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Quantum noise theory for quantum transport through nanostructures

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Abstract. We have developed a quantum noise approach for studying quantum transport through nanostructures. The nanostructures, such as quantum dots, are regarded as artificial atoms, subject to quasi-equilibrium fermionic reservoirs of electrons in biased leads. Noise operators characterizing the quantum fluctuation in the reservoirs are related to the damping and fluctuation of the artificial atoms through the quantum Langevin equation. The average current and current noise are derived in terms of the reservoir noise correlations. In the white-noise limit, we show that the current and current noise can be calculated exactly by the quantum noise approach, even in the presence of interactions such as Coulomb blockade. As a typical application, the average current and current noise through a single quantum dot are studied.

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1. Introduction

Quantum transport through nanostructures is of importance in nanoscience and nanotechnology. Many electronic devices based on nanostructures, such as single-electron transistors, have been studied in the past few decades for their potential in various applications. Recently, in an effort to achieve coherent control of single electrons or electron spins, quantum transport methods have been used to detect the quantized motion of electrons in nanostructures [1, 2].

To understand and analyze the quantum transport phenomena, various approaches have been developed. For the general theory, the Landauer–Büttiker formula has established the basic relationship between scattering amplitudes and currents through nanostructures [3, 4]. The non-equilibrium Green’s function (NEGF) method provides a perturbation theoretical scheme for dealing with the many-body interaction effects in quantum transport [5, 6]. In recent years, approaches based on notions in quantum optics have been developed for studying time-dependent quantum transport processes in solid-state structures [7]–[11].

In addition to the average current, current noises also contain useful information about the quantum dynamics in nanostructures [12]–[14]. Current noise was first analyzed semi-classically based on the rate equations [15]–[18], which gave the basic physical picture and predicted the non-trivial phenomena due to the presence of nanostructures. Full quantum mechanical theories of current noise have been developed in recent years [19]–[27]. Most of these theories use the (generalized) master equations for the density matrix in the Schrödinger picture. Two major approximations are always applied either independently or simultaneously, depending on the specific problem concerned: (i) the tunneling between the leads and

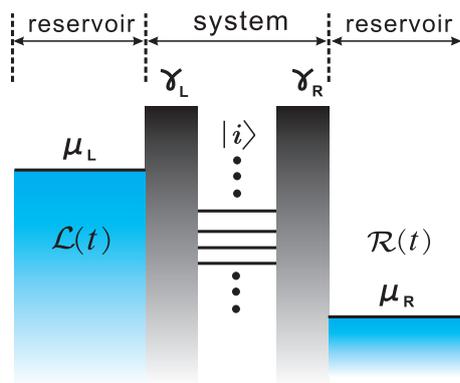


Figure 1. Schematic illustration of transport through a quantum dot. The whole system is divided into three parts: the central system and the left and right reservoirs. The central system is characterized by eigenstates $|i\rangle$ with discrete energies. Quantum noise operators $\mathcal{L}(t)$ and $\mathcal{R}(t)$ are introduced to describe the reservoirs.

nanostructures are treated as perturbations, which is valid in the weak tunneling regime, and (ii) the Markovian approximation for the noise correlation of the leads, which is justified in the large bias situation. In this paper, instead of the rate equations or the master equations, we will develop a quantum noise approach based on the quantum Langevin equation to the quantum transport problem. It will be shown that using the quantum Langevin equations, the transport problem under large bias conditions (Markovian noise) can be solved *exactly* without any further approximations.

Essentially, we recognize that a general quantum transport problem can be regarded as a system-plus-reservoir problem. In this sense, the total system is divided into several subsystems (see figure 1). The *central system* (system for short) is a nanostructure, such as a quantum dot or coupled quantum dots. This subsystem contains several discrete electronic energy levels, resembling an artificial atom. The electrons in the leads, which have a continuous energy spectrum and are kept in quasi-equilibrium, constitute the fermionic *reservoirs*. The electrons in the reservoirs can be treated as free quasi-particles with the screened Coulomb interaction taken into account as a renormalization of electron effective mass. The central system and the reservoirs are coupled together to each other through hopping across the barriers. With this observation, it is natural to treat the quantum transport problem in the framework of the quantum open system method, the quantum Langevin equation. This method is a standard approach in quantum optics for studying cavity photon decay and atom damping, and has also been used for analyzing the noise behaviors in various problems, such as the conductance fluctuations in mesoscopic systems [28].

As compared with the application in quantum optics, the quantum Langevin approach in the quantum transport problem has two features to be pointed out: (i) the reservoirs consist of electrons, which are fermions, while the baths in quantum optics are bosonic, and (ii) when finite biases are applied between different leads, the electronic reservoirs in different leads are in quasi-equilibrium with different chemical potentials but do not stay in equilibrium with each other. Our investigation in this paper will clarify these features. As illustrative applications of our approach, the resonant transport through a single quantum dot is investigated for both the single-level case and the Coulomb blockade case.

The quantum Langevin approach is a natural formalism for studying the noise spectroscopy of quantum dynamics in nanostructures [14], which is particularly interesting for small quantum systems where the signals are often much weaker than the shot noises. When the coupling between the leads and the nanostructures can be described in the Markovian approximation, which is justified in large bias cases, the quantum Langevin approach provides an exact treatment of the interaction within the nanostructure. Furthermore, the quantum Langevin equation establishes a fundamental relationship and analogy between photon emission and electron tunneling processes, providing better understanding of quantum transport phenomena with notions and methods from quantum optics.

This paper is organized as follows. In section 2, we introduce the basic concepts and the general formalism of the quantum noise approach to treat the quantum transport problem. In sections 3 and 4, we apply the quantum noise approach to transport through a single quantum dot containing a single level and double energy levels, respectively. In section 5, we show the relations between our approach and other quantum transport theories. The conclusion and an outlook for our approach are presented in section 6.

2. General formalism

2.1. Quantum Langevin equations for quantum transport

In general, the quantum transport problem of nanostructures can be modeled by the following Hamiltonian,

$$H = H_{\text{sys}}(a_i, a_i^\dagger) + H_{\text{lead}} + H_{\text{T}}, \quad (1)$$

where H_{sys} describes the nanostructure, such as a quantum dot, with multiple discrete energy levels. The leads, which play the role of reservoirs, are described by the Hamiltonian H_{lead} . The electron tunneling between the leads and the nanostructure is included in H_{T} . For the two-lead case, the leads Hamiltonian H_{lead} and the tunneling Hamiltonian H_{T} can be written as

$$H_{\text{lead}} = \sum_k \hbar\omega_k^{(\text{L})} b_k^\dagger b_k + \sum_j \hbar\omega_j^{(\text{R})} c_j^\dagger c_j, \quad (2a)$$

$$H_{\text{T}} = i\hbar \sum_{i,k} \xi_{ik} b_k^\dagger a_i + i\hbar \sum_{i,j} \zeta_{ij} c_j^\dagger a_i + \text{h.c.}, \quad (2b)$$

where b_k and c_j are the annihilation operators of the left and right leads with continuous spectra $\hbar\omega_k^{(\text{L})}$ and $\hbar\omega_j^{(\text{R})}$, respectively. The tunneling is characterized by the coefficients ξ_{ik} and ζ_{ij} . Note that we have neglected the interaction in the leads as a common approximation for a Fermi sea with the Coulomb interaction effectively taken into the renormalized quasi-particle spectra.

Now, we consider the Heisenberg equations of motion of the system and reservoir operators. For simplicity, we show equations of motion for the simplest single-level case, i.e. $a_i(t) = a(t)$, $\xi_{ik} = \xi_k$, $\zeta_{ij} = \zeta_j$ and $H_{\text{sys}} = \hbar\omega_0 a^\dagger a$. The multi-level case will be discussed later in this paper. Straightforward calculation gives

$$\dot{a}(t) = -i\omega_0 a - \sum_k \xi_k b_k - \sum_j \zeta_j c_j, \quad (3a)$$

$$\dot{b}_k(t) = -i\omega_k^{(L)} b_k + \xi_k a, \quad (3b)$$

$$\dot{c}_j(t) = -i\omega_j^{(R)} c_j + \zeta_j a. \quad (3c)$$

In the following, we try to eliminate the lead variables from the equation of motion (3a) of the system operator. To this end, the formal solution for $b_k(t)$ is written as

$$b_k(t) = e^{-i\omega_k t} b_k(0) + \xi_k \int_0^t dt' [e^{-i\omega_k(t-t')} a(t')]. \quad (4)$$

With this formal solution, the following relation is obtained,

$$\sum_k \xi_k b_k(t) = -\mathcal{L}_{\text{in}}(t) + \frac{\gamma_L}{2} a(t), \quad (5)$$

where the *input* noise operator $\mathcal{L}_{\text{in}}(t)$ due to the left lead is defined as

$$\mathcal{L}_{\text{in}}(t) = -\sum_k \xi_k e^{-i\omega_k^{(L)} t} b_k(0). \quad (6)$$

The damping term in equation (5) arises from the Markovian approximation [29] under the continuous limit,

$$\sum_k \xi_k^2 e^{-i\omega_k^{(L)}(t-t')} = \int d\omega_k [D(\omega_k) \xi^2(\omega_k) e^{-i\omega_k^{(L)}(t-t')}] = \gamma_L \delta(t-t'), \quad (7)$$

where $D(\omega_k)$ is the density of the states of the lead. Here, we have assumed that $D(\omega_k) \xi^2(\omega_k)$ is flat around the frequency ω_0 and

$$\gamma_L = 2\pi D(\omega_0) \xi^2(\omega_0) \quad (8)$$

is widely used as the tunneling rate in nanostructure quantum transport problems.

Similarly, for the right lead,

$$\sum_j \zeta_j c_j(t) = -\mathcal{R}_{\text{in}}(t) + \frac{\gamma_R}{2} a(t), \quad (9)$$

where the noise operator of the right lead $\mathcal{R}(t)$ is defined as

$$\mathcal{R}_{\text{in}}(t) = -\sum_j \zeta_j e^{-i\omega_j^{(R)} t} c_j(0) \quad (10)$$

and γ_R is the tunneling rate to the right lead. Using equations (5) and (9), we obtain the quantum Langevin equation for the system operator $a(t)$,

$$\dot{a}(t) = -i\omega_0 a(t) - \frac{\gamma_L + \gamma_R}{2} a(t) + \mathcal{L}_{\text{in}}(t) + \mathcal{R}_{\text{in}}(t). \quad (11)$$

The noise operators in equation (11) can be regarded as the quantum counterpart of the stochastic force in the classical Langevin equations. Similar to the cases in quantum optics, the two electronic leads, which play the role of fermionic reservoirs, induce the damping and the fluctuations through these noise operators.

2.2. Projection operator formalism for interacting systems

For an interacting system, the complexity of the quantum Langevin equations arises from the evolution induced by the system Hamiltonian H_{sys} . To deal with such complexity, we introduce the projection operators of the interacting system in this subsection.

Although there may be interaction between the electrons in the system Hamiltonian H_{sys} , the artificial atom can always be considered as consisting of a few discrete many-body energy levels. In other words, we can diagonalize the system Hamiltonian H_{sys} as

$$H_{\text{sys}}(a_i, a_i^\dagger) = \sum_{i=1}^N \hbar\omega_i |i\rangle\langle i| = \sum_{i=1}^N \hbar\omega_i \sigma_{ii}, \quad (12)$$

where $\sigma_{ij} = |i\rangle\langle j|$ is the projection operator from $|j\rangle$ to $|i\rangle$, with $|i\rangle$ being the eigenstate of H_{sys} of energy $\hbar\omega_i$.

In the general cases, determined by the system Hamiltonian H_{sys} , the fermionic operators a_i and a_i^\dagger can be written in terms of the projection operators as

$$a_i = \sum_{k,l} T_{kl}^{(i)} \sigma_{kl} \quad \text{and} \quad a_i^\dagger = \sum_{k,l} \tilde{T}_{kl}^{(i)} \sigma_{kl}, \quad (13)$$

where $T^{(i)}$ and $\tilde{T}^{(i)}$ are $N \times N$ matrices associated with the fermionic operators a_i and a_i^\dagger , and $\tilde{T}^{(i)} \equiv (T^{(i)})^\dagger$.

The commutative relation between the projection operators σ_{ij} can be calculated with $\sigma_{ij}\sigma_{kl} = \sigma_{il}\delta_{jk}$. In the following calculations, we also need to identify the commutative relation between the projection operators σ_{ij} and the reservoir operators, i.e. $b_k(b_k^\dagger)$ and $c_j(c_j^\dagger)$. Note that the eigenstate $|i\rangle$ is also an eigenstate of the electron number operator in the quantum dot $\hat{N} = \sum_i a_i^\dagger a_i$, i.e. $\hat{N}|i\rangle = N_i|i\rangle$. Thus, the projection operator σ_{ij} corresponds to a definite electron number change $N_i - N_j$, which is either odd or even. Consequently, σ_{ij} and $b_k(b_k^\dagger)$ have the following (anti-)commutative relation,

$$[\sigma_{ij}, b_k]_{g_{ij}} \equiv \sigma_{ij}b_k + g_{ij}b_k\sigma_{ij} = 0, \quad (14)$$

where the factor

$$g_{ij} = \begin{cases} 1, & \text{for } N_i - N_j = \text{odd}, \\ -1, & \text{for } N_i - N_j = \text{even}. \end{cases} \quad (15)$$

The Heisenberg equation for the σ_{ij} is

$$\begin{aligned} \dot{\sigma}_{ij}(t) &= \frac{i}{\hbar}[H_{\text{sys}}, \sigma_{ij}] + \frac{i}{\hbar}[H_{\text{T}}, \sigma_{ij}] = -i\Delta_{ij}\sigma_{ij}(t) \\ &\quad - \sum_{\alpha,k,m,n} [(\xi_{\alpha k} T_{mn}^{(\alpha)} b_k^\dagger \sigma_{mn} - \text{h.c.}), \sigma_{ij}] - \sum_{\alpha,j,m,n} [(\zeta_{\alpha j} T_{mn}^{(\alpha)} c_j^\dagger \sigma_{mn} - \text{h.c.}), \sigma_{ij}]. \end{aligned} \quad (16)$$

With the help of the definition of noise operators and the first Markovian approximation, we obtain the quantum Langevin equation for the projection operator σ_{ij} ,

$$\begin{aligned} \dot{\sigma}_{ij}(t) &= -i\Delta_{ij}\sigma_{ij}(t) - \frac{\gamma_{\text{L}}}{2} \sum_{m,m'} D_{mm'}^{ij} \sigma_{mm'}(t) + (\gamma_{\text{L}} \rightarrow \gamma_{\text{R}}) + \sum_{m,m'} C_{mm'}^{ij} \mathcal{L}_{\text{in}}^\dagger(t) \sigma_{mm'}(t) + (\mathcal{L}_{\text{in}}^\dagger \rightarrow \mathcal{R}_{\text{in}}^\dagger) \\ &\quad + \sum_{m,m'} \tilde{C}_{mm'}^{ij} \mathcal{L}_{\text{in}}(t) \sigma_{mm'}(t) + (\mathcal{L}_{\text{in}} \rightarrow \mathcal{R}_{\text{in}}), \end{aligned} \quad (17)$$

where $\Delta_{ij} = \omega_j - \omega_i$, and the coefficients $D_{mm'}^{ij}$, $C_{mm'}^{ij}$ and $\tilde{C}_{mm'}^{ij}$ are defined as follows,

$$D_{mm'}^{ij} = A_{mm'}^{ij} + \tilde{A}_{mm'}^{ij} + B_{mm'}^{ij} + \tilde{B}_{mm'}^{ij}, \quad (18a)$$

$$C_{mm'}^{ij} = \sum_{\alpha} (T_{mi}^{(\alpha)} \delta_{m'j} + g_{ij} T_{jm'}^{(\alpha)} \delta_{mi}), \quad (18b)$$

$$\tilde{C}_{mm'}^{ij} = \sum_{\alpha} (\tilde{T}_{mi}^{(\alpha)} \delta_{m'j} + g_{ij} \tilde{T}_{jm'}^{(\alpha)} \delta_{mi}), \quad (18c)$$

with

$$A_{mm'}^{ij} = \sum_{\alpha, \alpha'} g_{ij} \tilde{T}_{mi}^{(\alpha')} T_{jm'}^{(\alpha)}, \quad (19a)$$

$$\tilde{A}_{mm'}^{ij} = \sum_{\alpha, \alpha'} g_{ij} T_{mi}^{(\alpha')} \tilde{T}_{jm'}^{(\alpha)}, \quad (19b)$$

$$B_{mm'}^{ij} = \delta_{m'j} \sum_{v, \alpha, \alpha'} \tilde{T}_{mv}^{(\alpha')} T_{vi}^{(\alpha)}, \quad (19c)$$

$$\tilde{B}_{mm'}^{ij} = \delta_{m'j} \sum_{v, \alpha, \alpha'} T_{mv}^{(\alpha')} \tilde{T}_{vi}^{(\alpha)}. \quad (19d)$$

In principle, the quantum Langevin equation for the system operators is equivalent to a quantum stochastic equation if we introduce the quantum Wiener process [30], and the properties of their solution can be discussed by defining the quantum stochastic integration [30]. Thus, we point out that the quantum transport problem provides an experimentally accessible proving ground for the quantum stochastic theory. Instead of discussing the mathematical properties of equation (17) further, in this paper we will focus, through concrete models, on how to derive the observable quantities in quantum transport.

2.3. Boundary relation and causality

Besides the *input* noise operators $\mathcal{L}_{\text{in}}(t)$ and $\mathcal{R}_{\text{in}}(t)$, the *output* noise operators [30, 31] can be defined as

$$\mathcal{L}_{\text{out}}(t) = - \sum_k \xi_k e^{-i\omega_k^{(L)}(t-t_f)} b_k(t_f), \quad (20a)$$

$$\mathcal{R}_{\text{out}}(t) = - \sum_j \zeta_j e^{-i\omega_j^{(R)}(t-t_f)} c_j(t_f), \quad (20b)$$

where t_f is a time in the remote future. Similar to equations (5) and (9), the first Markovian approximation gives the following relations,

$$\sum_k \xi_k b_k(t) = -\mathcal{L}_{\text{out}}(t) - \frac{\gamma_L}{2} a(t). \quad (21)$$

According to equations (5) and (21), the ‘boundary relation’ between the noise operators and the system operator is [31]

$$\mathcal{L}_{\text{in}}(t) - \mathcal{L}_{\text{out}}(t) = \gamma_{\text{L}} a(t). \quad (22)$$

Similarly, for the right lead,

$$\mathcal{R}_{\text{in}}(t) - \mathcal{R}_{\text{out}}(t) = \gamma_{\text{R}} a(t). \quad (23)$$

According to the quantum Langevin equation (11), the fermionic system operator $d(t) \in \{a(t), a^\dagger(t)\}$ at time t only depends on the input noise operators at time $t' < t$. As a result, in the Markovian limit, the causality relation reads [30, 32]

$$[\mathcal{L}_{\text{in}}(t'), d(t)]_+ = 0, \quad \text{for } t' > t. \quad (24)$$

For a similar reason, the system operator at t is independent of the output noise operators at time $t' < t$,

$$[\mathcal{L}_{\text{out}}(t'), d(t)]_+ = 0, \quad \text{for } t' < t. \quad (25)$$

According to equations (22)–(25), the anti-commutators between the noise and system operator are converted to those between system operators [30, 32],

$$[\mathcal{L}_{\text{in}}(t'), d(t)]_+ = \gamma_{\text{L}} \theta(t - t') [a(t'), d(t)]_+, \quad (26a)$$

$$[\mathcal{R}_{\text{in}}(t'), d(t)]_+ = \gamma_{\text{R}} \theta(t - t') [a(t'), d(t)]_+, \quad (26b)$$

where the step function $\theta(t)$ is defined as

$$\theta(t) = \begin{cases} 1, & t > 0, \\ \frac{1}{2}, & t = 0, \\ 0, & t < 0. \end{cases} \quad (27)$$

For the multi-level case, this causality relation equation (26) can be generalized to the system projection operators σ_{ij} , i.e.

$$[\mathcal{L}_{\text{in}}(t'), \sigma_{ij}(t)]_{\pm} = \gamma_{\text{L}} \theta(t - t') [a(t'), \sigma_{ij}(t)]_{\pm}, \quad (28a)$$

$$[\mathcal{R}_{\text{in}}(t'), \sigma_{ij}(t)]_{\pm} = \gamma_{\text{R}} \theta(t - t') [a(t'), \sigma_{ij}(t)]_{\pm}. \quad (28b)$$

The choice of the commutative and anti-commutative relations in equation (28) is determined by the parity of the electron number change (see equation (15)).

In the following, to simplify the notation, we will omit the subscript ‘in’ from the input noise operators, unless stated otherwise.

2.4. The current and the current noise

For the quantum transport problem, we are interested in the average current and the current noise spectra. In this subsection, we will give the expressions for such quantities in terms of the noise operators.

We consider the current through the right lead as an example. For simplicity, let us first study the single-level case. The formula for the multi-level case with Coulomb blockade will be

discussed later. The current operator can be defined as the changing rate of the electron number on the right lead, i.e.

$$\hat{I}_R = \frac{d}{dt} \hat{N}_R = \sum_j \zeta_j c_j^\dagger a + \text{h.c.} \quad (29a)$$

$$= \gamma_R a^\dagger(t) a(t) - \mathcal{R}^\dagger(t) a(t) - a^\dagger(t) \mathcal{R}(t). \quad (29b)$$

The second line is obtained by noting the relations in equations (5) and (9). We point out that the current operator can be divided into two parts: (i) the *damping part* $\gamma_R a^\dagger(t) a(t)$, which is proportional to the level occupation and the escaping rate γ_R , and (ii) the *fluctuation part* (the last two terms), which is induced by the noise operators $\mathcal{R}(t)$ and $\mathcal{R}^\dagger(t)$.

For the average current, we take the average of the current operator \hat{I}_R over the thermal states of the leads,

$$\langle \hat{I}_R \rangle = \gamma_R \langle a^\dagger(t) a(t) \rangle - \langle \mathcal{R}^\dagger(t) a(t) \rangle - \langle a^\dagger(t) \mathcal{R}(t) \rangle. \quad (30)$$

And for the current noise, we first calculate the current–current correlation function,

$$g^{(2)}(\tau) = \lim_{t \rightarrow +\infty} \text{Re}[\langle \hat{I}_R(t) \hat{I}_R(t + \tau) \rangle] - \langle \hat{I}_R \rangle^2. \quad (31)$$

At steady state, its Fourier transformation gives the current noise spectrum [33],

$$S(\omega) = 4 \int_0^\infty g^{(2)}(\tau) \cos(\omega\tau) d\tau. \quad (32)$$

To calculate the correlation $\langle \hat{I}_R(t) \hat{I}_R(t + \tau) \rangle$ in $g^{(2)}(\tau)$, by the definition of \hat{I}_R in equation (29b), one needs to calculate the two-time correlations, such as

$$\langle a^\dagger(t) a(t) a^\dagger(t + \tau) a(t + \tau) \rangle, \quad (33a)$$

$$\langle a^\dagger(t) \mathcal{R}(t) a^\dagger(t + \tau) a(t + \tau) \rangle, \quad (33b)$$

$$\langle a^\dagger(t) \mathcal{R}(t) \mathcal{R}^\dagger(t + \tau) a(t + \tau) \rangle. \quad (33c)$$

In section 3, we will show that the fluctuation part in the current operator does not contribute to the average current, so the average current $\langle \hat{I}_R \rangle = \gamma_R \langle a^\dagger(t) a(t) \rangle$ is held. But the fluctuation terms will contribute to the current noise through the correlations in equation (33).

3. Application I: single-level transport

In this section, the general quantum Langevin formula is applied to the resonant transport through a quantum dot. As the first example, we consider a model in which only a single energy level in the quantum dot is relevant. The system Hamiltonian reads

$$H_{\text{sys}} = \hbar\omega_0 a^\dagger a. \quad (34)$$

We consider the large bias condition and assume that the single-particle energy level with energy $\hbar\omega_0$ is well within the bias window, i.e. $\mu_L - \omega_0, \omega_0 - \mu_R \gg \gamma_L, \gamma_R$, with $\mu_{L/R}$ being the chemical potentials of the left/right leads. According to the discussion in section 2.1, the quantum Langevin equation reads

$$\dot{\tilde{a}}(t) = -\frac{\gamma_L + \gamma_R}{2} \tilde{a}(t) + \tilde{\mathcal{L}}(t) + \tilde{\mathcal{R}}(t), \quad (35)$$

where $\tilde{a}(t) = a(t)e^{i\omega_0 t}$, $\tilde{\mathcal{L}}(t) = e^{i\omega_0 t} \mathcal{L}(t)$, and $\tilde{\mathcal{R}}(t) = e^{i\omega_0 t} \mathcal{R}(t)$ are defined in the rotating reference frame to single out the slow-varying dynamics. In the white-noise limit, the correlation between the noise operators can be written as (see [appendix](#))

$$\langle \tilde{\mathcal{L}}^\dagger(t) \tilde{\mathcal{L}}(t') \rangle = \gamma_L \delta(t - t'), \quad (36a)$$

$$\langle \tilde{\mathcal{R}}(t) \tilde{\mathcal{R}}^\dagger(t') \rangle = \gamma_R \delta(t - t'), \quad (36b)$$

$$\langle \tilde{\mathcal{L}}(t) \tilde{\mathcal{L}}^\dagger(t') \rangle = \langle \tilde{\mathcal{R}}^\dagger(t) \tilde{\mathcal{R}}(t') \rangle = 0. \quad (36c)$$

Using these relations, we calculate the average current and the current noise.

3.1. Average current

From equation (35), the system operator $\tilde{a}(t)$ in terms of the noise operators is

$$\tilde{a}(t) = e^{-\Gamma/2t} \tilde{a}(0) + \int_0^t e^{-\Gamma/2(t-t')} \tilde{\mathcal{L}}(t') dt' + \int_0^t e^{-\Gamma/2(t-t')} \tilde{\mathcal{R}}(t') dt', \quad (37)$$

where $\Gamma = \gamma_L + \gamma_R$. Multiplying by the noise operator $\tilde{\mathcal{L}}^\dagger(t)$ both sides of equation (37), we have

$$\begin{aligned} \langle \tilde{\mathcal{L}}^\dagger(t) \tilde{a}(t) \rangle &= e^{-\Gamma/2t} \langle \tilde{\mathcal{L}}^\dagger(t) \tilde{a}(0) \rangle + \int_0^t e^{-\Gamma/2(t-t')} \langle \tilde{\mathcal{L}}^\dagger(t) \tilde{\mathcal{L}}(t') \rangle dt' \\ &= \int_0^t dt' [e^{-\Gamma/2(t-t')} \gamma_L \delta(t - t')] = \frac{\gamma_L}{2}. \end{aligned} \quad (38)$$

Here, we have assumed that at initial time $t = 0$, the system and the reservoir are independent, i.e. $\langle \tilde{\mathcal{L}}^\dagger(t) \tilde{a}(0) \rangle = \langle \tilde{\mathcal{L}}^\dagger(t) \rangle \langle \tilde{a}(0) \rangle = 0$. Similarly, we obtain

$$\langle \tilde{a}^\dagger(t) \tilde{\mathcal{L}}(t) \rangle = \frac{\gamma_L}{2} \quad (39)$$

and

$$\langle \tilde{a}^\dagger(t) \tilde{\mathcal{R}}(t) \rangle = \langle \tilde{\mathcal{R}}^\dagger(t) \tilde{a}(t) \rangle = 0. \quad (40)$$

Thus, according to equation (29b), the fluctuation part of the current operator does not contribute to the average current, and the average current becomes

$$\langle I_R \rangle = \gamma_R \langle \tilde{a}^\dagger \tilde{a} \rangle. \quad (41)$$

To determine the mean occupation number $\langle \tilde{a}^\dagger \tilde{a} \rangle$, we use the equation of motion

$$\begin{aligned} \frac{d}{dt} \tilde{a}^\dagger \tilde{a} &= \dot{\tilde{a}}^\dagger \tilde{a} + \tilde{a}^\dagger \dot{\tilde{a}} \\ &= -\Gamma \tilde{a}^\dagger \tilde{a} + \tilde{a}^\dagger(t) \tilde{\mathcal{L}}(t) + \tilde{\mathcal{L}}^\dagger(t) \tilde{a}(t) + \tilde{a}^\dagger(t) \tilde{\mathcal{R}}(t) + \tilde{\mathcal{R}}^\dagger(t) \tilde{a}(t). \end{aligned} \quad (42)$$

The ensemble average leads to

$$\frac{d}{dt} \langle \tilde{a}^\dagger \tilde{a} \rangle = -\Gamma \langle \tilde{a}^\dagger \tilde{a} \rangle + \gamma_L. \quad (43)$$

Thus, the averaged population in the quantum dot is

$$\langle \tilde{a}^\dagger \tilde{a} \rangle = \frac{\gamma_L}{\gamma_L + \gamma_R} - \frac{\gamma_L}{\gamma_L + \gamma_R} e^{-(\gamma_L + \gamma_R)t}. \quad (44)$$

As a result, the average current at steady state for $t \rightarrow +\infty$ is

$$\langle \hat{I}_R \rangle_{ss} = \frac{\gamma_L \gamma_R}{\gamma_L + \gamma_R}, \quad (45)$$

which is the well-known result for the resonant tunneling transport [6, 8].

3.2. Current noise

To investigate the current noise, we calculate the current–current correlation $\langle \hat{I}_R(t) \hat{I}_R(t + \tau) \rangle$. With the definition of the current operator in equation (29b), the noise contains typically two-time correlations, such as

$$\langle a^\dagger(t) a(t) a^\dagger(t + \tau) a(t + \tau) \rangle \equiv \langle \hat{n}(t) \hat{n}(t + \tau) \rangle \quad (46)$$

and

$$\langle a^\dagger(t) \tilde{\mathcal{R}}(t) \tilde{\mathcal{R}}^\dagger(t + \tau) a(t + \tau) \rangle. \quad (47)$$

We will discuss such correlations one by one.

Noting that the electron number correlation function $\langle \hat{n}(t) \hat{n}(t + \tau) \rangle$ contains only the system operators, we use the quantum regression theorem [35] and equation (43) and obtain

$$\frac{d}{d\tau} \langle \hat{n}(t) \hat{n}(t + \tau) \rangle = -\Gamma \langle \hat{n}(t) \hat{n}(t + \tau) \rangle + \gamma_L \langle \hat{n}(t) \rangle. \quad (48)$$

This equation, together with the initial condition with respect to τ , i.e. for $\tau = 0$, $\langle \hat{n}(t) \hat{n}(t + \tau) \rangle = \langle \hat{n}(t) \hat{n}(t) \rangle = \langle \hat{n}(t) \rangle$, determines the occupation number fluctuation in the quantum dot. The steady state correlation is

$$\lim_{t \rightarrow +\infty} \langle \hat{n}(t) \hat{n}(t + \tau) \rangle = \frac{\gamma_L^2}{(\gamma_L + \gamma_R)^2} + \frac{\gamma_L \gamma_R}{(\gamma_L + \gamma_R)^2} e^{-(\gamma_L + \gamma_R)\tau}. \quad (49)$$

The other terms contain the correlations between the system and noise operators. Taking $\langle \tilde{a}^\dagger(t) \tilde{\mathcal{R}}(t) \tilde{\mathcal{R}}^\dagger(t + \tau) \tilde{a}(t + \tau) \rangle$, for example, according to equation (37), we have

$$\langle \tilde{a}^\dagger(t) \tilde{\mathcal{R}}(t) \tilde{\mathcal{R}}^\dagger(t + \tau) \tilde{a}(t + \tau) \rangle = \int_0^t dt_1 \int_0^{t+\tau} dt_2 e^{-\Gamma/2(t-t_1) - \Gamma/2(t+\tau-t_2)} G(t_1, t, t + \tau, t_2), \quad (50)$$

where the four-time noise correlation is defined as

$$G(t_1, t_2, t_3, t_4) = \langle \tilde{\mathcal{L}}^\dagger(t_1) \tilde{\mathcal{R}}(t_2) \tilde{\mathcal{R}}^\dagger(t_3) \tilde{\mathcal{L}}(t_4) \rangle. \quad (51)$$

According to the independent noise assumption and the white-noise approximation,

$$G(t_1, t_2, t_3, t_4) = \gamma_L \gamma_R \delta(t_1 - t_4) \delta(t_2 - t_3). \quad (52)$$

Thus, we have

$$\begin{aligned} \langle \tilde{a}^\dagger(t) \tilde{\mathcal{R}}(t) \tilde{\mathcal{R}}^\dagger(t + \tau) \tilde{a}(t + \tau) \rangle &= \int_0^t dt_1 \int_0^{t+\tau} dt_2 [e^{-\Gamma/2(2t+\tau-t_1-t_2)} \delta(t_1 - t_2) \delta(\tau)] \\ &= \frac{\gamma_L \gamma_R}{\gamma_L + \gamma_R} e^{-(\gamma_L + \gamma_R)/2\tau} \delta(\tau), \quad \text{for } t \rightarrow +\infty. \end{aligned} \quad (53)$$

Similarly,

$$\langle \tilde{a}^\dagger(t) \tilde{\mathcal{R}}(t) \tilde{a}^\dagger(t + \tau) \tilde{a}(t + \tau) \rangle = \frac{\gamma_L \gamma_R}{\gamma_L + \gamma_R} e^{-(\gamma_L + \gamma_R)\tau}, \quad \text{for } t \rightarrow +\infty. \quad (54)$$

It can be checked that all other terms in the current–current correlation function vanish. Consequently, the current–current correlation function is

$$g^{(2)}(\tau) = -\frac{\gamma_L^2 \gamma_R^2}{(\gamma_L + \gamma_R)^2} e^{-\Gamma\tau} + \frac{\gamma_L \gamma_R}{\gamma_L + \gamma_R} e^{-\Gamma/2\tau} \delta(\tau), \quad (55)$$

and its Fourier transformation gives the current noise spectra,

$$S(\omega) = 2e \langle \hat{I}_R \rangle_{ss} \frac{\gamma_L^2 + \gamma_R^2 + \omega^2}{(\gamma_L + \gamma_R)^2 + \omega^2}. \quad (56)$$

This result agrees with the ones derived from other approaches [26] and shows that the presence of the single-level quantum dot suppresses the zero-frequency current noise to half of the Poisson value $S_P = 2e \langle I_R \rangle_{ss}$ in the case of $\gamma_L = \gamma_R$.

It is worth emphasizing that, as clearly shown in our quantum noise approach, although the fluctuation part (see equation (29b)) of the current operator does not contribute to the average current, it does to the current noise. According to our approach, the current–current correlation originates from three different kinds of sources: (i) the on-site number–number correlation (equation (49)), which always contributes a positive correlation, (ii) the correlation between the fluctuation terms equation (53), which induces a white-noise correlation, and (iii) the correlation between the on-site number and the fluctuation term equation (54), which always provides a negative correlation. This classification of current–current correlation is also valid in the interacting case, as will be discussed below.

4. Application II: Coulomb blockade

4.1. Average current

Now we apply the general theory to the Coulomb blockade case. As an example, we consider that a single orbital level in the quantum dot is relevant (i.e. within the energy range of interest). The system Hamiltonian reads

$$H_{\text{sys}}(a_i, a_i^\dagger) = \hbar\omega_\uparrow a_\uparrow^\dagger a_\uparrow + \hbar\omega_\downarrow a_\downarrow^\dagger a_\downarrow + U a_\uparrow^\dagger a_\uparrow a_\downarrow^\dagger a_\downarrow, \quad (57)$$

where $\hbar\omega_{\uparrow,\downarrow}$ are the single-electron energy for spin-up and spin-down electrons in the quantum dot, and U is the Coulomb interaction strength between two electrons. To illustrate the application of the quantum Langevin approach, we focus on the average current and current noise in the regions near the Coulomb blockade peaks, and consider the large U limit, i.e. $\hbar\omega_\uparrow + U, \hbar\omega_\downarrow + U \gg \mu_L \gg \hbar\omega_\uparrow, \hbar\omega_\downarrow \gg \mu_R$. The higher-order (e.g. cotunneling) processes, which are dominant near the Coulomb blockade valleys at low temperatures, are not included in the example discussed below.

As has been discussed in section 2.2, although there is interaction between the electrons in the system Hamiltonian, H_{sys} is diagonalized as

$$H_{\text{sys}} = \hbar\omega_\uparrow \sigma_{\uparrow\uparrow} + \hbar\omega_\downarrow \sigma_{\downarrow\downarrow} + (\hbar\omega_\uparrow + \hbar\omega_\downarrow + U) \sigma_{dd}, \quad (58)$$

and the projection operators are related to the Fermion operators by

$$a_\uparrow = \sigma_{v\uparrow} - \sigma_{\downarrow d}, \quad (59a)$$

$$a_\downarrow = \sigma_{v\downarrow} + \sigma_{\uparrow d}, \quad (59b)$$

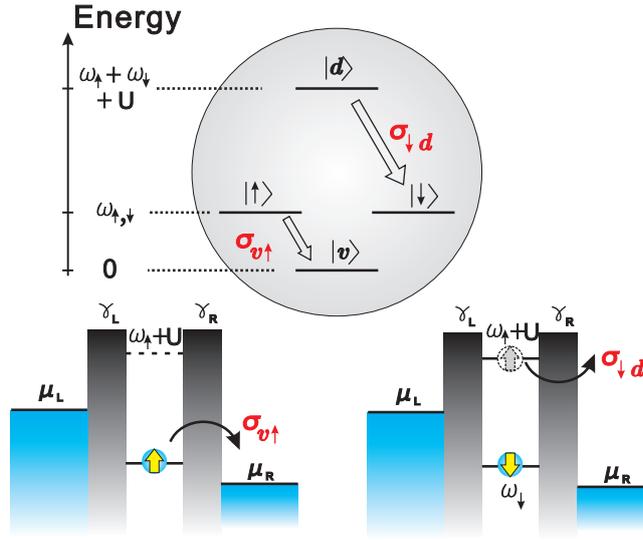


Figure 2. Schematic illustration of Coulomb blockade transport. The quantum dot can be regarded as a four-level artificial atom with $|v\rangle$, $|\uparrow\rangle$, $|\downarrow\rangle$ and $|d\rangle$ representing the vacuum, spin-up, spin-down and doubly occupied states. The lower panel shows the two elementary processes of a spin-up electron tunneling out of the quantum dot depending on whether the spin-down state is occupied or not.

where $\sigma_{ij} = |i\rangle\langle j|$ for $i, j = v, \uparrow, \downarrow$ and d . The subscripts v, \uparrow, \downarrow and d represent the vacuum, spin-up, spin-down and doubly occupied states, respectively (figure 2). Annihilating an electron with definite spin (say spin-up) from the quantum dot consists of two different projection processes depending on whether the spin-down level is occupied or not.

Here, we assume that the quantum dot is coupled to ferromagnetic leads. Thus, the electron with different spin can tunnel on and off the quantum dot with different rates. The quantum Langevin equations of the projection operators σ_{ij} in this Coulomb blockade case follow the general formula in section 2.2. The resultant equations for the diagonal elements are

$$\dot{\sigma}_{vv} = -\frac{\Gamma_{\uparrow} + \Gamma_{\downarrow}}{2}\sigma_{vv} + \frac{\Gamma_{\uparrow}}{2}\sigma_{\uparrow\uparrow} + \frac{\Gamma_{\downarrow}}{2}\sigma_{\downarrow\downarrow} - [\mathcal{F}_{\uparrow}^{\dagger}\sigma_{v\uparrow} + \mathcal{F}_{\downarrow}^{\dagger}\sigma_{v\downarrow} - \mathcal{F}_{\uparrow}\sigma_{\uparrow v} - \mathcal{F}_{\downarrow}\sigma_{\downarrow v}], \quad (60a)$$

$$\dot{\sigma}_{\uparrow\uparrow} = -\frac{\Gamma_{\uparrow} + \Gamma_{\downarrow}}{2}\sigma_{\uparrow\uparrow} + \frac{\Gamma_{\uparrow}}{2}\sigma_{vv} + \frac{\Gamma_{\downarrow}}{2}\sigma_{dd} + [\mathcal{F}_{\uparrow}^{\dagger}\sigma_{v\uparrow} - \mathcal{F}_{\downarrow}^{\dagger}\sigma_{\uparrow d} - \mathcal{F}_{\uparrow}\sigma_{\uparrow v} + \mathcal{F}_{\downarrow}\sigma_{d\uparrow}], \quad (60b)$$

$$\dot{\sigma}_{\downarrow\downarrow} = -\frac{\Gamma_{\uparrow} + \Gamma_{\downarrow}}{2}\sigma_{\downarrow\downarrow} + \frac{\Gamma_{\downarrow}}{2}\sigma_{vv} + \frac{\Gamma_{\uparrow}}{2}\sigma_{dd} + [\mathcal{F}_{\uparrow}^{\dagger}\sigma_{\downarrow d} + \mathcal{F}_{\downarrow}^{\dagger}\sigma_{v\downarrow} - \mathcal{F}_{\uparrow}\sigma_{d\downarrow} - \mathcal{F}_{\downarrow}\sigma_{\downarrow v}], \quad (60c)$$

$$\dot{\sigma}_{dd} = -\frac{\Gamma_{\uparrow} + \Gamma_{\downarrow}}{2}\sigma_{dd} + \frac{\Gamma_{\downarrow}}{2}\sigma_{\uparrow\uparrow} + \frac{\Gamma_{\uparrow}}{2}\sigma_{\downarrow\downarrow} - [\mathcal{F}_{\uparrow}^{\dagger}\sigma_{\downarrow d} - \mathcal{F}_{\downarrow}^{\dagger}\sigma_{\uparrow d} - \mathcal{F}_{\uparrow}\sigma_{d\downarrow} + \mathcal{F}_{\downarrow}\sigma_{d\uparrow}], \quad (60d)$$

and those for the off-diagonal elements are

$$\dot{\sigma}_{v\uparrow} = -i\Delta_{v\uparrow}\sigma_{v\uparrow} - \frac{\Gamma_{\uparrow} + \Gamma_{\downarrow}}{2}\sigma_{v\uparrow} - \frac{\Gamma_{\downarrow}}{2}\sigma_{\downarrow d} + [\mathcal{F}_{\uparrow}(\sigma_{vv} + \sigma_{\uparrow\uparrow}) + \mathcal{F}_{\downarrow}\sigma_{\downarrow\uparrow} + \mathcal{F}_{\downarrow}^{\dagger}\sigma_{vd}], \quad (61a)$$

$$\dot{\sigma}_{\downarrow d} = -i\Delta_{\downarrow d}\sigma_{\downarrow d} - \frac{\Gamma_{\uparrow} + \Gamma_{\downarrow}}{2}\sigma_{\downarrow d} - \frac{\Gamma_{\downarrow}}{2}\sigma_{v\uparrow} - [\mathcal{F}_{\uparrow}(\sigma_{\downarrow\downarrow} + \sigma_{dd}) - \mathcal{F}_{\downarrow}\sigma_{\downarrow\uparrow} - \mathcal{F}_{\downarrow}^{\dagger}\sigma_{vd}], \quad (61b)$$

$$\dot{\sigma}_{v\downarrow} = -i\Delta_{v\downarrow}\sigma_{v\downarrow} - \frac{\Gamma_{\uparrow} + \Gamma_{\downarrow}}{2}\sigma_{v\downarrow} - \frac{\Gamma_{\uparrow}}{2}\sigma_{\uparrow d} + [\mathcal{F}_{\downarrow}(\sigma_{vv} + \sigma_{\downarrow\downarrow}) + \mathcal{F}_{\uparrow}\sigma_{\uparrow\downarrow} - \mathcal{F}_{\uparrow}^{\dagger}\sigma_{vd}], \quad (61c)$$

$$\dot{\sigma}_{\uparrow d} = -i\Delta_{\uparrow d}\sigma_{\uparrow d} - \frac{\Gamma_{\uparrow} + \Gamma_{\downarrow}}{2}\sigma_{\uparrow d} + \frac{\Gamma_{\uparrow}}{2}\sigma_{v\downarrow} + [\mathcal{F}_{\downarrow}(\sigma_{\uparrow\uparrow} + \sigma_{dd}) - \mathcal{F}_{\uparrow}\sigma_{\uparrow\downarrow} + \mathcal{F}_{\uparrow}^{\dagger}\sigma_{vd}], \quad (61d)$$

$$\dot{\sigma}_{vd} = -i\Delta_{vd}\sigma_{vd} - \frac{\Gamma_{\uparrow} + \Gamma_{\downarrow}}{2}\sigma_{vd} + [\mathcal{F}_{\uparrow}(\sigma_{\uparrow d} + \sigma_{v\downarrow}) - \mathcal{F}_{\downarrow}(\sigma_{v\uparrow} - \sigma_{d\downarrow})], \quad (61e)$$

$$\dot{\sigma}_{\uparrow\downarrow} = -i\Delta_{\uparrow\downarrow}\sigma_{\uparrow\downarrow} - \frac{\Gamma_{\uparrow} + \Gamma_{\downarrow}}{2}\sigma_{\uparrow\downarrow} + [\mathcal{F}_{\uparrow}^{\dagger}(\sigma_{\uparrow d} + \sigma_{v\downarrow}) - \mathcal{F}_{\downarrow}(\sigma_{v\uparrow} - \sigma_{d\downarrow})], \quad (61f)$$

where $\Delta_{ij} = \omega_j - \omega_i$, $\mathcal{F}_s \equiv \mathcal{F}_s(t) = \mathcal{L}_s(t) + \mathcal{R}_s(t)$ for $s \in \{\uparrow, \downarrow\}$, and the spin-dependent noise operators $\mathcal{L}_s(t)$ and $\mathcal{R}_s(t)$ are defined as

$$\mathcal{L}_s(t) = - \sum_k \xi_{ks} e^{-i\omega_{ks}^{(L)}t} b_{ks}(0), \quad (62a)$$

$$\mathcal{R}_s(t) = - \sum_j \zeta_{js} e^{-i\omega_{js}^{(R)}t} c_{js}(0). \quad (62b)$$

The damping rate $\Gamma_s = \gamma_{L_s} + \gamma_{R_s}$, with the spin-dependent tunneling rates

$$\gamma_{L_s} = 2\pi \xi_s^2(\omega_s) D_s^{(L)}(\omega_s), \quad (63a)$$

$$\gamma_{R_s} = 2\pi \zeta_s^2(\omega_s) D_s^{(R)}(\omega_s), \quad (63b)$$

where ξ_s and ζ_s are the coupling amplitudes of the quantum dot to the left and right leads, and $D_s^{(L)}(\omega)$ and $D_s^{(R)}(\omega)$ are the spin-resolved density of states of left and right leads, respectively.

These Langevin equations of the system variables are analogous to the ones used to describe the quantum theory of laser [34, 35]. In the quantum theory of laser, the atoms are subject to bosonic reservoirs, while in our quantum transport case, the quantum dot is ‘pumped’ by a fermionic reservoir (the left lead) and output to another fermionic reservoir (the right lead).

In contrast to the non-interacting case (see equation (35)), the noise operators couple to the system projection operators in equations (60) and (61). The correlations between noise operators and projection operators, such as $\langle \mathcal{L}_s^{\dagger}(t) = \sigma_{ij}(t) \rangle$, are calculated according to the generalized causality relation equation (26). Taking $\langle \mathcal{L}_{\uparrow}^{\dagger}(t) = \sigma_{v\uparrow}(t) \rangle$, for example,

$$\begin{aligned} \langle \mathcal{L}_{\uparrow}^{\dagger}(t) \sigma_{v\uparrow}(t) \rangle &= \langle \tilde{\mathcal{L}}_{\uparrow}^{\dagger}(t) \tilde{\sigma}_{v\uparrow}(t) \rangle = \langle [\tilde{\mathcal{L}}_{\uparrow}^{\dagger}(t), \tilde{\sigma}_{v\uparrow}(t)]_+ \rangle - \langle \tilde{\sigma}_{v\uparrow}(t) \tilde{\mathcal{L}}_{\uparrow}^{\dagger}(t) \rangle \\ &= \frac{1}{2} \gamma_L \langle [a_{\uparrow}^{\dagger}(t), \sigma_{v\uparrow}(t)]_+ \rangle = \frac{1}{2} \gamma_L \langle \sigma_{vv}(t) + \sigma_{\uparrow\uparrow}(t) \rangle, \end{aligned} \quad (64)$$

where $\tilde{\sigma}_{v\uparrow}(t) = \sigma_{v\uparrow}(t) e^{i\omega_{\uparrow}t}$ is the slow-varying amplitude of projection operator, and $\tilde{\mathcal{L}}_{\uparrow}^{\dagger}(t) = \mathcal{L}_{\uparrow}^{\dagger}(t) e^{-i\omega_{\uparrow}t}$. The correlation $\langle \tilde{\sigma}_{v\uparrow}(t) \tilde{\mathcal{L}}_{\uparrow}^{\dagger}(t) \rangle$ in the second line of equation (64) vanishes when the noise operator $\tilde{\mathcal{L}}_{\uparrow}^{\dagger}(t)$ acts on the full-filled Fermi sea of the left lead (see appendix).

With these correlations, the ensemble average of equations (60) and (61) gives the ‘rate equations’ for the diagonal elements,

$$\langle \dot{\sigma}_{vv} \rangle = -(\gamma_{L\uparrow} + \gamma_{L\downarrow}) \langle \sigma_{vv} \rangle + \gamma_{R\uparrow} \langle \sigma_{\uparrow\uparrow} \rangle + \gamma_{R\downarrow} \langle \sigma_{\downarrow\downarrow} \rangle, \quad (65a)$$

$$\langle \dot{\sigma}_{\uparrow\uparrow} \rangle = -\gamma_{R\uparrow} \langle \sigma_{\uparrow\uparrow} \rangle + \gamma_{L\uparrow} \langle \sigma_{vv} \rangle + (\gamma_{L\downarrow} + \gamma_{R\downarrow}) \langle \sigma_{dd} \rangle, \quad (65b)$$

$$\langle \dot{\sigma}_{\downarrow\downarrow} \rangle = -\gamma_{R\downarrow} \langle \sigma_{\downarrow\downarrow} \rangle + \gamma_{L\downarrow} \langle \sigma_{vv} \rangle + (\gamma_{L\uparrow} + \gamma_{R\uparrow}) \langle \sigma_{dd} \rangle, \quad (65c)$$

$$\langle \dot{\sigma}_{dd} \rangle = -(\gamma_{L\uparrow} + \gamma_{L\downarrow} + \gamma_{R\uparrow} + \gamma_{R\downarrow}) \langle \sigma_{dd} \rangle, \quad (65d)$$

and for off-diagonal elements,

$$\langle \dot{\sigma}_{v\uparrow} \rangle = -(i\Delta_{v\uparrow} + \Gamma_1) \langle \sigma_{v\uparrow} \rangle - \frac{\gamma_{L\downarrow} + 2\gamma_{R\downarrow}}{2} \langle \sigma_{\downarrow d} \rangle, \quad (66a)$$

$$\langle \dot{\sigma}_{\downarrow d} \rangle = -(i\Delta_{\downarrow d} + \Gamma_2) \langle \sigma_{\downarrow d} \rangle - \frac{\gamma_{L\downarrow}}{2} \langle \sigma_{v\uparrow} \rangle, \quad (66b)$$

$$\langle \dot{\sigma}_{v\downarrow} \rangle = -(i\Delta_{v\downarrow} + \Gamma_3) \langle \sigma_{v\downarrow} \rangle + \frac{\gamma_{L\uparrow} + 2\gamma_{R\uparrow}}{2} \langle \sigma_{\uparrow d} \rangle, \quad (66c)$$

$$\langle \dot{\sigma}_{\uparrow d} \rangle = -(i\Delta_{\uparrow d} + \Gamma_4) \langle \sigma_{\uparrow d} \rangle + \frac{\gamma_{L\uparrow}}{2} \langle \sigma_{v\downarrow} \rangle, \quad (66d)$$

$$\langle \dot{\sigma}_{vd} \rangle = -(i\Delta_{vd} + 2\gamma_{L\uparrow} + 2\gamma_{L\downarrow} + \gamma_{R\uparrow} + \gamma_{R\downarrow}) \langle \sigma_{vd} \rangle, \quad (66e)$$

$$\langle \dot{\sigma}_{\uparrow\downarrow} \rangle = -(i\Delta_{\uparrow\downarrow} + \gamma_{R\uparrow} + \gamma_{R\downarrow}) \langle \sigma_{\uparrow\downarrow} \rangle, \quad (66f)$$

where $\Gamma_1 = (\gamma_{L\uparrow} + \gamma_{L\downarrow} + \gamma_{R\uparrow})/2$, $\Gamma_2 = (\gamma_{L\uparrow} + \gamma_{L\downarrow} + \gamma_{R\uparrow} + 2\gamma_{R\downarrow})/2$, $\Gamma_3 = (\gamma_{L\uparrow} + \gamma_{L\downarrow} + \gamma_{R\downarrow})/2$ and $\Gamma_4 = (\gamma_{L\uparrow} + \gamma_{L\downarrow} + 2\gamma_{R\uparrow} + \gamma_{R\downarrow})/2$. Equation (65) shows that the coherence between energy levels vanishes after a long time, i.e.

$$\langle \sigma_{ij}(t) \rangle = 0, \quad \text{for } i \neq j \quad \text{and } t \rightarrow +\infty. \quad (67)$$

This indicates that the two different spin channels are incoherent, which physically arises from the fact that noise operators with different spins are uncorrelated, i.e. $\langle \mathcal{L}_{\uparrow}^{\dagger}(t) \mathcal{L}_{\downarrow}(t') \rangle = 0$.

The rate equations (65) describe the population transfer between each energy level, and the steady-state populations are

$$\langle \sigma_{vv} \rangle = \frac{\gamma_{R\uparrow} \gamma_{R\downarrow}}{\gamma_{L\downarrow} \gamma_{R\uparrow} + \gamma_{L\uparrow} \gamma_{R\downarrow} + \gamma_{R\uparrow} \gamma_{R\downarrow}}, \quad (68a)$$

$$\langle \sigma_{\uparrow\uparrow} \rangle = \frac{\gamma_{L\uparrow} \gamma_{R\downarrow}}{\gamma_{L\downarrow} \gamma_{R\uparrow} + \gamma_{L\uparrow} \gamma_{R\downarrow} + \gamma_{R\uparrow} \gamma_{R\downarrow}}, \quad (68b)$$

$$\langle \sigma_{\downarrow\downarrow} \rangle = \frac{\gamma_{L\downarrow} \gamma_{R\uparrow}}{\gamma_{L\downarrow} \gamma_{R\uparrow} + \gamma_{L\uparrow} \gamma_{R\downarrow} + \gamma_{R\uparrow} \gamma_{R\downarrow}}, \quad (68c)$$

$$\langle \sigma_{dd} \rangle = 0. \quad (68d)$$

The average current is

$$\begin{aligned} \langle \hat{I}_R \rangle &= \langle \hat{I}_{R\uparrow} \rangle + \langle \hat{I}_{R\downarrow} \rangle = \gamma_{R\uparrow} \langle \sigma_{\uparrow\uparrow} \rangle + \gamma_{R\downarrow} \langle \sigma_{\downarrow\downarrow} \rangle \\ &= \frac{(\gamma_{L\uparrow} + \gamma_{L\downarrow}) \gamma_{R\uparrow} \gamma_{R\downarrow}}{\gamma_{L\downarrow} \gamma_{R\uparrow} + \gamma_{L\uparrow} \gamma_{R\downarrow} + \gamma_{R\uparrow} \gamma_{R\downarrow}}. \end{aligned} \quad (69)$$

The current vanishes if $\gamma_{R\uparrow} = 0$ or $\gamma_{R\downarrow} = 0$. This is because turning off a certain spin channel, say the spin-up channel, i.e. $\gamma_{R\uparrow} = 0$, will induce the accumulation of the spin-up electron on the quantum dot. Then the electron tunneling of both spin channels is blocked due to the strong Coulomb interaction. When the tunneling rates are spin independent, i.e. $\gamma_{L\uparrow} = \gamma_{L\downarrow} = \gamma_L$ and $\gamma_{R\uparrow} = \gamma_{R\downarrow} = \gamma_R$, the average current in equation (69) becomes $\langle \hat{I}_R \rangle = 2\gamma_L \gamma_R / (2\gamma_L + \gamma_R)$, which agrees with the results obtained by other methods [8, 9, 26].

4.2. Current noise

Now we turn to the current noise. Similar to the non-interacting case, the two-time correlations of the following form should be calculated,

$$\langle n_s(t)n_{s'}(t+\tau) \rangle, \quad (70a)$$

$$\langle a_s^\dagger(t)\mathcal{R}_s(t)\mathcal{R}_{s'}^\dagger(t+\tau)a_{s'}(t+\tau) \rangle, \quad (70b)$$

$$\langle a_s^\dagger(t)\mathcal{R}_s(t)\mathcal{R}_{s'}^\dagger(t+\tau)a_{s'}(t+\tau) \rangle. \quad (70c)$$

In the interacting case, the system projection operators cannot be expressed in terms of the simple integration of the noise operators as in the non-interacting case (see equation (37)). The causality relations introduced in section 2.3 provide us with a convenient way of converting the noise–system correlation to the system–system correlation. Thus, in the white-noise limit, as a powerful tool, the quantum regression theorem is applied to calculate the two-time system correlations.

Noting that the noise operator $\mathcal{R}_s(t)$ plays the role of ‘annihilation operator’, the correlations between noise and system operators can be calculated following the spirit of Wick’s theorem (see the appendix). Taking the spin-up component, for example, the correlation $\langle a_\uparrow^\dagger(t)\mathcal{R}_\uparrow(t)\mathcal{R}_\uparrow^\dagger(t+\tau)a_\uparrow(t+\tau) \rangle$ is

$$\begin{aligned} & \langle a_\uparrow^\dagger(t)\mathcal{R}_\uparrow(t)\mathcal{R}_\uparrow^\dagger(t+\tau)a_\uparrow(t+\tau) \rangle \\ &= \langle a_\uparrow^\dagger(t)[\mathcal{R}_\uparrow(t), \mathcal{R}_\uparrow^\dagger(t+\tau)]_+ a_\uparrow(t+\tau) \rangle - \langle a_\uparrow^\dagger(t)\mathcal{R}_\uparrow^\dagger(t+\tau)\mathcal{R}_\uparrow(t)a_\uparrow(t+\tau) \rangle \\ &= \gamma_R \langle a_\uparrow^\dagger(t)a_\uparrow(t+\tau) \rangle \delta(\tau). \end{aligned} \quad (71)$$

The second line of equation (71) is simplified by noting the fact that $[\mathcal{R}_\uparrow(t), \mathcal{R}_\uparrow^\dagger(t')]_+ = \gamma_R \delta(t-t')$, and the third line vanishes since $[a_\uparrow^\dagger(t), \mathcal{R}_\uparrow^\dagger(t+\tau)]_+ = 0$. This white-noise correlation provides a constant current noise background. Due to the $\delta(\tau)$ function, only equal-time correlation ($\tau = 0$) is relevant. By noting equation (69), this correlation is written as

$$\langle a_\uparrow^\dagger(t)\mathcal{R}_\uparrow(t)\mathcal{R}_\uparrow^\dagger(t+\tau)a_\uparrow(t+\tau) \rangle = \langle \hat{I}_{R\uparrow} \rangle \delta(\tau). \quad (72)$$

For the correlation $\langle a_\uparrow^\dagger(t)\mathcal{R}_\uparrow(t)a_\uparrow^\dagger(t+\tau)a_\uparrow(t+\tau) \rangle \equiv \langle a_\uparrow^\dagger(t)\mathcal{R}_\uparrow(t)n_\uparrow(t+\tau) \rangle$, it can be translated into the correlations between the system operators using the causality relations as

$$\begin{aligned} \langle a_\uparrow^\dagger(t)\mathcal{R}_\uparrow(t)n_\uparrow(t+\tau) \rangle &= \langle a_\uparrow^\dagger(t)[\mathcal{R}_\uparrow(t), n_\uparrow(t+\tau)] \rangle = \gamma_R \langle a_\uparrow^\dagger(t)[a_\uparrow(t), n_\uparrow(t+\tau)] \rangle \\ &= \gamma_R \langle n_\uparrow(t)n_\uparrow(t+\tau) \rangle - \gamma_R \langle a_\uparrow^\dagger(t)n_\uparrow(t+\tau)a_\uparrow(t) \rangle. \end{aligned} \quad (73)$$

The first term cancels out the contribution of equation (70a). As a result, the spin-up current–current correlation is

$$\begin{aligned} g_{\uparrow\uparrow}^{(2)}(\tau) &= \lim_{t \rightarrow +\infty} \langle \hat{I}_{R\uparrow}(t)\hat{I}_{R\uparrow}(t+\tau) \rangle \\ &= \lim_{t \rightarrow +\infty} \langle \hat{I}_{R\uparrow} \rangle \delta(\tau) + \gamma_{R\uparrow}^2 \langle a_\uparrow^\dagger(t)n_\uparrow(t+\tau)a_\uparrow(t) \rangle. \end{aligned} \quad (74)$$

The current–current correlations of different spin components are calculated similarly, and, in general, they can be expressed in terms of the correlations of system operators as

$$\begin{aligned} g_{ss'}^{(2)}(\tau) &= \lim_{t \rightarrow +\infty} \langle \hat{I}_{R_s}(t)\hat{I}_{R_{s'}}(t+\tau) \rangle \\ &= \lim_{t \rightarrow +\infty} \langle \hat{I}_{R_s} \rangle \delta(\tau) \delta_{ss'} + \gamma_{R_s} \gamma_{R_{s'}} \langle a_s^\dagger(t)n_{s'}(t+\tau)a_s(t) \rangle. \end{aligned} \quad (75)$$

Here, we have shown an analogous form of the current–current correlation to the second-order optical coherence function [35]. The last term of equation (75) can be calculated from the quantum regression theorem. By this theorem, the current–current correlation function is determined by the rate equations (65), and in the Coulomb blockade case, it does not show the effect of the quantum coherence terms in equation (66).

The total current correlation function is

$$\begin{aligned} g^{(2)}(\tau) &= \lim_{t \rightarrow +\infty} \langle \hat{I}_R(t) \hat{I}_R(t + \tau) \rangle - \langle \hat{I}_R \rangle^2 \\ &= g_{\uparrow\uparrow}^{(2)}(\tau) + g_{\downarrow\downarrow}^{(2)}(\tau) + g_{\uparrow\downarrow}^{(2)}(\tau) + g_{\downarrow\uparrow}^{(2)}(\tau) - \langle \hat{I}_R \rangle^2. \end{aligned} \quad (76)$$

Its Fourier transformation gives the current noise spectrum $S(\omega)$. In the spin-independent tunneling rate case, i.e. $\gamma_{L\uparrow} = \gamma_{L\downarrow} = \gamma_L$ and $\gamma_{R\uparrow} = \gamma_{R\downarrow} = \gamma_R$, the noise spectrum is

$$S(\omega) = 2e \langle \hat{I}_R \rangle \frac{4\gamma_L^2 + 3\gamma_L\gamma_R + \gamma_R^2}{(2\gamma_L + \gamma_R)^2 + \omega^2}. \quad (77)$$

This result deviates from the single-level case (see equation (56)), due to the presence of the Coulomb interaction.

Typical current noise spectra for the spin-dependent tunneling rate case are shown in figure 3(a). The Fano factor is

$$\begin{aligned} F &\equiv S(\omega = 0) / 2e \langle \hat{I}_R \rangle \\ &= 1 - \frac{\gamma_{R\uparrow}\gamma_{R\downarrow}(\gamma_{L\downarrow}\gamma_{R\uparrow} + \gamma_{L\uparrow}\gamma_{R\downarrow}) - \gamma_{L\uparrow}\gamma_{L\downarrow}(\gamma_{R\uparrow} - \gamma_{R\downarrow})^2}{2(\gamma_{L\downarrow}\gamma_{R\uparrow} + \gamma_{L\uparrow}\gamma_{R\downarrow} + \gamma_{R\uparrow}\gamma_{R\downarrow})^2}. \end{aligned} \quad (78)$$

It is found that super-Poissonian noise arises when the tunneling is spin dependent, which can be realized, e.g., by using magnetized barriers between the leads and the quantum dot. Super-Poissonian noise appears when the numerator of the second term becomes negative. The Fano factor as a function of the tunneling rate imbalance is shown in figure 3(b).

Physically, the super-Poissonian noise is the consequence of the dynamical channel blockade effect [21, 24]. The tunneling rate imbalance induces different average currents for the two spin channels. Thus, in addition to the noises of the channels themselves, the shot noise between the two channels gives rise to the low-frequency noise enhancement. Such a kind of shot noise is absent when $P_L = 1 - P_R$ since the two spin channels have the same current.

5. Relation to other theories

5.1. Relation to the Landauer–Büttiker formula

Here, we show that the Landauer–Büttiker formula can be reproduced by the quantum Langevin approach. For simplicity, let us consider the single energy level transport example.

According to equation (29b) and the boundary relations equation (23), the current operators can be expressed solely by the input and output noise operators. For example,

$$\hat{I}_R = \frac{1}{\gamma_R} (\tilde{\mathcal{R}}_{\text{out}}^\dagger(t) \tilde{\mathcal{R}}_{\text{out}}(t) - \tilde{\mathcal{R}}_{\text{in}}^\dagger(t) \tilde{\mathcal{R}}_{\text{in}}(t)). \quad (79)$$

Thus, it is clear that the average current is divided into the input current proportional to $\langle \tilde{\mathcal{R}}_{\text{in}}^\dagger(t) \tilde{\mathcal{R}}_{\text{in}}(t) \rangle$ and the output current proportional to $\langle \tilde{\mathcal{R}}_{\text{out}}^\dagger(t) \tilde{\mathcal{R}}_{\text{out}}(t) \rangle$.

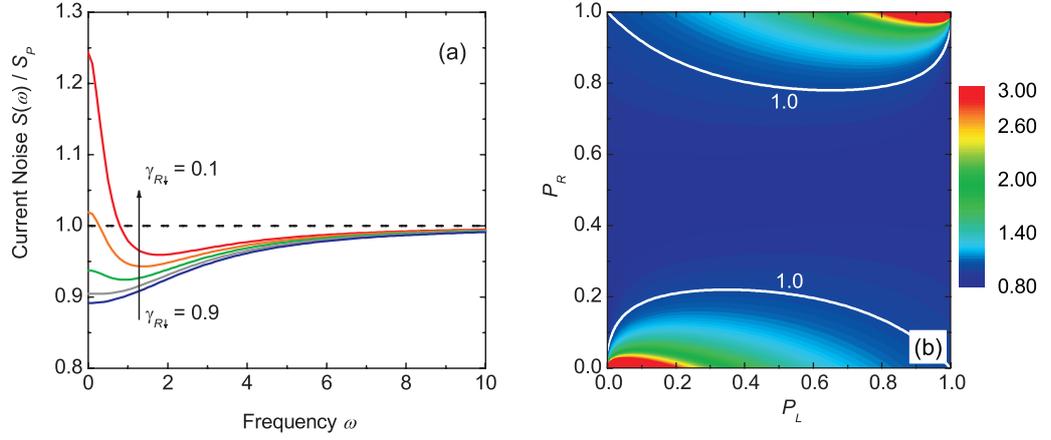


Figure 3. (a) Current noise spectra $S(\omega)$ (normalized by the Poisson value $S_p = 2e\langle\hat{I}_R\rangle$) for $\gamma_{R\downarrow} = 0.1, 0.3, \dots, 0.9$. Other parameters are chosen as $\gamma_{L\uparrow} = \gamma_{L\downarrow} = \gamma_{R\uparrow} = 1$. (b) The Fano factor as a function of the imbalance between spin-resolved tunneling rates P_L and P_R , which are defined as $P_L = \gamma_{L\uparrow}/(\gamma_{L\uparrow} + \gamma_{L\downarrow})$ and $P_R = \gamma_{R\uparrow}/(\gamma_{R\uparrow} + \gamma_{R\downarrow})$, for given total tunneling rates $\gamma_{L\uparrow} + \gamma_{L\downarrow} = \gamma_{R\uparrow} + \gamma_{R\downarrow} = 1$. The white thick lines are the boundary between sub-Poisson and super-Poisson regimes, i.e. $F = 1$.

Furthermore, defining the scattering matrix \mathbf{S} , the Fourier transformation of output noise operators is expressed in terms of the input operators as

$$\begin{pmatrix} \tilde{\mathcal{L}}_{\text{out}}(\omega) \\ \tilde{\mathcal{R}}_{\text{out}}(\omega) \end{pmatrix} = \mathbf{S} \begin{pmatrix} \tilde{\mathcal{L}}_{\text{in}}(\omega) \\ \tilde{\mathcal{R}}_{\text{in}}(\omega) \end{pmatrix}, \quad (80)$$

with

$$\tilde{\mathcal{L}}_{\text{in/out}}(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} \tilde{\mathcal{L}}_{\text{in/out}}(t) dt, \quad (81a)$$

$$\tilde{\mathcal{R}}_{\text{in/out}}(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} \tilde{\mathcal{R}}_{\text{in/out}}(t) dt, \quad (81b)$$

and

$$\begin{aligned} \mathbf{S}(\omega) &\equiv \begin{pmatrix} \mathbb{R}_{L\leftarrow L}(\omega) & \mathbb{T}_{L\leftarrow R}(\omega) \\ \mathbb{T}_{R\leftarrow L}(\omega) & \mathbb{R}_{R\leftarrow R}(\omega) \end{pmatrix} \\ &= \frac{2}{\gamma_L + \gamma_R - 2i\omega} \begin{pmatrix} \frac{\gamma_L - \gamma_R}{2} - i\omega & \gamma_L \\ \gamma_R & -\frac{\gamma_L - \gamma_R}{2} + i\omega \end{pmatrix}, \end{aligned} \quad (82)$$

where the functions $\mathbb{T}_{i\leftarrow j}(\omega)$ and $\mathbb{R}_{i\leftarrow j}(\omega)$ can be regarded as the energy-dependent transmission and reflection coefficients from lead j to lead i . The Fourier transformation of the average current is

$$\begin{aligned} \langle \hat{I}_R(\omega) \rangle &= \int \langle \hat{I}_R(t) \rangle e^{i\omega t} dt \\ &= \frac{1}{\gamma_R} \int_{-\infty}^{\infty} [\langle \tilde{\mathcal{R}}_{\text{out}}^\dagger(\omega') \tilde{\mathcal{R}}_{\text{out}}(\omega' + \omega) \rangle - \langle \tilde{\mathcal{R}}_{\text{in}}^\dagger(\omega') \tilde{\mathcal{R}}_{\text{in}}(\omega' + \omega) \rangle] \frac{d\omega'}{2\pi}. \end{aligned} \quad (83)$$

Noting the relation equation (80) and the correlations between the noise operators,

$$\langle \tilde{\mathcal{L}}_{\text{in}}^\dagger(\omega') \tilde{\mathcal{L}}_{\text{in}}(\omega' + \omega) \rangle = 2\pi \gamma_L \delta(\omega), \quad (84a)$$

$$\langle \tilde{\mathcal{R}}_{\text{in}}^\dagger(\omega') \tilde{\mathcal{R}}_{\text{in}}(\omega' + \omega) \rangle = 0, \quad (84b)$$

we obtain the Landauer–Büttiker-like formula of the average current,

$$\langle \hat{I}_{\text{R}} \rangle = \int_{-\infty}^{+\infty} T(\omega') \frac{d\omega'}{2\pi} = \frac{\gamma_L \gamma_R}{\gamma_L + \gamma_R}, \quad (85)$$

with the transmission spectrum,

$$T(\omega) = \frac{\gamma_L \gamma_R}{\left(\frac{\gamma_L + \gamma_R}{2}\right)^2 + \omega^2}. \quad (86)$$

5.2. Relation to non-equilibrium Green's function (NEGF) theory

Here we discuss the relation between the quantum Langevin approach and the NEGF for the quantum transport problems. In the NEGF theory, various Green's functions (e.g. retarded/advanced Green's function and lesser/greater Green's function) are defined. These Green's functions are related to different physical quantities (e.g. local density of states and level occupations). Using the quantum Langevin approach, the Green's functions and the related quantities could be calculated straightforwardly. Here we take the retarded Green's function, for example, which is defined as [36]

$$G_s(\tau) = -i\theta(\tau) \langle \{a_s(t + \tau), a_s^\dagger(t)\} \rangle, \quad (87)$$

for $s = \uparrow$ or \downarrow . The local density of states (LDOS) $\mathcal{D}_s(\omega)$ is given by

$$\mathcal{D}_s(\omega) = -\frac{1}{\pi} \text{Im}[G_s(\omega)], \quad (88)$$

where $\tilde{G}_s(\omega)$ is the Fourier transformation of $G_s(\tau)$. The LDOS contains the essential information about the system relevant to quantum transport. In the following, we take the Coulomb blockade example, and give the retarded Green's function and the LDOS using the quantum noise approach.

Noting that the definition of the retarded Green's function equation (87) only involves the system operators $a_s(t)$ and $a_s^\dagger(0)$, we apply the quantum regression theorem to calculate their correlations. The retarded Green's function can be expressed in terms of the two-time correlations between the projection operators. Consider the spin-up component, for example,

$$G_\uparrow(\tau) = -i\theta(\tau) \langle \{ \sigma_{v\uparrow}(t + \tau) - \sigma_{d\downarrow}(t + \tau), \sigma_{\uparrow v}(t) - \sigma_{d\downarrow}(t) \} \rangle. \quad (89)$$

The equations of motion for these projection operators are given in equation (66). By the quantum regression theorem, the two-time correlations are determined by

$$\frac{d}{d\tau} \begin{pmatrix} \langle \sigma_{\uparrow v}(t) \sigma_{v\uparrow}(t + \tau) \rangle \\ \langle \sigma_{\uparrow v}(t) \sigma_{d\downarrow}(t + \tau) \rangle \end{pmatrix} = \mathbf{M} \begin{pmatrix} \langle \sigma_{\uparrow v}(t) \sigma_{v\uparrow}(t + \tau) \rangle \\ \langle \sigma_{\uparrow v}(t) \sigma_{d\downarrow}(t + \tau) \rangle \end{pmatrix}, \quad (90)$$

with the initial condition for $\tau = 0$

$$\begin{pmatrix} \langle \sigma_{\uparrow v}(t) \sigma_{v\uparrow}(t) \rangle \\ \langle \sigma_{\uparrow v}(t) \sigma_{d\downarrow}(t) \rangle \end{pmatrix} = \begin{pmatrix} \langle \sigma_{\uparrow\uparrow}(t) \rangle \\ 0 \end{pmatrix}, \quad (91)$$

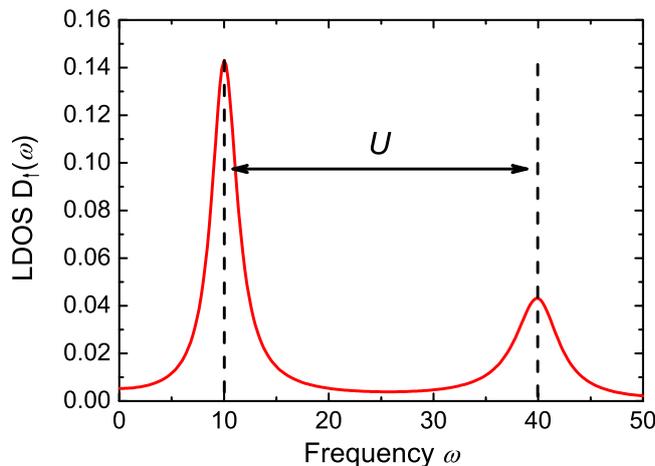


Figure 4. LDOS for the Coulomb blockade case obtained by the quantum Langevin approach. Parameters are $\omega_{\uparrow} = 10$, $U = 30$ and $\gamma = 1$. LDOS shows a double-peak structure for $U \gg \gamma$.

where the coefficient matrix \mathbf{M} is defined as

$$\mathbf{M} = \begin{pmatrix} -3\gamma/2 - i\omega_{\uparrow} & -3\gamma/2 \\ -\gamma/2 & -5\gamma/2 - i(\omega_{\uparrow} + U) \end{pmatrix}. \quad (92)$$

Here, $\gamma_{L\uparrow} = \gamma_{L\downarrow} = \gamma_{R\uparrow} = \gamma_{R\downarrow} = \gamma$ is assumed for simplicity. The other correlations involved in equation (89) can be similarly calculated.

Thus, the retarded Green's function is

$$G_{\uparrow}(\tau) = -i\theta(\tau)e^{-2\gamma\tau}(W_{+}e^{-i\omega_{+}\tau} + W_{-}e^{-i\omega_{-}\tau}), \quad (93)$$

with the renormalized frequencies

$$\omega_{\pm} = \omega_{\uparrow} + \frac{U}{2} \pm \frac{1}{2}\sqrt{U^2 - 2i\gamma U - 4\gamma^2} \quad (94)$$

and the weight factors

$$W_{\pm} = \frac{1}{2} \pm \frac{U/6 - i\gamma}{2\sqrt{U^2 - 2i\gamma U - 4\gamma^2}}. \quad (95)$$

The Fourier transformation of the Green's function gives the LDOS (see figure 4). It is obvious that, for the large U case considered in this paper, the LDOS consists of two Lorentz-shaped peaks, centered around ω_{\uparrow} and $\omega_{\uparrow} + U$. The two peaks separate from each other by U , which is a signature of the Coulomb blockade [6].

6. Conclusions and outlook

In this paper, we have developed a quantum noise approach to treat the quantum transport through a nanostructure such as a quantum dot. We formulate the average current and the current noise in terms of the correlations between the noise operators. The quantum noise approach is applied to a paradigmatic example, namely transport through a single quantum dot under large biases and both the non-interacting and Coulomb blockade cases are investigated. With the

Markovian approximation for the tunneling processes, the electron–electron interaction in the quantum dot can be treated exactly.

The quantum noise approach provides a bridge between quantum optics and quantum transport. Thus the notions and methods in quantum optics could be adopted to study quantum transport through nanostructures. Although we show the application of the quantum noise approach by a single quantum dot example, the theory is not limited to this simple case. On the one hand, the system could be generalized to more complicated ones, such as coupled quantum dots, multi-end nano-circuits or systems with spin interaction. On the other hand, the reservoirs of other kinds, such as phonon baths or spin baths, could be included to explore how such reservoirs would affect the current and current noise, providing a method of studying the bath dynamics via current noises. The Markovian approximation may also be released with colored noise correlation functions of the reservoir used in lieu of the white-noise model adopted in this paper.

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Appendix. Properties of the noise operators

In this [appendix](#), we give the correlations between noise operators. We consider the single-level case here. The physical quantities of interest are determined by the noise correlations, such as $\langle \tilde{\mathcal{L}}^\dagger(t) \tilde{\mathcal{L}}(t') \rangle$. According to the definition of the noise operators,

$$\begin{aligned} \langle \tilde{\mathcal{L}}^\dagger(t) \tilde{\mathcal{L}}(t') \rangle &= \sum_{k,k'} \xi_k \xi_{k'} e^{i(\omega_k - \omega_0)t - i(\omega_{k'} - \omega_0)t'} \langle b_k^\dagger b_{k'} \rangle \\ &= \sum_k \xi_k^2 e^{i(\omega_k - \omega_0)(t-t')} n_{\text{th}}^{(L)}(\omega_k) \\ &= \int_0^\infty \xi^2(\omega_k) D(\omega_k) n_{\text{th}}^{(L)}(\omega_k) e^{i(\omega_k - \omega_0)(t-t')} d\omega_k, \end{aligned} \quad (\text{A.1})$$

where $D(\omega_k)$ is the density of states in the leads, and

$$n_{\text{th}}^{(L)}(\omega) \equiv \frac{1}{1 + e^{(\hbar\omega - \mu_L)/k_B T}} \quad (\text{A.2})$$

is the thermal occupation number of the lead in quasi-equilibrium. The Markovian approximation requires two assumptions. First assumed is the ‘flat band’ condition that the relative change in the effective density of states around the resonant ω_0 over a range of the characteristic damping rate γ_L is much less than unity, i.e.

$$\left(\frac{\partial \ln \bar{D}(\omega_k)}{\partial \omega_k} \right)^{-1} \gg \gamma_L, \quad (\text{A.3})$$

where $\bar{D}(\omega_k) \equiv \xi^2(\omega_k)D(\omega_k)$. Under this condition, $\bar{D}(\omega_k)$ can be replaced by its value at ω_0 , and the correlation becomes

$$\langle \tilde{\mathcal{L}}^\dagger(t) \tilde{\mathcal{L}}(t') \rangle = \bar{D}(\omega_0) \int_0^{\mu_L} e^{i(\omega_k - \omega_0)(t-t')} d\omega_k. \quad (\text{A.4})$$

Here, the zero-temperature case has been considered for simplicity. Secondly, under the large bias condition, the resonant level ω_0 is far away from the Fermi energy and the conduction band bottom (chosen as the energy origin), i.e.

$$\mu_L - \omega_0, \omega_0 \gg \gamma_L. \quad (\text{A.5})$$

In this case, the integration over ω_k is extended to $\pm\infty$ and finally results in the white-noise correlation,

$$\langle \tilde{\mathcal{L}}^\dagger(t) \tilde{\mathcal{L}}(t') \rangle = \gamma_L \delta(t - t'), \quad (\text{A.6})$$

where $\gamma_L = 2\pi \xi^2(\omega_0)D(\omega_0)$.

Similarly, for the right lead,

$$\langle \tilde{\mathcal{R}}(t) \tilde{\mathcal{R}}^\dagger(t') \rangle = \gamma_R \delta(t - t'). \quad (\text{A.7})$$

Here, we use the fact that the thermal occupation number $n_{\text{th}}^{(\text{R})}(\omega_j) = 0$ for the right lead around the resonant level ω_0 . In the same way, one can show that other noise correlations vanish, i.e.

$$\langle \tilde{\mathcal{L}}(t) \tilde{\mathcal{L}}^\dagger(t') \rangle = \langle \tilde{\mathcal{R}}^\dagger(t) \tilde{\mathcal{R}}(t') \rangle = 0. \quad (\text{A.8})$$

Note that equation (A.8) implies that the noise operators $\tilde{\mathcal{L}}^\dagger(t)$ and $\tilde{\mathcal{R}}(t)$ play the role of ‘annihilation operators’, since they always give zero correlations when they stand on the rightmost position. With this observation, the normal-ordered product of noise operators can be defined by placing $\tilde{\mathcal{L}}^\dagger(t)$ and $\tilde{\mathcal{R}}(t)$ on the rightmost position, and the expectation value of the normal-ordered product vanishes identically. Thus, Wick’s theorem is generalized to the noise operators and the current and current noise can be exactly calculated in the white-noise limit.

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