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Decoherence in a single trapped ion due to an engineered reservoir

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Abstract

The decoherence in a trapped ion induced by coupling the ion to the engineered reservoir is studied in this paper. The engineered reservoir is simulated by random variations in the trap frequency, and the trapped ion is treated as a two-level system driven by a far-off-resonant plane wave laser field. The dependence of the decoherence rate on the amplitude of the superposition state is given.

Keywords: Engineered reservoir, decoherence

According to quantum mechanics [1], a system can exist in a superposition of distinct states, whereas such superposition states seem not to appear in the macroscopic world. One possible explanation of this paradox [2] is that systems are never completely isolated but interact with the surrounding environment, which contains a large number of degrees of freedom. The environment influences system evolution which continuously decoheres and transforms system superposition into statistical mixtures which behave classically [2, 3]. There are many assumptions involved in modelling the coupling of the system to its environment. For example, the nature of the coupling of a system to its environment is generally taken to be a linear [4] or a nonlinear [5] function of the position operator of the object. Assumptions are also made about the environment. One example is to treat the environment as a reservoir of quantum oscillators, each of which interacts with the quantum system in question. Such an environment is extremely difficult to control due to lack of knowledge about the environment and its coupling to a system. A recent work [6] extends the investigations of the decoherence beyond the ambient reservoirs and engineers the state of the reservoir, as well as the form of the system–reservoir coupling. In the paper, noisy potentials are applied to the trap electrodes to simulate a hot reservoir, the system of the trapped ion then behaves like a controllable reservoir, through which quantum superpositions are decohered into a state which behaves classically.

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In this paper, we present a theoretical study on the decoherence of quantum superpositions of a single trapped ion through coupling to the reservoir which is simulated by a varying trap frequency. We tackle the variations by treating them as a white noise, which results in decoherence of the trapped ion as will be shown. Before introducing an engineered reservoir, we first consider a single $^9Be^+$ ion confined in an rf(Paul) trap interacting with a plane wave laser field. The Hamiltonian of this system may be written as (with $\hbar = 1$)

$$H = \frac{p^2}{2m} + V(r) + \frac{\omega_{eg}}{2}(|e\rangle\langle e| - |g\rangle\langle g|) + \frac{\Omega_L}{2} e^{i\omega_L t - ik_L r}|g\rangle\langle e| + \text{h.c.}, \tag{1}$$

where the first three terms describe the free motion of the ion with two levels $|e\rangle$ and $|g\rangle$ in a trap $V(r) = \frac{1}{2}m\omega^2 r^2$, whereas the last two terms denote the coupling of the ion to the plane wave laser field. Ω_L denotes the coupling constant of the trapped atom to the plane laser wave with frequency ω_L and wavevector k_L. For a trapped ion $^9Be^+$, the two levels $|e\rangle$ and $|g\rangle$ correspond to the hyperfine states $|F = 2, m_F = -2\rangle$ and $|F = 1, m_F = -1\rangle$, respectively. Based on these kind of models, nonclassical motional states such as thermal, Fock, coherent, squeezed and the Schrödinger cat state are created [7, 8], opening up new possibilities of studying the decoherence of quantum superposition [6]. For the plane wave excitation along the x-axis, the motional effects along the y- and z-directions are unperturbed. In other words, the plane laser wave along the x-axis excites only the x-axis freedom. In this case our Hamiltonian equation (1) can be simplified to

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a one-dimensional model

$$\tilde{H} = \frac{p_x^2}{2m} + V(x) + \frac{\delta}{2}(|e\rangle\langle e| - |g\rangle\langle g|) + \frac{\Omega_L}{2}(|e\rangle\langle g| + |g\rangle\langle e|) + \frac{p_x k_{Lx}}{m}(|e\rangle\langle e| - |g\rangle\langle g|), \quad (2)$$

where a unitary transformation

$$U_1 = e^{-\frac{i}{2}(\omega_L t - k_{Lx}x)}|g\rangle\langle g| + e^{\frac{i}{2}(\omega_L t - k_{Lx}x)}|e\rangle\langle e| \quad (3)$$

has been made. In detuning $\delta = \omega_{eg} - \omega_L$, the recoil shift $\frac{k_L^2}{2m}$ is ignored. The last term in equation (2) results from the Doppler effect, which leads to the coupling between the nearest neighbour motional states [9] and the decoherence of a single trapped atomic/ionic qubit [10]. In the following, we concentrate on the situation where the detuning δ and the coupling strength Ω_L is large, notably, where the coupling strength Ω_L is much larger than the coupling of the system to the ambient reservoir, the latter condition is relevant to the engineered environment experiment in which the ambient environment that leads to natural decoherence is negligible [6]. Although the coupling strength (∼kHz) is large, it is smaller than the trap frequency (∼MHz) so that the internal spin dynamics and the external motion of the ion in the trap occur on two different timescales. Therefore, it is useful to go to a rotating frame that eliminates the fourth term in equation (2), which describes the Rabi oscillations between the two internal states. In the rotating frame, we will be able to understand more clearly how the last term in equation (2), which couples the motional and the internal dynamics, leads to decoherence when the trap frequency ω is modulated randomly.

We go to the rotating frame by making a unitary transformation

$$U_2 = e^{-i(\frac{\Omega_L}{2}\sigma_x + \frac{\delta}{2}\sigma_z)t}, \quad (4)$$

where $\sigma_x = |g\rangle\langle e| + |e\rangle\langle g|$ and $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ are Pauli matrices. In the rotating frame, the system evolves according to

$$i\frac{\partial}{\partial t}|\psi^I(t)\rangle = H_I(t)|\psi^I(t)\rangle, \quad (5)$$

where $H_I(t)$ is given by

$$H_I(t) = \frac{p_x^2}{2m} + V(x) + \frac{p_x k_{Lx}}{m}(\alpha_x(t)\sigma_x + \alpha_y(t)\sigma_y + \alpha_z(t)\sigma_z). \quad (6)$$

The time-varying coefficients $\alpha_x(t), \alpha_y(t), \alpha_z(t)$ are given by [11]

$$\alpha_x(t) = \frac{\Omega_L\delta}{\Omega_e^2}(1 - \cos(\Omega_e t)),$$

$$\alpha_y(t) = \frac{\Omega_l}{\Omega_e}\sin(\Omega_e t),$$

$$\alpha_z(t) = \frac{\delta^2}{\Omega_e^2} + \frac{\Omega_L^2}{\Omega_e^2}\cos(\Omega_e t),$$

where $\Omega_e = \sqrt{\Omega_L^2 + \delta^2}$. In what follows, we make two simplifications in order to extract the dominant behaviours of the system. Noticing the coefficients $\alpha_i(t), i = x, y, z$ oscillate rapidly, we expect the system in the rotating frame to evolve on a much slower timescale than the period $2\pi/\Omega_e$. In this sense we can simplify the Hamiltonian $H_I(t)$ given in equation (6) by taking average values of the coefficients $\alpha_i(t), i = x, y, z$, this is equivalent to coarse graining equation (6). The coefficients $\alpha_i(t)$ then become time independent and are reduced to $\alpha_x = \frac{\delta\Omega_L}{\Omega_e^2}$, $\alpha_y = 0$, $\alpha_z = \frac{\delta^2}{\Omega_e^2}$. Furthermore, we make the assumption that the system is being driven far-off-resonance, i.e., $\delta \gg \Omega_L$. We therefore have

$$H_I = \frac{p_x^2}{2m} + V(x) + \frac{p_x k_{Lx}}{m}\frac{\delta^2}{\Omega_e^2}\sigma_z, \quad (7)$$

and we note $p_x = i\sqrt{\frac{m\omega}{2}}(a^\dagger - a)$ with $a(a^\dagger)$ the annihilation (creation) operator for the motional state $|n\rangle$ which satisfies $(\frac{p_x^2}{2m} + V(x))|n\rangle = \omega(n + \frac{1}{2})|n\rangle$, therefore the Hamiltonian (7) can be rewritten as

$$H_I = \omega a^\dagger a + ih\sigma_z(a^\dagger - a), \quad (8)$$

where $h = \sqrt{\frac{\omega}{2m}}\frac{\delta^2}{\Omega_e^2}k_{Lx}$. H_I couples the nearest neighbours of the motional state $|n\rangle$ with a same internal level $|g\rangle$ or $|e\rangle$. In the following we will show that the second term in H_I leads to decoherence while the reservoir is applied. To show this, we first show the time evolution operator in the rotating frame

$$U_I(t) = e^{-i\omega a^\dagger a t}(|g\rangle\langle g|e^{f(t)}e^{A(t)a^\dagger}e^{B(t)a} + |e\rangle\langle e|e^{f(t)}e^{-A(t)a^\dagger}e^{-B(t)a}), \quad (9)$$

where

$$A(t) = \frac{ih}{\omega}(e^{i\omega t} - 1), \qquad B(t) = -A^*(t),$$

$$f(t) = -i\frac{h^2}{\omega}t + \frac{h^2}{\omega^2}(1 - e^{-i\omega t}).$$

We next consider an initial state of the form in the rotating frame

$$|\psi^I(0)\rangle = c_g|g\rangle \otimes |\alpha_g\rangle + c_e|e\rangle \otimes |\alpha_e\rangle, \quad (10)$$

where $|\alpha_i\rangle (i = g, e)$ denotes a coherent state, c_g and c_e are constants satisfying $|c_g|^2 + |c_e|^2 = 1$. This kind of state may be created during Raman transitions [7]. Equations (3), (4) and (9) together govern the evolution of the system. With these equations, we can analytically evolve the initial state (10) to obtain

$$|\psi(t)\rangle = e^{\frac{i}{2}\omega_L t - i\omega a^\dagger a t}(\alpha_1(t)c_g(t)|\alpha_g^-(t)\rangle + \alpha_2(t)c_e(t)|\alpha_e^-(t)\rangle) \otimes |g\rangle + e^{-\frac{i}{2}\omega_L t - i\omega a^\dagger a t}(\alpha_2(t)c_g(t)|\alpha_g^+(t)\rangle + \alpha_1^*(t)c_e(t)|\alpha_e^+(t)\rangle) \otimes |e\rangle, \quad (11)$$

where

$$\alpha_1(t) = \cos\frac{\Omega_e}{2}t - \frac{i\delta}{\Omega_e}\sin\frac{\Omega_e}{2}t, \qquad \alpha_2(t) = -i\frac{\Omega_L}{\Omega_e}\sin\frac{\Omega_e}{2}t,$$

$$c_g(t) = c_g e^{f(t) - \frac{1}{2}|A(t)|^2},$$

$$c_e(t) = c_e e^{f(t) - \frac{1}{2}|A(t)|^2},$$

$$\alpha_g^{\pm}(t) = \alpha_g + A(t) \pm \frac{i}{2}k_{Lx}\sqrt{\frac{1}{2m\omega}}e^{-i\omega t},$$

$$\alpha_e^{\pm}(t) = \alpha_e - A(t) \pm \frac{i}{2}k_{Lx}\sqrt{\frac{1}{2m\omega}}e^{-i\omega t}.$$

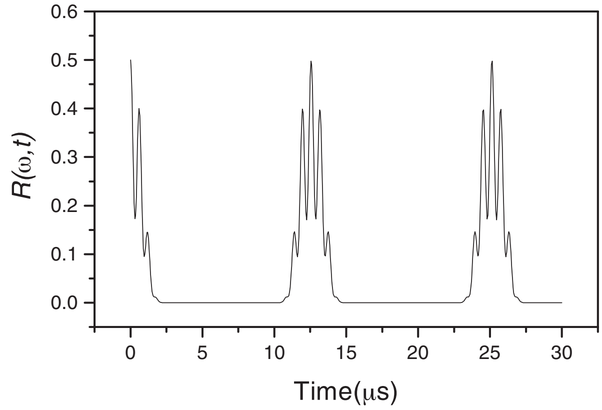

Figure 1. The module $R(\omega, t)$ of the off-diagonal element of the density operator is plotted as a function of time with a fixed trap frequency. The parameters chosen are $\alpha_g = \alpha_e = 3$, $\omega = 2\pi \times 11.3$ MHz, $\delta = 4$ GHz, $\Omega_L = 10$ kHz.

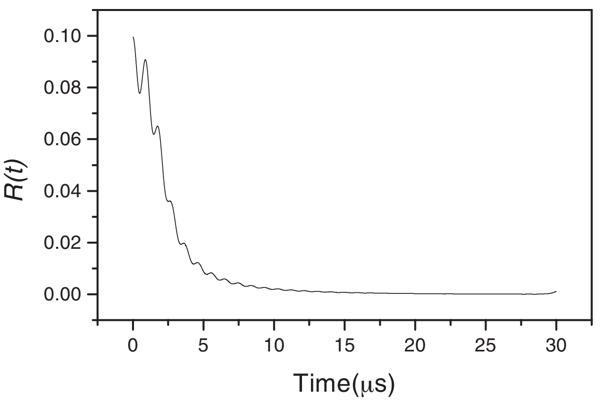

Figure 2. This plot shows the decoherence in the trapped ion induced by the reservoir. The module $R(t)$ of the off-diagonal element of the density operator is calculated by the master equation. The parameters chosen are the same as in figure 1 and $\Gamma = 1$ kHz.

Equation (11) is the main result of our study, and by using it we can explain the essential properties of the system. As equation (11) shows, the initial motional states $|\alpha_g\rangle$ and $|\alpha_e\rangle$ are displaced to become $|\alpha_g^{\pm}(t)\rangle$ and $|\alpha_e^{\pm}(t)\rangle$, respectively. Their displacements depend on the internal state of the trapped ion. We are now interested in the coefficient of the off-diagonal element of the density operator $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$ in the ionic internal space and its module which represents qualitatively decoherence of the system is

$$\begin{aligned}R(\omega, t) = \mathrm{Mod}\{&\alpha_1^*(t)\alpha_2(t)|c_g|^2\langle\alpha_g^-(t)|\alpha_g^+(t)\rangle \mathrm{e}^{-\mathrm{i}\omega\alpha_g^+(t)\alpha_g^{-*}(t)t}\\ &+\alpha_1^{*2}(t)c_g^* c_e\langle\alpha_g^-(t)|\alpha_e^+(t)\rangle \mathrm{e}^{-\mathrm{i}\omega\alpha_e^+(t)\alpha_g^{-*}(t)t}\\ &+|\alpha_2(t)|^2 c_e^* c_g\langle\alpha_e^-(t)|\alpha_g^+(t)\rangle \mathrm{e}^{-\mathrm{i}\omega\alpha_e^-(t)\alpha_g^{+*}(t)t}\\ &+\alpha_1^*(t)\alpha_2^*(t)|c_e|^2\langle\alpha_e^-(t)|\alpha_e^+(t)\rangle\\ &\times \mathrm{e}^{-\mathrm{i}\omega\alpha_e^-(t)\alpha_e^{+*}(t)t}\}\mathrm{e}^{-4\frac{h^2}{\omega^2}(1-\cos\omega t)}\end{aligned} \tag{12}$$

where $\mathrm{Mod}\{\cdots\}$ denotes the module of the term in the braces. We take a special case $\alpha_g = \alpha_e = 2$ with ω fixed at $\omega = 2\pi \times 11.3$ MHz as an example to illustrate $R(\omega, t)$ versus time shown in figure 1. As figure 1 shows, $R(\omega, t)$ is a periodic function of time t, whose period depends on the chosen parameters. During the first few Rabi cycles, the coherent state parameters $\alpha_g^{\pm}(t) \simeq \alpha_g$ $\alpha_e^{\pm}(t) \simeq \alpha_e$, so that the internal and external degrees of freedom appear to be decoupled and the system simply oscillates rapidly between its internal states as shown in the first three peaks of figure 1. However, for long timescales ($t \gg 1/\omega_{eg}$), the coupling between the internal and external states results in a modulation of the Rabi oscillations. Up to now, the engineered reservoir is not in place and the results as illustrated in figure 1 indicate that without the engineered reservoir no decoherences occur in the system.

An engineered reservoir coupled to the trapped ion is simulated by variations in the trap frequency, oscillating near the ion's original trapped frequency. Physically, decoherence in this case arises from random perturbations of the Hamiltonian. In what follows, we want to model the effects of the variations in the trap frequency with the same formalism as in [12]. Our main idea is to treat the variations in the trap frequency as fluctuations. For an ion in a linear Paul trap [6], Wineland and coworkers achieved variations using a random voltage noise source applied to the trap electrodes, where the noise source was passed through a low-pass filter network with a cut-off frequency well below the trap frequency. The atom then 'sees' a harmonic potential with a fluctuating spring constant. The Hamiltonian for a trapped ion in a harmonic potential with a fluctuating spring constant in the rotating frame is

$$H_I = \frac{p_x^2}{2m} + V(x) + \frac{p_x k_{Lx}}{m}\frac{\delta^2}{\Omega_e^2}\sigma_z + \frac{1}{2}m\omega^2\varepsilon(t)x^2. \tag{13}$$

This Hamiltonian is just equation (7) plus a term which describes the fluctuations in the trap frequency. If we take the fluctuations to be a white noise, i.e.

$$\varepsilon(t)\,\mathrm{d}t = \sqrt{\Gamma}\,\mathrm{d}W(t), \tag{14}$$

and set

$$X = \sqrt{\frac{m\omega}{2}}x, \qquad P_x = (2m\omega)^{-\frac{1}{2}}p_x,$$

the Hamiltonian (14) becomes

$$\begin{aligned}H &= \omega(P_x^2 + X^2) + hP_x\sigma_z + \sqrt{\Gamma}\omega X^2\,\mathrm{d}W(t)\\ &= H_0 + \sqrt{\Gamma}\omega X^2\,\mathrm{d}W(t).\end{aligned} \tag{15}$$

Here, $\mathrm{d}W(t)$ is the increment of a real Wiener process [13], h is as before and Γ scales the fluctuations. For a single run with a known behaviour of the fluctuations in time, we use a stochastic Schrödinger equation in the Ito formalism [14]

$$\begin{aligned}\mathrm{d}\rho_I(t) = &-\frac{\mathrm{i}}{\hbar}[H_0, \rho_I]\,\mathrm{d}t - \frac{\mathrm{i}\sqrt{\Gamma}}{\hbar}[X^2, \rho_I]\,\mathrm{d}W(t)\\ &-\frac{\Gamma}{2}[X^2[X^2, \rho_I]]\,\mathrm{d}t\end{aligned} \tag{16}$$

to describe the time evolution of the density operator ρ_I in the rotating frame. Here, we are not interested in the effects of the fluctuation over a short timescale and so we may take an average over the fluctuation to get the master equation for the average density operator ρ^a:

$$\frac{\mathrm{d}\rho^a}{\mathrm{d}t} = -\mathrm{i}[H_0, \rho^a] - \Gamma[X^2[X^2, \rho^a]]. \tag{17}$$

We want to determine the off-diagonal element of the density operator $\rho = U_2\rho^a U_2^\dagger$ in the internal space spanned by $|e\rangle$ and $|g\rangle$, where U_2 is given by equation (4). To do this, we first derive a system of equations for $\rho_{ij}^a = \langle i|\rho^a|j\rangle (i, j = g, e)$:

$$\frac{\mathrm{d}\rho_{ge}^a}{\mathrm{d}t} = 2\mathrm{i}hP_x\rho_{ge} - \frac{\Gamma^2}{2}\omega^2[X^2[X^2, \rho_{ge}^a]],$$
$$\frac{\mathrm{d}\rho_{eg}^a}{\mathrm{d}t} = -2\mathrm{i}hP_x\rho_{eg} - \frac{\Gamma^2}{2}\omega^2[X^2[X^2, \rho_{eg}^a]], \tag{18}$$
$$\frac{\mathrm{d}\rho_{ii}^a}{\mathrm{d}t} = -\frac{\Gamma^2}{2}\omega^2[X^2[X^2, \rho_{ii}^a]] \qquad (i = e, g).$$

It is easy to show that the off-diagonal element of the density operator ρ can be represented as

$$\langle g|\rho|e\rangle = \alpha_1^*(t)\alpha_2^*(t)\rho_{gg}^a(t) + [\alpha_1^*(t)]^2\rho_{ge}^a(t) + |\alpha_2(t)|^2\rho_{eg}^a(t) + \alpha_1^*(t)\alpha_2(t)\rho_{ee}^a(t). \tag{19}$$

Over a short timescale ($\Gamma t \ll 1$), $\langle n|\rho_{ij}^a(t)|m\rangle \sim 0$, for $m \neq n$, the module $R(t)$ of the off-diagonal element which characterizes the decoherence is given by tracing over the external degree of freedom

$$R(t) = \mathrm{Mod}\{\mathrm{Tr}_e\langle g|\rho|e\rangle\} \tag{20}$$
$$= \mathrm{Mod}\Bigg\{\sum_{n=0}^{\infty}[\alpha_1^*(t)\alpha_2^*(t)\langle n|\rho_{gg}^a(0)|n\rangle + [\alpha_1^*(t)]^2\langle n|\rho_{ge}^a(0)|n\rangle + |\alpha_2(t)|^2\langle n|\rho_{eg}^a(0)|n\rangle + \alpha_1^*(t)\alpha_2(t)\langle n|\rho_{ee}^a(0)|n\rangle]\mathrm{e}^{-\Gamma\omega^2(n^2+n+1)t}\Bigg\}, \tag{21}$$

where $\alpha_1(t)$, $\alpha_2(t)$ and $|n\rangle$ are the same as mentioned earlier. As equation (21) shows, the decoherence rate $\Gamma\omega^2(n^2 + n + 1)$ depends on the character of the fluctuations, the trap frequency and the motional state of the trapped ion. Physically, the trap frequency plays the role of coupling the system to the reservoir, so the larger the trap frequency, the larger the decoherence rate (decay rate). The fact that the decoherence rate depends on the motional state of the trapped ion was observed in [7]. Again, we consider the state given by equation (10) to be an initial state. The numerical results of equation (20) are illustrated in figure 2 and it can be seen that there are a few Rabi cycles initially. However, for longer timescales the oscillations disappear, it is evident that decoherence occurs in the system. A single $^9\mathrm{Be}^+$ ion trapped in a linear Paul trap is suitable for this modelled experiment. As described in [15], the ion is cooled using sideband cooling with stimulated Raman transition between the $^2\mathrm{S}_{1/2}(F = 2, m_\mathrm{F} = -2)$ and $^2\mathrm{S}_{1/2}(F = 1, m_\mathrm{F} = -1)$ hyperfine states, which we denoted by $|e\rangle$ and $|g\rangle$, respectively. Far-off-resonance is applied to drive the transition between $|e\rangle$ and $|g\rangle$ (in a real experiment, this process may be achieved with the help of other hyperfine levels). Meanwhile, random uniform electric fields are applied along the axis of the trap. The measurement of the transition between $|e\rangle$ and $|g\rangle$ will show a decoherence in the trapped ion system. At the end of this paper, we would like to note that the far-off-resonance laser wave plays an important role in the decoherence process, without which we could not achieve decoherence. Mathematically, in the case where there is no far-off-resonance laser wave, equation (17) becomes

$$\frac{\mathrm{d}\rho^a}{\mathrm{d}t} = -\mathrm{i}[H_n, \rho^a] - \Gamma[X^2[X^2, \rho^a]], \tag{22}$$

where

$$H_n = \frac{p_x^2}{2m} + V(x) + \frac{\omega_{eg}}{2}\sigma_z.$$

It is obvious from equation (22) that the final (at time t) density matrix has the following form:

$$\rho_a(t) = \rho_e^a(x)|e\rangle\langle e|, \tag{23}$$

if we prepare initially the trapped atom in state $|e\rangle$, equation (23) indicates that there is no transition between the internal states $|e\rangle$ and $|g\rangle$ at any time. This is not relevant to the experiments conducted recently [6].

To sum up, decoherence in a two-level trapped ion is studied in this paper. The decoherence is induced by coupling the ion to the reservoir, which is simulated by random variations in the trap frequencies. Without this reservoir, the transitions between the ionic internal levels manifest modulated Rabi transitions, whereas the transition was suppressed when the engineered reservoir was in place. The suppressed transitions indicate that there is decoherence in the trapped ion system.

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Note added. When this paper was being revised the authors became aware of the paper written by Turchette *et al* [16], who have studied the decoherence of a trapped ion from another aspect, i.e., by treating the reservoir as a set of harmonic oscillators or as a varying variable.

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