Quantum information storage and state transfer based on spin systems

Z. Song*

Department of Physics, Nankai University, Tianjin 300071, China

C. P. Sun†

Department of Physics, Nankai University, Tianjin 300071, China; Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing, 100080, China

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The idea of quantum state storage is generalized to describe the coherent transfer of quantum information through a coherent data bus. In this universal framework, we comprehensively review our recent systematical investigations to explore the possibility of implementing the physical processes of quantum information storage and state transfer by using quantum spin systems, which may be an isotropic antiferromagnetic spin ladder system or a ferromagnetic Heisenberg spin chain. Our studies emphasize the physical mechanisms and the fundamental problems behind the various protocols for the storage and transfer of quantum information in solid state systems. © *2005 American Institute of Physics.* [DOI: 10.1063/1.2008129]

I. INTRODUCTION

The current development of quantum information science and technology demands optimal systems serving as long-lived quantum memories, through which the quantum information carried by a quantum system with short decoherence time can be coherently transferred.[1] In this sense a quantum channel or a quantum data bus is needed for perfect transmission of quantum states. In this article, we will demonstrate that both the quantum information storage and the quantum state transfer can be uniquely described in a universal framework.

There exist some schemes[2–5] concerning the quantum storage of photon states, and there are also some efforts devoted to the universal quantum storage for a qubit (a basic two-level system) state, which is necessary in quantum computation. For example, most recently an interesting protocol[6–8] was presented to reversibly map the electronic spin state onto the collective spin state of the surrounding nuclei. Because of the long decoherence time of the nuclear spins, the information stored in them can be robustly preserved. It was found that,[9] only under two homogeneous conditions with low excitations, such many-nuclei system approximately behaves as a single-mode boson to serve as an efficient quantum memory.

The low-excitation condition requires a ground state with all spins oriented, which can be prepared by applying a magnetic field polarizing all spins along the same direction. With the concept of spontaneous symmetry breaking (SSB), one can recognize that a ferromagnetic Heisenberg spin chain usually has a spontaneous magnetization, which naturally offers a ground state of this kind. In event of SSB, the intrinsic interaction between spins will strongly correlate with the nuclei to form the magnon, a collective mode of spin wave, even without any external magnetic field. With these considerations, Wang, Li, Song, and Sun[10] explored the possibility of using a ferromagnetic quantum spin system, instead of the free nuclear ensemble, to serve as a robust quantum memory. A protocol was presented to implement a quantum storage element for the electronic spin state in a ring array of interacting nuclei. Under appropriate control of both the electron and the external magnetic field, an arbitrary quantum state of the electronic spin qubit, either a pure or a mixed state, can be coherently stored in the nuclear spin wave and then read out in the reverse process.

On the other hand, designed for a more realistic quantum computing, a scalable architecture of quantum network should be based on the solid state system.[11,12] However, the intrinsic feature of solid state based channels, such as the finiteness of the correlation[13,14] and the environment induced noise (especially the low-frequency noise) may block this scalability. Fortunately, analytical study shows that a spin system possessing a commensurate structure of energy spectrum matched with the corresponding parity can ensure the perfect state transfer.[15–17] Based on this fact, an isotropic antiferromagnetic spin ladder system can be pre-engineered as a novel robust kind of quantum data bus.[18] Because the effective coupling strength between the two spins connected to a spin ladder is inversely proportional to the distance of the two spins, the quantum information can be transferred between the two spins separated by a longer distance. Another example of the near-perfect transfer of quantum information was given to illustrate an application of the theorem. The proposed protocol of such near-perfect quantum state transfer utilizes a ferromagnetic Heisenberg chain with uniform coupling constant but in an external parabolic magnetic field.[17]

The present paper will give a broad overview of the present situation of the our investigations mentioned above on quantum state storage and quantum information coherent transfer based on quantum spin systems. We will understand the physical mechanisms and the fundamental problems behind these protocols in the view of a unified conception, the generalized quantum information storage.

II. GENERALIZED QUANTUM STORAGE AS A DYNAMIC PROCESS

For the dynamic process recording and reading quantum information carried by quantum states, we first describe the idea of generalized quantum storage, which was also introduced in association with the Berry's phase factor.[19] Let M be a quantum memory possessing a subspace spanned by $|M_n\rangle$ $(n=1,2,\ldots,d, \langle M_n|M_m\rangle=\delta_{nm})$, which can store the quantum information of a system S with basis vectors $|S_n\rangle$, $n=1,2,\ldots,d$. If there exists a controlled time evolution interpolating between the initial state $|S_n\rangle\otimes|M\rangle$ and the final state $|S\rangle\otimes|M_n\rangle$ for each index n and arbitrarily given states $|S\rangle$ and $|M\rangle$, we define the usual quantum storage by using a factorized evolution of time T_m

$$|\Phi(T_m)\rangle=U(T_m)|\Phi(0)\rangle=|S\rangle\otimes|M_n\rangle, \tag{1}$$

starting from the initial state $|\Phi(0)\rangle=|S_n\rangle\otimes|M\rangle$. The corresponding readout process is an inverse evolution of time T_f $(>T_m)$

$$|\Phi(T_f)\rangle=U(T_f)|\Phi(0)\rangle=|S_n\rangle\otimes|M\rangle. \tag{2}$$

In this sense, writing an arbitrary state $|S(0)\rangle=\Sigma_n c_n|S_n\rangle$ of S into M with the initial state $|M\rangle$ of quantum memory can be realized as a controlled evolution from time $t=0$ to $t=T_m$

$$\sum_n c_n|S_n\rangle\otimes|M\rangle\rightarrow|S\rangle\otimes\sum_n c_n|M_n\rangle. \tag{3}$$

The readout process from M is another controlled evolution from time $t=T_m$ to $t=T_f$:

$$|S\rangle\otimes\sum_n c_n|M_n\rangle\rightarrow\sum_n c_n|S_n\rangle\otimes|M\rangle. \tag{4}$$

Obviously, the combination of these two processes forms a cyclic evolution in which a state totally returns to the initial one.

However, in the view of the decoding approach, one does not need "totally return" to revive the information of initial state, and a difference is allowed by the *n-independent* unitary transformation $W=W_S\otimes 1$, namely,

$$|S\rangle\otimes W_M\sum_n c_n|M_n\rangle\rightarrow(W_S\sum_n c_n|S_n\rangle)\otimes|M\rangle. \tag{5}$$

This is a quantum dynamic process (QDP) for recording and reading, which defines a quantum storage. Because the factor W_S is known to be independent of the initial state, it can be easily decoded from $W_S\Sigma_n c_n|S_n\rangle$ by the inverse transformation of W_S. We notice that the quantum storage usually relates to two quantum subsystems.

We will show as follows that the quantum state transfer can be understood as a generalized quantum storage with three subsystems, the input one with the Hilbert space S^A, the data bus with D, and the output one with S^B. As illustrated in Fig. 1, the two subsystems S^A and S^B are located at two distant locations A and B, respectively. Then the Hilbert space of the total system can be written as

$$S_T=S^A\otimes D\otimes S^B\equiv S^A\otimes M, \tag{6}$$

where $M=D\otimes S^B$ can be regarded as the generalized quantum memory with the memory space spanned by $|M_n\rangle$ $=|D\rangle\otimes U_B|S_n^B\rangle$. Here $|D\rangle$ is a robust state of the data bus and U_B represents some local unitary transformations with respect to B, which are independent of the initial state. With this notation, the quantum state transfer indeed can be regarded as a generalized QDP.

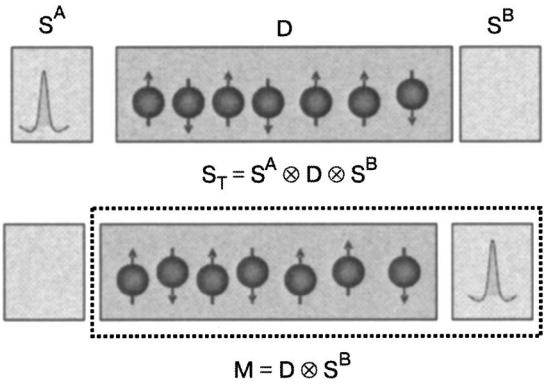

FIG. 1. Demonstration of quantum state transfer as a process of generalized quantum information storage by grouping the data bus D and the target subsystem S^B as a generalized quantum memory.

In fact, if one inputs a state of $|S^A\rangle=\Sigma_n c_n|S_n^A\rangle$ localized at A at $t=0$, the initial state of whole system can be written as

$$|\psi(0)\rangle=\sum_n c_n|S_n^A\rangle\otimes|M\rangle, \tag{7}$$

where $|M\rangle=|D\rangle\otimes U_B|S^B\rangle$. The quantum state transfer can be usually described as a factorized time evolution at time $t=T_f$

$$|\psi(T_f)\rangle=|S\rangle\otimes|D\rangle\otimes\sum_n c_n U_B|S_n^B\rangle=|S\rangle\otimes\sum_n c_n|M_n\rangle \tag{8}$$

with $|M_n\rangle=|D\rangle\otimes U_B|S_n^B\rangle$. The above equations just demonstrate that the quantum state transfer is essentially a generalized quantum memory with $W_M=(1\otimes U_B)$. In this sense the revisable quantum state transfer can be regarded as a general readout process.

Now we would like to remark on the differences between generalized quantum state storage and other two types of quantum processes, quantum teleportation and quantum copy. In fact, quantum teleportation is theoretically perfect, yielding an output state which revives the input with a fidelity $F=1$. Actually one of necessary procedure in teleportation is to measure the Bell state at location A, which will induce wave packet collapse. On the other way around, the quantum state storage process is always on time evolution without any measurement. As for quantum copy the initial state remains unchanged during its copy and can be generated in a dynamic process.

III. QUANTUM STATE TRANSFER IN SPIN SYSTEMS

A robust quantum information processing based on solid state system is usually implemented in a working space spanned by the lowest states, which are well separated from other dense spectra of high excitations. In this sense the energy gap of the solid state system is an important factor we should take into account. The decoherence induced by the

environmental noise can also destroy the robustness of quantum information processing, such as the low-frequency (e.g., $1/f$) noise prevalent in solid state devices.

People believe that the gap of the data bus can suppress the stay of transferred state in the middle way in order to enhance the fidelity, but the large gap may result in a shorter correlation length. The relationship between correlation length and the energy gap is usually established in the system with translational symmetry. So we need to consider some modulated-coupling systems or artificially engineered irregular quantum spin systems where the strong correlation between two distant site can be realized.

A. Theorem for the perfect quantum state transfer

Quantum mechanics shows that perfect state transfer is possible. To sketch our central idea, let us first consider a single-particle system with the usual spatial reflection symmetry (SRS) in the Hamiltonian H. Let P be the spatial reflection operator. The SRS is implied by $[H,P]=0$. Now we prove that at time π/E_0 any state $\psi(\mathbf{r})$ can evolve into the reflected state $\pm\psi(-\mathbf{r})$ if the eigenvalues ε_n match the parities p_n in the following way:

$$\varepsilon_n=N_nE_0, p_n=\pm(-1)^{N_n} \tag{9}$$

for arbitrary positive integer N_n and

$$H\varphi_n(\mathbf{r})=\varepsilon_n\varphi_n(\mathbf{r}), \quad P\varphi_n(\mathbf{r})=p_n\varphi_n(\mathbf{r}). \tag{10}$$

Here $\varphi_n(\mathbf{r})$ is the common eigen wave function of H and P, $\mathbf{r}$ is the position of the particle. We call Eq. (9) the spectrum-parity matching condition (SPMC). The proof of the above rigorous conclusion is a simple but heuristic exercise in basic quantum mechanics. In fact, for the spatial reflection operator, $P\psi(\mathbf{r})=\pm\psi(-\mathbf{r})$. For an arbitrarily given state at $t=0$, $\psi(\mathbf{r},t)|_{t=0}$, it evolves to

$$\psi(\mathbf{r},t)=\exp(-iHt)\psi(\mathbf{r})=\sum_n C_n\exp(-iN_nE_0t)\varphi_n(\mathbf{r}) \tag{11}$$

at time t, where $C_n=\langle\varphi_n|\psi\rangle$. Then at time $t=\pi/E_0$, we have

$$\psi\left(\mathbf{r},\frac{\pi}{E_0}\right)=\sum_n C_n(-1)^{N_n}\varphi_n(\mathbf{r})=\pm P\psi(\mathbf{r}) \tag{12}$$

that is $\psi(\mathbf{r},\pi/E_0)=\pm\psi(-\mathbf{r})$. This is just the central result[20] discovered for quantum spin system that the evolution operator becomes a parity operator $\pm P$ at some instant $t=(2n+1)\pi/E_0$, that is $\exp(-iH\pi/E_0)=\pm P$. From the above arguments we have the consequence that if the eigenvalues $\varepsilon_n=N_nE_0$ of a 1D Hamiltonian H with spatial reflection symmetry are odd-number spaced, i.e., N_n-N_{n-1} are always odd, any initial state $\psi(x)$ can evolve into $\pm\psi(-x)$ at time $t=\pi/E_0$. In fact, for such 1D systems, the discrete states alternate between even and odd parities. Consider the odd-number-spaced eigenvalues $\varepsilon_n=N_nE_0$. The next-nearest level must be even-number spaced; then the SPMC is satisfied. Obviously, the 1D SPMC is more realizable for the construction of the model Hamiltonian to perform perfect state transfer.

Now, we can directly generalize the above analysis to many particle systems. For the quantum spin chain, one can identify the above SRS as the middle inversion of spins with respect to the center of the quantum spin chain. As the discussion in Ref. 20, we write spin inversion operation

$$P\Psi(s_1,s_2,\ldots,s_{N-1},s_N)=\Psi(s_N,s_{N-1},\ldots,s_2,s_1) \tag{13}$$

for the wave function $\Psi(s_1,s_2,\ldots,s_{N-1},s_N)$ of the spin chain. Here, $s_n=0,1$ denotes the spin values of the nth qubit.

B. Perfect state transfer in modulated coupling system

Based on the above analysis, in principle, perfect quantum state transfer is possible in the framework of quantum mechanics. According to SPMC, many spin systems can be pre-engineered for perfect quantum states transfer. For instance, two-site spin-1/2 Heisenberg system is the simplest example which meets the SPMC. Recently, Christandl *et al.*[15,16] proposed an N-site XY chain with an elaborately designed modulated coupling constants between two nearest-neighbor sites, which ensures a perfect state transfer. It is easy to find that this model corresponds the SPMC for the simplest case $N_n=n$. A natural extension of the application of the theorem leads to discover other models with $N_n\neq n$. Following this idea, a new class of different models whose spectrum structures obey the SPMC exactly were proposed for perfect state transfer. Consider an N-site spin-1/2 XY chain with the Hamiltonian

$$H=2\sum_{i=1}^{N-1} J_i[S_i^xS_{i+1}^x+S_i^yS_{i+1}^y], \tag{14}$$

where S_i^x, S_i^y, and S_i^z are Pauli matrices for the ith site, and J_i is the coupling strength for the nearest-neighbor interaction. For the open boundary condition, this model is equivalent to the spinless fermion model. The equivalent Hamiltonian can be written as

$$H=\sum_{i=1}^{N-1} J_i^{[k]}a_i^\dagger a_{i+1}+\text{h.c.}, \tag{15}$$

where $a_i^\dagger$, a_i are the fermion operators. This describes a simple hopping process in the lattice. According to the SPMC, we can present different models (labeled by different positive integer $k\in 0,1,2,\ldots$) through pre-engineering of the coupling strength as $J_i=J_i^{[k]}=\sqrt{i(N-i)}$ for even i and $J_i=J_i^{[k]}=\sqrt{(i+2k)(N-i+2k)}$ for odd i. By a straightforward calculation, one can find the k-dependent spectrum $\varepsilon_n=-N+2(n-k)-1$ for $n=1,2,\ldots,N/2$, and $\varepsilon_n=-N+2(n+k)-1$ for $n=N/2+1,\ldots,N$. The corresponding k-dependent eigenstates are

$$|\varphi_n\rangle=\sum_{i=1}^N c_{ni}|i\rangle=\sum_{i=1}^N c_{ni}a_i^\dagger|0\rangle, \tag{16}$$

where the coefficients c_{ni} can be explicitly determined by the recurrence relation presented in Ref. 18.

It is obvious that the model proposed in Ref. 15 is just the special case of our general model with $k=0$. For arbitrary k, one can easily check that it meets the our SPMC by a straightforward calculation. Thus we can conclude that these spin systems with a set of pre-engineered couplings $J_i^{[k]}$ can serve as the perfect quantum channels that allow the qubit information transfer.

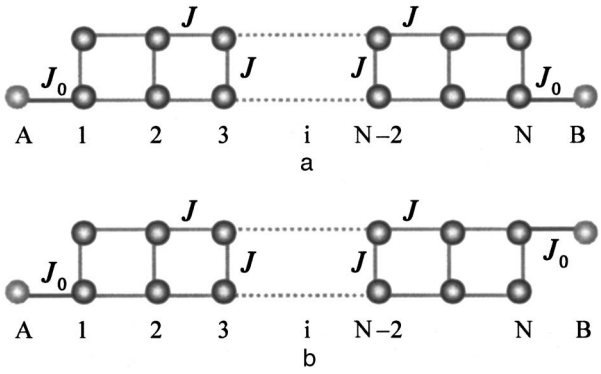

FIG. 2. Two qubits A and B connect to a $2\times N$-site spin ladder. The ground state of H with a-type connection (a) is singlet (triplet) when N is even (odd), while for the b-type connection (b), one should have the opposite result.

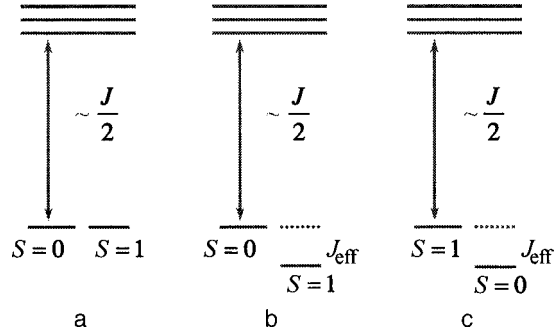

FIG. 3. Schematic illustration of the energy levels of the system. When the connections between two qubits and the medium are switched off ($J_0=0$), the ground states are degenerate (a). When J_0 switches on, the ground state(s) and the first excited state(s) are either singlet or triplet. This is approximately equivalent to that of two coupled spins (b), (c).

C. Near-perfect state transfer

In real many-body systems, the dimension of the Hilbert space increases exponentially with the size N. For example, for an N-site spin-1/2 system the dimension is $D=2^N$, and the symmetry of the Hamiltonian cannot help so much. So it is almost impossible to obtain a model to be exactly engineered. In the above arguments we just show the possibility to implement the perfect state transfer of any quantum state over arbitrary long distances in a quantum spin chain. It sheds light into the investigation of near-perfect quantum state transfer. There is a naive way that one selects some special states to be transported, which is a coherent superposition of commensurate part of the whole set of eigenstates.

For example, we consider a truncated Gaussian wave packet for an anharmonic oscillator with lower eigenstates to be harmonic. It is obvious that such system allows some special states to transfer with high fidelity. We can implement such approximate harmonic system in a natural spin chain without the pre-engineering of couplings but in the presence of a modulated external field. Another way to realize near perfect state transfer is to achieve the entangled states and fast quantum states transfer of two spin qubits by connecting two spins to a medium which possesses a spin gap. A perturbation method, the Fröhlich transformation, shows that the interaction between the two spins can be mapped to the Heisenberg type coupling.

Spin ladder. We sketch our idea with the model illustrated in Fig. 2. The whole quantum system we consider here consists of two qubits (A and B) and a $2\times N$-site two-leg spin ladder. In practice, this system can be realized by an engineered array of quantum dots.[21] The total Hamiltonian $H=H_m+H_q$ contains two parts, the medium Hamiltonian

$$H_M=J\sum_{\langle ij\rangle_\perp}\mathbf{S}_i\cdot\mathbf{S}_j+J\sum_{\langle ij\rangle_\parallel}\mathbf{S}_i\cdot\mathbf{S}_j \tag{17}$$

describing the spin-1/2 Heisenberg spin ladder consisting of two coupled chains, and the coupling Hamiltonian

$$H_q=J_0\mathbf{S}_\Lambda\cdot\mathbf{S}_L+J_0\mathbf{S}_B\cdot\mathbf{S}_R \tag{18}$$

describing the connections between qubits A, B and the ladder. In the term H_M, i denotes a lattice site on which one electron sits, $\langle ij\rangle_\perp$ denotes nearest-neighbor sites on the same rung, and $\langle ij\rangle_\parallel$ denotes nearest neighbors on either leg of the ladder. In the term H_q, L and R denote the sites connecting to the qubits A and B at the ends of the ladder. There are two types of connection between $\mathbf{S}_A$ ($\mathbf{S}_B$) and the ladder, which are illustrated in Fig. 2. According to Lieb's theorem,[22] the spin of the ground state of H with the connection of type a is zero (one) when N is even (odd), while for the connection of type b, one should have the opposite result. For the two-leg spin ladder H_M, analytical analysis and numerical results have shown that the ground state and the first excited state of the spin ladder have spin 0 and 1, respectively.[13,14] It is also shown that there exists a finite spin gap $\Delta=E_1^M-E_g^M\sim J/2$ between the ground state and the first excited state (see Fig. 3). This fact has been verified by experiments[13] and is very crucial for our present investigation.

Thus, it can be concluded that the medium can be robustly frozen to its ground state to induce the effective Hamiltonian $H_{\text{eff}}=J_{\text{eff}}\mathbf{S}_A\cdot\mathbf{S}_B$ between the two end qubits. With the effective coupling constant J_{eff} to be calculated in the following, this Hamiltonian depicts the direct exchange coupling between two separated qubits. As the famous Bell states, H_{eff} has singlet and triplet eigenstates $|j,m\rangle_{AB}$: $|0,0\rangle=1/\sqrt{2}(|\uparrow\rangle_A|\downarrow\rangle_B-|\downarrow\rangle_A|\uparrow\rangle_B)$ and $|1,1\rangle=|\uparrow\rangle_A|\uparrow\rangle_B$, $|1,-1\rangle=|\downarrow\rangle_A|\downarrow\rangle_B$, $|1,0\rangle=1/\sqrt{2}(|\uparrow\rangle_A|\downarrow\rangle_B+|\downarrow\rangle_A|\uparrow\rangle_B)$, which can be used as a channel to share entanglement for a perfect quantum communication in a longer distance.

The above central conclusion can be proved with both analytical and numerical methods as follows. To deduce the above effective Hamiltonian we use $|\psi_g\rangle_M$ ($|\psi_\alpha\rangle_M$) and E_g (E_α) to denote ground (excited) states of H_M and the corresponding eigenvalues. The zero-order eigenstates $|m\rangle$ can then be written in a joint way as

$$\begin{aligned}&|j,m\rangle_g=|j,m\rangle_{AB}\otimes|\psi g\rangle_M,\\&|\psi_\alpha^{jm}(s^z)\rangle=|j,m\rangle_{AB}\otimes|\psi_\alpha\rangle_M.\end{aligned} \tag{19}$$

Here we have considered that the z component $S^z=S_M^z+S_A^z+S_B^z$ of the total spin is conserved with respect to the connection Hamiltonian H_q. Since S_M^z and S_M^2 commute with H_M, we can label $|\psi_g\rangle_M$ as $|\psi_g(s_M,s_M^z)\rangle_M$, and then $s^z=m+s_M^z$ can characterize the non-coupling spin state $|\psi_\alpha^{jm}(s^z)\rangle$.

When the connections between the two qubits and the medium are switched off, i.e., $J_0=0$, the degenerate ground states of H are just $|j,m\rangle_g$ with the degenerate energy E_g and spin 0, 1, respectively, which is illustrated in Fig. 3a. When the connections between the two qubits and the medium are switched on, the degenerate states with spin 0,1 (Ref. 23) should split as illustrated in Fig. 3b, c. In the case with $J_0 \ll J$ at lower temperature $kT<J/2$, the medium can be frozen to its ground state, and then we have the effective Hamiltonian

$$H_{\text{eff}} \sim \sum_{j',m',j,m,s^z} \frac{|g\langle j,m|H_q|\psi_\alpha^{j'm'}(s^z)\rangle|^2}{E_g-E_\alpha}|j,m\rangle_{gg}\langle j,m|$$
$$= J_{\text{eff}}.\text{Diag.}\left(\frac{1}{4},\frac{1}{4},\frac{1}{4},-\frac{3}{4}\right)+\varepsilon \tag{20}$$

where

$$J_{\text{eff}}=\sum_\alpha \frac{J_0^2[L(\alpha)R^*(\alpha)+R(\alpha)L^*(\alpha)]}{E_g-E_\alpha},$$
$$\varepsilon=\sum_\alpha \frac{3J_0^2[|L(\alpha)|^2+|R(\alpha)|^2]}{4(E_g-E_\alpha)}. \tag{21}$$

This just proves the above effective Heisenberg Hamiltonian (5). Here the matrix elements of interaction $K(\alpha)$ $={}_M\langle\psi_g|S_K^z|\psi_\alpha(1,0)\rangle_M$ $(K=S,L)$ can be calculated only for the variables of the data bus medium. We also remark that, because S^z and S^2 are conserved for H_q, the off-diagonal elements in the above effective Hamiltonian vanish.

To sum up so far, we have shown that at lower temperature $kT<J/2$, H can be mapped to the effective Hamiltonian H_{eff}, which seemingly depicts the direct exchange coupling between two separated qubits. Notice that the coupling strength has the form $J_{\text{eff}}\sim g(L)J_0^2/J$, where $g(L)$ is a function of $L=N+1$, the distance between the two qubits concerned. Here we take the $N=2$ case as an example. According to Eq. (21) one can get $J_{\text{eff}}=-(1/4)J_0^2/J$ and $(1/3)J_0^2/J$ when A and B connect the plaquette diagonally and adjacently, respectively. This result is in agreement with the theorem[22] about the ground state and the numerical result when $J_0 \ll J$. In general cases, the behavior of $g(L)$ versus L is very crucial for quantum information since $L/|J_{\text{eff}}|$ determines the characteristic time of quantum state transfer between the two qubits A and B. In order to investigate the profile of $g(L)$, a numerical calculation is performed for the systems $L=4,5,6,7,8,10$, with $J=10,20,40$ and $J_0=1$. The spin gap(s) between the ground state(s) and first excited state(s) is (are) calculated, corresponding to the magnitude of J_{eff}. The numerical result is plotted in Fig. 4, which indicates that $J_{\text{eff}} \sim 1/(LJ)$. It implies that the characteristic time of quantum state transfer linearly depends on the distance and then guarantees the possibility of realizing the entanglement of two separated qubits in practice.

In order to verify the validity of the effective Hamiltonian H_{eff}, we need to compare the eigenstates of H_{eff} with those reduced states from the eigenstates of the whole system. In general the eigenstates of H can be written formally as

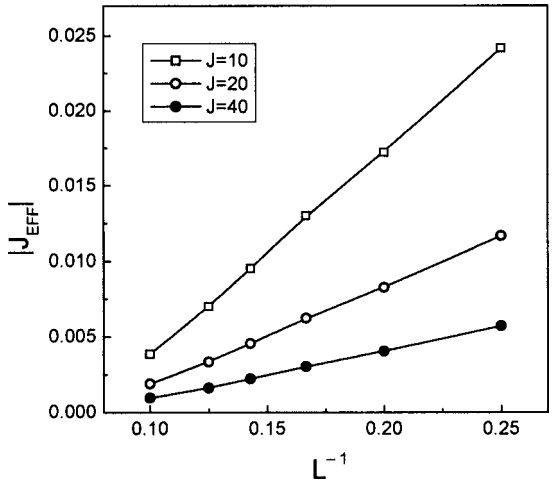

FIG. 4. The spin gaps obtained by numerical method for the systems $L=4,5,6,7,8,10$ with $J=10,20,40$ and $J_0=1$ are plotted, corresponding to the magnitude of J_{eff}. It indicates that $J_{\text{eff}}\sim 1/(LJ)$.

$$|\psi\rangle=\sum_{jm} c_{jm}|j,m\rangle_{AB}\otimes|\beta_{jm}\rangle_M \tag{22}$$

where $\{|\beta_{jm}\rangle_M\}$ is a set of vectors of the data bus, which is not necessarily orthogonal. Then we have the condition $\Sigma_{jm}|c_{jm}|_M^2\langle\beta_{jm}|\beta_{jm}\rangle_M=1$ for normalization of $|\psi\rangle$. In this sense the practical description of the $A-B$ subsystem of two quits can only be given by the reduced density matrix

$$\rho_{AB}=\text{Tr}_M(|\psi\rangle\langle\psi|)=\sum_{jm}|c_{jm}|^2|j,m\rangle_{AB}\langle j,m|$$
$$+\sum_{j'm'\neq jm} c_{j'm'}^* c_{jmM}\langle\beta_{j'm'}|\beta_{j,m}\rangle_M|j,m\rangle_{AB}\langle j',m'| \tag{23}$$

where Tr_M means the trace over the variables of the medium. By a straightforward calculation we have

$$|c_{11}|^2=|c_{1-1}|^2=\left\langle\psi\left|\left(\frac{1}{4}+S_A^z\cdot S_B^z\right)\right|\psi\right\rangle,$$
$$|c_{00}|^2=\left\langle\psi\left|\left(\frac{1}{4}-\mathbf{S}_A\cdot\mathbf{S}_B\right)\right|\psi\right\rangle, \tag{24}$$
$$|c_{10}|^2=1-2|c_{11}|^2-|c_{00}|^2.$$

Now we need a criterion to judge how close the practical reduced eigenstate is to the pure state for the effective two-site coupling H_{eff}. As we noticed, it has the singlet and triplet eigenstates $|j,m\rangle_{AB}$ in the subspace spanned by $|0,0\rangle_{AB}$ with $S^z=S_A^z+S_B^z=0$, we have $|c_{11}|^2=|c_{10}|^2=|c_{1-1}|^2=0$, $|c_{00}|^2=1$; for the triplet eigenstate $|1,0\rangle_{AB}$ we have $|c_{11}|^2$ $=|c_{1-1}|^2=|c_{00}|^2=0$, $|c_{10}|^2=1$. With the practical Hamiltonian H, the values of $|c_{jm}|^2$, $i=1,2,3,4$ are calculated numerically for the ground state $|\psi_g\rangle$ and first excited state $|\psi_1\rangle$ of finite system(s) $L=4,5,6,7,8,10$ with $J=10,20,40$ $(J_0=1)$ in the $S^z=0$ subspace, which are listed in the Table 1a, b, c of Ref. 17. It shows that, at lower temperature, the realistic interaction leads to results for $|c_{jm}|^2$ which are very close to that described by H_{eff}, even if J is not so large in comparison with J_0.

We assert that the above tables reflect all the facts distinguishing the difference between the results about the entanglement of two end qubits generated by H_{eff} and H. Though we have ignored the off-diagonal terms in the reduced density matrix, the calculation of the fidelity $F(|j,m\rangle)\equiv{}_M\langle j,m|\rho_{AB}|j,m\rangle_M=|c_{jm}|^2$ further confirms our observation that the effective Heisenberg type interaction of two end qubits can approximate the realistic Hamiltonian very well. Then the quantum information can be transferred between the two ends of the $2\times N$-site two-leg spin ladder, that can be regarded as the channel to share entanglement with separated Alice and Bob. Physically, this is just due to a large spin gap existing in such a perfect medium, whose ground state can induce a maximal entanglement of the two end qubits. We also pointed out that our analysis is applicable for other types of medium systems as data buses, which possess a finite spin gap. Since $L/|J_{\text{eff}}|$ determines the characteristic time of quantum state transfer between the two qubits, the dependence of J_{eff} upon L becomes important and relies on the appropriate choice of the medium.

In conclusion, we have presented and studied in detail a protocol for quantum state transfer. Numerical results show that the isotropic antiferromagnetic spin ladder system is a perfect medium through which the interaction between two separated spins is very close to the Heisenberg type of coupling, with a coupling constant inversely proportional to the distance, even if the spin gap is not so large compared to the couplings between the input and output spins and the medium.

Spin chain in modulated external magnetic field. Let us consider the Hamiltonian of a $(2N+1)$-site spin-1/2 ferromagnetic Heisenberg chain

$$H=-J\sum_{i=1}^{2N}\mathbf{S}_i\cdot\mathbf{S}_{i+1}+\sum_{i=1}^{2N+1}B(i)S_i^z \tag{25}$$

with the uniform coupling strength $-J<0$, but in the parabolic magnetic field

$$B(i)=2B_0(i-N-1)^2 \tag{26}$$

where B_0 is a constant. In single-excitation invariant subspace with the fixed z component of the total spin, $S^z=N-1/2$, this model is equivalent to the spinless fermion hopping model with the Hamiltonian

$$H=-\frac{J}{2}\sum_{i=1}^{2N}(a_i^\dagger a_{i+1}+\text{h.c.})+\frac{1}{2}\sum_{i=1}^{2N+1}B(i)a_i^\dagger a_i \tag{27}$$

where for simplicity we have neglected a constant in the Hamiltonian. For the single-particle case with the basis set $\{|n\rangle=|0,0,\ldots,1^{n'\text{th}},0,\ldots\rangle,n=1,2,\ldots\}$, which is just the same as that of the Hamiltonian of a Josephson junction in the Cooper-pair number basis[24] for $E_J=J$, $E_c=2B_0$, analytical analysis and numerical results have shown that the lower energy spectrum is indeed quasi-harmonic in the case $E_J\gg E_c$ (Ref. 24). Although the eigenstates of the Hamiltonian (25) do not satisfy the SPMC precisely, especially in the high-energy range, there must exist some Gaussian wave packet states expanded by the lower eigenstates. This kind of state can be transferred with high fidelity.

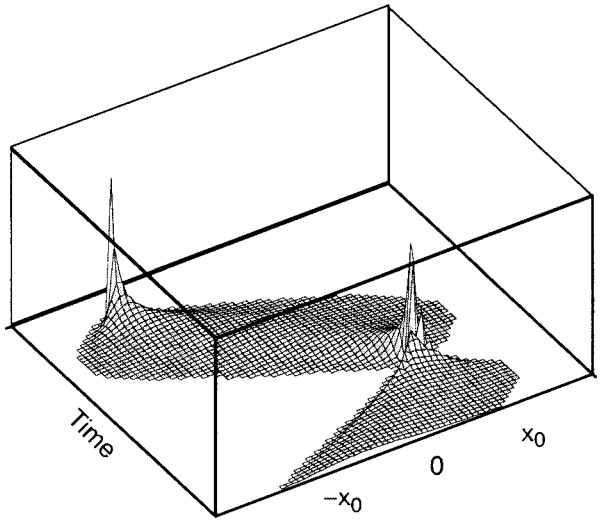

FIG. 5. Schematic illustration of the time evolution of a Gaussian wave packet. It shows that the near-perfect state transfer over a long distance is possible in the quasi-harmonic system.

We consider a Gaussian wave packet at $t=0$, $x=N_A$ as the initial state

$$|\psi(N_A,0)\rangle=C\sum_{i=1}^{2N+1}\exp\left(-\frac{1}{2}\alpha^2(i-N_A-1)^2\right)|i\rangle \tag{28}$$

where $|i\rangle$ denotes the state with $2N$ spins in the down state and only the ith spin in the up state, and C is the normalization factor. The coefficient $\alpha^2=4\ln 2/\Delta^2$ is determined by the width Δ of the Gaussian wave packet. The state $|\psi(0)\rangle$ evolves to $|\psi(t)\rangle=\exp(-iHt)|\psi(N_A,0)\rangle$ at time t, and the fidelity for the state $|\psi(0)\rangle$ transferring to the position N_B is defined as

$$F(t)=|\langle\psi(N_B,0)|\exp(-iHt)|\psi(N_A,0)\rangle|. \tag{29}$$

In Fig. 5 the evolution of the state $|\psi(0)\rangle$ is illustrated schematically. From the investigation of Ref. 25, we know that for small $N_A=-N_B=-x_0$, where N_B is the mirror counterpart of N_A, but in the large Δ limit, if we take $B_0=8(\ln 2/\Delta^2)^2$, $F(t)$ has the form

$$F(t)=\exp\left[-\frac{1}{2}\alpha^2N_A^2\left(1+\cos\frac{2t}{\alpha^2}\right)\right] \tag{30}$$

which is a periodic function of t with the period $T=\alpha^2\pi$ and has maximum of 1. This is in agreement with our above analysis. However, in quantum communication, what we are concerned with is the behavior of $F(t)$ in the case of the transfer distance $L\gg\Delta$, where $L=2|N_A|=2|N_B|$. For this purpose the numerical method is performed for the case $L=500$, $\Delta=2,4,6$ and $B_0=8(\ln 2\Delta^2)^2\lambda$. The factor λ determines the maximum fidelity, and then the optimal field distribution can be obtained numerically. In Fig. 2a, b, c of Ref. 18 the functions $F(t)$ are plotted for different values of λ. It shows that for the given wave packets with $\Delta=2$, 4, and 6 there exists a range of λ within which the fidelities $F(t)$ are up to 0.748, 0.958, and 0.992, respectively. For finite distance, the maximum fidelity decreases as the width of Gaussian wave packet increases. On the other hand, the strength of the external field also determines the value of the optimal fidelity for a given wave packet. There exists an optimal

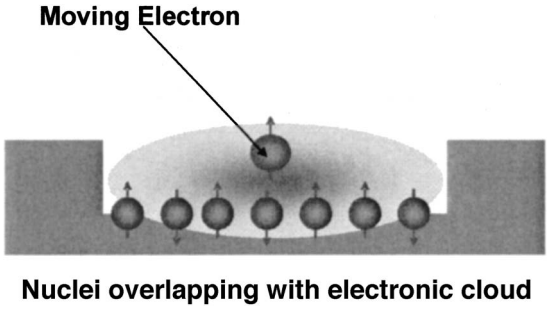

FIG. 6. The electronic spin state onto the collective spin state of the surrounding nuclei ensemble in a quantum well.

external field to obtain maximal fidelity, while the period of $F(t)$ is close to $T=\alpha^2\pi$. This shows a difference from the ideal system, i.e., continuous harmonic systems, in which the fidelity is independent of the strength of the external field. Numerical results indicate that it is possible to realize near-perfect quantum state transfer over a longer distance in a practical ferromagnetic spin chain system.

In summary, we have shown that a perfect quantum transmission can be realized through a universal quantum channel provided by a quantum spin system with spectrum structure, in which each eigenenergy is commensurate and matches with the corresponding parity. According to this SPMC for the a mirror inversion symmetry,[20] we can implement the perfect quantum information transmission with several novel pre-engineered quantum spin chains. For more practical purpose, we prove that an approximately commensurate spin system can also realize near-perfect quantum state transfer in a ferromagnetic Heisenberg chain with uniform coupling constant in an external field. The fidelity for the system in a parabolic magnetic field has been studied by a numerical method. The external field plays a crucial role in the scheme. It induces a lower quasi-harmonic spectrum, which can drive a Gaussian wave packet from the initial position to its mirror counterpart. The fidelity depends on the initial position (or distance L), the width Δ of the wave packet, and the magnetic field distribution $B(i)$ via the factor λ. Thus for given L and Δ, proper selection of the factor λ can achieve the optimal fidelity. Finally, we conclude that it is possible to implement near-perfect Gaussian wave packet transmission over a longer distance in many-body system.

IV. QUANTUM STORAGE BASED ON THE SPIN CHAIN

Recently a universal quantum storage protocol[6–8] was presented to reversibly map the electronic spin state onto the collective spin state of the surrounding ensemble of nuclei in a quantum well (see Fig. 6). Because of the long decoherence time of the nuclear spins, the information stored in them can be robustly preserved.

When all nuclei (with spin operators $I_x^{(i)}$, $I_y^{(i)}$, $I_z^{(i)}$) of spin I_0 are coupled with a single electron spin with strength g_i, a pair of collective operators[9]

$$B=\frac{\Sigma_{i=1}^{N} g_i I_-^{(i)}}{\sqrt{2I_0\Sigma g_j^2}} \tag{31}$$

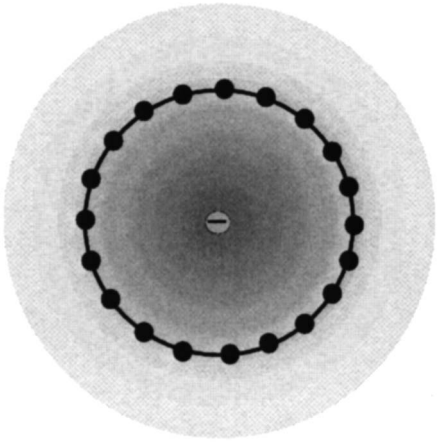

FIG. 7. The configuration geometry of the nuclei-electron system. The nuclei are arranged in a circle within a quantum to form a ring array. To turn on the interaction one can push a single electron towards the center of the circle along the axis perpendicular to the plane.

and its conjugate B^+ are introduced to depict the collective excitations in ensemble of nuclei with spin I_0 from its polarized initial state

$$|G\rangle=|-NI_0\rangle=\prod_{i=1}^{N}|-I_0\rangle_i$$

which denotes the saturated ferromagnetic state of ensemble of nuclei. There is an intuitive argument that if the g_i have different values, while the distribution is "quasi-homogeneous," B and $B^\dagger$ can also be considered as boson operators satisfying $[B,B^+]\to 1$ approximately.

Song, Zhang, and Sun analyzed the universal applicability of this protocol in practice.[9] It was found that only under two homogeneous conditions with low excitations does the many-nuclei system behave approximately as a single-mode boson and can its excitation serve as an efficient quantum memory. The low-excitation condition requires a ground state with all spins oriented, which can be prepared by applying a magnetic field polarizing all spins along a single direction. With consideration of the spontaneous symmetry breaking for all spins oriented, a protocol for a quantum storage element was proposed utilizing a ferromagnetic quantum spin system, instead of the free nuclear ensemble, to serve as a robust quantum memory.

The configuration of the quantum storage element is illustrated in Fig. 7. The nuclei are arranged in a circle within a quantum to form a spin ring array. A single electron is just localized at the center of the ring array, surrounded by the nuclei. The interaction of the nuclear spins is assumed to exist only between the nearest neighbors while the external magnetic field B_0 threads through the spin array. Then the electron-nuclei system can be modeled by a Hamiltonian $H=H_e+H_n+H_{\rm en}$. It contains the electronic spin Hamiltonian $H_e=g_e\mu_B B_0\sigma^z$, the nuclear spin Hamiltonian

$$H_n=g_n\mu_n B_0\sum_{l=1}^{N} S_l^z-J\sum_{l=1}^{N}\mathbf{S}_l\cdot\mathbf{S}_{l+1} \tag{32}$$

with the Zeeman splitting and the ferromagnetic interaction $J>0$, and the interaction between the nuclear spins and the electronic spin

$$H_{\mathrm{en}}=\frac{\lambda}{2N}\sigma^{+}\sum_{l=1}^{N} S_l^{-}+\mathrm{h.c.} \tag{33}$$

Here g_e (g_n) is the Lande g factor of the electron (nuclei), and μ_B (μ_N) is the Bohr magneton (nuclear magneton). The Pauli matrices S_l^{-} and σ^{+} represent the nuclear spin of the lth site and the electronic spin, respectively. The denominator N in Eq. (33) originates from the envelope normalization of the localized electron wave function.[6–8] The hyperfine interactions between nuclei and electron are proportional to the envelope function of localized electron. The electronic wave function is supposed to be cylindrically symmetric, e.g., the s-wave component. Thus the coupling coefficient $\lambda \propto|\psi(\mathbf{r})|^2$ is homogeneous for all the N nuclei in the ring array.

To consider the low spin-wave excitations, the discrete Fourier transformation defines the bosonic operators

$$b_k=\frac{1}{\sqrt{N}}\sum_{l=1}^{N} \exp\left(i\frac{2\pi kl}{N}\right)S^{-l}, \tag{34}$$

in the large N limit. Then one can approximately diagonalize the Hamiltonian (32) as

$$H_T=H_N+\sum_{k=1}^{N-1} \omega_k b_k^{\dagger}b_k$$

where H_N is a Jaynes-Cummings (JC) type Hamiltonian

$$H_N=\omega_N b_N^{\dagger}b_N+\frac{\Omega}{2}\sigma^{z}+\lambda\sqrt{\frac{s}{2N}}(\sigma^{+}b_N+\sigma^{-}b_N^{\dagger}). \tag{35}$$

Then we obtain the dispersion relation for the magnon or spin-wave excitation

$$\omega_k=g_n\mu_n B_0+2Js-2Js\cos\frac{2\pi k}{N}. \tag{36}$$

The above results show that H_T contains only the interaction of the Nth magnon with the electronic spin, and the other $N-1$ magnons decouple with it. Here the frequency of the boson $\omega_N=g_n\mu_n B_0$ and the two-level spacing $\Omega=2g^{*}\mu_B B_0$ can be modulated by the external field B_0 simultaneously.

The process of quantum information storage can be implemented in the invariant subspace of the electronic spin and the Nth magnon. Now we can describe the quantum storage protocol based on the above spin-boson model. Suppose the initial state of the total system is prepared so that there is no excitation in the N nuclei at all, while the electron is in an arbitrary state $\rho_e(0)=\Sigma_{n,m=\pm}\rho_{nm}|n\rangle\langle m|$, where $|+\rangle$ ($|-\rangle$) denotes the electronic spin up (down) state. The initial state of the total system can then be written as

$$\rho(0)=\rho_b(0)\otimes|0_N\rangle\langle 0_N|\otimes\rho_e(0) \tag{37}$$

in terms of $\rho_b(0)=|\{0\}\rangle_{N-1}\langle\{0\}|$, where $|n_1,n_2,\ldots,n_{N-1}\rangle \equiv|\{n_k\}\rangle_{N-1}$ ($k=1,2,\ldots,N-1$) denotes the Fock state of the other $N-1$ magnons. If we set $B_0=0$, at $t=T\equiv(\pi/\lambda)\sqrt{N/2s}$, the time evolution from $\rho(0)$ is just described as a factorized state

$$\rho(T)=\rho_b(0)\otimes w_F\otimes|-\rangle\langle-|, \tag{38}$$

where $w_F=\Sigma_{n,m=0,1}w_{nm}|n_N\rangle\langle m_N|$ is the storing state of the Nth magnon with

$$w_{nm}=\rho_{nm}\exp\left[\frac{i}{2}(m-n)\pi\right]. \tag{39}$$

Here, to simplify our expression, we have denoted $\rho_{++}\equiv\rho_{00}$, $\rho_{\pm}\equiv\rho_{01}$, $\rho_{-+}\equiv\rho_{10}$, $\rho_{-}\equiv\rho_{11}$. The difference between w_F and $\rho_e(0)$ is only an unitary transformation independent of the stored initial state $\rho_e(0)$.

So far we have discussed the ideal case with homogeneous coupling between the electron and the nuclei, that is, the coupling coefficients are the same constant λ for all the nuclear spins. However, the inhomogeneous effect of coupling coefficients has to be taken into account if what our concern extends beyond the s-wave component, where the wave function is not strictly cylindrically symmetric. In this case, the quantum decoherence induced by the so-called quantum leakage has been extensively investigated for the atomic ensemble based quantum memory.[26] We now discuss the similar problems for the magnon-based quantum memory.

In the general case the $\lambda_l\propto|\psi(\mathbf{r}_l)|^2$, where $\psi(\mathbf{r}_l)$ is the envelope function of the electron at site $\mathbf{r}_l$, vary with the positions of the nuclear spins. In this case, the Hamiltonian contains terms other than the interaction between the spin and Nth mode boson, that is, the inhomogeneity induced interaction

$$V=\lambda\sqrt{\frac{s}{2N}}\left(\sigma_{+}\sum_{k=1}^{N-1}\chi_k b_k+\mathrm{h.c.}\right) \tag{40}$$

should be added in our model Hamiltonian H_T, where

$$\chi_k=\sum_{l=1}^{N}\frac{\lambda_l}{\lambda N}\exp(i2\pi kl/N).$$

For a Gaussian distribution of λ_l, e.g., $\lambda_l=(\lambda/\sqrt{2\pi}\sigma)\exp\times(-(l-1)^2/(2\sigma^2))$ with width σ and $\lambda_1=\lambda$, the corresponding inhomogeneous coupling is described by

$$\chi_k=\frac{1}{N}\sum_{l-1=0}^{N-1}\frac{1}{\sqrt{2\pi}\sigma}\exp\left(\frac{-(l-1)^2}{2\sigma^2}+i\frac{2\pi kl}{N}\right). \tag{41}$$

Figure 8 shows the magnitude of χ_k for different Gaussian distributions of λ_l with different widths σ. It indicates that the modes near 1 and $N-1$ have a stronger coupling with the electron. When the interaction gets more homogenous (with larger σ) the coupling coefficients χ_k for all the modes from 1 to $N-1$ become smaller. When the distribution is completely homogeneous, all the couplings with the $N-1$ magnon modes vanish, and then we obtain the Hamiltonian H_T.

In the following we will adopt a rather direct method to analyze the decoherence problem of our protocol resulting from dissipation. If N is so large that the spectrum of the quantum memory is quasicontinuous, this model is similar to the ''standard model'' of quantum dissipation for the vacuum induced spontaneous emission.[27] The $N-1$ magnons will induce the quantum dissipation of the electronic spin with a decay rate

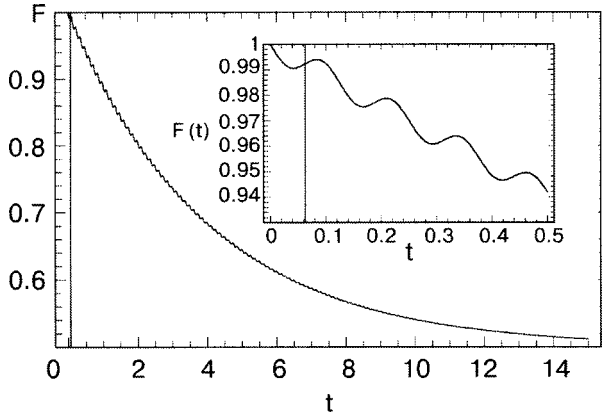

FIG. 8. The fidelity $F(t)$ in the large N limit. The vertical line indicates the instant $\pi/2g$ at which the quantum storage is just implemented. Here $\gamma/g = 1/50$. The inset shows the decaying oscillation with details of $F(t)$ in a small region near the instant $\pi/2g$.

$$\gamma = 2\pi \sum_{k=1}^{N} \frac{\lambda^2 s |\chi_k|^2}{2N} \delta\left(\omega_k - 2\lambda \sqrt{\frac{s}{2N}}\right). \tag{42}$$

Let $|\Psi\rangle$ be the ideal evolution governed by the expected Hamiltonian H_T without dissipation and $|\Psi'\rangle$ be the realistic evolution governed by the Hamiltonian with dissipation. Supposeing that the initial state of the electron is $(|+\rangle + |-\rangle)/\sqrt{2}$, we can analytically calculate the fidelity

$$\begin{aligned} F(t) &= |\langle \Psi | \Psi' \rangle| \\ &= \frac{1}{2}\left[1 + \exp\left(-\frac{\gamma}{2}t\right)\right] \sec\phi (\cos gt \cos(\Delta_1' t + \phi) \\ &\quad + \sin gt \sin \Delta_1' t), \end{aligned} \tag{43}$$

where

$$\phi = \arcsin \sqrt{2N_\gamma^2/\lambda^2 s}, \quad g = \lambda\sqrt{s/2N} \quad \text{and} \quad \Delta_1' = \sqrt{g^2 - \gamma^2}.$$

Figure 8 shows the curve of the fidelity $F(t)$ changing with time t. We can see that the fidelity exhibits a exponential decay behavior with a sinusoidal oscillation. At the instance when we have just implemented the quantum storage process, the fidelity is about $1 - \pi\gamma/8$. Therefore, the deviation from the ideal case with homogeneous couplings is very small for $\gamma/g \ll 1$. Since the ring-shape spin array with inhomogeneous coupling is just equivalent to an arbitrary Heisenberg spin chain in the large N limit, the above arguments means that an arbitrary Heisenberg chain can be used for quantum storage following the same strategy addressed above if γ/g is small, i.e., the inhomogeneous effect is not very strong.

On the other hand, if N is small, the spectrum of the quantum memory is discrete enough to guarantee the adiabatic elimination of the $N-1$ magnon modes, i.e., $\lambda\sqrt{s/2N}\chi_k/|\omega_k| \ll 1$ for the $N-1$ magnon modes. As a consequence of this adiabatic elimination, quantum decoherence or dephasing can result from the mixing of different magnon modes.

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*E-mail: songtc@nankai.edu.cn
†E-mail: suncp@itp.ac.cn

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