Quantum many-body theory of coherent output of Bose-Einstein condensation (I)

——Factorization and atom coherence*

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Abstract Based on the Bogoliubov approximation in the thermodynamical limit, a linear quantum many-body theory is presented to deal with the output of the bosonic atoms to a free propagating mode from a magnetic trap. It is shown that, when atoms are prepared initially in the Bose-Einstein condensation inside a trap, the output will be described by a macroscopic quantum state, a many-body coherent state. This is due to the factorization structure of the total wave function, showing that the output atom beam is coherent. The general expressions for the spectral density of the output and its spatial distribution are given in terms of our linear quantum many-body theory

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Since light wave can be amplified coherently to form a laser output, it is quite natural to imagine if one can get the coherent output of matter wave of particles similar to laser. This was quite an intuitive thought, until the early days of 1997 that the MIT (Massachusetts Institute of Technology) group realized the coherent output of atoms for the first time[1, 2]. This experiment is mainly based on the realization of the Bose-Einstein Condensation (BEC) of atoms in the last two years[3—5]. In the MIT experiment, the untrapped state couples with the trapped state by a radiation frequency (r.f) field coherently. This process leads the BEC atoms in the trap to a propagating mode in the untrapped state and gets a coherent output of atoms in a single quantum state[1]. They examined the coherent character of this coupled output in an interference experiment as well[2]. Due to the large rate of turning the atoms from the trapped state to the untrapped one, the influence of the spontaneous emission of the single atom can be ignored reasonably. In fact, as the MIT group pointed out, physically, the key point of the experiment lies in the factorization in the evolution of the system: one part of the atoms initially prepared at BEC state will keep their original situation, while the other part will be at a coherent state, the coupled output.

Starting from the second quantization of the MIT two level model[1], we will first consider the

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factorization structure and its dynamical generating process systemically. Then, we generalize it for the many-mode case relevant to the spatial coordinate. Theoretically, the initial state of the total system can bedescribed as

$$|\psi(0)\rangle = |\alpha\rangle_p \otimes |\sqrt{N_c}\rangle_b \otimes |0\rangle,$$

where the vacuum state $|0\rangle = \prod_k \otimes |0\rangle_k$ means that there is no atom at any propagating mode $|k\rangle$, while the coherent states $|\sqrt{N_c}\rangle_b$ and $|\alpha\rangle_p$ represent the atomic BEC state and the quasi-classical state of the r.f field respectively. As shown below, in the thermodynamical limit[6], i.e. $N_c \to \infty$, the evolution of the total system will keep the factorization structure[7]

$$|\psi(t)\rangle = |\alpha(t)\rangle_p \otimes |\sqrt{N_c}\rangle_b \otimes \prod_k \otimes |\alpha_k(t)\rangle,$$

where $|\alpha_k(t)\rangle$ is the many-body coherent state for the mode $|k\rangle$. So the coupled output remains in coherence. At the same time the spectral density in momentum space can be defined by $|\alpha_k(t)|^2$, whose Fourier transformation determines the spatial distribution of the output atoms.

1 Concept of factorization and coherent output

The model of MIT group is a two level system. Its ground state $|g\rangle$ is a trapped state and the excited state $|e\rangle$ is an untrapped state. When the system couples with the r.f field of frequency ω, the Hamiltonian of a single atom is

$$H = \hbar\omega_a |e\rangle\langle e| + \hbar\omega a^\dagger a + \hbar\omega_R(|e\rangle\langle g|a + |g\rangle\langle e|a^\dagger), \tag{1.1}$$

where $\hbar\omega_a$ is the energy level difference, $a^\dagger$ and a are the creation and annihilation operators of the field respectively; $\hbar\omega_R = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}} \equiv \frac{\hbar g}{\sqrt{V}}$ is the dipole matrix element, ε_0 the vacuum dielectric constant and V the effective mode volume. As shown in ref. [1], the motion of mass center has been ignored. So it is of no use to give the concrete form of the trap potential.

As to a system consisting of many atoms without interatomic interaction, we consider the second quantization of the Hamiltonian (1.1)

$$H = \hbar\omega_a b_e^\dagger b_e + \hbar\omega a^\dagger a + (\hbar\omega_R b_e^\dagger b_g a + h.c), \tag{1.2}$$

where $b_e^\dagger$, $b_g^\dagger$ and b_e, b_g represent the creation and annihilation operators of $|e\rangle$ and $|g\rangle$ respectively.

When the BEC occurs, a lot of atoms are condensed in the ground state $|g\rangle$. With Bogoliubov approximation[6], we can replace annihilation b_g and creation operator $b_g^\dagger$ with a c-number $\sqrt{N_c}$. Then the effective Hamiltonian is obtained as

$$\mathcal{H}_e = \hbar\omega_a b_e^\dagger b_e + \hbar\omega a^\dagger a + \hbar\omega_R \sqrt{N_c}(a b_e^\dagger + b_e a^\dagger). \tag{1.3}$$

It can be proved that there is a close relationship between the Bogoliubov approximation and the dynamic process of the system in the thermodynamical limit.

As described in eq. (1.3), two harmonic oscillators are coupled in the rotation wave approximation (RWA) so that its Heisenberg motion equation is linear. This results in a linear solution that both $b_e(t)$ and $a(t)$ are the linear combinations of their initial values $b_e(0)$ and $a(0)$, i.e.

$$a(t) = \alpha_a(t)a(0) + \beta_a(t)b_e(0),\ b_e(t) = \alpha_b(t)a(0) + \beta_b(t)b_e(0), \tag{1.4}$$

where

$$\begin{aligned} &\alpha_a(t) = f(\omega_a),\ \beta_b(t) = f(\omega), \\ &\alpha_b(t) = \beta_a(t) = \frac{\sqrt{N_c}}{\Omega}\omega_R(e^{-i\omega_+ t} - e^{-i\omega_- t}), \\ &f(\lambda) = \frac{1}{\Omega}\{(\lambda - \omega_-)e^{-i\omega_+ t} - (\lambda - \omega_+)e^{-i\omega_- t}\}, \\ &\Omega = \sqrt{(\omega_a - \omega)^2 + 4N_c\omega_R^2},\ \omega_\pm = \frac{1}{2}(\omega_a + \omega \pm \Omega). \end{aligned} \tag{1.5}$$

The above linear structure of the Heisenberg operators will directly lead to the factorization of the Schrödinger wave function.

Assume that the initial state of the total system is $|\psi(0)\rangle = |0\rangle \otimes |\alpha\rangle_p$, where $|\alpha\rangle_p$ is a coherent state of the r.f field and $|0\rangle$ is the state with no atom in the excited state at the beginning. Due to the property of the evolution operator $U(t)$ of the system, $c(t) = U^\dagger(t)c(0)U(t)$, $(c = a, b)$, the initial state will evolve into

$$\begin{aligned} |\psi(t)\rangle &= U(t)|\psi(0)\rangle \\ &= [U(t)e^{\alpha a^\dagger(0) - \alpha^* a(0)}U^\dagger(t)]U(t)|0\rangle \otimes |0\rangle_p \\ &= e^{\alpha a^\dagger(-t) - h.c}|0\rangle \otimes |0\rangle_p \end{aligned}$$

according to the method presented in refs. [8, 9]. From eq. (2.4), we obtain the wave function at time t

$$\begin{aligned} |\psi(t)\rangle &= e^{\alpha[\alpha_a(-t)a^\dagger(0) + \beta_a(-t)b^\dagger(0)] - h.c}|0\rangle \otimes |0\rangle_p \\ &= |\alpha\beta_a(-t)\rangle \otimes |\alpha\alpha_a(-t)\rangle. \end{aligned} \tag{1.6}$$

The meaning of the above factorized solution is physically clear: the atoms initially at the BEC state will mostly output to the free state $|e\rangle$ by the stimulation of a coherent r.f field. They form a macroscopic many-body quantum state——the coherent state $|\alpha\alpha_a(-t)\rangle$. The number of atoms at this state

$$\langle b_e^{\dagger}(0) b_e(0) \rangle = |\alpha|^2 |\beta_a(-t)|^2 = \frac{4\omega_R^2 N_c^2}{(\omega_a - \omega)^2 + 4N_c\omega_R^2} \sin^2\left(\frac{1}{2}\Omega t\right) \tag{1.7}$$

behaves as macroscopic Rabi oscillation with certain frequency Ω and amplitude. And there exists the maximum amplitude value N_c at resonance. In this sense, the coupled output possesses the character of laser, so is called "atom laser".

2 The many-body model of coupled output

The above simplified model has clarified the physical implication of the factorization for the coherent output of atoms. To consider further the dynamics of the coordinate and momentum distributions of output BEC atoms, we will generalize the above model to the practical case relevant to spatial coordinate in this section.

In the trap, the spatial motion of atoms can be depicted by the eigen-state $|n\rangle$ with a potential $V(x)$. The eigenvalue E_n is the energy level of the quantization of atomic mass center accordingly. While the atoms are untrapped, their spatial motions are described by the momentum eigen-state $|k\rangle$ of free particles. With the complete basis $\{|e_k\rangle = |e\rangle \otimes |k\rangle, |g_n\rangle = |g\rangle \otimes |n\rangle\}$, and their corresponding creation and annihilation operators $b_{ek}^{\dagger}$, b_{ek} and $b_{gn}^{\dagger}$, b_{gn}, the second quantized many-body Hamiltonian is written as

$$H = \sum_n E_n b_{gn}^{+} b_{gn} + \sum_k (e_k + \hbar\omega_a) b_{ek}^{+} b_{ek} + \hbar\omega a^{+} a$$

$$+ \hbar\omega_R \sum_{n,k} (\langle n | k - q \rangle b_{gn}^{+} b_{ek} a^{+} + h.c). \tag{2.1}$$

Here, the interaction part results from the single particle $(\hbar\omega_R |e\rangle\langle g| e^{iqx} + h.c)$ in the RWA; q is the wave vector of the single mode r.f field and $e_k = \frac{\hbar^2 k^2}{2M}$. Meanwhile, the state $|0\rangle$ is the ground state of the single atom in the trap.

In the thermodynamical limit, we can replace operator b_{g0} and $b_{g0}^{\dagger}$ with the c-number $\sqrt{N_c}$ according to the Bogoliubov approximation. As the part including b_{g0} and $b_{g0}^{\dagger}$ in the Hamiltonian is proportional to $\hbar\omega_R \sqrt{N_c} = \hbar g \sqrt{n_c}$, it is finite. The other terms proportional to $\hbar\omega_R \sim \frac{1}{\sqrt{V}}$ is so small that they can be ignored[11]. Then, we get the effective Hamiltonian in the Bogoliubov approximation

$$H = \sum_k \hbar\Omega_k b_k^{+} b_k + \hbar\omega a^{+} a + \hbar\sqrt{N_c} \sum_k (g(k) b_k a^{\dagger} + h.c), \tag{2.2}$$

where $\Omega_k = \omega_a + e_k/\hbar$, and $b_k = b_{ek}$. The coupling coefficient $g(k) = \omega_R \langle 0 | k - q \rangle$ is defined by the spatial shape of the ground state in the trap. When BEC happens, atoms are mostly condensed in the bottom of the trap. Thus, we can describe the ground state of the atoms by the ground state of the oscillator.

It isn't difficult to observe that the fundamental Hamiltonian (2.1) is bilinear in the atomic op-

erators b_{gn}^{+}, b_{ek}^{+} and b_{gn}, b_{ek}, and thus the total number of stoms is conserved. This fact originates from that the Hamiltonian is invariant under the global $U(1)$ gauge transformation[11]

$$U(\theta) = \exp\left[- i\theta\left(\sum_{n} b_{gn}^{+} b_{gn} + \sum_{k} b_{ek}^{+} b_{ek} \right) \right]. \tag{2.3}$$

It is obvious that the Bogoliubov approximation will lead to a symmetry breaking, thus violates the conservation of the number of atoms. It is trivial because we ignored the infinite background formed by the BEC atoms in the ground state. The above effective Hamiltonian (2.2) with Bogoliubov approximation only characterizes the atomic excitation in the infinite background. This broken symmetry is the key to produce a macroscopic quantum effect. It is interesting to notice that there is also another symmetry in association with the unitary transformation

$$U_{ab}(\theta) = \exp\left[- i\theta\left(a^{+} a + \sum_{k} b_{ek}^{+} b_{ek} \right) \right] \tag{2.4}$$

which is maintained in the Bogoliubov approximation. This symmetry implies a conservation of the number of output atoms and photons. In fact, we have assumed infinite BEC atomic states and then the atoms in other modes filled into them can create infinite photons. This mechanism physically guarantees the factorization structure of the total wave function.

3 Dynamic equation and its formal solution

The Hamiltonian (2.2) looks like an ordinary dissipative quantum system, which comprises a harmonic oscillator (the single mode r.f. photon) plus the bath of many oscillators (the modes of atoms in the untrapped states). However, there are some substantial differences between the physics of the present problem and those in the quantum dissipative systems. The Wigner-Wisskopf (WW) approach as well as the Markov approximation can not work well for the present model. This is because: (i) our model involves a strong coupling proportional to $\sqrt{N_c}$ in the BEC case, but the WW approximation essentially is a second order approximation theory for small coupling constant g; (ii) the exact solvable approaches such as those in refs. [8, 9] for the quantum dissipative system require the spectral distribution of the bath, which is determined by the basic dynamics in our model; (iii) the Markov approximation for the quantum Langevin equation concerns the localization of a memory function[11], which makes the equation at a given time t independent of the history of the dynamic variables before this time. Thus, this can be achieved only when the coupling constants and the bath spectral distribution take specific forms. In the present problem, however, the coupling constants as well as the spectral density has been determined by the shape of a given trap. So the Markov approximation does not work well.

In this section, we use the Laplace transformation to describe the factorization of the dynamic process in the formal solution. Consider the Heisenberg equations

$$\dot{b}_k(t) = - i\Omega_k b_k(t) - i\sqrt{N_c} g^{*}(k) a(t),$$

$$\dot{a}(t) = -\mathrm{i}\omega a(t) - \mathrm{i}\sqrt{N_c}\sum_k g(k)b_k(t) \tag{3.1}$$

of bosonic operator $b_k(t)$ and $a(t)$. By the Laplace transformation and its involution calculation, the linear solution is formally obtained as

$$\begin{aligned} a(t) &= u(t)a(0) + \sqrt{N_c}\sum_k u_k(t)b_k(0), \\ b_k(t) &= \mathrm{e}^{-\mathrm{i}\Omega_k t}b_k(0) - \mathrm{i}\sqrt{N_c}w_k(t)a(0) - \mathrm{i}N_c\sum_{k'} g(k)^* v_{k'k}(t)b_{k'}(0). \end{aligned} \tag{3.2}$$

Here, $a(t)$ is determined by the Langevin equation

$$\dot{a}(t) = -\mathrm{i}\omega a(t) - \mathrm{i}\sqrt{N_c}\sum_k g(k)\mathrm{e}^{-\mathrm{i}\Omega_k t}b_k(0) - N_c\int_0^t M(t-\tau)a(\tau)\mathrm{d}\tau, \tag{3.3}$$

where

$$M(t) = \int_{-\infty}^{\infty} |g(k)|^2 \mathrm{e}^{-\mathrm{i}\Omega_k t}\mathrm{d}k \tag{3.4}$$

is the so-called memory function. Let $F[p]$ be the Laplace transformation $L(F(t))$ of $F(t)$. Then we get

$$\begin{aligned} u(t) &= L^{-1}\left(\frac{1}{p + \mathrm{i}\omega + N_c M[p]}\right), \\ u_k(t) &= L^{-1}\left(\frac{-\mathrm{i}g(k)}{(p + \mathrm{i}\omega + N_c M[p])(p + \mathrm{i}\Omega_k)}\right), \\ w_k(t) &= \int_0^t g^*(k)u(\tau)\mathrm{e}^{-\mathrm{i}\Omega_k(t-\tau)}\mathrm{d}\tau, \\ v_{k'k}(t) &= \int_0^t u_{k'}(\tau)\mathrm{e}^{-\mathrm{i}\Omega_k(t-\tau)}\mathrm{d}\tau. \end{aligned} \tag{3.5}$$

In the traditional theory for quantum dissipation, the memory function is localized based on the Winger-Wisskopf (WW) approximation in the case of weak coupling or with the special form of $g(k)$, i.e. $M(t-\tau) \sim -\frac{\gamma}{N_c}\delta(t-\tau)(\gamma > 0)$. This leads to a dissipative term $-\gamma a(t)$ appearing in the quantum Langevin equation. This localization makes the evolution of $a(t)$ depend only on the state at given time t, but not on the history of dynamic variables before this time. Due to the stronger interaction, the memory function can not be localized. However, as the Heisenberg equation (3.1) is essentially linear, its solution must be a linear combination of its initial states, whatever is approximate or rigorous. This linear structure will automatically lead to a factorization structure of Schrödinger wave function, which characterizes the coherence of output atom correctly.

4 Coherent coupled output of atoms

Based on the method developed in refs. [8, 9], the dynamic process of the coherent output of atoms can be pictured by calculating the Schrödinger wave function from the Heisenberg operators. If all atoms are initially at the BEC states, the initial state for the effective equation will be described theoretically to be a direct product of the coherent state $|\alpha\rangle$ of r.f field and the many-body vacuum state $|0_k\rangle$ of propagating mode of k i.e.

$$|\psi(0)\rangle = |\alpha\rangle \otimes \prod_k |0_k\rangle. \tag{4.1}$$

The evolution matrix $U(t)$ of the total system transforms into the operator from time $t = 0$ to t, i.e. $b_k(t) = U^\dagger(t)\, b_k(0)\, U(t)$. Since $a(t) = U^\dagger(t)\, a(0)\, U(t)$ and $|\alpha\rangle = \exp[\alpha a^\dagger(0) - \alpha^* a(0)]|0\rangle$, the Schrödinger wave function at time t

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle = U(t)\exp[\alpha a^\dagger(0) - h.c]U^\dagger(t)U(t)|0\rangle \tag{4.2}$$

can be constructed from eq. (3.2). Here, $|0\rangle = |0\rangle \otimes \prod_{k=1}^{N} |0_k\rangle$. From $U(t)|0\rangle = |0\rangle$, we get

$$\begin{aligned} |\psi(t)\rangle &= \exp[\alpha a^\dagger(-t) - h.c]|0\rangle \\ &= \exp\left[\alpha\left\{u^*(-t)a(0) + \sqrt{N_c}\sum_k u_k^*(-t)b_k(0)\right\} - h.c\right]|0\rangle \\ &= |\alpha u^*(-t)\rangle \otimes \prod_k \left|\alpha\sqrt{N_c}u_k^*(-t)\right\rangle. \end{aligned} \tag{4.3}$$

It can be observed from the above equation that the atomic component $\left|\alpha\sqrt{N_c}u_k^*(-t)\right\rangle$ defines a coherent output pulse of atoms in each propagating modes $|k\rangle$ of untrapped states. Its norm square of amplitude represents the number of atoms in this mode $|k\rangle$

$$\rho(t) = \left\langle \alpha\sqrt{N_c}u_k^*(-t)\right| b_k^\dagger(0)b_k(0)\left|\alpha\sqrt{N_c}u_k^*(-t)\right\rangle = |\alpha|^2 N_c |u_k(t)|^2 \tag{4.4}$$

which represents the spectral distribution of the output atoms at a given time t. When the ground state of the harmonic oscillator

$$\psi_0(x) = \langle x|0\rangle = \left(\frac{2}{\pi a^2}\right)^{\frac{1}{4}} e^{-x^2/a^2} \tag{4.5}$$

is chosen to be the ground state of the trap, the k-dependent coupling $g(k)$

$$g(k) = \omega_R\left(\frac{a^2}{2\pi\hbar^2}\right)^{\frac{1}{4}} e^{-\frac{a^2(k-q)^2}{4\hbar^2}} \tag{4.6}$$

determines the memory function as

$$M(t) = \int_{-\infty}^{\infty} |g(k)|^2 e^{-i\Omega_k t} dk$$

$$= \omega_R^2 \frac{a}{\hbar} \frac{e^{-i\omega_a t - i\frac{\hbar q^2}{2M}t}}{\sqrt{\frac{a^2}{\hbar^2} - \frac{\hbar t}{iM}}} e^{-\left(\frac{\hbar q t}{M}\right)^2 \left(\frac{a^2}{\hbar^2} - i\frac{\hbar t}{M}\right) \Big/ \left[2\left(\frac{a^4}{\hbar^4} + \frac{\hbar^2 t^2}{M^2}\right)\right]} \tag{4.7}$$

analytically. The function obviously represents the evolution of a Gauss wave packet from time $t = 0$ to time t. The non-localization part $\int_0^t M(t-\tau)a(\tau)d\tau$ in the Langevin equation means that the result at time t comes from the involution superpositions of wave packets just before this time. So the spectral distribution of output atoms $|u_k(t)|^2$ has a direct relation to the memory function.

Because the solution of eq. (4.2) is described as a direct product of some coherent states, and the output component has been proved especially to have the factorized structure, the output atoms are shown to have good coherence. If we decompose the field operator of bosonic atoms in propagating modes into two parts

$$\hat{\psi}(x) = \hat{\psi}^+(x) + \hat{\psi}^-(x),$$

$$\hat{\psi}^+(x) = \sum_k \frac{1}{\sqrt{V_k}} e^{ikx} b_k(0), \quad \hat{\psi}^-(x) = (\hat{\psi}^+(x))^\dagger, \tag{4.8}$$

the Glauber coherence function of order n

$$G^{(n)}(x_1 x_2 \cdots x_n, x_{n+1} \cdots x_{2n})$$

$$= \langle \psi(t) | \psi^-(x_1)\psi^-(x_2)\cdots\psi^-(x_n)\psi^+(x_{n+1})\cdots\psi^+(x_{2n}) | \psi(t) \rangle$$

$$= Q(x_1)Q(x_2)\cdots Q(x_n)Q^*(x_{n+1})\cdots Q^*(x_{2n}) \tag{4.9}$$

is completely factorized, where

$$Q(x) = \sum_k \frac{\alpha^* \sqrt{N_c}}{\sqrt{V_k}} e^{ikx} u_k^*(t). \tag{4.10}$$

This result shows the complete coherence of the output atoms.

5 Discussion

In this paper, we study only the ideal case ignoring the interatomic interaction. Just as the MIT group has pointed out, the interaction, in fact, can be ignored in the coherent output problem in present experiments. This is because the output atoms scatter continuously and the interatomic interaction becomes weaker and weaker. As for the higher order approximation case, the non-linear part

$\frac{4\pi\hbar^2 a}{M} b_k^\dagger b_k^\dagger b_k b_k$ [12], which is decided by the scattering length a of S-partial wave in the two-body scattering problem, will result in the non-linear Heisenberg equation. Starting from the exact solution shown in this paper, we can solve the non-linear effect in the weak coupling case with a perturbation theory.

With the expressions of $b_k(t)$, we can calculate the output eq. (4.3) directly in the Heisenberg picture

$$\begin{aligned}\rho(t) &= \langle \psi(0) | b_k^\dagger(t) b_k(t) | \psi(0) \rangle \\ &= N_c \langle \alpha | a^\dagger(0) a(0) | \alpha \rangle | W_k(t) |^2 = N_c |\alpha|^2 | u_k(t) |^2. \end{aligned} \tag{5.1}$$

Here, we have used the involution formula to prove that $| W_k(t) | = | u_k(t) |$. This discussion can be extended to the cases with arbitrary initial states of the r.f. field. Assume the initial state of the field to be $| \phi_p \rangle$, the initial state of the effective system is $| \psi(0) \rangle = | \phi_p \rangle \otimes \prod_k | 0_k \rangle$. Then

$$\rho(t) = N_c \overline{N_p} | u_k(t) |^2, \tag{5.2}$$

where $\overline{N_p} = \langle \phi_p | a^\dagger(0) a(0) | \phi_p \rangle$ represents the average number of photons of the initial state. Physically, this result shows that the collective Rabi oscillation of output atoms have no direct relationship to the details of the r.f field. As a function of time t, this behavior of collective oscillation depends only on the physics properties of the atomic system.

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