y. Li, Z. Song, and C. P. Sun, *Phys. Rev. A* 72, 062329 (2005)
4. Z. Song, P. Zhang, T. Shi, and C.-P. Sun, *Phys. Rev. B* 71, 205314 (2005)
5. T. Shi, Y. Li, Z. Song, and C.-P. Sun, *Phys. Rev. A* 71, 032309 (2005)
6. Y. Li, T. Shi, B. Chen, Z. Song, and C.-P. Sun, *Phys. Rev. A* 71, 022301 (2005)
7. R. Xin, Z. Song and C.P. Sun, *Physics Letters A*, 342,, 30 (2005)

Review Article:

Song Z, Sun CP, LOW TEMPERATURE PHYSICS 31 (8-9): 686 (2005)

Lecture1 - Overviews:

Quantum Information Processing

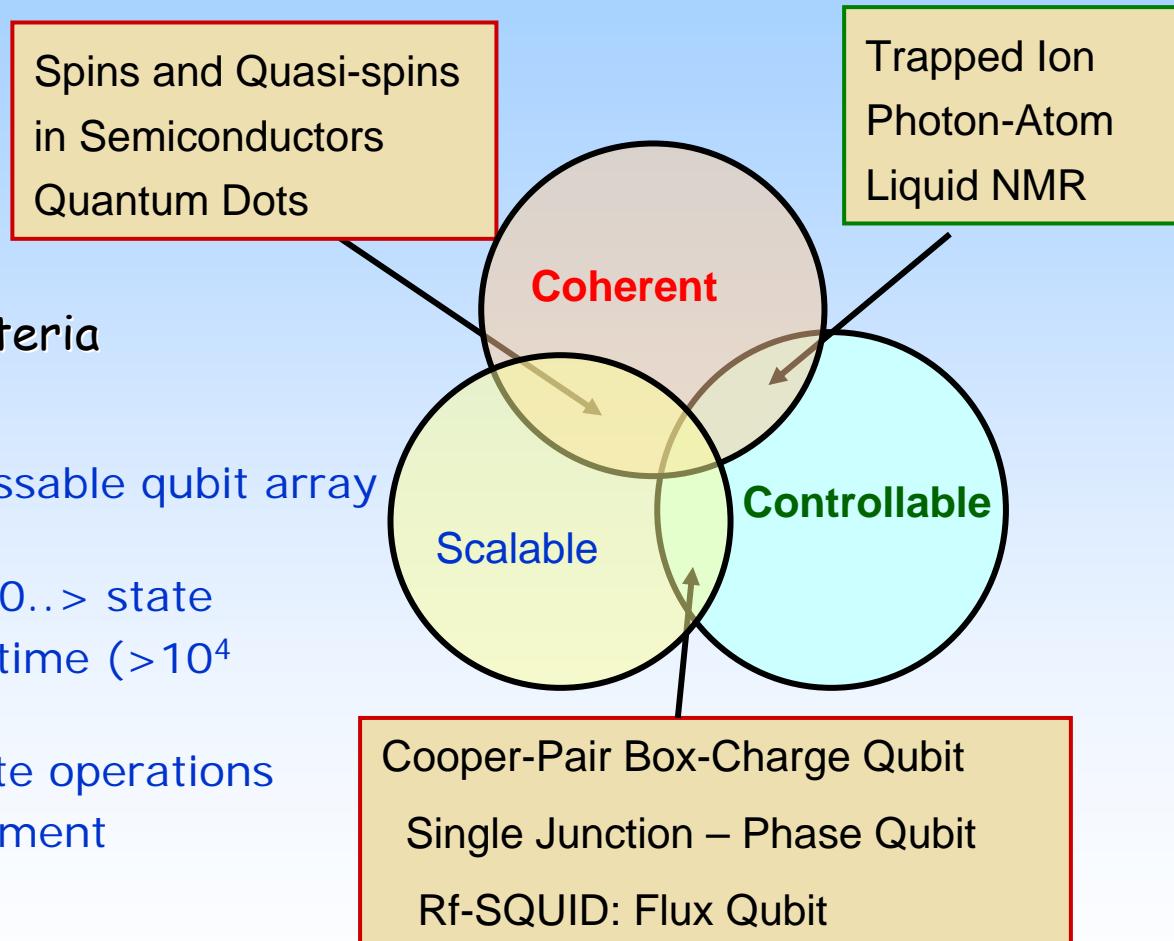
based on Solid State Systems

---Superconducting Quantum Circuits and Nano-Mechanical Systems

Physical Realizations of Quantum Bits (Qubits) Require Specially Designed Two-Level Systems:

DiVincenzo Five criteria

1. Well defined addressable qubit array
stable memory
2. Initialize in the $|000..\rangle$ state
3. Long decoherence time ($>10^4$
operation time)
4. Universal set of gate operations
5. Single-bit measurement



Main Obstacles and Challenges For QC

quantum decoherence, which can be significantly enhanced in a system of many qubits,

a. In General

One Qubit Dephasing

$$e^{-\gamma t}$$

N Qubit Dephasing

$$e^{-N^2 \gamma t}$$

Sun et al., PRA (1993,1998)

b. An Atomic Ensemble

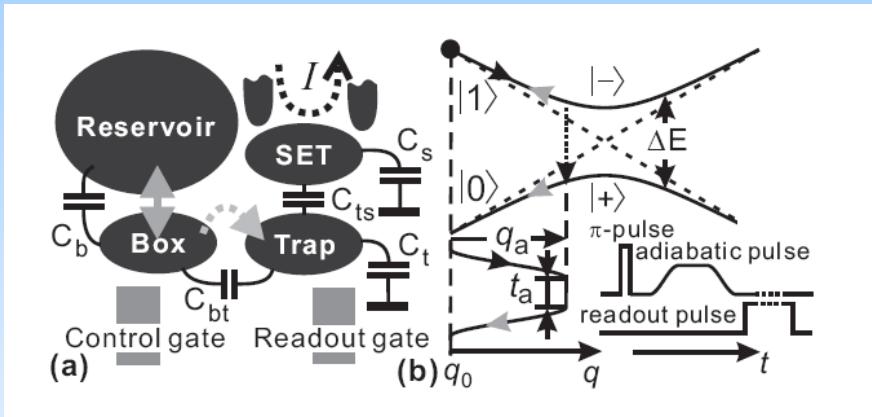
$$H = \sum_{k=1}^N g_k \sigma_+^{[k]} + h.c \xrightarrow{?} g \sqrt{N} b^+ + \dots$$

$$\sum_{k=1}^N \sigma_+^{[k]} = S_+ \Rightarrow \sqrt{N} b^+$$

Achieve a \sqrt{N} enhanced effective coupling, but induce a \sqrt{N} enlarged decoherence

Sun, Yi, Li, and You, PRA (2002,2003)

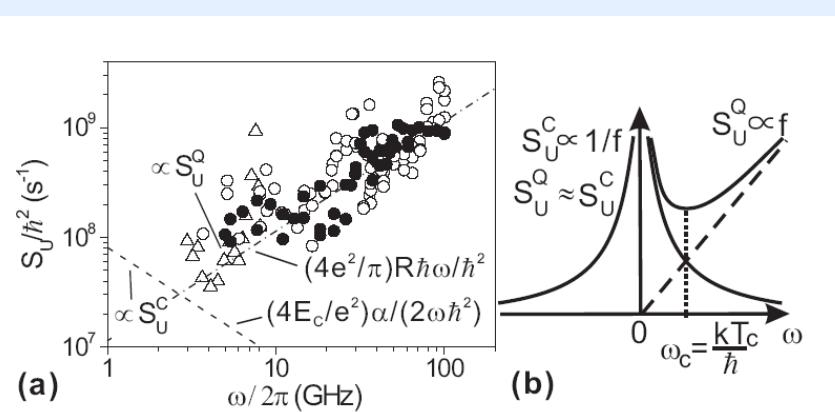
Low Frequency Noise 1/f



NEC experiment 2005

It seems to be a non-principle difficulty , but it challenges all present technologies

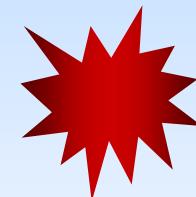
Nobody know its origins physically !!!



Quantum Information Meets
Condensed Matter Physics ?

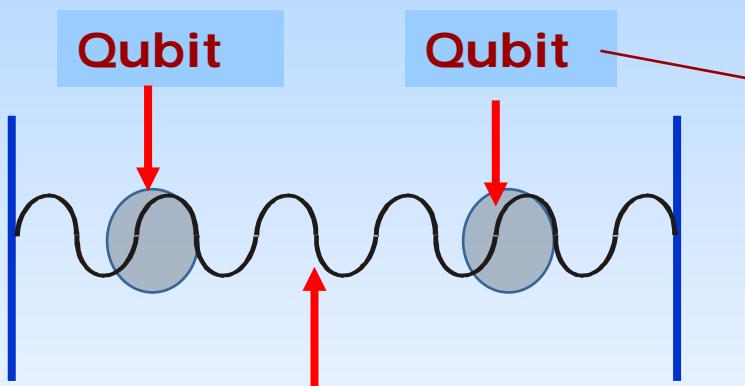
Or

Condensed Matter Physics Beats
Quantum Information ?



Integrating Various Qubits for QIP

Physics: Creating Distant Entanglement?



Atom, Spin, Ion, or
Artificial Atoms :
Charge, Flux , Quantum Dots

Data Bus: A Quantized Field

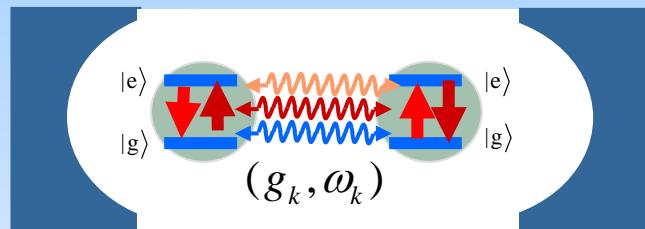
Quantized Electromagnetic Field
Large Josephson Junction (2004, Exp)
Quantum Transmission Line (2004, Exp)
“Phonons”: GHz. Nano Mechanical Beam

Examples

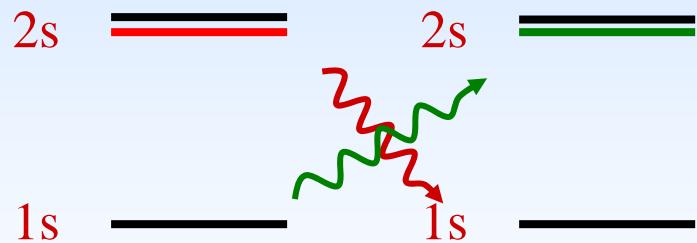
(I+N) : Tian, Zoller, PRL **93**, 266403 (04)
(C+N) : Irish , Schwab, PRB, **68**, 155311 (03)
(C+JJ): Wang, Sun, PRB, **70**, 224515 (04)

Artificial Cavity QED =
Spin-Boson Model

Cavity QED: Virtual Photon Exchange Induces Qubit-Qubit coupling



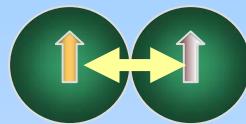
Effective Inter-Qubit Coupling



$$J_{eff} = \sum_k \frac{|g_k|^2}{\omega_k - \omega_a}$$

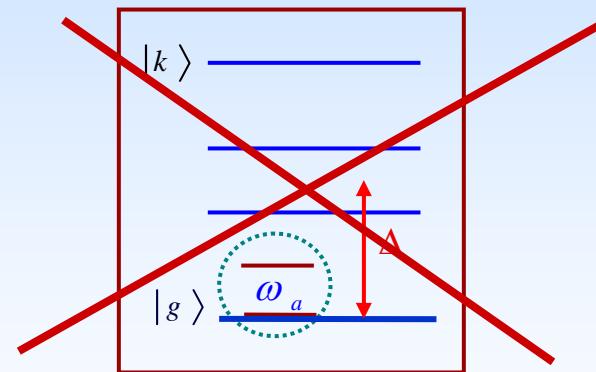
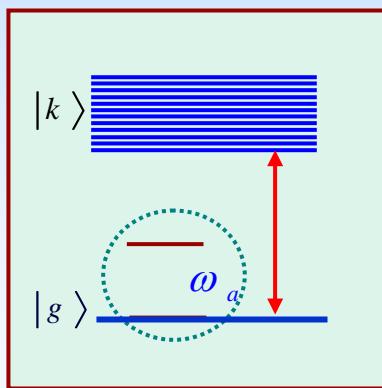
See the details in Lecture 2

Finite Effective Interaction Requires Gapped Systems



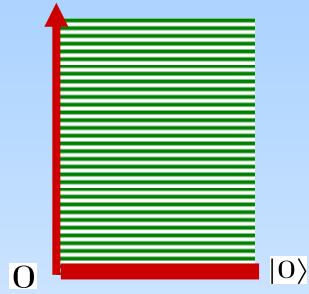
Effective Interaction

$$J_{eff} = \sum_k \frac{|g_k|^2}{\omega_k - \omega_a}$$

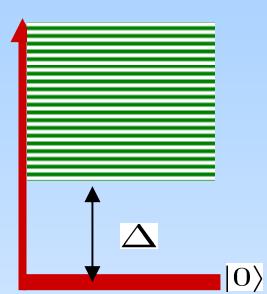


Circuit QED :

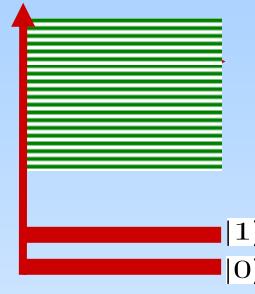
Superconducting Transmission Line + Charge Qubit



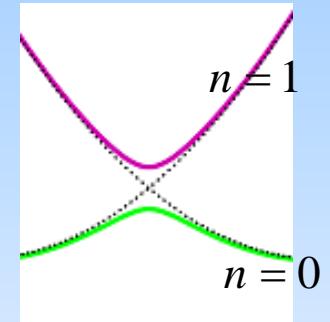
Normal Metal



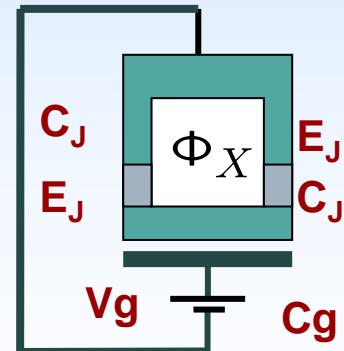
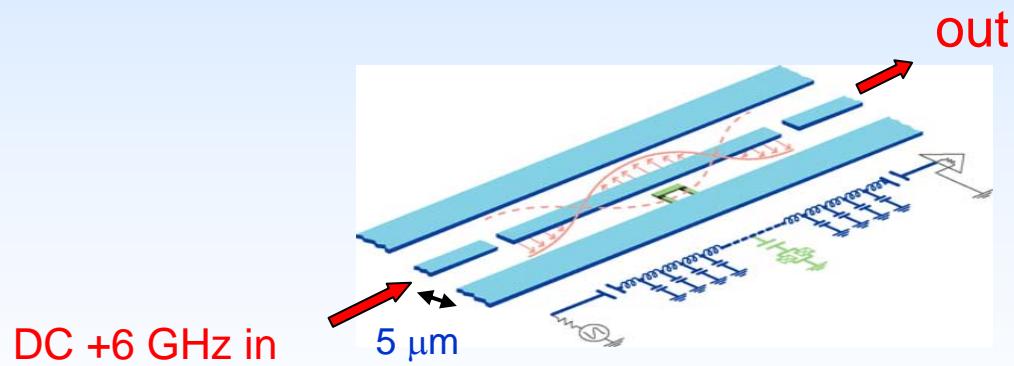
Supercond.



with Qubit?



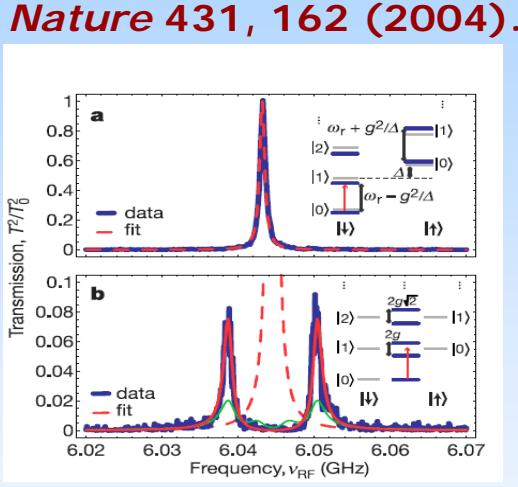
二能级系统



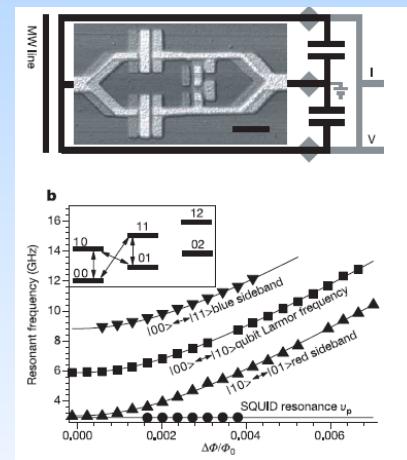
Circuit QED Based on Superconductors

$$H = \hbar\omega a^\dagger a + \frac{1}{2} \hbar\omega \sigma_z + g(a\sigma_+ + a^\dagger\sigma_-)$$

Experiments



Nature 431, 159 (2004).



Wang, Zhang, Zhou, and Sun, Phys. Rev. B 70, 224515 (2004)

A Earlier Proposal: Circuit QED based on Large junction by us (2002)

Large Junction $V = \cos(\phi) \simeq 1 - \frac{1}{2}\phi^2$

Strong Couplings by Circuit QED

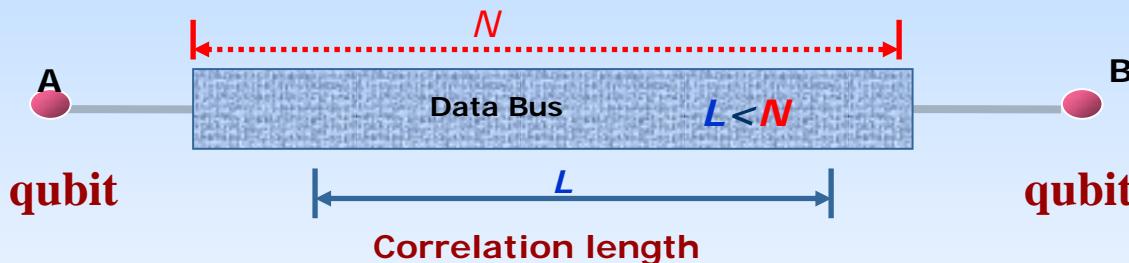
Parameter	Symbol	Optical cQED with Cs atoms	Microwave cQED	circuit QED
Dipole moment	d/ea_0	1	1,000	20,000
Vacuum Rabi frequency	g/π	220 MHz	47 kHz	100 MHz
Cavity lifetime	$1/\kappa; Q$	1 ns; 3×10^7	1 ms; 3×10^8	160 ns; 10^4
Atom lifetime	$1/\gamma$	60 ns	30 ms	$> 2 \mu\text{s}$
Atom transit time	t_{transit}	$> 50 \mu\text{s}$	100 μs	Infinite
Critical atom #	$N_0 = 2\gamma\kappa/g^2$	6×10^{-3}	3×10^{-6}	6×10^{-5}
Critical photon #	$m_0 = \gamma^2/2g^2$	3×10^{-4}	3×10^{-8}	1×10^{-6}
# of vacuumRabi oscillations	$n_{\text{Rabi}} = 2g/(\kappa + \gamma)$	10	5	100

Blais, A., et al., Phys. Rev.A 69, 062320 (2004)

Other Condensed Matter System seems to Beat Quantum Information ?

Finite Correlated Length vs. Decaying Entanglement
Due to the decreasing effective coupling

QI Transfer from A to B = Dynamic Entangling A and B States



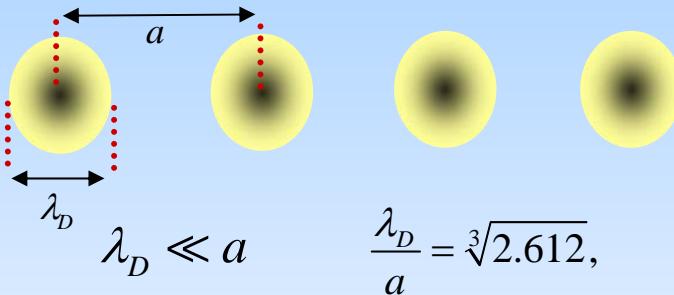
a long range order? $\langle \varphi | \varphi_0 \rangle \sim 1 \text{ or } 1/N^\delta$

$$\langle \Psi(x) \Psi(x') \rangle \propto \exp(-\frac{|x - x'|}{L}), \quad L \approx \zeta \Delta$$

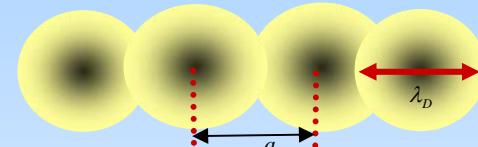
BEC : Long distance correlation of Boson System

Off-diagonal long range order (ODLRO)

High temperature ($T > T_c$):



Low temperature ($T < T_c$):



Coherent Overlap :
Macro-Atoms

$$\rho(x, y) = \frac{n_0}{V} + \int n_k e^{ik(x-y)} dk \rightarrow \frac{n_0}{V} + \frac{mkT}{2\hbar^2 r} \exp\left[-\frac{r}{R}\right]$$

$$r = |x - y| \rightarrow \infty$$

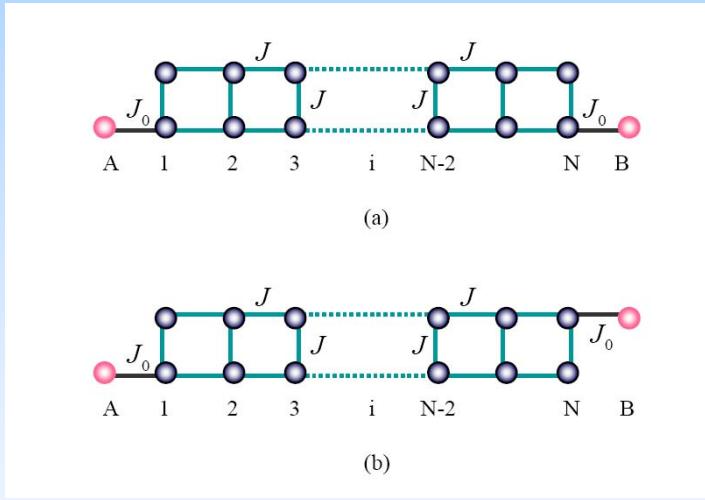
$$\rho(x, y) \rightarrow \frac{n_0}{V}$$

$$\lambda_D = \sqrt{\frac{2\pi\hbar^2}{m k_B T}}$$

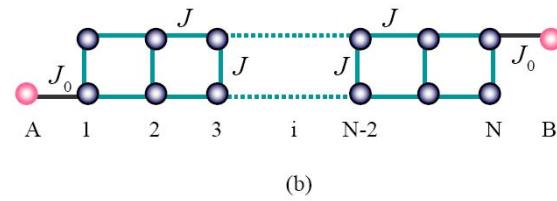
Thermal Wave Length

Quantum state transfer via Spin Ladder

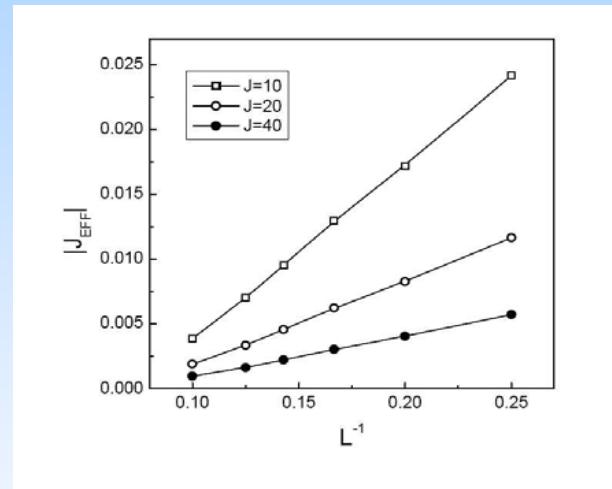
A typical gapped system



(a)



(b)



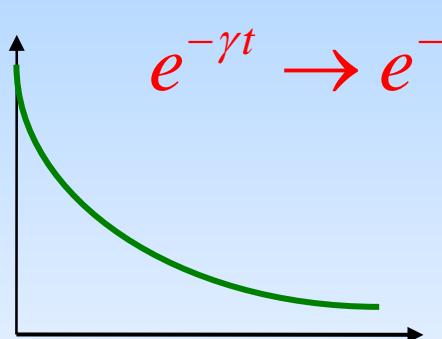
$$H_I = J_0 \vec{S}_A \cdot \vec{S}_1 + J_0 \vec{S}_B \cdot \vec{S}_N$$

$$H_{\text{int}} = J_0^2 G^{zz} (\omega = 0, x) \vec{S}_A \cdot \vec{S}_B$$

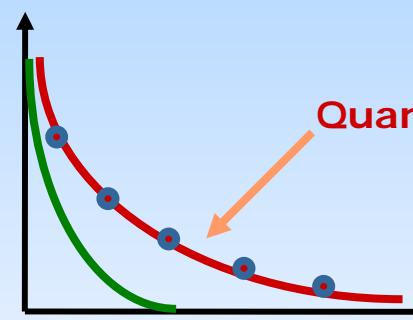
Li, Shi, Chen, Song, Sun, Phys. Rev. A 71, 022301 (2005)

De-entanglement vs. Decoherence

Yu, Eberly, Phys. Rev. Lett. **93**, 140404 (2004)



Decoherence of single qubit



Concurrence=entanglement

- What characterize the quantum coherence of many body system?
- Entanglement or Correlation? Are there differences?

Overcome the difficulty by Engineered Spin Chain With Always-On Interactions

S. Bose, 2003;
M. Christandl, PRL, 92, (2004)

$$H = \theta \sum_{i=1}^{i=N-1} J_i (S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+)$$

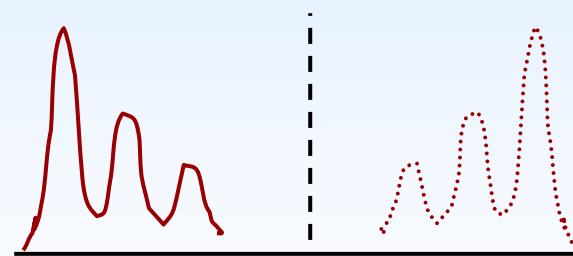
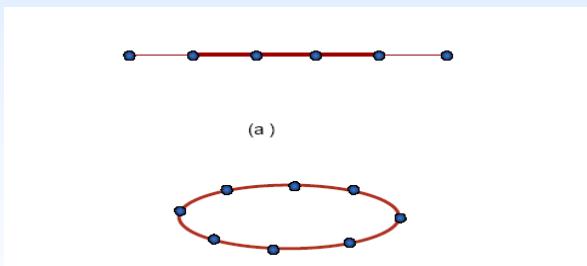
$$J_i = \sqrt{i(N-i)}$$

Our discovery :
Commensurate Spectrum

$$U(t_0) = P$$

Shi, Li, Song, and Sun, Phys. Rev. A 71, 032309 (2005)

Time evolution=Parity Reflection

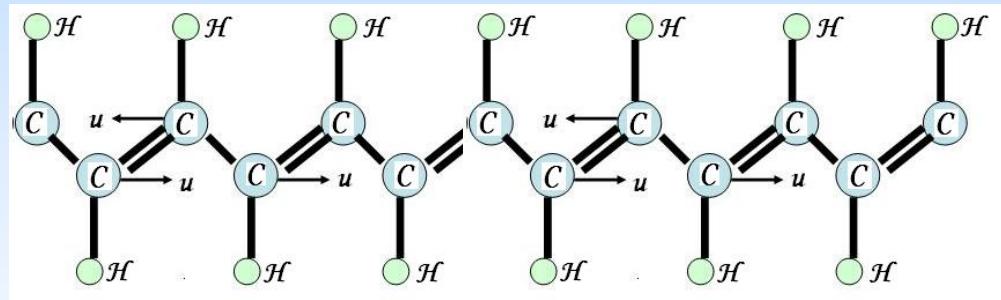


For the details see [Lecture 3](#):

Peierls Distorted Chain for QST

M.X. Huo, Ying Li, Z. Song , C.P. Sun, 2006 in preparation

More Realistic System with spatially-varying Hamiltonian

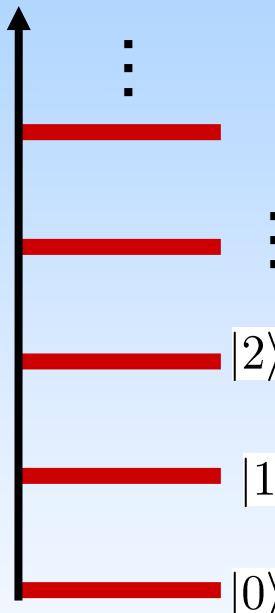


Dimmerization : SSH model

Nano-Mechanical Resonator (NAMR)

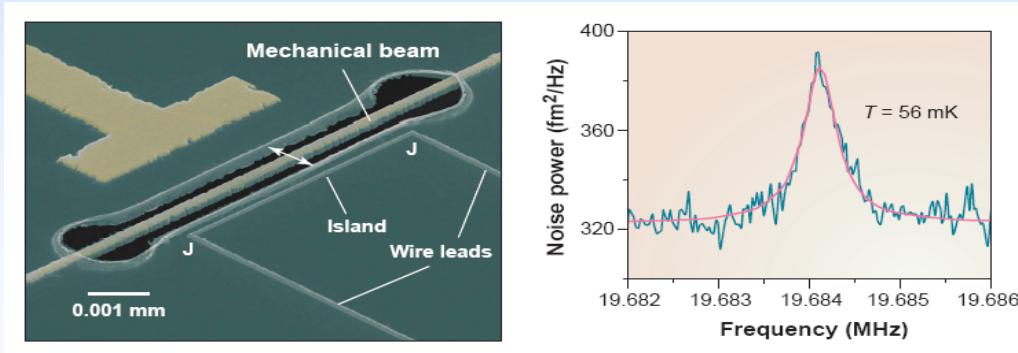
Quantization of Single Model Phonon

$$k_B T = 48mK \cdot k_B \approx \hbar \sqrt{E / \rho} (hL) \approx \hbar \cdot 1GHz$$



Knobel and Cleland, Nature, 2003
La Haye et al., *Science*, 304, 74
(2004)

$$H = \hbar \omega a^\dagger a$$



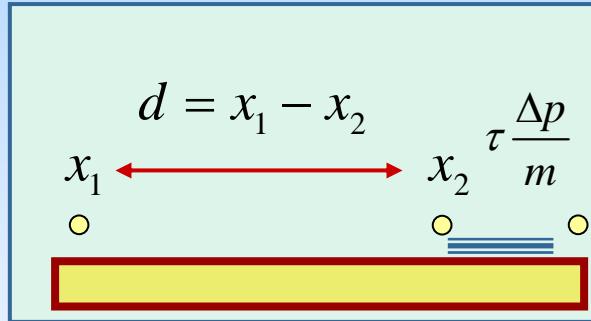
Standard Quantum Limit (SQL):

Standard Quantum Limit (SQL):

Measurement of length Limited by Uncertain Relation

Free Particle

$$\Delta p \Delta x_1 \geq \frac{\hbar}{2}$$



$$\begin{aligned}\Delta d &= \sqrt{(\Delta x_1)^2 + (\Delta x_2)^2 + \left(\tau \frac{\Delta p}{m}\right)^2} \\ &\geq \sqrt{2(\Delta x_1)\left(\tau \frac{\Delta p}{m}\right) + (\Delta x_2)^2} \\ &= \sqrt{\frac{\hbar \tau}{m} + (\Delta x_2)^2} \sim \sqrt{\frac{\hbar \tau}{m}}\end{aligned}$$

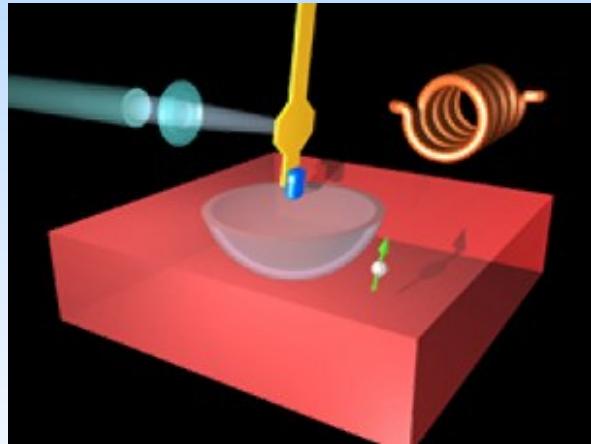
Harmonic Oscillator

$$x(t) = x(0) \cos(\omega t) + \frac{p(0)}{m\omega} \sin(\omega t)$$

$$\Delta x(t) = \sqrt{(\Delta x)^2 \cos^2(\omega t) + \frac{(\Delta p)^2}{m^2 \omega^2} \sin^2(\omega t)} \geq \sqrt{\frac{\hbar}{m\omega}}$$

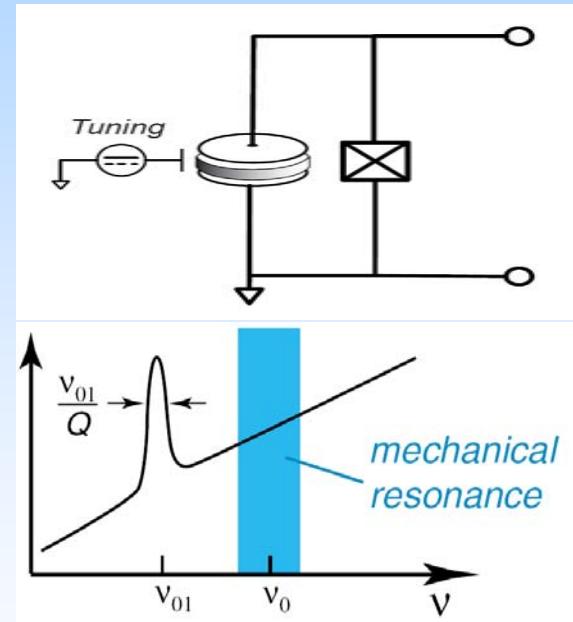
Nano-Mechanical QED (NM-QED)Constructs

1. NAMR + Spins



Nature 430, 329 (2004)

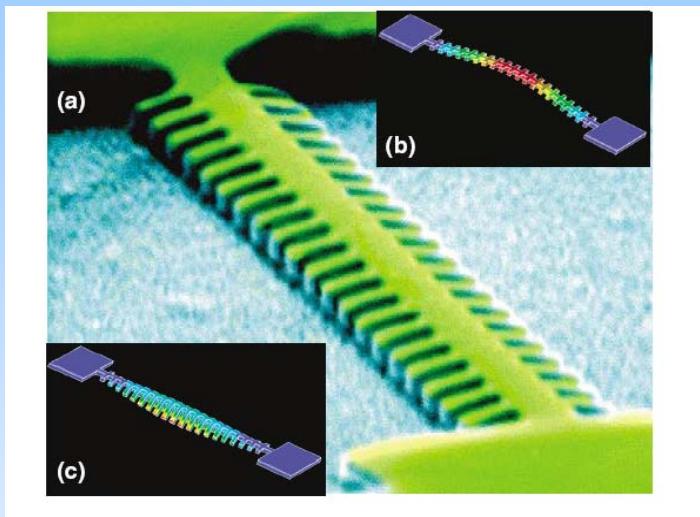
2. NAMR + Josephson Qubit



Armour, et al *Phys. Rev. Lett.* **88**, 148301 (2002).

Wang, Gao and Sun. *Eur. Phys. Jour. B* **40**, 321-326 (2004).

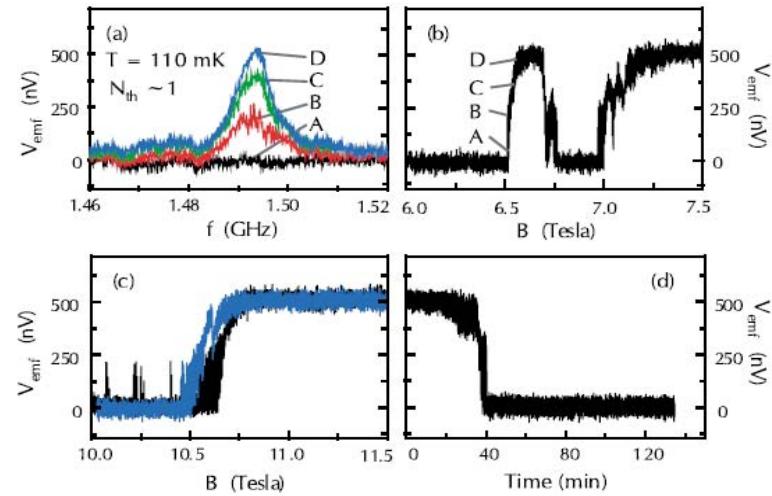
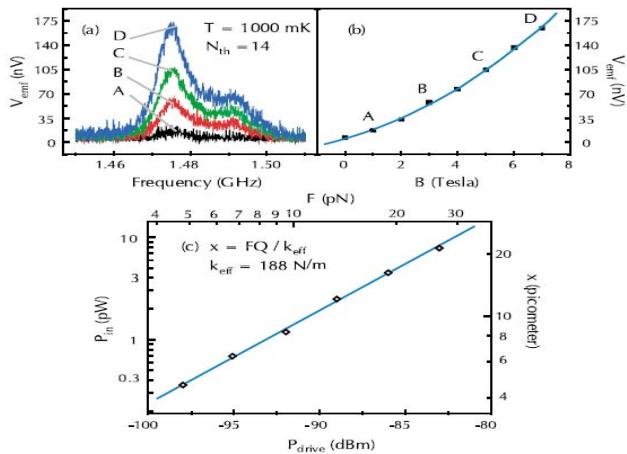
Direct Observation of Quantization of NAMR



Gaidarzhy et al, PRL. 94, 030402 (2005)

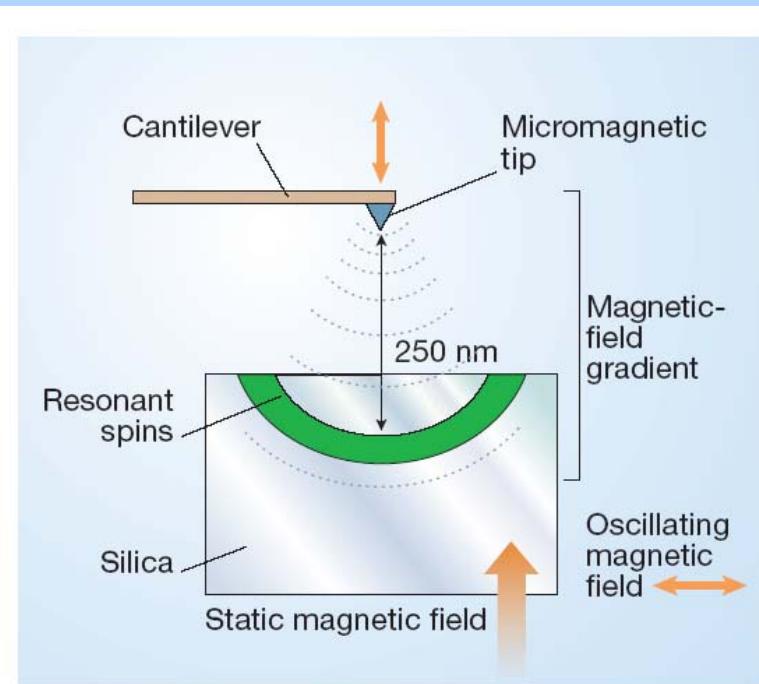
金电极上的电压

$$V = \frac{i\omega\xi L^2 B^2 / m}{\omega^2 - \omega^2 + i\omega\omega/Q} I(\omega)$$

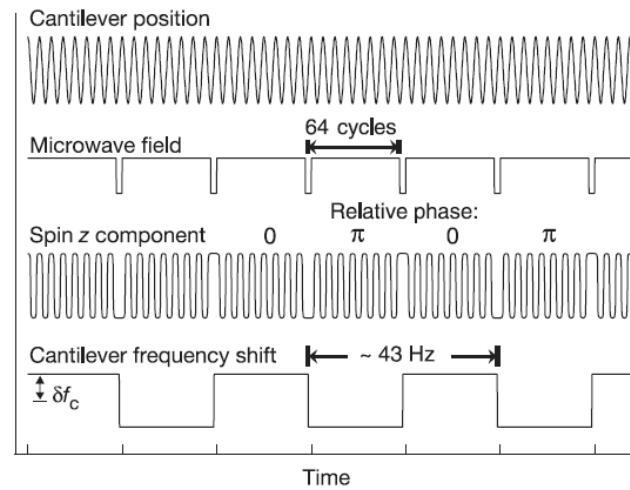


Detection Of Single Spin

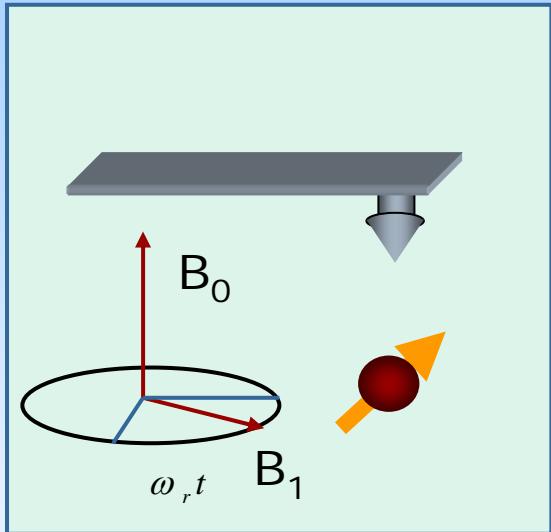
Nature 430, 329 - 332 (15 Jul 2004)



$$\delta f_c = \pm \frac{2f_c G \mu_B}{\pi k x_{\text{peak}}}, \quad (1)$$



Nano-Mechanical QED: Single Mode Phonon



$$H = \frac{P_z^2}{2M} + \frac{1}{2} M \omega_c^2 z^2 + \gamma \hbar \vec{B} \cdot \vec{S}$$

$$\vec{B} = B_1 \cos(\omega_r t) \hat{e}_x + B_1 \sin(\omega_r t) \hat{e}_y + B_0 \hat{e}_z + \vec{B}_{tip}(z)$$

$$B_{tip} = \frac{\mu_0}{4\pi} \frac{3(\vec{n} \cdot \vec{m})\vec{n} - \vec{m}}{d^3}$$

$$\vec{B}_{tip}(z) \approx (\xi_0 - \xi_1 z) \hat{e}_x + (\eta_0 - \eta_1 z) \hat{e}_z$$

$$\mathbf{H} = (p_x^2 + x^2)/2 + \varepsilon S_x + 2\eta x S_z,$$

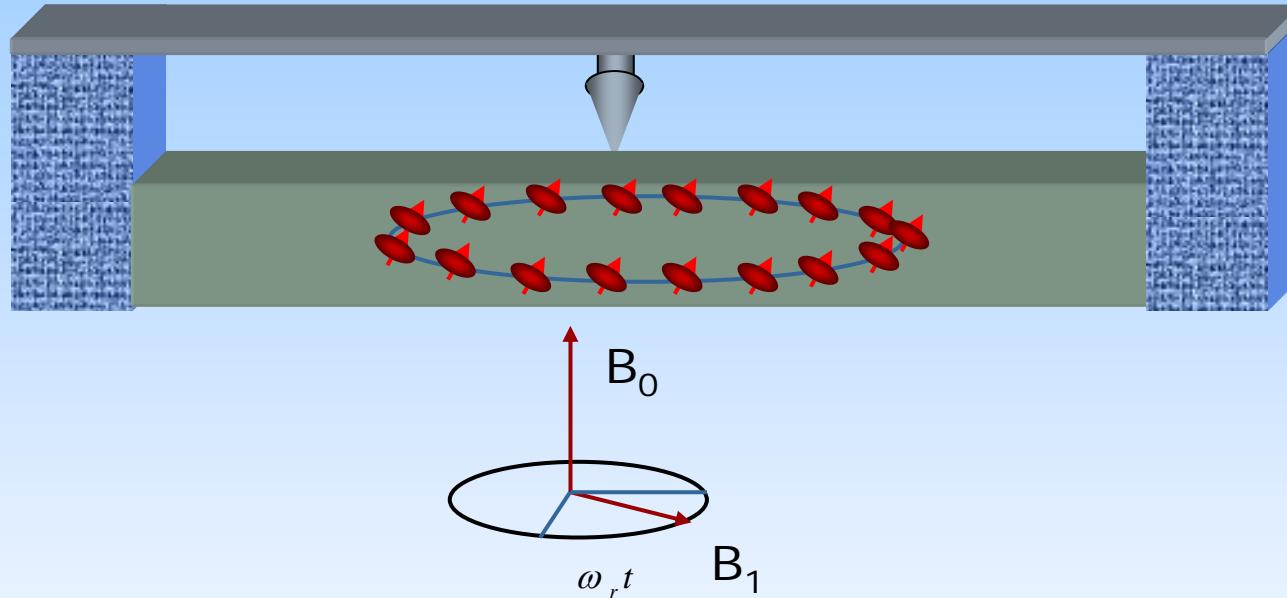
Adiabatic Quantum Measurement

Sun, et al, PHYS REV A 6301 : 2111(2001); EUR PHYS J D 17 :85(2001)

$$(\varepsilon S_x + 2\eta x S_z) |n(x)\rangle = V_n(x) |n(x)\rangle, \quad n = \pm$$
$$V_{\pm}(x) = \pm \sqrt{\varepsilon^2 + 4\eta^2 x^2} \approx \pm \varepsilon \pm 2\eta^2 x^2 / \varepsilon,$$

$$\begin{aligned}\mathbf{H}_{\text{eff}} &= (p_x^2 + x^2)/2 + V_{\pm} \\ &= (\omega \pm \delta)a^+a\end{aligned}$$

Probe for Spin Wave : Dressed Boson



F. Xue, L. Zhong, C.P. Sun, 2006, PRB

Artificial Cavity QED Construct

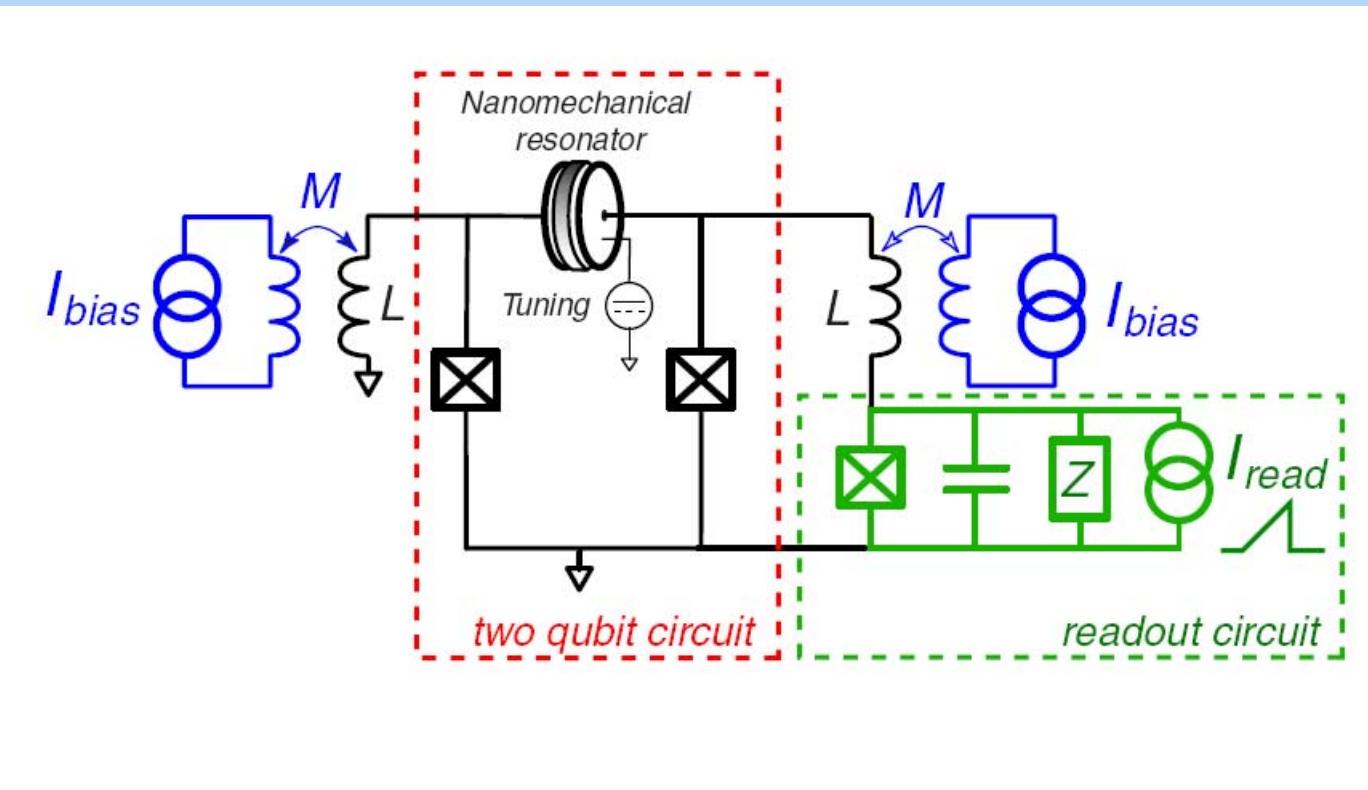
2-Level Artificial Atoms

Quantum data bus

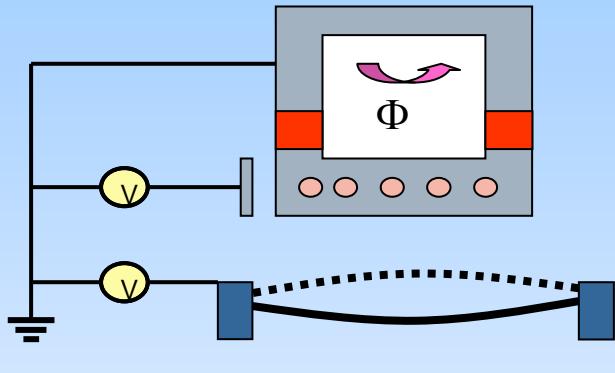
	Natural Atom,Ions	Superconducting Qbits	Nuclear Spin,Excitation	Quantum Dots
Q-EM field	CavityQED	Semi-Clas.	Quant.	Quant.
S-TLR	T	Circuit QED	×	×
NAMR	T	NM-QED	Semi-Clas	T
Large JJ	T	Circuit QED	×	×
Collective Spin	×	×	SW.QED	SW.QE D

T=Theoretical Protocols

Cleland, Geller, Cond-
mat/0311007



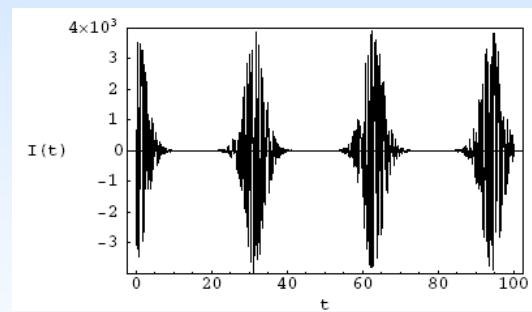
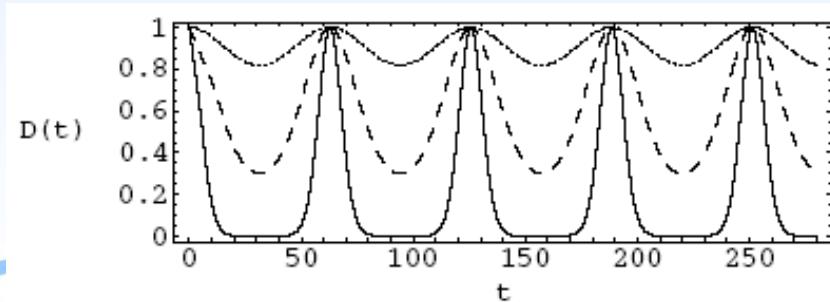
Decoherence showed by Nano MQED



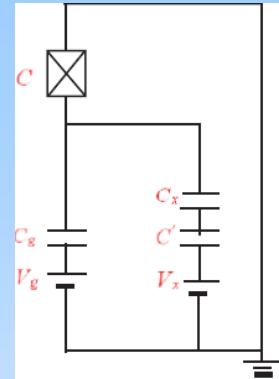
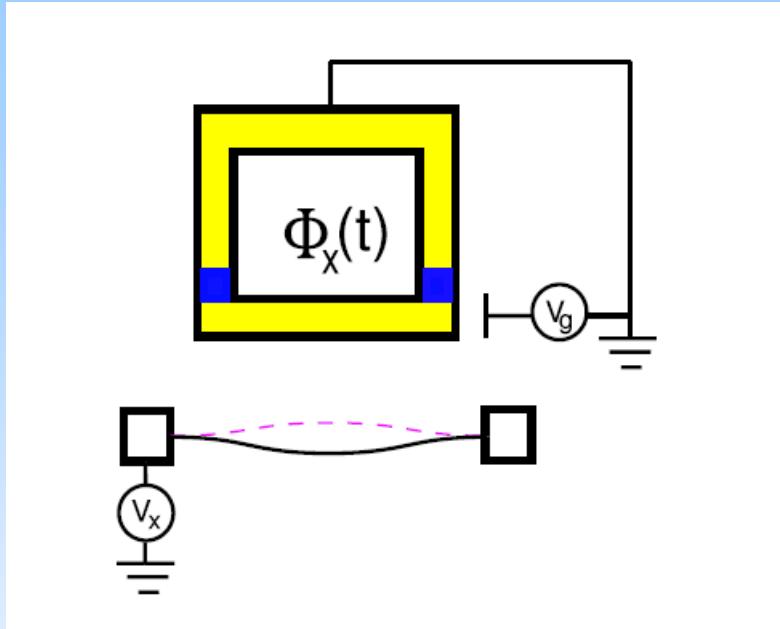
$$H_c = \frac{2e^2 (\hat{n} - n(\hat{x}))^2}{C_\Sigma}$$

$$\begin{aligned} n_g(\hat{x}) &= \frac{1}{2e} (C_g V_g + C_x(\hat{x}) V_x) \\ &= n_g + \delta n(\hat{x}) \end{aligned}$$

$$H = 4E_c \delta n \sigma_z - \frac{E_J}{2} \sigma_x - \lambda (a + a^\dagger) \sigma_z + \hbar \omega_0 a^\dagger a$$



Y. D. Wang, Y. B. Gao and C. P. Sun,
Eur. Phys. Jour. B 40, 321-326 (2004).



$$H_c = \frac{2e^2 (\hat{n} - n(\hat{x}))^2}{C_{\Sigma}}$$

$$n_g(\hat{x}) = \frac{1}{2e} (C_g V_g + C_x(\hat{x}) V_x)$$

$$= n_g + \delta n(\hat{x})$$

$$C_x(\hat{x}) \cong C_x(1 - \frac{x}{d})$$

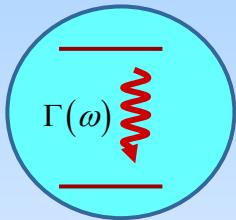
$$\delta n(x) = -n_x \frac{\Delta_x}{d} (\hat{a} + \hat{a}^\dagger)$$

$$H = 4E_c \delta n \sigma_z - \frac{E_J}{2} \sigma_x - \lambda(a + a^\dagger) \sigma_z + \hbar \omega_0 a^\dagger a$$

Cooling Nano-Mechanical Resonator

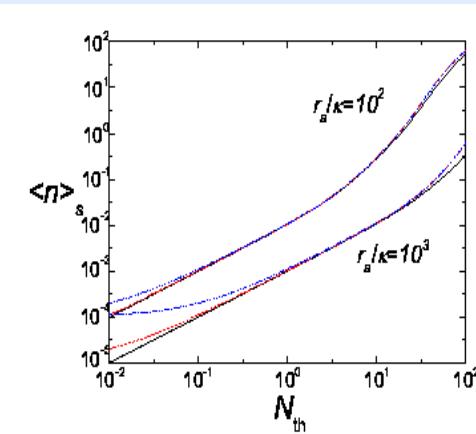
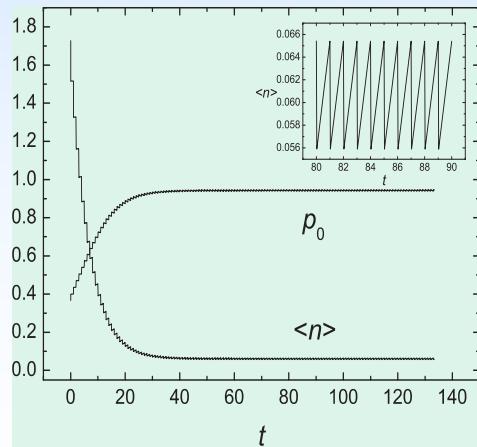
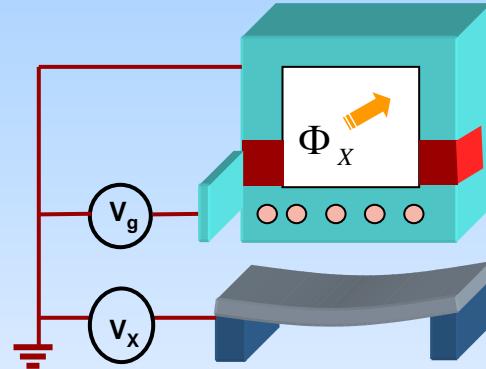
By "Bang-bang" couplings to Charge Qubit

Zhang, Wang, Sun, PRL, 097204 (2005),



$$\Gamma(\omega) = \frac{\pi}{2} \alpha_g \omega \left[\coth\left(\frac{\omega}{2k_B T}\right) + 1 \right]$$

$$\omega = E_J \cos\left(\pi \frac{\Phi_x}{\Phi_0}\right)$$

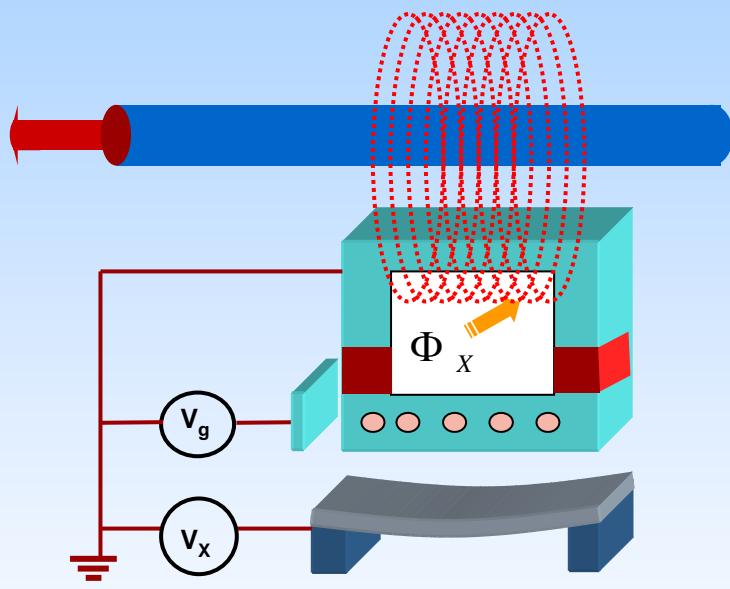


$N_{th} \approx 1.7$;
 $g\tau = \pi/8$;
 $r_a/\kappa = 133$

Our result : Cooling efficiency vs. initial state

Quantum transducers:

Sun, Wei, Liu, Nori, PRA, 2006

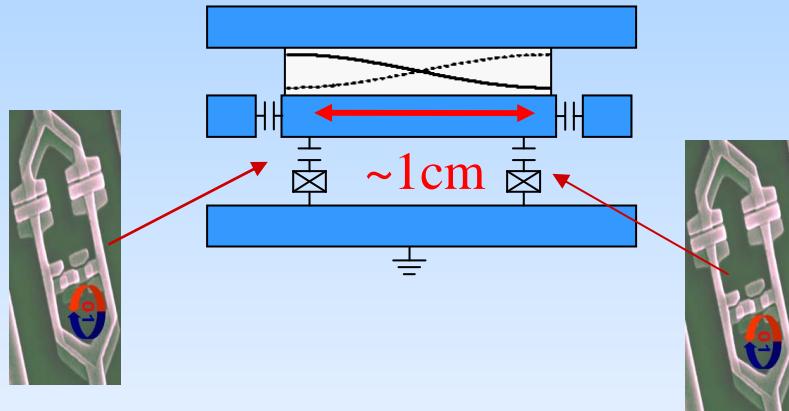


$$\Phi_q = i \sum_k \phi_k (a_k - a_k^\dagger),$$

$$H = \frac{\omega}{2} \sigma_z' + \omega_b b^\dagger b + \lambda(b^\dagger + b)\sigma_z'$$
$$- \frac{E_J}{2} \cos\left(\frac{\pi\Phi_x}{\Phi_0}\right) \sigma_x'$$

Mechanical QED and Circuit QED

—link Qubits with quantum Data Bus



Jaynes-Cummings (JC) Model

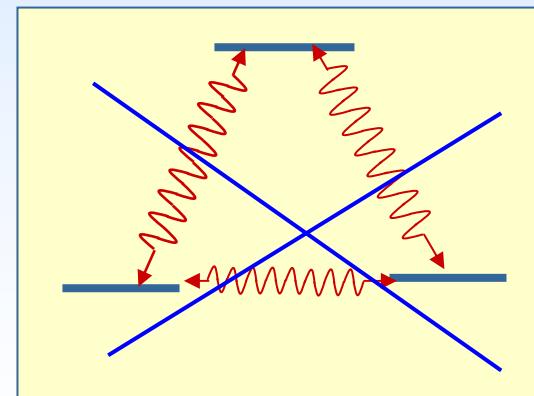
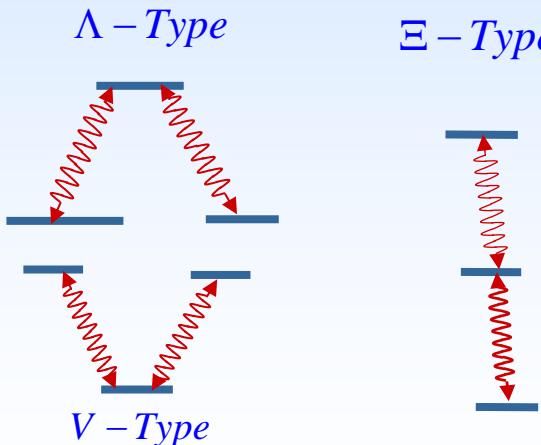
What is New Physics ?

Exotic features of Superconducting Artificial Atoms

Liu, You, Wei , Sun, and Nori, PRL, (2005)

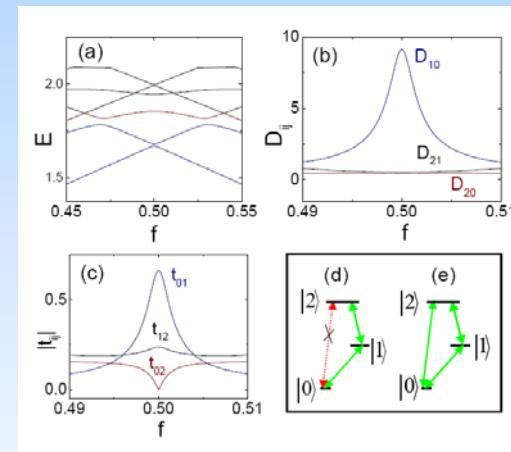
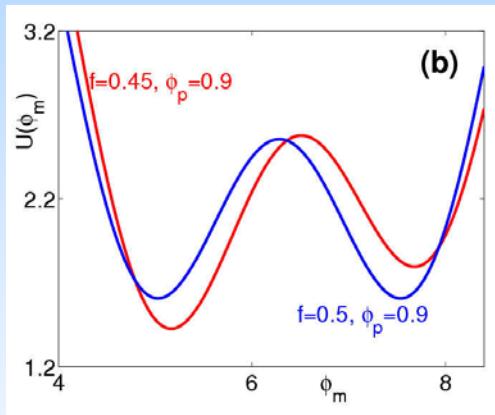
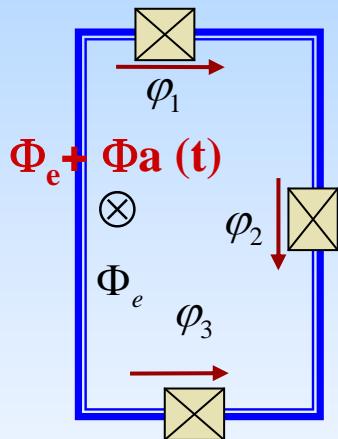
No Δ -Type Natural Atoms in Electric Dipole Transitions

Winger –Ekert Theorem with SO(3) or SO(4) Symmetry

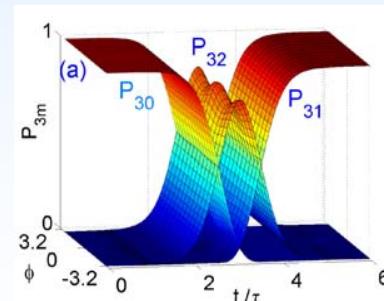


Things change dramatically for the Artificial Atoms in Symmetry Breaking

Liu, You, Wei , Sun, and Nori, PRL, (2005,)

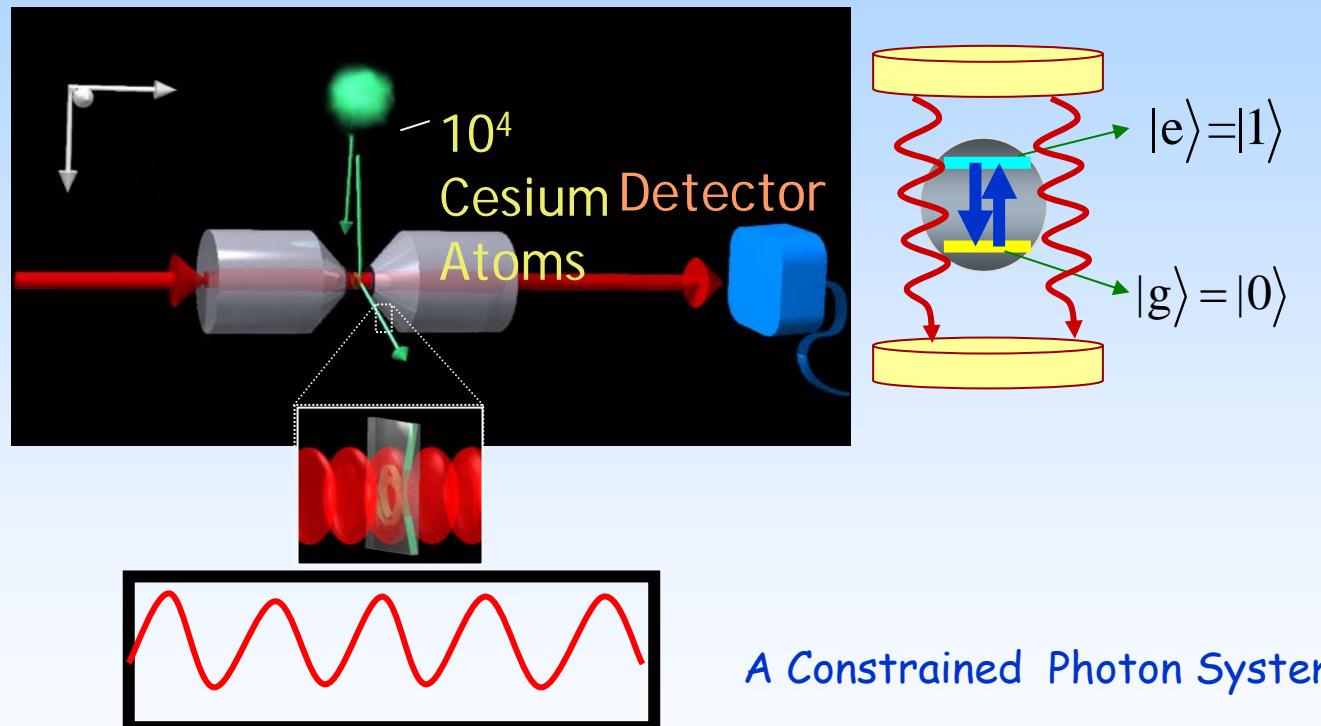


Total Phase Sensitiveness
for Adiabatic Manipulations



Thank you!

Appendix 1: Cavity QED for Atom-Photon system



Model and Effects of Cavity QED :

Jaynes-Cummings (JC) Model = Spin-Boson Model

$$H = \hbar\omega a^\dagger a + \frac{1}{2} \hbar\omega_a \sigma_z + g(a\sigma_+ + a^\dagger\sigma_-)$$

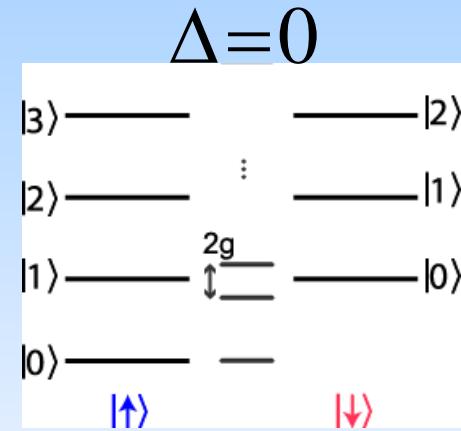
$$[a, a^\dagger] = 1$$

1. Enhancement and suppression spontaneous emission
2. Vacuum Rabi Splits in Spectrum
3. Collapse and revivals of Atomic Population
4. Coherent Forces for Atoms entering the cavity

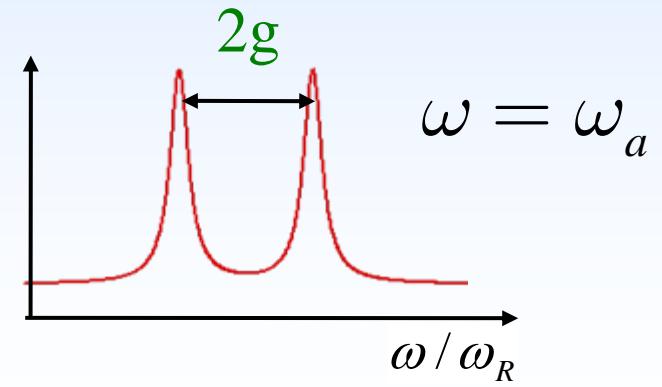
Energies of Dressed States

$$\begin{aligned} |+, n\rangle &= \sin \theta_n |g, n+1\rangle + \cos \theta_n |e, n\rangle \\ |-, n\rangle &= \cos \theta_n |g, n+1\rangle - \sin \theta_n |e, n\rangle \end{aligned}$$

$$\Delta = \omega_a - \omega \quad \cos 2\theta_n = \frac{\Delta}{\sqrt{4g^2(n+1) + \Delta^2}}$$



$$E_{\pm} = \hbar\omega\left(n + \frac{1}{2}\right) \pm \frac{\hbar}{2}\sqrt{\Delta^2 + 4g^2(n+1)}$$

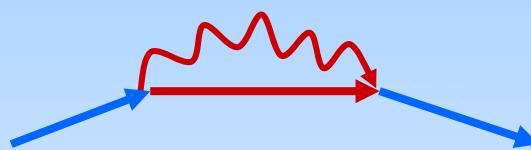


Single Qubit - Single Model Virtual Photon Process

$$H = H_0 + V :$$

$$H_0 = \frac{1}{2} \omega_a \sigma_z + \omega a^\dagger a,$$

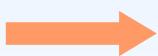
$$V = g a \sigma_+ + h.c$$



2'nd Order Process

$$H_e \sim H_0 + V_{eff} = H_0 + \frac{1}{2} [V, S],$$

$$0 = V + [H_0, S]$$

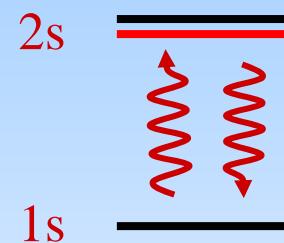


$$S = \frac{g}{\omega - \omega_a} a \sigma_+ - h.c$$

The Lamb Shifts :

$$V_f = \xi aa^+ \sigma_3 \equiv \frac{1}{2} \frac{g^2}{\omega - \omega_a} aa^+ \sigma_3$$

$$H_{\text{eff}} \approx \left(\omega_r - \frac{g^2}{\Delta} \sigma_z \right) a^\dagger a - \frac{1}{2} \left(\omega_{01} + \frac{g^2}{\Delta} \right) \sigma_z$$



Atom AC Stark shift

Optical AC Stark Effect

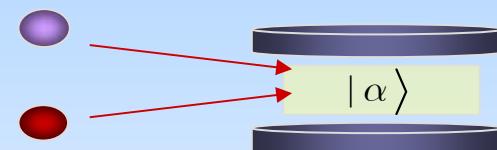
$$E_\pm \approx \hbar(\omega \pm \frac{g^2}{\Delta})(n + \frac{1}{2}) \pm \frac{\hbar}{2}(\Delta \mp \frac{g^2}{\Delta})$$

Decoherence due to Lamb Shifts :

Large detuning

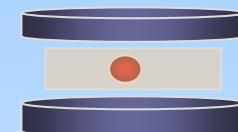
$$|e\rangle \sim$$

$$|g\rangle \sim$$



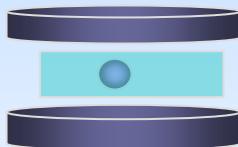
$$\frac{g^2}{|\omega - \omega_a|^2} \ll 1$$

$$|\alpha\rangle \otimes (b|e\rangle + c|g\rangle)$$



- $|e^{-it\kappa_-} \alpha\rangle \otimes |g\rangle$

$$\kappa_{\pm} = \omega \pm \xi$$

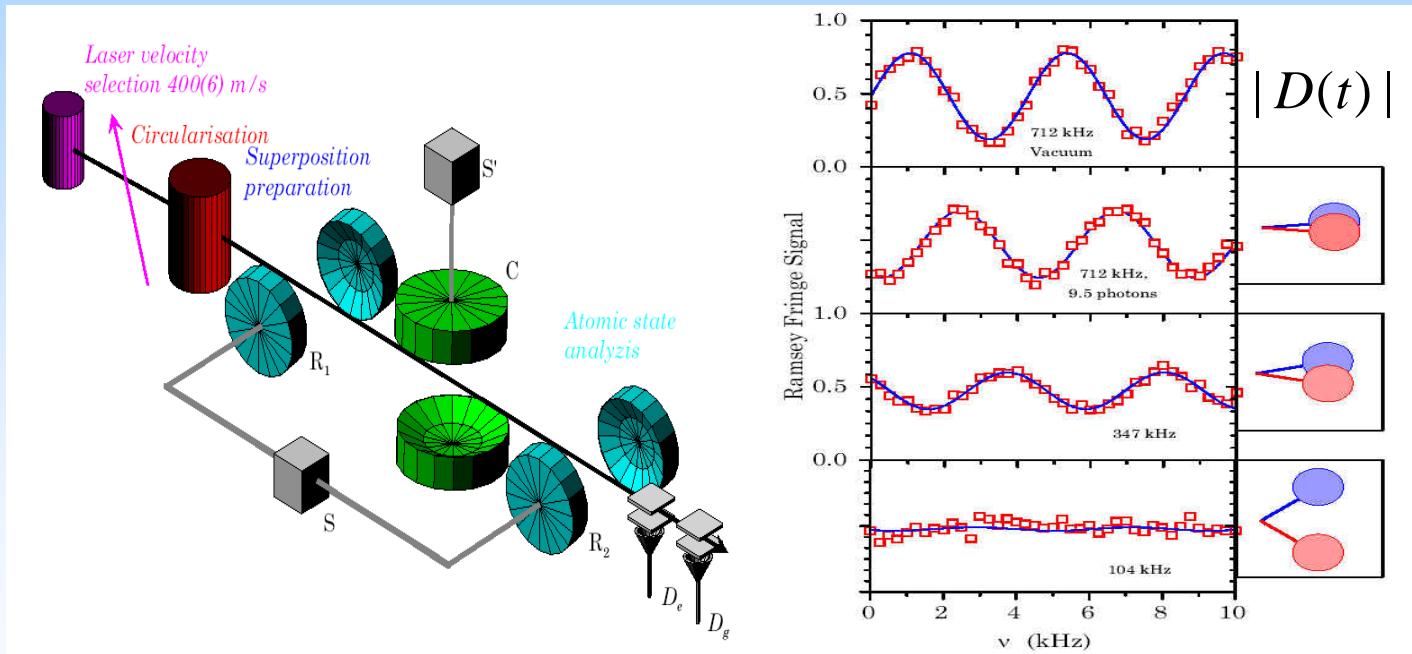


- $|e^{-it\kappa_+} \alpha\rangle \otimes |e\rangle$

$$\rho_f(t) = |a|^2 |g\rangle\langle g| + |b|^2 |e\rangle\langle e| + ab * D(t) |e\rangle\langle g| + h.c$$

Progressive Decoherence Experiment

$$D(t) = \langle e^{-it\kappa_+} \alpha | e^{-it\kappa_-} \alpha \rangle$$



Haroch's Group

Phys. Rev. Lett. 77, 4887, (1996)

Entanglement due to Virtual Photon Exchange

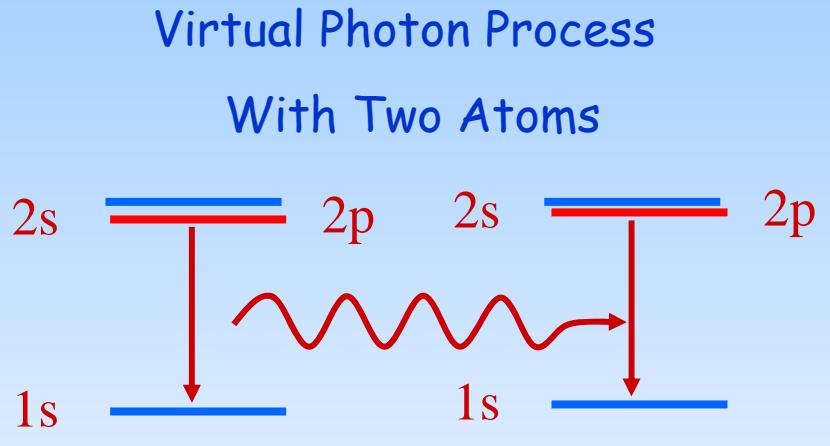
$$H = H_0 + V :$$

$$H_0 = \frac{1}{2} \omega_a (\sigma_z^{(1)} + \sigma_z^{(2)}) + \omega a^+ a$$

$$V = g a (\sigma_+^{(1)} + \sigma_+^{(2)}) + g a^+ (\sigma_-^{(1)} + \sigma_-^{(2)})$$

$$H_f \approx -\frac{g^2}{\omega - \omega_a} (\sigma_+^{(2)} \sigma_-^{(1)} + \sigma_+^{(1)} \sigma_-^{(2)})$$

$$+ \frac{g^2}{\omega - \omega_a} (a^+ a + 1/2) [\sigma_z^{(1)} + \sigma_z^{(2)}]$$



Large detuning

$$\frac{g^2}{|\omega - \omega_a|^2} \ll 1$$

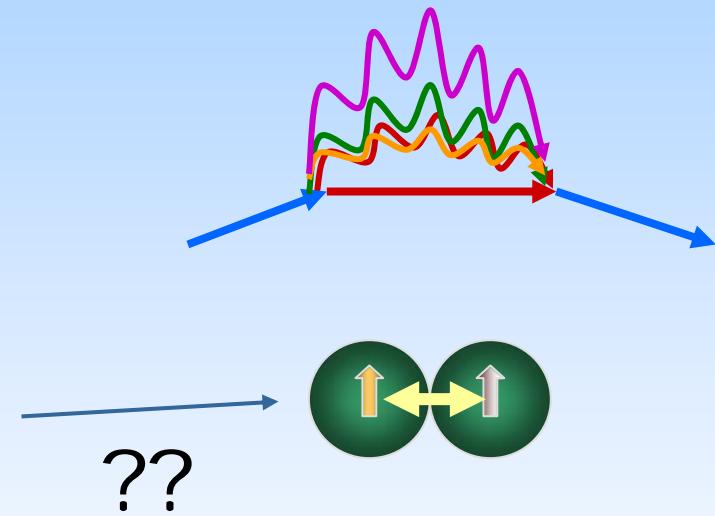
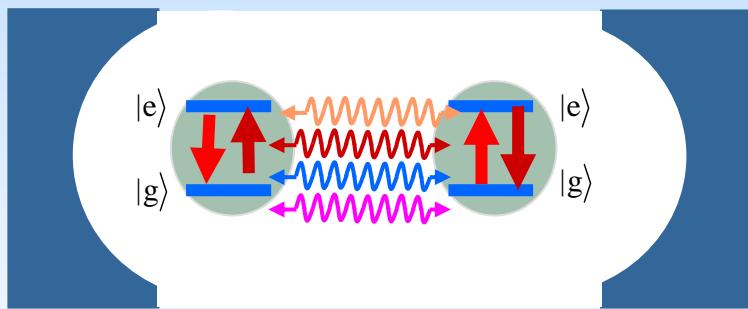
Small $g_{eff} = g^2 / |\omega - \omega_a|$ means a slow process

Multi-model virtual photon Process

$$H = H_0 + V :$$

$$H_0 = \frac{1}{2} \omega_a (\sigma_z^{(1)} + \sigma_z^{(2)}) + \sum_k \omega_k a_k^+ a_k$$

$$V = \sum_k g_k a_k^+ (\sigma_-^{(1)} + \sigma_-^{(2)}) + hc$$



Second order perturbation

$$H_e \sim H_0 + V_{eff} = H_0 + \frac{1}{2}[V, S],$$

$$S = \sum_k \frac{g_k}{\omega_k - \omega_a} a_k^+ (\sigma_-^{(1)} + \sigma_-^{(2)}) - hc$$

$$V_{eff} = \dots + J_{eff} (\sigma_-^{(1)} \sigma_+^{(2)} + hc) + \frac{1}{2} [\omega_a + \delta(a_k^+ a_{k'})] (\sigma_Z^{(1)} + \sigma_Z^{(2)})$$

$$J_{eff} = \sum_k \frac{|g_k|^2}{\omega_k - \omega_a} \quad \delta(a_k^+ a_{k'}) = \sum \frac{a_k^+ a_{k'} g_k g_{k'}}{(\omega_k - \omega_a)}$$

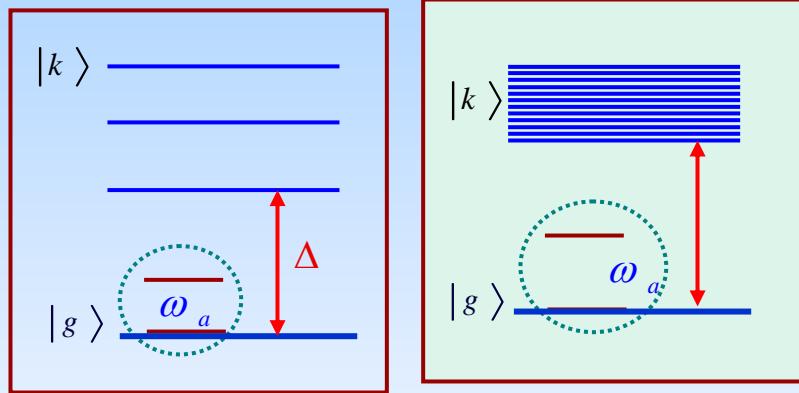
Off-diagonal Lamb shift

Condition for Finite Effective Coupling

$$J_{eff} = \sum_k \frac{|g_k|^2}{\omega_k - \omega_a}$$

二阶微扰成立条件

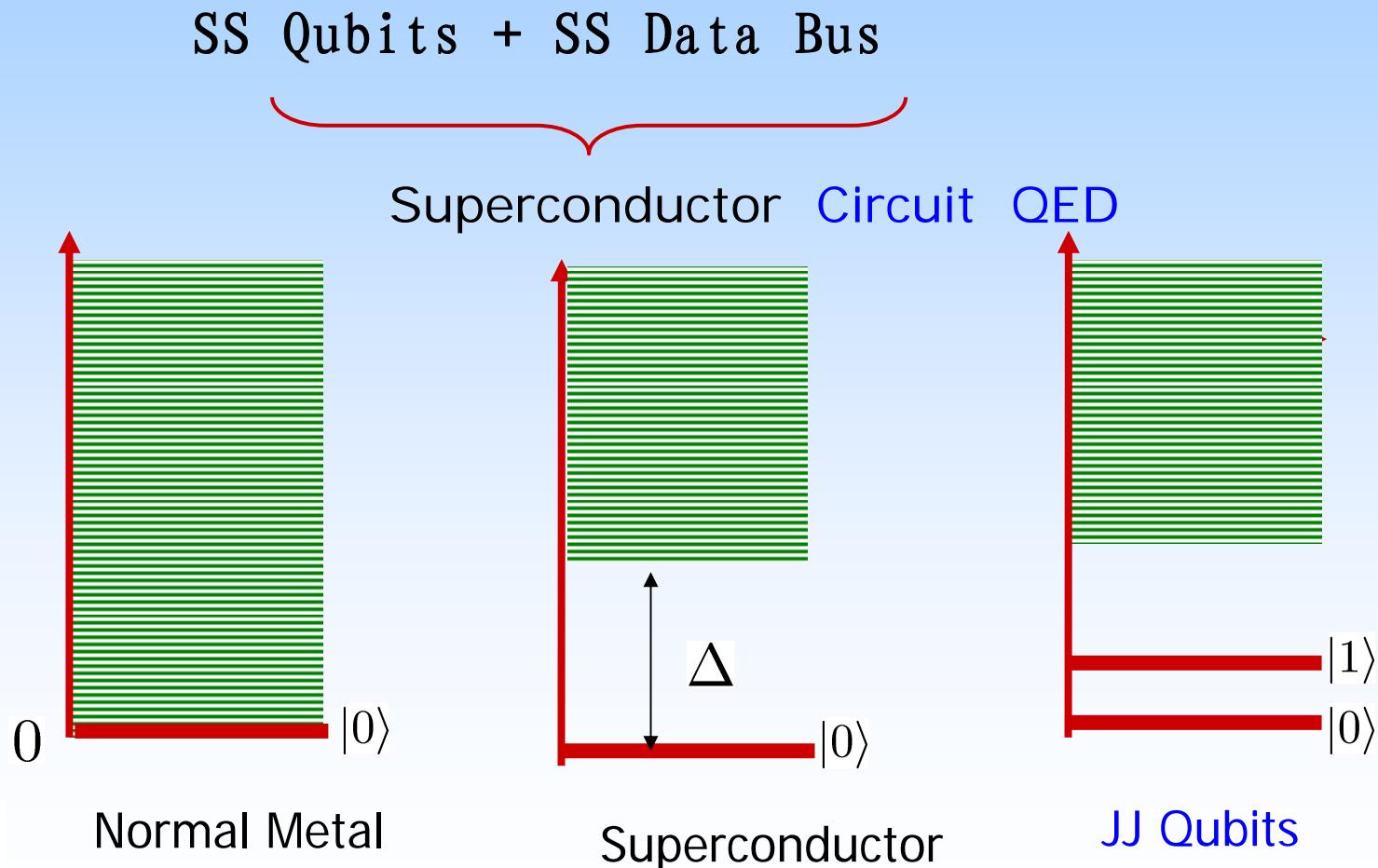
$$\frac{|g_k|^2}{|\omega_k - \omega_a|^2} \ll 1$$



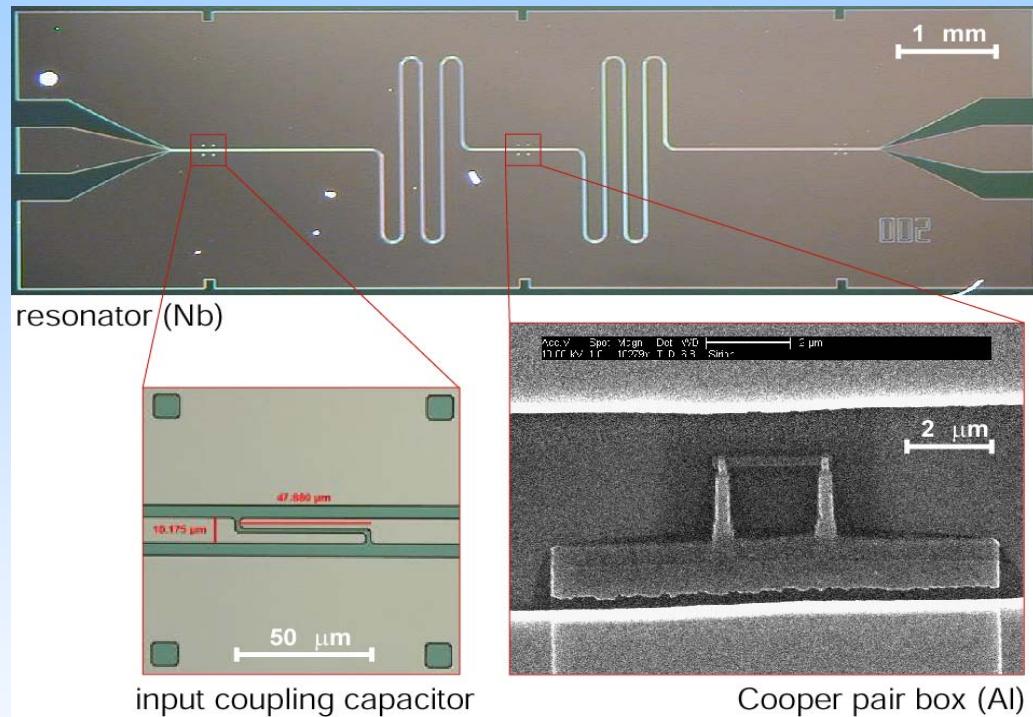
Gapped System is needed !!!

In Quantum Computing, fast operation requires
that the sum $\sum_k \frac{|g_k|^2}{\omega_k - \omega_a}$ is not too small

Towards to Fully Solid State Cavity QED



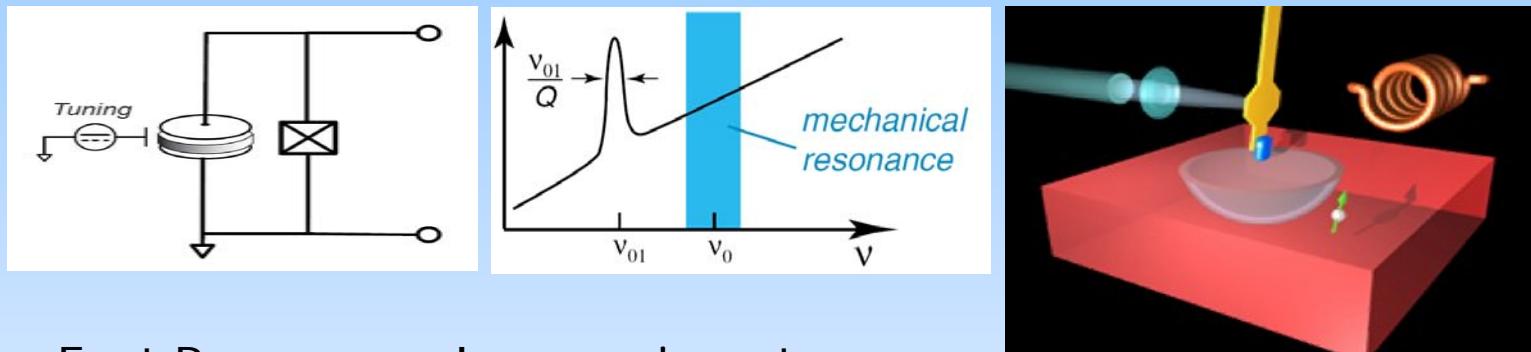
Circuit QED based Josephson Junction



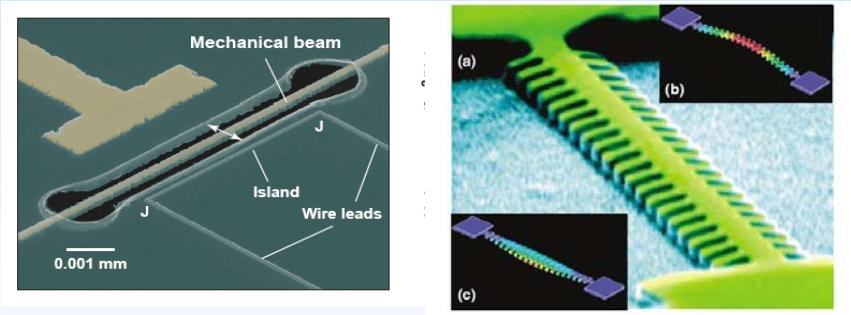
Nature 431 162 (2004)

Nano-Mechanic (NM)- QED

Integrating Josephson Qubit or Spin with A nano -beam

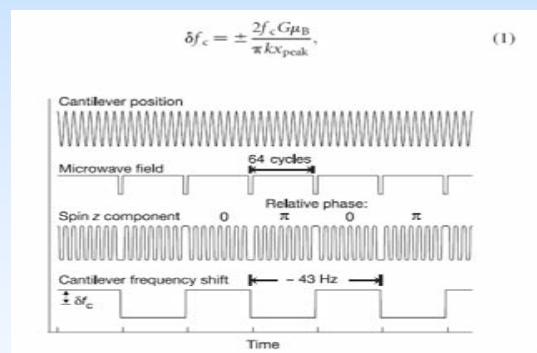


Fast Progresses In experiments



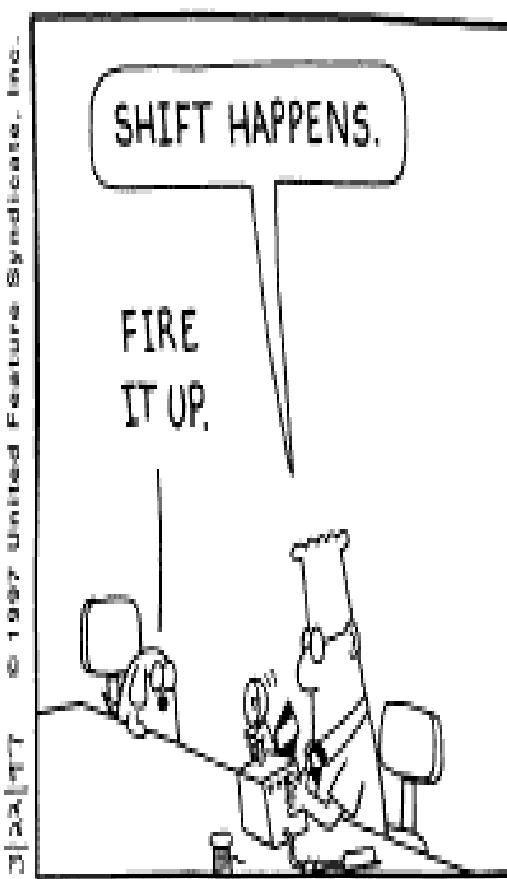
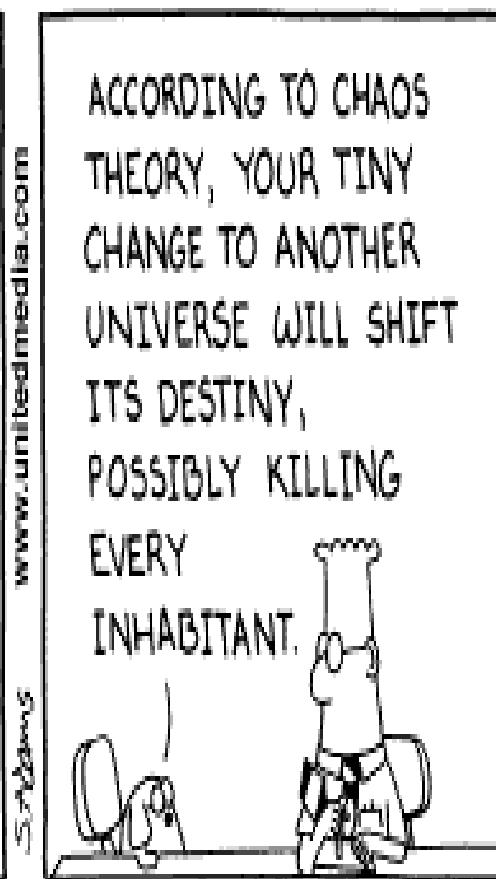
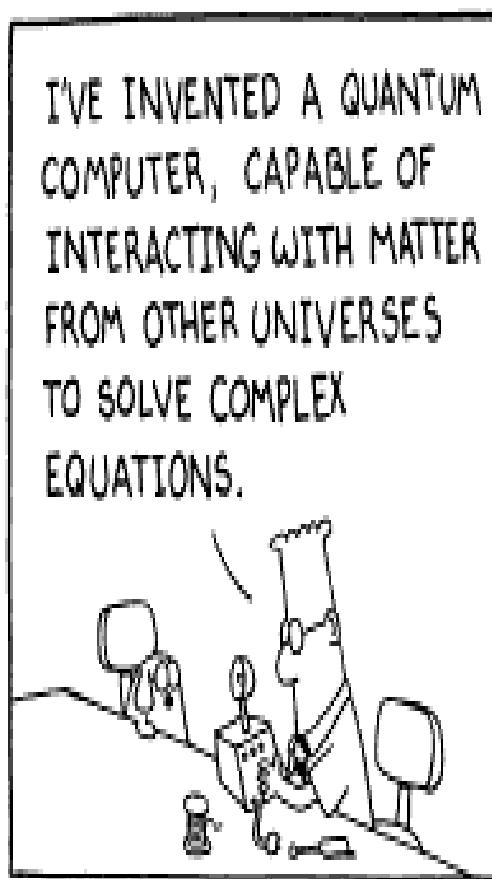
Knobel et al , Nature, 2003
LaHaye et al., *Science* (2004),

Gaidarzhy et al, PRL.
94, 030402 (2005)



Nature 430, 329 (2004)

Quantum Information is Fragile



Physics of Quantum Information:

