

Dynamic realization of quantum measurements in a quantized Stern–Gerlach experiment

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Abstract. A novel dynamic model for wavefunction collapse in cavity quantum electrodynamics (QED) is proposed associated with the gedanken Stern–Gerlach experiment of the quantized magnetic field. It is used to describe and test the fundamental problems of quantum measurements, such as wavefunction collapse, Schrödinger entanglement states (cats) and quantum decoherence. This model also manifests rich phenomena in atomic optics, such as the splitting of atomic beams, and the generation of even- and odd-coherent states. It is also shown that this model possesses a factorizable structure which has been found in the other dynamic models for wavefunction collapse.

1. Introduction

The interpretation of quantum mechanics is a fundamental problem which no physicist can completely avoid. A bridge linking the mathematical framework of quantum mechanics to physical reality is the description of the process of quantum measurement [1–3]. In order to answer the question of what is the result of a measurement and what is the wavefunction after a measurement if a wavefunction is given before the measurement, the wavefunction collapse (WFC) was introduced to close the system of basic laws in quantum mechanics as an extra assumption so that the mathematical formalism of quantum measurement could be interpreted correctly in physics. Mathematically, the WFC represents a reduction process where the wavefunction $|\psi\rangle = \sum_n c_n |n\rangle$ becomes its single branch $|n\rangle$, once a well determined result a_n has actually been achieved by the measurement of the observable $\hat{A}$, with eigenvectors $|n\rangle$ ($n = 1, 2, \dots, n$) and corresponding eigenvalues a_n . This is because a second measurement repeated immediately after the first one must give the same result a_n . According to von Neumann [1], the rigorous description of WFC in mathematics is an evolution from quantum probability amplitudes to the classical one, i.e.

$$\rho = |\psi\rangle\langle\psi| = \sum_{n,m} c_n c_m^* |n\rangle\langle m| \rightarrow \rho_r = \sum_n |c_n|^2 |n\rangle\langle n|. \quad (1.1)$$

That is, the measurement results in vanishing of the off-diagonal elements of the density matrix of a pure state or quantum decoherence [3].

However, the original WFC postulate introduces some non-quantum elements to quantum mechanics. Here, a classical measuring instrument (detector) must be used to decohere the coherent superposition beyond quantum mechanics [1–3]. Since one hopes

that quantum mechanics is a complete theory for our world, it should be valid for both the detector and the system. For this reason exactly soluble dynamic models are needed to treat the WFC as an evolution governed by the Schrödinger equation [4–8]. In such a dynamic theory, the measured system is considered as an open subsystem embedded in the closed total system formed by the measured system plus the detector. The Schrödinger evolution of the total system ‘projected’ onto this subsystem (by taking the partial trace over the variables of the detector) results in the reduced density matrix ρ_r . Under certain limits, such as the classical limit and the macroscopic limit, this reduced density matrix could approach the decohered density matrix without the off-diagonal elements. This is just the dynamic realization of the WFC in quantum measurement.

A novel realization (now called the Hepp–Coleman (HC) model [4]) of such a dynamic model was presented by Hepp, motivated by a communication with Coleman. He showed that the decoherence (or WFC) of the two-state system could appear dynamically as the particle number in the detector approaches infinity. Many generalizations of the HC model were proposed by several authors concerning the introduction of the energy exchange effect between the system and detector [9, 10] and the construction of a real classical limit of a detector with a larger quantum number [11–13]. Although the HC model and its generalizations depend on the specific forms of Hamiltonians of the system and the detector, it was found by one (CPS) of the present authors that the essence of these concrete models lies in the factorizability of the effective evolution matrix for the total system [13]. All previous generalizations of the HC model are special realizations of this factorizable structure and many extensive generalizations of the HC model can easily be given based on the observation of the factorizability. Notice that the previous investigations of the dynamic realization only concern some toy models such as the ultra-relativistic systems which cannot be verified experimentally even by a gedanken experiment. To find a possibility for realizing the dynamic model experimentally, it is necessary to seek a nearly realistic example of the dynamic model of WFC in physics.

The studies presented in this paper are devoted to taking a step towards this goal. We propose a dynamic model for WFC in cavity quantum electrodynamics (QED) associated with a gedanken Stern–Gerlach experiment on the quantized magnetic field. This model is used to describe and test the fundamental problems of quantum measurements, such as wavefunction collapse, Schrödinger entanglement states (cats) and quantum decoherence. This model also manifests rich phenomena in atomic optics, such as the splitting of an atomic beam, and generation of even and odd coherent states. It is also shown that this model possesses the factorizable structure which was found in the previous dynamic models for wavefunction collapse.

This paper is organized as follows. In section 2, the origin of our dynamic model is traced to cavity QED [14–18] and the dynamic generation of the Schrödinger entangled states [19] is discussed associated with the pure Schrödinger evolution in the strong-coupling limit in section 3. In section 4 the WFC in quantum measurement is analysed for various cases and thereby the relevant atomic optical problems are studied associated with the motion of the atomic centre of mass in section 5. Here, it is shown that the high-temperature excitation will cause an ideal decoherence of the state of the system and thereby the Schrödinger entangled states [19]. Finally, in section 6 we note that the factorizable structure is also implied in our model.

2. A model of WFC in cavity QED

Cavity QED provides a modern quantum theory with a plausible laboratory to test many novel ‘pure’ quantum effects such as the cavity vacuum effect producing the adiabatic force, the Casimir effect [16], Lamb shifts, quantum non-demolition (QND) measurement [20, 21] and so on. It is also accepted that cavity QED can give a practical realization of the WFC in quantum measurement.

Our model is described as follows. In a ring cavity with a quantized magnetic field of a single mode

$$\vec{B} = iB_0(a e^{ikx} - a^\dagger e^{-ikx})\hat{e}_z \quad (2.1)$$

along the axis $\hat{e}_z$, the interaction between a two-level Zeeman atom (or a spin $\frac{1}{2}$) [22] and the field is given in the representation of diagonal σ_z by

$$H_I(x) = \mu \vec{B} \cdot \hat{\sigma} = i\hbar g (|e\rangle\langle e| - |g\rangle\langle g|) (a e^{ikx} - a^\dagger e^{-ikx}) \quad (2.2)$$

where $g = \mu B_0/\hbar$ is the coupling constant proportional to the strength of the magnetic field, $|g\rangle$ and $|e\rangle$ the ground and excited states of the atom, respectively, and $\vec{\sigma}$ are the quasi-spin operators, i.e. $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$ and $\sigma_3 = |e\rangle\langle e| - |g\rangle\langle g|$. The whole Hamiltonian

$$H = \frac{p^2}{2m} + \hbar\omega a^\dagger a + H_I(x) \quad (2.3)$$

is just the kinetic energy of the atom plus the internal atom-field energy. The frequency of the field is related to the wavevector by $k = \omega/c$. Notice that the above model is only a space-dependent generalization of the generic one for the precession of spin $\frac{1}{2}$, e.g. for the ground silver atoms with spin $\frac{1}{2}$ or for a two-level Zeeman atom, the interacting part in our model is only an alternative expression in different representation for the Jaynes–Cummings (JC) model for the effect of quantization on spin resonance described in [22].

Because the kinetic part $p^2/2m$ does not commute with H_I , it is somewhat difficult to diagonalize the total Hamiltonian. But, we can hope that a trick can be used to diagonalize H approximately. To this end, a unitary transformation [23]

$$W(x) = e^{ikxa^\dagger a} \quad (2.4)$$

is presented to define an effective Hamiltonian

$$H_{\text{eff}} = W^\dagger(x) H W(x) = \frac{p^2}{2m} + i\hbar g \sigma_3 (a - a^\dagger) + \hbar\omega a^\dagger a \quad (2.5)$$

where we have neglected the photon recoil term $(\hbar^2 k^2/2m)(a^\dagger a)^2$ and

$$\omega = \omega(p) = \omega + \frac{pk}{m} \quad (2.6)$$

is a modified frequency with a Doppler shift pk/m . It should be emphasized that H_{eff} and H have equivalent dynamics within the approximation by neglecting the photon recoil term, because a unitary transformation of operator $\hat{O}$ has the same spectrum as that of $\hat{O}$. Since the kinetic part in (2.5) commutes with the remaining part

$$H_e = \hbar\omega a^\dagger a + i\hbar g \sigma_3 (a - a^\dagger) = \begin{pmatrix} H_+ & 0 \\ 0 & H_- \end{pmatrix} \quad (2.7)$$

where

$$H_\pm = \hbar\omega a^\dagger a \pm i\hbar g (a - a^\dagger) \quad (2.8)$$

we should focus on the dynamics of H_e . It is a direct sum of the Hamiltonians of two forced harmonic oscillators with opposite forces, respectively.

Also note that since the unitary transformation does not change the rank of the density matrix, the properties of the WFC will be the same for a Hamiltonian and its unitary transformation. Therefore, except for the approximation ignoring the recoil terms, the Hamiltonians H_e as well as H_{eff} , define a dynamic model as reasonable as the original one of H in the real context of cavity QED.

3. The dynamic generalization of state entanglement

Physically, the process of a measurement is a scheme using the macroscopic counting number of the detector to manifest the state of the measured system. The state entanglements produced by measurement will just exhibit this manifestation. The state entanglement was also highlighted by Schrödinger as the ‘living cat–dead cat’ paradox [19–22] when a component of the entangled state is ‘macroscopically distinguishable’ while another is ‘microscopic’. Usually, the entangled state has a more general meaning than that of the cat state as a special case. In the present discussion, the cavity field is imagined to be a macroscopic detector with many photons to ‘measure’ the state of the atom which is considered as a measured system. The correlation between the macroscopic and microscopic states will be shown to be that between the coherent cavity field with very large amplitude and the internal atomic state.

The evolution governed by the Hamiltonian (2.7) is realized by an evolution matrix in the interaction picture

$$U(t) = D(A(t))|e\rangle\langle e| + D(-A(t))|g\rangle\langle g| \quad (3.1)$$

with two components, where

$$D(A(t)) = e^{A(t)a^\dagger - A(t)^*a} \quad (3.2)$$

with

$$A(t) = ig \frac{e^{i\omega t} - 1}{\omega} \quad (3.3)$$

is the evolution operator of the forced harmonic oscillator with the Hamiltonian H_+ . The above time evolution determines an entangled state

$$|\psi_I(t)\rangle = c_e|e\rangle \otimes D(A(t))|\phi\rangle + c_g|g\rangle \otimes D(-A(t))|\phi\rangle \quad (3.4)$$

when the atom and the cavity are prepared in the initial pure states

$$|\psi_a(0)\rangle = c_g|g\rangle + c_e|e\rangle \quad |\psi_f(0)\rangle = |\phi\rangle \quad (3.5)$$

respectively.

The ideal entanglement state of the system with the detector is an orthogonal decomposition of the states $|s_i\rangle$ ($i = 1, 2, 3, \dots$) of the measured system with respect to the states $|D_i\rangle$ of the detector with certain macroscopic differences. The process of entanglement is expressed as

$$|\psi(0)\rangle = \sum c_i |s_i\rangle \otimes |D\rangle \xrightarrow{\text{evolution}} |\psi(t)\rangle = \sum_{i=1} c_i |s_i\rangle \otimes |D_i\rangle \quad (3.6)$$

then we can read out the states $|s_k\rangle$ once we have determined the state $|D_k\rangle$ of the detector. Such a scheme of a ‘reading state’ is ideal if the states $|D_i\rangle$ ($i = 1, 2, \dots, N$) are orthogonal to each other, i.e. $\langle D_i | D_j \rangle = \delta_{ij}$.

For our situation with a specific initial state $|\psi\rangle = |\alpha\rangle$ (coherent state) of the detector,

$$|\psi(t)\rangle = c_e|e\rangle \otimes |\psi_e\rangle + c_g|g\rangle \otimes |\psi_g\rangle \quad (3.7)$$

where

$$|\psi_e\rangle = D(A)|\alpha\rangle \quad |\psi_g\rangle = D(-A)|\alpha\rangle. \quad (3.8)$$

The overlap of detector components in the entangled state has a non-zero overlap determined by

$$|\langle\psi_e|\psi_g\rangle|^2 = e^{-4|A(t)|^2} = e^{(-4g^2/\omega^2)4\sin^2\omega t/2} = e^{-4\sin^2(\lambda gt)/\lambda^2} \quad (3.9)$$

in the general case. However, in the limit $2\lambda = \omega/g \rightarrow 0$ of strong coupling, an ideal entanglement appears for two particular cases. The first one, similar to the Cini model [5], essentially is that ω approaches zero for fixed g . In this limiting case, $A(t) \rightarrow gt$ and hence the overlap becomes an exponential decaying factor

$$|\langle\psi_e|\psi_g\rangle|^2 = e^{-4g^2t^2}. \quad (3.10)$$

This phenomenon of exponential decay is illustrated in figure 1, which results mathematically from the very large period of oscillation at the strong-coupling limit. Another case is that as an effective coupling constant g approaches infinity for fixed ω , which can be realized physically at high temperature (see the following). In this gradually changing process from weak coupling to strong coupling, the rapid revival and collapse of coherence happens in the weak coupling with periodic points $\omega t/2 = n\pi$, $n = 1, 2, \dots$, but the ideal decoherence happens for the strong coupling except for very narrow and sharp quantum jumps. This phenomenon will be illustrated in figure 2 in the next section for the finite temperature case where g is replaced by an effective coupling depending on temperature.

For a vacuum cavity the initial state is $|\psi_f(0)\rangle = |0\rangle$. With strong coupling the entangled state

$$|\psi_c(t)\rangle = c_e|e\rangle \otimes |gt\rangle + c_g|g\rangle \otimes |-gt\rangle \quad (3.11)$$

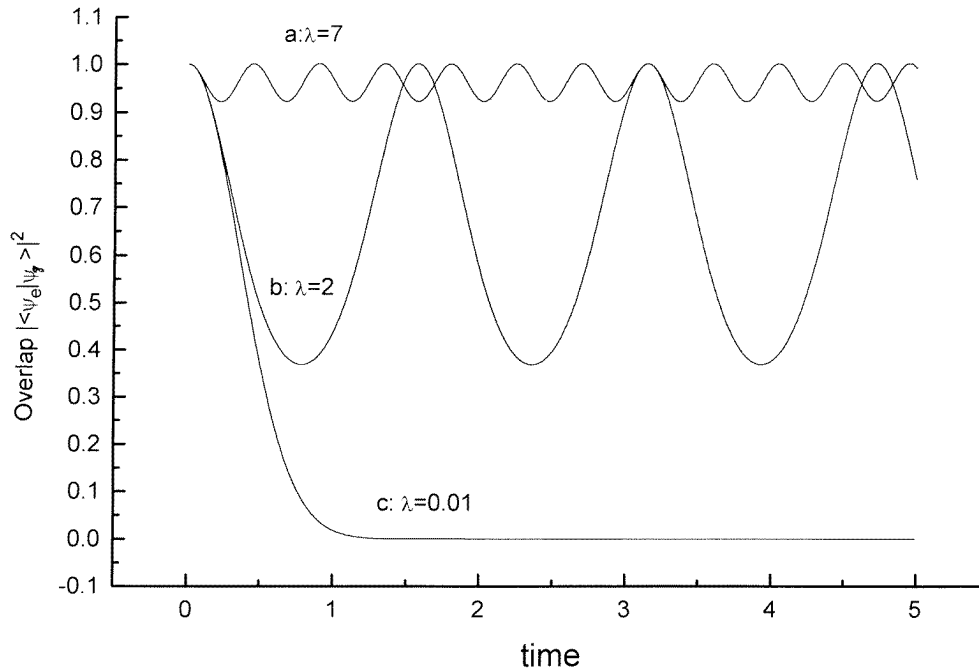


Figure 1. The appearance of the exponential decaying behaviour of the overlap in the limit of strong coupling for fixed g .

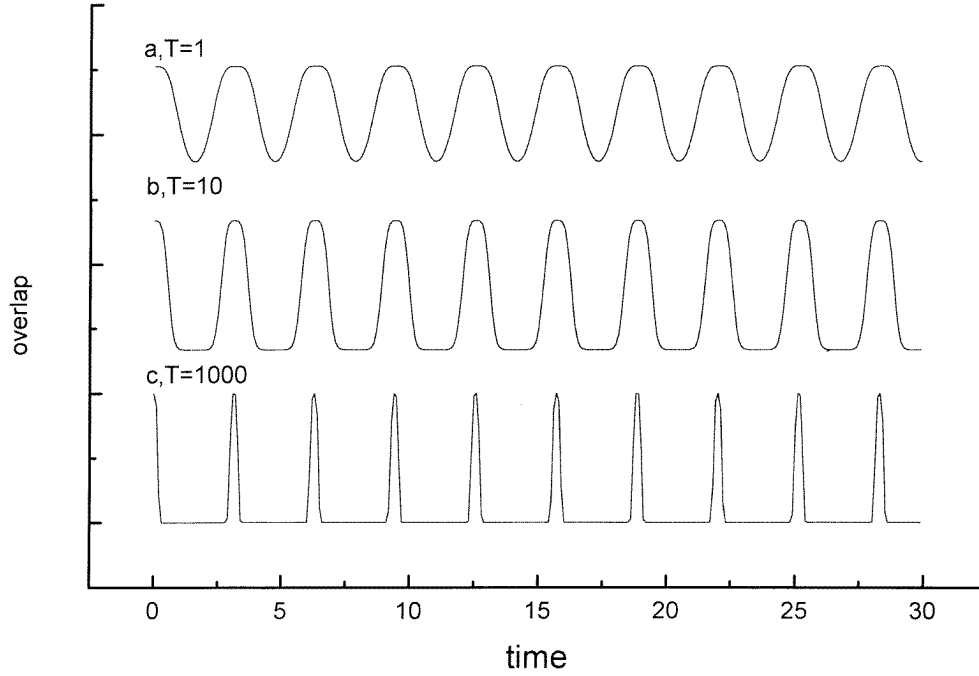


Figure 2. The accompanying overlap factors for different temperatures. This case is equivalent to the process approaching the strong coupling, i.e. g goes to infinity.

is just associated with the Schrödinger cat state, which is a quantum superposition of two coherent states 180° out of phase with each other [19–22] and with very large amplitudes $|\alpha| = gt$ ($\alpha = gt$). Generally, such a superposition of coherent states $|\alpha\rangle$ and $|\alpha\rangle$ is macroscopically distinguishable, namely, there is little overlap between the states. Roughly speaking, the coherent state with very large amplitude $|\alpha|$ is usually referred to a macroscopic object. The coherence between the macroscopic ‘living cat’ correlated to microscopic state $|e\rangle$ and the macroscopic ‘dead cat’ to microscopic states $|g\rangle$ is described, in the case of effective strong coupling, by the overlap

$$|\langle gt| - gt\rangle|^2 = e^{-4g^2t^2} \xrightarrow{gt \rightarrow \infty} 0. \quad (3.12)$$

Therefore, we may say that the ‘living cat’ does not interfere with the ‘dead cat’ for a long enough time. In this sense, though we cannot confirm whether the cat in a black box is living or dead, we may infer the final result according to a classical probability without coherence rather than to the quantum probability with coherence. Notice that the state (3.11) is not an exact cat state of the light field, but a pure Schrödinger cat state can be further prepared from it if classical light acting only on the internal states is applied to produce a Rabi rotation of the atomic internal states $|e\rangle$ and $|g\rangle$:

$$\begin{aligned} |e\rangle &\rightarrow e^{-iH_c T/\hbar} |e\rangle = \frac{1}{\sqrt{2}}(|e\rangle - |g\rangle) \\ |g\rangle &\rightarrow e^{-iH_c T/\hbar} |g\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle) \end{aligned} \quad (3.13)$$

where

$$H_c = i\hbar\mu (|e\rangle\langle g| - |g\rangle\langle e|) \quad T = \frac{1}{\mu} \left(2n + \frac{1}{4}\right)\pi. \quad (3.14)$$

Therefore, when $c_e = c_g = 1/\sqrt{2}$, the entangling state (3.11) is transformed into

$$e^{-iH_c T/\hbar} |\psi_c(t)\rangle = \frac{1}{2} |e\rangle \otimes (|gt\rangle + |-gt\rangle) - \frac{1}{2} |g\rangle \otimes (|gt\rangle - |-gt\rangle). \quad (3.15)$$

It can be imagined that once we find the atom in the state, $|e\rangle$, the light field is prepared in the pure cat state $\frac{1}{\sqrt{2}}(|gt\rangle + |-gt\rangle)$.

4. Wavefunction collapse

For the case with the pure detector state, the orthogonal decomposition (i.e., the ideal entanglement) implies the WFC of the reduced density matrix. For the general case in (3.6), the reduced density matrix can be obtained by taking a partial trace over the variables of the detector

$$\begin{aligned} \rho_r &= \text{Tr}_D(|\psi(t)\rangle\langle\psi(t)|) = \sum_i \langle D_i | \psi(t) \rangle \langle \psi(t) | D_i \rangle \\ &= \sum_i |c_i|^2 |s_i\rangle\langle s_i| + \sum_{i \neq j} c_i^* c_j \langle D_i | D_j \rangle |s_i\rangle\langle s_j| = \sum_i |c_i|^2 |s_i\rangle\langle s_i| \end{aligned} \quad (4.1)$$

since $\langle D_i | D_j \rangle = \delta_{i,j}$. For the case with strong coupling and an initial state $|\alpha\rangle$ for the photon field in our model,

$$\rho_r(t) = |c_e|^2 |e\rangle\langle e| + |c_g|^2 |g\rangle\langle g| + \langle \alpha | D(2A(t)) | \alpha \rangle (c_e c_g^* |e\rangle\langle g| + c_e^* c_g |g\rangle\langle e|) \quad (4.2)$$

and

$$|\langle \alpha | D(2A(t)) | \alpha \rangle|^2 = e^{-4|A(t)|^2} \xrightarrow{\omega/g \rightarrow 0} e^{-4g^2 t^2}. \quad (4.3)$$

The off-diagonal terms decay exponentially to zero as time tends to infinity such that the WFC can be realized dynamically.

Another interesting situation is when the cavity is prepared exactly in a Fock number state

$$|n\rangle = \frac{1}{\sqrt{n!}} (a^\dagger)^n |0\rangle. \quad (4.4)$$

Note that it is very difficult to realize such a situation in a practical experiment since the Fock state has a very large fluctuation of the electric or magnetic field. In this situation, the reduced density matrix is

$$\rho_r(t) = |c_g|^2 |g\rangle\langle g| + |c_e|^2 |e\rangle\langle e| + \langle n | D(2A(t)) | n \rangle (c_g c_e^* |g\rangle\langle e| + c_e c_g^* |e\rangle\langle g|). \quad (4.5)$$

The accompanying factor $\langle n | D(2A(t)) | n \rangle$ of the off-diagonal elements of $\rho_r(t)$ can be expressed as

$$\langle n | D(2A(t)) | n \rangle = e^{-2|A(t)|^2} L_n(|2A(t)|^2) \xrightarrow{\omega/g \rightarrow 0} e^{-g^2 t^2} L_n(|2gt|^2) \quad (4.6)$$

in terms of the Laguerre polynomial $L_n(z)$. According to the theory of special functions, $L_n(z)$ approaches the zero-order Bessel function $J_0(4\sqrt{n}gt)$ when $n \rightarrow \infty$, hence,

$$\langle n | D(2A(t)) | n \rangle \xrightarrow{\omega/g \rightarrow 0} \langle n | D(2gt) | n \rangle \xrightarrow{n \rightarrow \infty} e^{-2g^2 t^2} J_0(4\sqrt{n}gt). \quad (4.7)$$

Since the Bessel function with real variables is a decaying oscillating function, it approaches zero as n tends to infinity. Therefore, when the cavity is occupied by large amount of photons, the macroscopic feature of the detector (photon field) decoheres the initial pure state of the atom and hence the WFC is realized dynamically.

The third situation is that the cavity is initially in a thermal equilibrium state since a measurement is always carried out at finite temperature, which is described by the density matrix

$$\rho_D = \frac{\exp[-\beta\hbar\omega a^\dagger a]}{\text{Tr}[\exp(-\beta\hbar\omega a^\dagger a)]} \quad (4.8)$$

then the density matrix for the initial state of the total system is a simple product

$$\rho(0) = \sum_{\lambda, \lambda'=e, g} c_\lambda c_{\lambda'}^* |\lambda\rangle\langle\lambda'| \otimes \rho_D. \quad (4.9)$$

In fact, at high temperature the excitation of the cavity field will make the decoherence or the WFC more enhanced than at zero temperature. In this case, the evolution of the reduced density matrix of the atom takes the form

$$\rho_r(t) = \text{Tr}_D(U(t)\rho(0)U^\dagger(t)) = |c_e|^2 |e\rangle\langle e| + |c_g|^2 |g\rangle\langle g| + (c_g^* c_e |e\rangle\langle g| + \text{HC}) F(t). \quad (4.10)$$

Using the coherent-state representation

$$\rho_D = \int \rho(\alpha) |\alpha\rangle\langle\alpha| d^2\alpha \quad (4.11)$$

with the diagonal elements

$$\rho(\alpha) = \frac{1}{\pi \langle n \rangle} e^{-|\alpha|^2 / \langle n \rangle} \quad (4.12)$$

where $\langle n \rangle = (e^{\beta\hbar\omega} - 1)^{-1}$ is the average photon number in the cavity, the accompanying overlap factor

$$F(t) = \text{Tr}_D(\rho_D D(2A(t))) \quad (4.13)$$

is calculated

$$\begin{aligned} F(t) &= \frac{1}{\pi \langle n \rangle} \int_{-\infty}^{\infty} e^{-|\alpha|^2 / \langle n \rangle} \langle \alpha | D(2A(t)) | \alpha \rangle d^2\alpha \\ &= \frac{1}{\pi \langle n \rangle} \int_{-\infty}^{\infty} e^{-|\alpha|^2 / \langle n \rangle} \exp(-2|A(t)|^2 + 2A(t)\alpha^* - 2A(t)^*\alpha) d^2\alpha \\ &= \exp(-2|A(t)|^2(1 + 2\langle n \rangle)). \end{aligned} \quad (4.14)$$

As the temperature increases, the vanishing of the off-diagonal elements described by the accompanying factor $F(t)$ is illustrated in figure 2. If we define the effective coupling constant

$$g[\langle n \rangle] = g\sqrt{1 + 2\langle n \rangle} \quad (4.15)$$

it is observed that the effective strong coupling $g[\langle n \rangle]/\omega \rightarrow \infty$ can be achieved at very high temperature since $\langle n \rangle$ is a monotonically increasing function of temperature T . It can be demonstrated more clearly by considering the strong-coupling limit, and in this case the accompanying overlap factor is obtained

$$F(t) \approx \exp[-2(g^2(1 + 2\langle n \rangle))t^2] = \exp[-2(g[\langle n \rangle]t)^2]. \quad (4.16)$$

Since $\langle n \rangle$ increases from zero to infinity as the temperature changes from zero to infinity, we claim that the thermal effect must enhance the decay of the off-diagonal terms in the reduced density matrix. Figure 2 shows the gradually changing process of the accompanying overlap factor from weak coupling to strong coupling as the temperature increases. During this process the collapse and revival oscillate rapidly in the weak coupling. In the strong coupling the ideal decoherence happens with very narrow and sharp quantum jumps. Coherence only

appears at the top points of these jumps. Now, it can be concluded that the low-temperature condition is not needed for the dynamic realization of the WFC in the context of cavity QED. In fact, the excitation of photons by the thermal effect can result in the WFC at very high temperature. For this reason, it can be claimed that it is possible to realize the present model in further experiments since it is not too difficult to get strong coupling in the high-temperature limit.

5. Atom-optical effects

All of the above discussions do not concern the influence of exchange of momentum between the photon and the centre of mass of the atom directly. The generalized Jaynes–Cummings model including the motion of the atomic centre of mass has already been discussed everywhere, for example, in the spontaneous emission of the atoms as well as laser cooling [24–26]. This influence can also cause atom-optical effects, such as diffraction and splitting of an atom beam, the optical Stern–Gerlach experiment and so on. Transforming back to the original laboratory ‘frame of reference’ through the unitary operator

$$W(x) = e^{ikxa^\dagger a} \quad (5.1)$$

one can find the transfer of momentum due to the entangling action

$$W(x)|\vec{p}\rangle \otimes |n\rangle = |\vec{p} + n\hbar\vec{k}\rangle \otimes |n\rangle \quad (5.2)$$

for the direct product state of the composite system of the atomic centre of mass and photon field. It is emphasized that it does not change the motion of the atomic centre of mass if the cavity is in the vacuum state $|0\rangle$.

We now consider the case of strong coupling. At time $t = 0$, the Zeeman atom with a certain momentum $\vec{p}$ is injected into a vacuum cavity. The joint state of the total system is then initially

$$|\psi(0)\rangle = |\vec{p}\rangle \otimes (c_e|e\rangle + c_g|g\rangle) \otimes |0\rangle. \quad (5.3)$$

Its time evolution gives the wavefunction at time t

$$\begin{aligned} |\psi(t)\rangle &= W[|\vec{p}\rangle \otimes (c_e|e\rangle \otimes |gt\rangle + c_g|g\rangle \otimes |-gt\rangle)] \\ &= \sum_{n=0}^{\infty} \frac{(gt)^n}{\sqrt{n!}} |\vec{p} + n\hbar\vec{k}\rangle \otimes (c_e|e\rangle + (-1)^n c_g|g\rangle) \otimes |n\rangle. \end{aligned} \quad (5.4)$$

The above expression manifests a phenomenon of splitting of the atomic beam. If the direction of motion of atom is vertical with respect to the wavevector $\vec{k}$ of the cavity, the atom will absorb the momentum $n\hbar\vec{k}$ ($n = 1, 2, \dots$) along $\vec{k}$ and the atomic beam will split with different momenta $\vec{p} + m\vec{k}$.

To understand the influence of the motion of the atomic centre of mass on the WFC, we first write down $\psi(t)$ in the x -representation

$$|\psi(x, t)\rangle = \langle x|\psi(t)\rangle = e^{ipx/\hbar} (c_e|e\rangle \otimes |gte^{ikx}\rangle + c_g|g\rangle \otimes |-gte^{ikx}\rangle) \quad (5.5)$$

which leads to the reduced density matrix

$$\begin{aligned} \rho(x) &= \text{Tr}_D(|\psi(x, t)\rangle\langle\psi(x, t)|) = |c_e|^2|e\rangle\langle e| + |c_g|^2|g\rangle\langle g| \\ &\quad + (c_e c_g^* \langle gte^{ikx} | -gte^{ikx} \rangle |e\rangle\langle g| + \text{HC}). \end{aligned} \quad (5.6)$$

The accompanying factor in the off-diagonal element

$$\langle gte^{ikx} | -gte^{ikx} \rangle = e^{-2g^2 t^2} \quad (5.7)$$

is independent of x and also decays exponentially. Thus, the motion of atomic centre of mass does not affect the realization of WFC.

Finally, the influence of the motion of the atomic centre of mass will generate the even and odd coherent states dynamically [27]. In fact, the x -representation of wavefunction (5.5) can be rewritten as

$$\begin{aligned} |\psi(x, t)\rangle &= e^{ipx - |gt \exp(ikx)|^2/2} \left\{ \sum_{m=0}^{\infty} \frac{(gte^{ikx})^{2m}}{(2m)!} (c_e|e\rangle + c_g|g\rangle) \otimes |2m\rangle \right. \\ &\quad \left. + \sum_{m=0}^{\infty} \frac{(gte^{ikx})^{2m+1}}{(2m+1)!} (c_e|e\rangle - c_g|g\rangle) \otimes |2m+1\rangle \right\} \\ &= e^{ipx} [(c_e|e\rangle + c_g|g\rangle) \otimes |gte^{ikx}\rangle_e + (c_e|e\rangle - c_g|g\rangle) \otimes |gte^{ikx}\rangle_o] \end{aligned} \quad (5.8)$$

where the definitions of even and odd coherent states are

$$|\alpha\rangle_e = e^{-|\alpha|^2/2} \sum_{m=0}^{\infty} \frac{(\alpha)^{2m}}{\sqrt{2m!}} |2m\rangle \quad (5.9)$$

$$|\alpha\rangle_o = e^{-|\alpha|^2/2} \sum_{m=0}^{\infty} \frac{(\alpha)^{2m+1}}{\sqrt{(2m+1)!}} |2m+1\rangle \quad (5.10)$$

respectively. Actually, when $c_e = c_g = \sqrt{\frac{1}{2}}$ initially, the above discussion implies a procedure of generating even- and odd-coherent states in a dynamic process with a measurement for the coherent superpositions states $\sqrt{\frac{1}{2}}(|e\rangle \pm |g\rangle)$. If one measures the operator $\sigma_1 = |e\rangle\langle g| + |g\rangle\langle e|$ and obtains a certain result 1 or -1, the wavefunction will collapse to the reduced state $\sqrt{\frac{1}{2}}(|e\rangle + |g\rangle) \otimes |gte^{ikx}\rangle_e$ or $\sqrt{\frac{1}{2}}(|e\rangle - |g\rangle) \otimes |gte^{ikx}\rangle_o$. Such a kind of entangling collapse will produce even- and odd-coherent states!

6. Comments

The dynamic model discussed in this paper not only realizes the WFC in quantum measurement as a process of Schrödinger evolution, but also manifests rich phenomena in atomic optics. These contexts are directly related to the fundamental aspects of quantum mechanics. Before concluding this paper, we should emphasize its relations to the factorizable structure which is the essence of the dynamic realization of WFC in previous models. In this paper, all of the dynamic features come from a basic Hamiltonian

$$H = i\hbar g \sigma_3 (a - a^\dagger) \quad (6.1)$$

for the strong-coupling limit and

$$H = \hbar \omega a^\dagger a + i\hbar g \sigma_3 (a - a^\dagger) \quad (6.2)$$

in the high-temperature limit. The high-spin correspondence, $J_+/\sqrt{2j} \rightarrow a^\dagger$, $J_-/\sqrt{2j} \rightarrow a$, $J_3 \rightarrow \hat{N}/j$ as $j = N/2 \rightarrow \infty$, shows its equivalence to a simple spin-coupling model $H_{\text{spin}} = 2g'\sigma_3 J_2$ where $J_\pm = J_1 \pm iJ_2$ and J_α ($\alpha = 1, 2, 3$) are the $SO(3)$ angular momentum operators. In terms of the spinor representation of $SO(3)$: $J_\alpha = \frac{1}{2} \sum_{i=1}^N \sigma_\alpha(i)$ where $\sigma_\alpha(i)$ are Pauli matrices assigned to different sites in a lattice and commute with each other of different sites, it is proved that our model is equivalent to the HC model

$$H_{\text{HC}} = g'\sigma_3 \sum_{i=1}^N \sigma_2(i). \quad (6.3)$$

This means that, as well as all of the previous dynamic models, our model also possesses an intrinsic factorizable structure. The above correspondences among the different representations of $SO(3)$ were even used to prove the equivalence between the HC model and Cini model by Nakazato and Pascazio [28] recently and our comments here are motivated by their work.

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