ELSEVIER

NUCLEAR PHYSICS B

Dark information of black hole radiation raised by dark energy

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Abstract

The "lost" information of black hole through the Hawking radiation was discovered being stored in the correlation among the non-thermally radiated particles (Parikh and Wilczek, 2000 [31], Zhang et al., 2009 [16]). This correlation information, which has not yet been proved locally observable in principle, is named by dark information. In this paper, we systematically study the influences of dark energy on black hole radiation, especially on the dark information. Calculating the radiation spectrum in the existence of dark energy by the approach of canonical typicality, which is reconfirmed by the quantum tunneling method, we find that the dark energy will effectively lower the Hawking temperature, and thus makes the black hole has longer life time. It is also discovered that the non-thermal effect of the black hole radiation is enhanced by dark energy so that the dark information of the radiation is increased. Our observation shows that, besides the mechanical effect (e.g., gravitational lensing effect), the dark energy rises the stored dark information, which could be probed by a non-local coincidence measurement similar to the coincidence counting of the Hanbury–Brown–Twiss experiment in quantum optics.

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1. Introduction

Dark energy and dark matter are mysterious to compose our universe together with various material particles [1,2], which can form the generic black holes in some circumstance. While dark matter are now widely thought to be some form of massive exotic particles, such as primordial black hole formed within the first several second of our universe [3], it seems that black holes have nothing to do with dark energy because dark energy is not affected by gravity, no matter what size the black hole is and no matter where the dark energy locates around the horizon of the black hole. However, dark energy can causes some "opposite gravity effect" so that the universe will expand faster and faster, thus it makes everything in the universe become more distant so that the gravitational lensing effect can be revised by dark energy [4]. In other hand, dark energy can push the black hole horizon away from its center in increase the horizon area (entropy) and thus and in turn it can exert significant influences on both the black hole radiation and the corresponding information loss. In this paper we will investigate these influences based on statistical mechanics about entropy.

It is well known that, dark energy composes 69% of our universe while the dark matter and ordinary matter constitute the remaining 31% [1]. Many theories about dark energy have been proposed [5,6], including a constant energy density filled in space, or some scalar fields named quintessence [7]. However, dark energy is still a hypothesis used to explain the accelerating expansion of the universe [8], and a convincing theory for dark energy is lacked yet. Recently, LIGO and Virgo have made a breakthrough to detect gravitational waves [9], the provided experimental data showed new possible evidence for dark energy [10–13]. Besides the recently discovered gravitational lensing effect [4] and the estimated proportion, more detailed properties of the dark energy and its influence on other objects, such as thermodynamic influences, have not yet been comprehensively understood and studied, both in theory and experiment.

In view of the expanding of the universe, as a result of the repulsion effect provided by dark energy, the celestial bodies in the universe will be stretched by the expansion of the space, thus, their geometry are changed. Let us now pay our attention to the influences of dark energy on black hole. As a kind of celestial body, one of its geometric features, the surface area, will be affected by the dark energy. It follows from the black hole thermodynamics that the surface area of a black hole, namely the horizon area, corresponds to the Bekenstein–Hawking (B–H) entropy of black hole [14]. In addition, as we have shown, the Hawking radiation spectrum can be straightforwardly derived form the explicit expression of the B–H entropy with mass, charge and angular momentum as three hair variables [15]. Therefore, the dark energy may influence the black hole radiation spectrum. As a result, the change of the radiation spectrum of the black hole will effect on various properties of the black hole and its radiation, such as the temperature of the radiation field, namely the Hawking temperature, the life time of the black hole and so on.

Considering the non-thermal effect of the black hole radiation, Zhang et al. [16] found that there exists information correlation among the radiated particles from the black hole. The two or more particle correlation can be regarded as a kind of hidden information of the black hole because once it were ignored, the paradox of black hole information loss would appear. In other words, the information we used to believe being lost in the Hawing radiation process is actually stored as radiated particles' correlation in the radiation field [17–19]. Now we recognized that the correlation information is dark since it can not be observed locally even though the Hawking radiation can be finally measured experimentally. Actually, to explore quantum or classical correlation, two or more non-local probes are needed to make a coincidence measurement similar to the coincidence counting of the Hanbury–Brown–Twiss experiment in quantum optics [20]. In

Table 1
Correction of dark energy on Black hole radiation.

	Without dark energy	With dark energy
Hawking temperature T_H	$(8\pi M)^{-1}$	$(8\pi M)^{-1}\left(1 - 16M^2\Lambda/3\right)$
Black hole life time t	$5120\pi M^3$	$5120\pi M^3\left(1 + 56M^2\Lambda/5\right)$
Dark information I	$8\pi E_a E_b$	$8\pi E_a E_b\left(1 + 16M^2\Lambda\right)$

this paper we are interested in whether dark energy will increase or reduce the dark information characterized by the mutual entropy.

All in all, the study of the black hole radiation with dark energy will enlighten us to comprehend the influences of dark energy on the thermodynamic properties of black hole and its "lost" information. To this end, we first derive the correction to the radiation spectrum of black hole with the existence of dark energy. The universal approach based on the canonical typicality [21–23] to achieve the black hole radiation spectrum is confirmed by the quantum tunneling approach with the Schwarzschild black hole as an example. According to the corrected radiation spectrum with dark energy, the Hawking temperature, the life time of black hole, and the dark information in the black hole radiation are obtained for the Schwarzschild black hole of mass M with non-vanishing cosmological constant Λ and the state parameter w [4].

The main results of this paper are illustrated in Tab. 1, where E_a and E_b are the energy of two radiated particles a and b, respectively; the natural units with $\hbar = G = k_B = c = 1$ are adopted thereafter for simplicity. It should be mentioned here that the results shown in Tab. 1 are hold only in the cosmological constant model, where the state parameter $w = -1$. For an arbitrary w, the corresponding results are more complicated and will be demonstrated in the main text of this paper as Eqs. (4.4), (4.13), and (5.2). Our results show that the dark energy makes the Hawking radiation colder, the life time of the black hole longer, and raises more dark information of Hawking radiation.

This paper is organized as follows. In Sec. 2, we first review the canonical typicality based general approach we developed to derive the radiation spectrum of the black hole. Then, we briefly introduce the quantum tunneling method for obtaining the radiation spectrum dynamically. In Sec. 3, we calculate the horizon radius of a Schwarzschild black hole by taking account of the effect of dark energy. Then, we use it to obtain the radiation spectrum from our statistical mechanical approach. We also make a verification of this spectrum with the quantum tunneling method. Furthermore, according to the corrected radiation spectrum, we consider the black hole's evaporation and give the corrected Hawking temperature and black hole life time in Sec. 4. In Sec. 5, we study the influence of dark energy on dark information among Hawing radiation. We will conclude and make some discussions in Sec. 6. In Appendix, we provide the detailed derivation of the canonical typicality-based approach.

2. General approach to radiation spectrum of black hole

The radiation spectrum of black hole describes the statistical distribution of particle's energy after the particles cross the horizon through the Hawking process. From the microscopic point of view, Hawking process is composed of the following two steps: (i) quantum fluctuations make a large number of particle pairs created and annihilated near event horizon; (ii) while a positive energy particle from a pair created virtually inside the horizon escapes out of the horizon through

tunneling, the Hawking radiation happens. This means that the Hawking process is a dynamic process. Thus, it seems that the dynamic analysis of radiated particles is needed to derive the Hawking radiation spectrum. With the help of the curved space-time quantum field theory, the radiation spectrum of black hole was first obtained by Hawking, and it was found to obey the thermal distribution [24,25].

In fact, this thermal spectrum implies that the entropy will increase through the Hawking process, which leads to the black hole information paradox. In order to resolve this paradox, different schemes have been put forward [26–35], such as modifying the radiation spectrum itself [31]. When the constraint of energy conservation is considered, the black hole radiation spectrum is shown to be not perfectly thermal. This non-thermal radiation spectrum have been proved to satisfy the requirement of information conservation [16].

2.1. Canonical typicality based approach

In our recent study [15], we obtained the non-thermal black hole radiation spectrum by a purely statistical mechanical approach based on canonical typicality. It is emphasized that this approach do not refer to any dynamics of particle tunneling. In this section, we will briefly review this general approach (for the details please see Appendix). It follows from the canonical typicality [15,21–23] or the micro-canonical hypothesis that the density matrix of an arbitrary black hole B with three "hairs", namely mass M, charge Q, and angular momentum J, reads

$$\rho_{\mathrm{B}} = \sum_i \frac{1}{\Omega_{\mathrm{B}}(M, Q, J)} |M, Q, J\rangle_i \langle M, Q, J|. \tag{2.1}$$

Here, $|M, Q, J\rangle_i$ is the ith eigenstate of B, and $\Omega_B(M, Q, J)$ is B's number of microstate. When the black hole evaporates, the system evolves into two components, the radiation field R and the remaining black hole B′. By taking into consideration the fact that the black hole "hairs" are conserved quantities, then tracing over all the degree of freedom of B′, the density matrix of R is obtained as

$$\rho_{\mathrm{R}} = \sum_{\omega, q, j} p(\omega, q, j, M, Q, J) |\omega, q, j\rangle \langle \omega, q, j|, \tag{2.2}$$

where $|\omega, q, j\rangle$ is the eigenstate of R, which has mass ω, charge q, and angular momentum j. The distribution probability of the radiation is given by

$$p(\omega, q, j, M, Q, J) = \mathrm{e}^{-\Delta S_{\mathrm{BB}'}(\omega, q, j, M, Q, J)}, \tag{2.3}$$

with

$$\Delta S_{\mathrm{BB}'}(\omega, q, j, M, Q, J) = S_{\mathrm{B}}(M, Q, J) - S_{\mathrm{B}'}(M - \omega, Q - q, J - j) \tag{2.4}$$

being the entropy difference between B and B′. Equations (2.2)–(2.4) offer us a straightforward approach to calculate the radiation spectrum of black hole. Only when the entropy linearly depends on energy, the spectrum will be perfectly thermal. In fact, in the thermodynamic limit, the higher orders of energy in the distribution probability are always ignored for a large black hole, thus the thermal equilibrium distribution appears. However, when the system being studied is not large enough, the higher-order terms of energy in the distribution probability need to be preserved. Then the emergent non-thermal spectrum means the correlation inside the system due to the energy (charge, angular momentum) conservation law. It follows from this observation that we have found the non-thermal distributions for some specific systems [36,37].

It can be seen from Eq. (2.3) that the radiation spectrum can be given from the entropy of black hole, the Bekenstein–Hawking (B–H) entropy. According to so-called black hole area law [14], the B–H entropy is proportional to the area of horizon. Without loss of generality, we can express the black hole entropy as,

$$S_{\mathrm{B}}\left(M, Q, J\right) = S_{BH}\left(M, Q, J\right) = \frac{A_H\left(M, Q, J\right)}{4} = \pi R_H^2\left(M, Q, J\right), \qquad (2.5)$$

where, $A_H\left(M, Q, J\right)$ and $R_H\left(M, Q, J\right)$ are the area and radius of the horizon, respectively. With Eqs. (2.3) and (2.5), we re-write the radiation spectrum as the function of horizon radius as

$$p\left(\omega, q, j, M, Q, J\right) = \mathrm{e}^{-\pi\left[R_H^2(M,Q,J) - R_H^2(M-\omega, Q-q, J-j)\right]}. \qquad (2.6)$$

As an illustration, we apply Eq. (2.6) to a Schwarzschild black hole with horizon radius being $R_H = 2M$. We then straightforward obtain the distribution probability for a particle with energy ω as

$$p\left(\omega, M\right) = \exp\left[4\pi\left(M - \omega\right)^2 - 4\pi M^2\right] = \exp\left[-8\pi\omega\left(M - \frac{\omega}{2}\right)\right]. \qquad (2.7)$$

This non-thermal spectrum is exactly the same as Eq. (10) of Ref. [31],

$$\Gamma = \mathrm{e}^{-8\pi\omega(M-\omega/2)}, \qquad (2.8)$$

which even was derived through the quantum tunneling perspective.

Obviously, when the mass of a black hole is large, i.e., $M \to \infty$, Eq. (2.7) can be approximated as

$$p\left(\omega, M\right) \approx \exp\left(-8\pi M\omega\right). \qquad (2.9)$$

This is the original thermal radiation spectrum obtained by Hawking. Therefore, we can say that we have developed an effective method to calculate the non-thermal radiation spectrum of the black holes, and it does not need to analysis the tunneling dynamics for the particle crossing the horizon. Generally, to find the black hole radiation spectrum, one can only derive the horizon radius by analyzing the singularity of the metric for a given black hole at first. Secondly, by making use of Eq. (2.6), the specific form of the radiation spectrum is obtained.

2.2. Quantum tunneling based approach

In the aforementioned derivation, we only make use of the statistical properties of the black hole. This method obtains the black hole radiation spectrum without referring the particle's dynamics, thus is only of the statistical mechanics. Another approach obtaining black hole radiation spectrum is the quantum tunneling method, which was first introduced by Parikh and Wilczek to derive the non-thermal radiation spectrum of Schwarzschild black hole [31]. This method was even used for different types of black holes to obtain their radiation spectrum successfully [38–48]. These spectra have been proved to meet the requirement of information conservation, and are exactly consistent with our result of Eq. (2.5) obtained statistical-mechanically.

We now brief introduce this tunneling approach. Since there is a strong gravitational potential near the horizon, the particles escape from the horizon through the Hawking process can be considered as a quantum tunneling process. With the semi-classical approximation, the tunneling probability for the particle can be written as [31]

$$\Gamma = \exp\left[-2\mathrm{Im}(I)\right], \qquad (2.10)$$

where

$$\mathrm{Im}(I) = \int_{r_{in}}^{r_{out}} p_r dr \tag{2.11}$$

is the imaginary part of the action I for a positive energy particle tunneling outside crossing the horizon. r_{in} (r_{out}) is the horizon radius before (after) the particle escaping from the horizon, and p_r is the radial momentum of the particle. Making use of the Hamilton's equation

$$\dot{r} = \frac{dH}{dp_r}, \tag{2.12}$$

we can eliminate the momentum by energy in the action as

$$\mathrm{Im}\,(I) = \mathrm{Im} \int_{H_i}^{H_f} \int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}} dH. \tag{2.13}$$

Here, $\dot{r}$ is the radial null geodesics, and it can be obtained from the black hole metric, and H_i (H_f) is the energy of black hole before (after) the particle escaping from the horizon. With the specific metric form of a given black hole, we can derive the corresponding radial null geodesics $\dot{r}$, then we finish the integral in Eq. (2.13) and thus the tunneling probability is obtained. The detailed calculation of this approach in the case with dark energy is presented in the Sec. 3.2.

Moreover, the statistical mechanical approach allows us to calculate the distribution probability in the radiation process without analyzing the dynamics of particle tunneling. The obtained results are exactly the same as that given by the tunneling approach. This indicates that the statistical mechanical properties of the radiation spectrum may have an intrinsic relationship with the tunneling dynamics of the particles through the Hawing process. Some relevant studies also suggest this, for example, the number of micro-state for black holes can be calculated directly from the tunneling spectrum [49], and the radiation spectra of the black holes have been proved to be independent of the black holes' geometry [50].

3. Black hole radiation spectrum with dark energy

In this section, we are going to derive the radiation spectrum of the Schwarzschild black hole in the existence of the dark energy from both the statistical mechanical approach and the tunneling approach. To obtain the specific form of the spectrum, both of these two methods need to express the horizon radius as the function of the black hole mass. Therefore, we first derive the horizon radius from the metric of the black hole. When the effect of the dark energy is taken into account, the line element for the Schwarzschild black hole reads [4,51]

$$ds^2 = \left[1 - \frac{2M}{r} - 2\left(\frac{r_o}{r}\right)^{3w+1}\right] dt^2 - \frac{dr^2}{\left[1 - \frac{2M}{r} - 2\left(\frac{r_o}{r}\right)^{3w+1}\right]} - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{3.1}$$

where M is the mass of the black hole. r_o is a scale factor, and it can be represented by the cosmological constant Λ as $r_o = \sqrt{6/\Lambda}$. The state parameter w is a constant within the range $-1 < w < -1/3$. According to the definition of the black hole horizon, its radius satisfies the following equation

$$1 - \frac{2M}{R_H} - 2\left(\frac{r_o}{R_H}\right)^{3w+1} = 1 - \frac{2M}{r} - 2\left(\frac{6}{\Lambda R_H^2}\right)^{\frac{3w+1}{2}} = 0. \tag{3.2}$$

Assuming that the dark energy affects the geometry of the black hole slightly, we can solve Eq. (3.2) perturbatively by taking $R_H = R_H^0 + \delta$. Here, $\delta/R_H^0 \ll 1$, and $R_H^0 = 2M$ is the horizon radius of the Schwarzschild black hole in the case that the dark energy does not exist. Then we have

$$\frac{\delta}{2M} = 2(2M)^{2\xi}\left(\frac{\Lambda}{6}\right)^{\xi}, \tag{3.3}$$

where $\xi \equiv -(3w + 1)/2 \in (0, 1)$ is a re-defined positive constant. Finally, the horizon radius of the black hole is obtained as

$$R_H = 2M + 4M^{2\xi+1}\left(\frac{2\Lambda}{3}\right)^{\xi} \equiv 2M + 4M^{2\xi+1}f \tag{3.4}$$

with

$$f = f(\xi, \Lambda) = \left(\frac{2\Lambda}{3}\right)^{\xi}, \tag{3.5}$$

where we have only kept the first order of f. It is seen from Eq. (3.4) that the dark energy will increase the horizon radius. In other words, the black hole becomes bigger in the existence of dark energy. This can be easily understood from the repulsion effect of dark energy to the universe. In comparison with the attraction effect of the gravitation among normal matters, dark energy produces the repulsive effect, which is considered as the reason for the expansion of the universe. Our result shows that the black hole is indeed extended by the dark energy. The quantitative dependence description of this extension effect is given in Eq (3.4).

3.1. Radiation spectrum from statistical mechanical approach

From the correction of the dark energy to the horizon radius, we will further study the influences of the dark energy on the black hole radiation. Through the statistical mechanical method we introduced in Sec. 2.1, we substitute the horizon radius [Eq. (3.4)] into Eq. (2.6), and then the corrected radiation spectrum of Schwarzschild black hole is obtained as

$$\begin{aligned} p(\omega, M) = \exp\Big\{ &-8\pi M\left[1 + 4(\xi + 1)M^{2\xi}f\right]\omega \\ &+ 4\pi\left[1 + 4(\xi + 1)(2\xi + 1)M^{2\xi}f\right]\omega^2\Big\}, \end{aligned} \tag{3.6}$$

where we have kept the second order of ω, and the higher orders $O(\omega^2)$ are ignored. This indicates that the radiation spectrum or the tunneling probability is ξ and Λ-dependent. In the case of the cosmological constant model of the dark energy, we have $\xi = 1$ with the state parameter $w = -1$, and Eq. (3.6) reduces to a more concise form as

$$p(\omega, M) = \exp\left[-8\pi M(1 + 8M^2 f)\omega + 4\pi\left(1 + 24M^2 f\right)\omega^2\right]. \tag{3.7}$$

If the effect of the dark energy approaches 0, the above probability would reduce to $p(\omega, M) = \exp[-8\pi\omega(M - \omega/2)]$, which is exactly consistent with the Parikh–Wilczek (P–W) spectrum. We then plot Eq. (3.7) in Fig. 1, where the black hole mass $M = 1$, and the P–W spectrum is
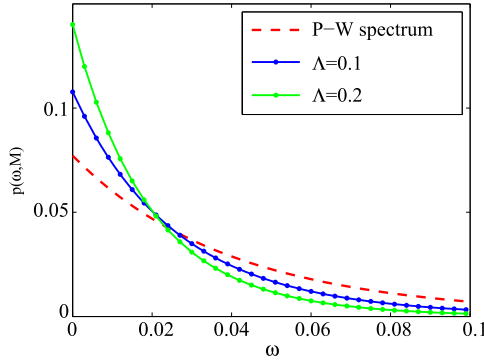
Fig. 1. (Color online.) Radiation distribution probability as the function of the particle energy. The red dash line is the Parikh–Wilczek spectrum, while the blue dotted line (green dotted line) represents the radiation spectrum with the dark energy, where the corresponding cosmological constant is taken as $\Lambda = 0.1$ ($\Lambda = 0.2$).

also potted as a comparison. It can be seen from Fig. 1 that the radiation spectrum is sharply affected by the dark energy. As the influence of the dark energy becomes lager (Λ increasing), the distribution of the low energy parts in the radiation increases, and the proportion of high energy radiation decreases. This implies that the temperature of the radiation becomes lower due to the existence of the dark energy, which agrees with the direct theoretical analysis in Sec. 4.

3.2. The verification from quantum tunneling method

To confirm the result obtained through our statistical mechanical approach, we calculate the radiation spectrum by utilizing the tunneling approach in the case with the dark energy. In the quantum tunneling method, we need to analyze the dynamic behavior of the particle when it is crossing the horizon. Unfortunately, the Schwarzschild coordinates, as we introduced in Eq. (3.1), are singular at the horizon. Thus, it is necessary to choose a new coordinate system to eliminate this singularity at the horizon. One of the suitable choice, first given by Painlevé [52,53], is to introduce a time coordinate, in our case, as follows

$$dt = dt_s + \frac{\sqrt{2M + r_o \left(\frac{r_o}{r}\right)^{3w}} \sqrt{r}}{2M + r_o \left(\frac{r_0}{r}\right)^{3w} - r} dr, \tag{3.8}$$

where t_s is the Schwarzschild time. According to Eq. (3.1), the line element is re-written in the new coordinates as

$$ds^2 = -\left[1 - \frac{2M}{r} - \left(\frac{r_o}{r}\right)^{3w+1}\right] dt^2 + 2\sqrt{\frac{2M}{r} + \left(\frac{r_o}{r}\right)^{3w+1}} dr dt + dr^2 + r^2 d\Omega^2. \tag{3.9}$$

The above equation shows that there is no singularity at the horizon. Then, from Eq. (3.9), we obtain the radial null geodesics

$$\dot{r} \equiv \frac{dr}{dt} = \pm 1 - \sqrt{\frac{2M}{r} + \left(\frac{r_o}{r}\right)^{3w+1}}, \tag{3.10}$$

where the upper (lower) sign is corresponding to the outgoing (ingoing) geodesics. Consequently, the imaginary part of the action I for an s-wave outgoing positive energy particle crossing the horizon can be expressed as

$$\text{Im}\,(I) = \text{Im} \int_M^{M-\omega} \int_{r_{in}}^{r_{out}} \frac{dr}{1 - \sqrt{\frac{2M'}{r} + \left(\frac{r_o}{r}\right)^{3w+1}}} \left(dM'\right), \tag{3.11}$$

where we have used the replacement $dH \to dM'$. $r_{in}\,(r_{out})$ is the horizon radius before (after) the Hawking process corresponding to the particle with energy ω, and can be derived from Eq. (3.4). Letting

$$x(M') = \sqrt{\frac{2M'}{r} + \left(\frac{r_o}{r}\right)^{3w+1}} \tag{3.12}$$

and exchanging the order of the integral in Eq. (3.11), we have

$$\text{Im}\,(I) = \text{Im} \int_{r_{in}}^{r_{out}} \int_{x_i}^{x_f} \frac{xr}{1-x} dx dr, \tag{3.13}$$

where $x^i = x\,(M)$, and $x^f = x\,(M - \omega)$. Note that $x_i < 1 < x_f$, so that the integral contain a singular point $x = 1$. With the contour evaluated via the prescription $x \to x - i\epsilon$, we have the imaginary part of the action

$$\text{Im}\,(I) = \int_{r_{in}}^{r_{out}} -\pi r dr = \frac{1}{2}\pi \left(r_{in}^2 - r_{out}^2\right) \tag{3.14}$$

$$= 2\pi\omega^2 - 4\pi M\omega - 16\pi\,(\xi + 1)\,M^{2\xi+1} f\omega + 8\pi\,(\xi + 1)\,(2\xi + 1)\,M^{2\xi} f\omega^2. \tag{3.15}$$

As an ingoing negative energy particle contributes the same value, we obtain

$$\Gamma = e^{-2\text{Im}(I)} = \exp\left\{-8\pi M \left[1 + 4\,(\xi + 1)\,M^{2\xi} f\right]\omega \right. $$
$$\left. + 4\pi \left[1 + 4\,(\xi + 1)\,(2\xi + 1)\,M^{2\xi} f\right]\omega^2\right\}, \tag{3.16}$$

which exactly agrees with Eq. (2.3) from our statistical mechanical approach.

Until now, the Black hole radiation spectrum have been obtained from the perspective of the statistical physics and the quantum tunneling, respectively. As shown in Eqs. (3.6) and (3.16), these two methods provide the same result, indicating that they are mutually corroborated. Besides, we can clearly perceive that using the statistical mechanical way to get the black hole radiation spectrum is much more succinct than using the tunneling method, while the latter one need to analyze the particle's dynamics in the Hawking process.

4. Evaporation of the Schwarzschild black hole with dark energy

In this section, with the dark energy based radiation spectrum, we will study the evaporation process of the black hole. Specifically, as the two characteristics of the black hole evaporation, the Hawking temperature and the black hole life time will be re-investigated with the correction of the dark energy.

The Hawking radiation [24,25] implies that the black hole has temperature. Using the curved space-time quantum field theory, Hawking obtained the thermal radiation spectrum of the black hole. Therefore, he explained that the temperature of the black hole i.e., $T_H = 1/(8\pi M)$, is the

true temperature. This is the so-called Hawking temperature, and was first introduced according to the similarity between the black hole laws and the thermodynamics laws. For a black hole with a large mass, i.e., $M \gg 1$, the non-thermal part of the radiation spectrum can be ignored. Thus, from Eqs. (2.3) and (3.6), we have the approximated thermal distribution probability

$$p(\omega, M) = \exp\left\{-8\pi M \left[1 + 4(\xi + 1) M^{2\xi} f\right] \omega\right\}. \tag{4.1}$$

When the effect of the dark energy vanishes, namely $f \to 0$, the above distribution will reduce to the Hawking radiation spectrum

$$p(\omega, M) = e^{-8\pi M \omega}, \tag{4.2}$$

4.1. Hawking temperature

It is directly obtained from Eq. (4.1) that the inverse radiation temperature

$$\beta_H = 8\pi M \left[1 + 4(\xi + 1) M^{2\xi} f\right], \tag{4.3}$$

thus the Hawking temperature with the existence of dark energy reads

$$T_H = \frac{1}{8\pi M} \left[1 - 4(\xi + 1) M^{2\xi} \left(\frac{2\Lambda}{3}\right)^\xi\right], \tag{4.4}$$

where we only kept the first order of f. By introducing a the modification factor

$$\lambda = 4(\xi + 1) M^{2\xi} \left(\frac{2\Lambda}{3}\right)^\xi, \tag{4.5}$$

the Hawking temperature can be further simplified as

$$T_H = T_H^0 (1 - \lambda), \tag{4.6}$$

where $T_H^0 = 1/(8\pi M)$ is the Hawking temperature without the dark energy. Equation (4.3) shows that the dark energy makes the Hawking radiation colder.

In Fig. 2, we show that the result of Eq. (4.6)

can also be understood from the black hole thermodynamics, where the temperature of the black hole is positively correlated to the gravity on black hole's surface. As we have shown in Sec. 3 that black hole can be extended by the dark energy, i.e., $R_H \uparrow$, leading to the black hole surface gravity $g_H = M/R_H^2$ decreases, i.e., $g_H \downarrow$. Therefore, the Hawing temperature $T_H = g_H/(2\pi)$ becomes lower, namely $T_H \downarrow$. The phenomenon that the dark energy decreases the Hawking temperature through expanding the space is named by the dark energy based cooling mechanism (DECM).

In addition, the modification factor λ decreases with M, meaning that the influence of dark energy on the Hawking temperature increases with the black hole mass. In the cosmological constant model of the dark energy, we have the state parameter $w = -1$ and $\xi = 1$. The radiation temperature of Eq. (4.4) becomes

$$T_H = \frac{1}{8\pi M} \left(1 - \frac{16M^2 \Lambda}{3}\right). \tag{4.7}$$

Until now, we have obtained the Hawking radiation temperature with the existence of dark energy. According to the theory of black-body radiation, the lower the temperature is, the smaller
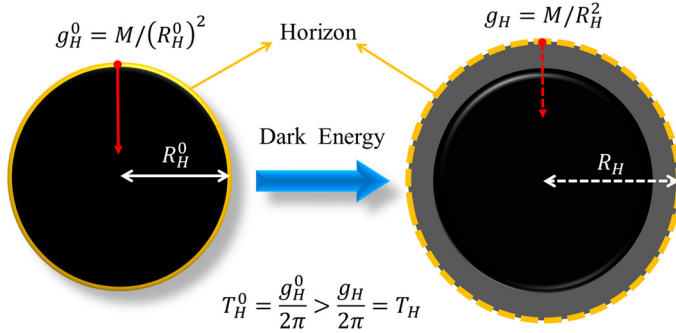
Fig. 2. (Color online.) The dark energy based cooling mechanism (DECM) for the black hole. The black circular areas represent the black hole of mass M, R_H and g_H (R_H^0 and g_H^0) are the radius and surface gravity of the black hole with (without) dark energy, respectively. T_H (T_H^0) is the Hawking temperature with (without) dark energy. The repulsion effect provided by dark energy increases the radius of the black hole, so that the surface gravity of the black hole decreases. In the black hole thermodynamics, the Hawking temperature is linearly dependent on the black hole surface gravity, and is thus decreased by dark energy.

the radiation power is. Intuitively, when a black hole is completely radiated, the time for this process will be longer while the total energy is a constant. Next, we will focus on the influence of the dark energy on the life time of the black hole.

4.2. Life time of the black hole

It follows from the Stefan–Boltzmann power law that the radiation power of the black hole with temperature T_H is

$$P = A_H \sigma T_H^4. \tag{4.8}$$

Here, $A_H = 4\pi R_H^2$ is the area of the horizon, and $\sigma = \pi^2/60$ is the Stefan constant. Using the energy conservation law for the black hole and its radiation, one has

$$\frac{dM}{dt} + P = 0. \tag{4.9}$$

With the help of Eqs. (4.3) and (4.8), the above equation is explicitly expressed as

$$-\frac{dM}{dt} = 4\pi \left(2M + 4M^{2\xi+1} f\right)^2 \left(\frac{\pi^2}{60}\right) \left\{\frac{1}{8\pi M}\left[1 - 4(\xi+1) M^{2\xi} f\right]\right\}^4. \tag{4.10}$$

Eq. (4.10) is simplified as

$$-\frac{dM}{dt} = \frac{1}{15360\pi M^2}\left[1 - (16\xi + 12) M^{2\xi} f\right], \tag{4.11}$$

by keeping the first order of f. Thus, the time for the black hole evaporating from mass M to $M(t)$ is

$$t = -\int\limits_M^{M(t)} 15360\pi M^2 \left[1 + (16\xi + 12) M^{2\xi} f\right] dM. \tag{4.12}$$
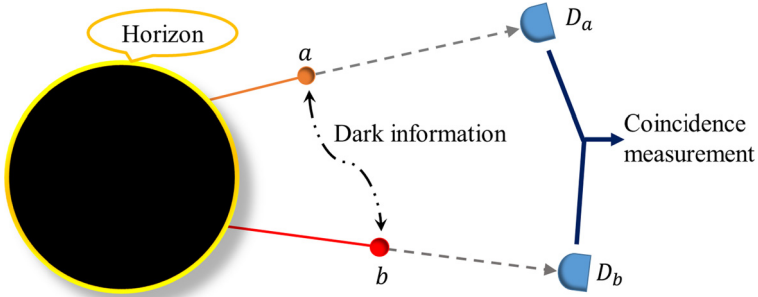
Fig. 3. (Color online.) Probing the dark information of the black hole radiation. Two radiated particles a and b escape from the black hole horizon. The energy distribution of these two particles are not independent of each other due to the non-thermal radiation spectrum, implying that there exist correlation between them. Because the particles may be radiated out from any position of the horizon, and the directions of their momentum are random, it is almost impossible to detect them at the same location. Suppose we have two detectors D_a and D_b to detect these two particles separately in the space. If one only focus on D_a (D_b), the result of its detection is an average over the distribution of b (a), thus the correlation between a and b is hidden and can not be observed. To show this correlation information, the coincidence measurement should be taken among a and b through the two detectors. Therefore, we can conclude that this correlation can not be probed through local measurement. Namely, the information is dark.

When $M(t) = 0$, the black hole evaporates all its energy, and the evaporation time in this case is just the life time of the black hole, namely

$$t = t_0 \left[1 + \frac{3(16\xi + 12)}{2\xi + 3} \left(\frac{2\Lambda}{3} \right)^{\xi} M^{2\xi} \right]. \tag{4.13}$$

Here, $t_0 = 5120\pi M^3$ is the life time for the Schwarzschild black hole in the absence of the dark energy. Equation (4.13) tells us that the dark energy makes the black hole's life time longer. This can be seen as a reflection of the cooling effect that the dark energy on the Hawking radiation. When the radiation temperature drops, i.e., $T_H \downarrow$, the power of the black hole radiation process decreases, i.e., $P \downarrow$, meaning that the energy released per unit time becomes less. Therefore, it takes more time for the black hole to release all its energy, namely $t \uparrow$. In the cosmological constant model of the dark energy, we have Eq. (4.13) been reduced to

$$t = 5120\pi M^3 \left(1 + \frac{56}{5} M^2 \Lambda \right). \tag{4.14}$$

It is found that the dark energy behaves like a "refrigerator" placed around the black hole, which will make the radiation of the black hole colder and slow down the Hawing radiation process. Consequently, the black hole's life time is prolonged by the dark energy.

5. Dark information added by dark energy

As the black hole evaporates so that it becomes smaller and smaller, the non-thermal effect of its radiation increases gradually. The non-thermal radiation has been proved to be the origin of the information correlation [16] between the emissions that being radiated out from the black hole's horizon. In this section, we will study the influences of the dark energy on this correlation.

As shown in Fig. 3., this information correlation can not be observed locally even though the Hawking radiation can be measured experimentally. In this sense the information stored in the

correlation is named by dark information. When the dark information is taken into account, the sum of the information of the radiation field and the remaining black hole gives the total information of the black hole system, and thus is conserved [15,16,50] in the Hawking radiation process. Therefore, the dark information caused by the non-canonical statistic behavior of the Hawking radiation will results in a possible solution to the black hole information paradox. Besides, the dark information of Hawking depends on the radiation spectrum of the black hole, and more accurately, it depends on the specific form of the non-thermal part of the radiation spectrum. From the distribution probability given in Eq. (3.6) we can clearly see that the existence of the dark energy changes the radiation spectrum of the black hole, and it may also affect the dark information of the radiation. In this section, we consider the influence of the dark energy on the dark information, and show that the dark energy will increase the dark information of black hole radiation.

For the two non-independent events a and b, they are correlated with each other, or in other words, there exists a information correlation between them. From the point of view of the information theory, this correlation can be described quantitatively with the mutual information [54]

$$I(a, b) = \sum_{a,b} p_{a,b} \ln\left(\frac{p_{a,b}}{p_a p_b}\right), \tag{5.1}$$

where p_i is the probability for event i $(i = a, b)$, and $p_{a,b}$ is the joint probability of these two events. Obviously, when a and b are independent with each other, we have $p_{a,b} = p_a p_b$. Thus, the mutual information vanishes, $I(a, b) = 0$. Because the original radiation spectrum of the black hole discovered by Hawking is perfectly thermal, one can easily check that the correlation among radiations is zero. This implies that there is no dark information in the thermal black hole radiation.

To find the correlation among the non-thermal black hole radiations, we choose the two events as the radiation process of the two particles a, b with energy ω_a and ω_b, respectively. In this case, the probability for each particle is given by Eq. (3.6) as $p_i = p(\omega_i, M)$ $(i = a, b)$, and the joint probability $p_{a,b} = p(\omega_a + \omega_b, M)$. Substituting Eq. (3.6) into Eq. (5.1), we obtain the dark information as (for detailed calculation, please see Appendix B)

$$I(a, b) = 8\pi \left[1 + 4(\xi + 1)(2\xi + 1) M^{2\xi} f\right] E_a E_b, \tag{5.2}$$

which can be further simplified as

$$I(a, b) = 8\pi (1 + \mu) E_a E_b, \tag{5.3}$$

where

$$\mu = 4(\xi + 1)(2\xi + 1) M^{2\xi} \left(\frac{2\Lambda}{3}\right)^{\xi} \tag{5.4}$$

is a positive modification factor, and $E_i = \langle \omega_i \rangle = \sum \omega_i p_i$ is the internal energy of ω_i. Since the dark information without dark energy is shown to be $I_0(a, b) = 8\pi E_a E_b$ [16], we can conclude that the dark energy will increase the dark information of the Hawking radiation. It can be seen from Eq. (5.2) that when the cosmological constant $\Lambda \to 0$, the modification by the dark energy on the dark information vanishes, i.e., $\mu \to 0$, then the result is naturally back to the dark information I_0 in the case without dark energy. We can further use the difference between I and I_0 to measure the amount of the dark information added by dark energy, which reads

$$\Delta I_\Lambda (a, b) \equiv I - I_0 = 32\pi \, (\xi + 1) \, (2\xi + 1) \, M^{2\xi} \left(\frac{2\Lambda}{3}\right)^\xi E_a E_b. \tag{5.5}$$

In the cosmological constant model of dark energy, $\xi = 1$, we have

$$I(a, b) = 8\pi \left(1 + 16M^2\Lambda\right) E_a E_b, \tag{5.6}$$

so that

$$\Delta I_\Lambda (a, b) = 128\pi M^2 \Lambda E_a E_b. \tag{5.7}$$

Equation (5.7) tells us that the dark information of the black hole radiation added by the dark energy increases as the mass of the black hole increases. When we pay attention to the Hawking radiation process, we can infer that with the evaporation of black hole, the black hole mass is decreasing, i.e., $M \downarrow$. As a result, the dark information added by dark energy is also decreasing, i.e., $\Delta I_\Lambda (a, b) \downarrow$, for the case with E_a and E_b remain unchanged.

The results we obtained in this section show that the dark energy not only has a gravitational effect, but also has an influence on the information.

6. Remarks and conclusion

In this paper, we revealed various influences of the dark energy on the black hole and its radiation. Firstly, a general canonical typicality based approach is developed to derive the non-thermal black hole radiation spectrum without referring the dynamics of the particle tunneling. This statistical mechanical approach only need to know the horizon radius as the specific expression of the three hairs of the black hole (mass, charge, angular momentum). With the Schwarzschild black hole as an illustration, we calculated the black hole radiation spectrum with the existence of the dark energy. The result is confirmed with the "standard" approach—the quantum tunneling based approach. From the corrected radiation spectrum, the dark energy is showed to make the black hole radiation colder as given by Eq. (4.6), and the lifetime of black hole longer in Eq. (4.13). Moreover, according to the black hole thermodynamics, we put forward the dark energy based cooling mechanism (DECM) to explain the decreasing of the Hawking temperature physically. When the non-thermal effect of the black hole radiation is taken into account, we found that the dark energy can increase the dark information of the black hole radiation, as shown in Eq. (5.7). This dark information is introduced to describe the locally unobservable correlation among the black hole radiations.

Our investigation in this paper can be extended to other types of black holes to find their radiation spectra as well as the influences of the dark energy on their radiation. To this end, the follows two steps are needed: (i) obtain the metric of the black hole with the existence of dark energy, and then use it to derive the corrected horizon radius; (ii) use the canonical typicality based approach to get the black hole radiation spectrum. Once the radiation spectrum is obtained, all the thermodynamic properties of the black hole radiation field can be obtained from it. This study shows that the dark energy has an influence on information (for black hole radiation, the dark information increases), not just the mechanical effect of accelerating the expansion of the universe.

Note added   After we finished this study and the paper was completed, we note that the Hawing temperature in the case with dark energy has also been discussed in Ref. [55], where the authors treated a charged black hole as a heat engine and investigated the effect that the dark energy on

the heat engine's performance. The result they obtained seems consistent with the dark energy based cooling mechanism we presented.

Acknowledgements

Appendix A. Radiation spectrum of black hole from canonical typicality

The universe U, which contains the system of interest B (e.g., black hole) and the environment O, is assumed to be in an arbitrary pure state

$$|\Psi\rangle = \sum_b \sum_o \frac{C(b,o)}{\sqrt{\Omega_U}} |b\rangle \otimes |o\rangle, \tag{A.1}$$

where $\Omega_U = \Omega_U(E_U)$ is the total number of microstates for the universe with energy E_U, and $C(b,o)$ is the coefficient of state $|b\rangle \otimes |o\rangle$. And $|b\rangle$ and $|o\rangle$ are the eigen-states of B and O, respectively. The orthogonal conditions are $\langle b_i|b_j\rangle = \delta_{ij}$, and $\langle o_k|o_l\rangle = \delta_{kl}$. Using the normalization condition of state $|\Psi\rangle$, we have

$$\frac{\sum_b \sum_o |C(b,o)|^2}{\Omega_U} = 1, \tag{A.2}$$

which means the average value of $|C(b,o)|^2$

$$\left\langle |C(b,o)|^2 \right\rangle = 1. \tag{A.3}$$

Let $|\Psi_b\rangle = \sum_o C(b,o)|o\rangle$. Then the universe state can be re-expressed as

$$|\Psi\rangle = \frac{\sum_b |b\rangle |\Psi_b\rangle}{\sqrt{\Omega_U}}. \tag{A.4}$$

As a result, by tracing over the environment O, the reduced density matrix of B is obtained as

$$\rho_B = \text{Tr}_O(|\Psi\rangle\langle\Psi|) = \frac{1}{\Omega_U} \sum_{b_i,b_j} \left\langle \Psi_{b_i}|\Psi_{b_j} \right\rangle |b_i\rangle\langle b_j|, \tag{A.5}$$

where $\left\langle \Psi_{b_i}|\Psi_{b_j} \right\rangle = \delta_{ij} \langle\Psi_b|\Psi_b\rangle$ [22]. For the environment O supported in a high-dimension Hilbert space, according to central limit theorem, we have

$$\langle\Psi_b|\Psi_b\rangle = \sum_o |C(b,o)|^2 = \Omega_O(E_U - E_b), \tag{A.6}$$

where $\Omega_O(E)$ is the number of O's micro-states with energy $E = E_U - E_b$. The reduced density matrix of B is thus simplified as

$$\rho_B = \sum_b \frac{\Omega_O(E_U - E_b)}{\Omega_U} |b\rangle\langle b|. \tag{A.7}$$

For B is a macroscopic object, the eigen-energy of its macrostate $|b\rangle$ can be approximated as $E_b \approx E$, with the fluctuation being about $\Delta E_b \sim E/N$, where N is the micro-degrees of freedom of B. Obviously, the energy fluctuation vanishes in the thermodynamic limit, i.e., $N \to \infty$, thus we have

$$\rho_{\mathrm{B}} = \frac{\sum_b |b\rangle \langle b|}{\Omega_{\mathrm{B}}(E)}, \tag{A.8}$$

with

$$\Omega_{\mathrm{B}}(E) \equiv \frac{\Omega_{\mathrm{U}}}{\Omega_{\mathrm{O}}(E_{\mathrm{U}} - E)}. \tag{A.9}$$

This means that B obeys the micro-canonical distribution. When look at B's subsystem R, the rest of B is denoted as $B' = B - R$. The reduced density matrix of B can be rewritten as

$$\rho_{\mathrm{B}} = \frac{\sum_{r,b'} |r, b'\rangle \langle r, b'|}{\Omega_{\mathrm{B}}(E)}, \tag{A.10}$$

where $|r\rangle$ and $|b'\rangle$ are the eigenstates of R and B' with eigen-energies E_r and $E_{b'}$, respectively. $E_r + E_{b'} = E$ is the constraint condition given by energy conservation. The reduced density matrix of R

$$\rho_{\mathrm{R}} = \mathrm{Tr}_{\mathrm{B}'}(\rho_{\mathrm{B}}) = \sum_r \frac{\Omega_{\mathrm{B}'}(E - E_r)}{\Omega_{\mathrm{B}}(E)} |r\rangle \langle r|, \tag{A.11}$$

is obtained by tracing over B'. Here, $\Omega_{\mathrm{B}'}(E - E_r)$ is the number of micro-states of the system B' with energy $E - E_r$. Making use of the Boltzmann entropy

$$S_{\mathrm{B}}(E) = \ln[\Omega_{\mathrm{B}}(E)], \tag{A.12}$$

and

$$S_{\mathrm{B}'}(E - E_r) = \ln[\Omega_{\mathrm{B}'}(E - E_r)], \tag{A.13}$$

Eq. (A.11) can be further written as

$$\rho_{\mathrm{R}} = \mathrm{Tr}_{\mathrm{B}'}(\rho_{\mathrm{B}}) = \sum_r \mathrm{e}^{-\Delta S_{\mathrm{BB}'}(E_r, E)} |r\rangle \langle r|, \tag{A.14}$$

where $\Delta S_{\mathrm{BB}'}(E_r, E) \equiv S_{\mathrm{B}}(E) - S_{\mathrm{B}'}(E - E_r)$ is the difference in entropy between B and B'. We can clearly see from Eq. (A.14) that only when $\Delta S_{\mathrm{BB}'}$ is linearly dependent on E_r, the spectrum of R is perfectly thermal. What should be mentioned here is that we do not expand $\Delta S_{\mathrm{BB}'}$ only up to the first order of E_r, as done in the most studies in the thermodynamic limit.

Now we apply the above result to black holes. For an arbitrary black hole, there are three macro parameters to describes its geometry and number of microstate. These parameters, known as hairs, are mass M, charge Q, and angular momentum J. Considering the conservation laws for hairs, we can generalize Eq. (A.11) as

$$\rho_{\mathrm{R}} = \sum_{\omega, q, j} \frac{\Omega_{\mathrm{B}'}(M - \omega, Q - q, J - j)}{\Omega_{\mathrm{B}}(M, Q, J)} |\omega, q, j\rangle \langle \omega, q, j|, \tag{A.15}$$

where $|\omega, q, j\rangle$ is the eigenstate of R with mass ω, charge q, and angular momentum j. Then, by expressing the number of microstate with entropy, and making use of the Bekenstein–Hawking (B–H) entropy for B and B'

$$S_B (M, Q, J) = S_{BH} (M, Q, J) = \frac{A_H (M, Q, J)}{4} = \pi R_H^2 (M, Q, J),$$ (A.16)

and

$$\begin{aligned} S_B (M - \omega, Q - q, J - j) &= S_{BH} (M - \omega, Q - q, J - j) \\ &= \pi R_H^2 (M - \omega, Q - q, J - j), \end{aligned}$$

we have

$$\rho_R = \sum_{\omega, q, j} \exp\left[\pi R_H^2 (M - \omega, Q - q, J - j) - \pi R_H^2 (M, Q, J) \right] |\omega, q, j\rangle \langle \omega, q, j|.$$ (A.17)

Here, $A_H (M, Q, J)$ and $R_H (M, Q, J)$ is the area and radius of the horizon, respectively. Therefore, we obtain the distribution probability of $|\omega, q, j\rangle$

$$p (\omega, q, j, M, Q, J) = e^{-\pi \left[R_H^2 (M, Q, J) - R_H^2 (M - \omega, Q - q, J - j) \right]}.$$ (A.18)

Appendix B. Dark information of black hole radiation

From Eq. (3.6), one has the distribution possibility for particles a and b as

$$p_a = p (\omega_a, M) = \exp\left(-\beta_H \omega_a + \chi \omega_a^2 \right),$$ (B.1)

$$p_b = p (\omega_b, M) = \exp(-\beta_H \omega_b + \chi \omega_b^2),$$ (B.2)

and the joint possibility

$$p_{a,b} = p (\omega_a + \omega_b, M) = \exp\left[-\beta_H (\omega_a + \omega_b) + \chi (\omega_a + \omega_b)^2 \right],$$ (B.3)

where the inverse radiation temperature

$$\beta_H = 8\pi M \left[1 + 4 (\xi + 1) M^{2\xi} f \right],$$ (B.4)

and the second order coefficient

$$\chi = 4\pi \left[1 + 4 (\xi + 1) (2\xi + 1) M^{2\xi} f \right].$$ (B.5)

Following from Eqs. (B.1), (B.2), and (B.3), we find

$$\frac{p_{a,b}}{p_a p_b} = e^{2\chi \omega_a \omega_b} \neq 0,$$ (B.6)

which implies that these two radiated particles have correlation. Substituting the above equation into Eq. (5.1) and then the dark information is obtained as

$$I (a, b) = 2\chi \sum_{a,b} p_{a,b} \omega_a \omega_b.$$ (B.7)

Note that the joint possibility satisfies $p_{a,b} = p (\omega_a, M) p (\omega_b, M - \omega_a)$, thus, with the replacement $M - \omega_a \to M'$,

$$I (a, b) = 2\chi \left[\sum_{\omega_a=0}^{\omega_a=M} p (\omega_a, M) \omega_a \right] \left[\sum_{\omega_b=0}^{\omega_b=M'} p (\omega_b, M') \omega_b \right]$$ (B.8)

Here,

$$\left[\sum_{\omega_a=0}^{\omega_a=M} p\left(\omega_a, M\right) \omega_a \right] = \langle \omega_a \rangle \equiv E_a \tag{B.9}$$

and

$$\left[\sum_{\omega_b=0}^{\omega_b=M'} p\left(\omega_b, M'\right) \omega_b \right] = \langle \omega_b \rangle \equiv E_b \tag{B.10}$$

are the internal energy of particles a and b, respectively. Finally, we obtain the dark information of black hole radiation with the correction of dark energy as

$$I\left(a, b\right) = 2\chi E_a E_b = 8\pi \left[1 + 4\left(\xi + 1\right)\left(2\xi + 1\right) M^{2\xi} f \right] E_a E_b, \tag{B.11}$$

this is what we illustrated in Eq. (5.2).

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