

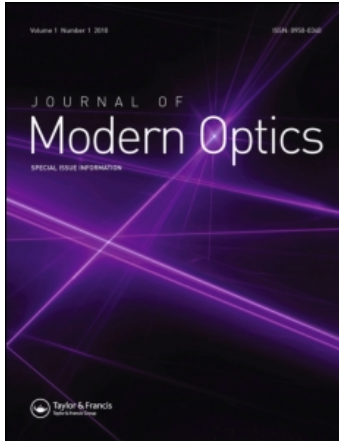
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Effects of a moving mass centre on atomic dynamics in locally inhomogeneous quantized cavity field

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Abstract. By invoking an exactly solvable model as the generalization of the Jaynes-Cummings model, the influence of spatial motion of atomic centre on the dynamics of a single-mode cavity-two-level atom system are studied for various initial conditions. These investigations show that the Doppler effect exercised by the motion of atom in a locally inhomogeneous cavity field can lead to the phenomenon of oscillation collapse and revival in the transition probability and the atomic population inversion.

Recently, new developments in the techniques of cavity quantum electrodynamics have made it possible to control the dynamical behaviour of atoms constrained in a resonant cavity with an extremely small size comparable with the wavelength of atomic emissions [1, 2]. It has been manifested in both experimental and theoretical investigations that, in a cavity with such a small, many distinctive phenomena of atomic motion appear to be very attractive to either the interest in fundamental theory or the advances in new techniques. These phenomena mainly concern the suppression and enhancement of atomic spontaneous radiation by the cavity [1], the deflection of atoms in connection with the potential realization of atomic interferometers [3-5], and cooling atoms by an adiabatically decaying cavity [6-8]. It is obvious that, behind the above-mentioned features of the atom-field interaction system, the effect of the spatial motion of the atomic centre on the 'internal energy level' plays a crucial role through an inhomogeneous cavity field on the scale of an atom.

To study such effects of spatial motion analytically, one should first pay attention to the simplest case, namely the two-level atom interacting with a single-mode cavity field. For this case the famous Jaynes-Cummings (J-C) model is certainly a good candidate for describing the dynamics of the atom-cavity system. Note that the original J-C model [9] (for a review see [10]) characterizes only direct interaction between the atomic levels and the cavity field by assuming that the atom only 'sees' a homogeneous cavity on its own scale. However, motivated by the above considerations about the atomic and optical techniques and its consequent theoretical developments, this paper incorporates the original J-C atomic dynamics with a spatial coupling to construct an exactly solvable model describing the back action of the atomic mass centre on both the cavity field and the internal levels. By solving the evolution equation of this model analytically, some new dynamical features of the atom-cavity system caused by the spatial motion of the mass centre, such as the phenomenon of oscillation collapse and

revival in atomic transition probability and population inversion, are investigated for the cases with intensity-dependent and intensity-independent couplings.

The Hamiltonian corresponding to a two-level atom interacting with one mode of the cavity electromagnetic field $E(x) \propto a \exp(ikx) + a^+ \exp(-ikx)$ is given under the rotating-wave approximation [11]:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{\hbar\omega_0}{2} (|2\rangle\langle 2| - |1\rangle\langle 1|) + \hbar\omega a^+ a + \hbar g(a^+ |1\rangle\langle 2| \exp(-ikx) + a |2\rangle\langle 1| \exp(ikx)), \quad (1)$$

where \hat{p} is the operator of the momentum conjugate to the position x along the propagating axis of the cavity field, a and a^+ denote the annihilation and creation operators of the bosonic cavity mode with frequency ω , $|1\rangle$ and $|2\rangle$ are the energy eigenstates (ground and excited states) with the energy difference $\hbar\omega_0$, and k is the wavenumber for the running wave of the cavity. It should be noted that a Hamiltonian similar to equation (1) has been used by Sleator and Wilkens [12] to study the quantum non-destructive measurement of atomic momentum through the adiabatic solution. Note that there may be a second counterpropagating running-wave mode with wave-vector $-k$ in such a ring cavity, which must influence the atom-field dynamics and contributes to the interaction with a term

$$\hbar g(a^+ |1\rangle\langle 2| \exp(ikx) + a |2\rangle\langle 1| \exp(-ikx)).$$

However, in the same circumstance as in [12], we consider that an atom only traverses an arm of the optical ring cavity and the cavity mode is excited by an external laser. In this sense, it is reasonable to consider a special case only with a running wave along one direction of the optical axis. Now, let us try to find an exact solution for the dynamical evolution of the atom-cavity system governed by the Hamiltonian (1). To this end, we first factorize the evolution operator for \hat{H} into a product as

$$\hat{U}(t) = \hat{W}(x) \hat{U}_e(t) \hat{W}^+(x), \quad (2)$$

where the first factor as an unitary transformation,

$$\hat{W}(x) = \exp\left(\frac{ikx}{2}\right) |2\rangle\langle 2| + \exp\left(-\frac{ikx}{2}\right) |1\rangle\langle 1|, \quad (3)$$

concerns the coupling of the internal level with the spatial degree of freedom of the atom in the cavity. The other factor $U_e(t)$ to be determined is easily proved to satisfy an effective Schrödinger equation governed by an effective Hamiltonian

$$\begin{aligned} \hat{H}_e &= \hat{H}_0 + \hat{H}_1, \\ \hat{H}_0 &= \frac{\hat{p}^2}{2m} + \frac{\hbar^2 k^2}{8m}, \\ \hat{H}_1 &= \frac{\hbar\Omega}{2} (|2\rangle\langle 2| - |1\rangle\langle 1|) + \hbar\omega a^+ a + \hbar g(a^+ |1\rangle\langle 2| + a |2\rangle\langle 1|). \end{aligned} \quad (4)$$

Obviously, the above effective Hamiltonian contains two commuting parts: the effective translated kinetics \hat{H}_0 and the J-C Hamiltonian \hat{H}_1 . However, what we must emphasize is the effective energy difference operator

$$\hat{\Omega} = \Omega(\hat{p}) = \omega_0 + \frac{\hat{p}k}{m}, \tag{5}$$

which modifies the original operator by a detuning dependent on the momentum operator. This will result precisely in the Doppler effect.

If it is borne in mind that $\hat{\Omega} = \Omega(\hat{p})$ is an operator commuting with \hat{p} , the J-C Hamiltonian can be diagonalized in an invariant subspace spanned by $|n+1, 1\rangle = |n+1\rangle \otimes |1\rangle$, and $|n, 2\rangle = |n\rangle \otimes |2\rangle$, where $|n\rangle$ is the Fock state of cavity. Then, we obtained operator-valued J-C eigenstates [9]

$$\begin{aligned} |\chi_+, n\rangle &= \cos\left(\frac{\alpha_n}{2}\right) |n\rangle \otimes |2\rangle + \sin\left(\frac{\alpha_n}{2}\right) |n+1\rangle \otimes |1\rangle, \\ |\chi_-, n\rangle &= \sin\left(\frac{\alpha_n}{2}\right) |n\rangle \otimes |2\rangle - \cos\left(\frac{\alpha_n}{2}\right) |n+1\rangle \otimes |1\rangle, \end{aligned} \tag{6}$$

with the eigenvalues (operator valued)

$$E_{\pm} = (n + \frac{1}{2})\hbar\omega \pm \hbar A_n(\hat{p}), \tag{7}$$

where

$$\begin{aligned} A_n(\hat{p}) &= \left(\frac{\hat{\Delta}^2}{4} + g^2(n+1)\right)^{1/2}, \\ \hat{\Delta} = \Delta(\hat{p}) &= \Omega(\hat{p}) - \omega, \\ \alpha_n = \alpha_n(\hat{p}) &= \tan^{-1}\left(\frac{2g(n+1)^{1/2}}{\hat{\Delta}}\right). \end{aligned} \tag{8}$$

Note that the above operator-valued objects $f(\hat{p})$, for example the states $|\chi_{\pm}, n\rangle$ or the eigenvalues, should take their own eigenvalues $f(p)$ on the eigenstate $|p\rangle$ of operator \hat{p} . An exact solution immediately follows from the above observation as

$$\begin{aligned} |\psi(t)\rangle &= \int dp \sum_{n=0}^{\infty} \exp\left(\frac{-i\hbar^2 k^2 t}{8m}\right) \left[\left(\exp\left(\frac{-i(p - \hbar k/2)^2 t}{2m}\right) \exp[-i(n + \frac{1}{2})\omega t] C_2(p, n) \right. \right. \\ &\quad \times \left\{ \left[\cos^2\left(\frac{\alpha'_n}{2}\right) \exp(iA'_n t) + \sin^2\left(\frac{\alpha'_n}{2}\right) \exp(iA'_n t) \right] |n, 2\rangle \otimes |p\rangle \right. \right. \\ &\quad \left. \left. - i \sin \alpha'_n \sin(A'_n t) |n+1, 1\rangle \otimes |p - \hbar k\rangle \right\} \right) \\ &\quad + \left(\exp\left(\frac{-i(p + \hbar k/2)^2 t}{2m}\right) \exp[-i(n - \frac{1}{2})\omega t] C_1(p, n) \right. \\ &\quad \times \left\{ \left[\cos^2\left(\frac{\alpha''_n}{2}\right) \exp(iA''_n t) + \sin^2\left(\frac{\alpha''_n}{2}\right) \exp(-iA''_n t) \right] |n+1, 1\rangle \otimes |p\rangle \right. \\ &\quad \left. \left. - i \sin \alpha''_n \sin(A''_n t) |n, 2\rangle \otimes |p + \hbar k\rangle \right\} \right) \Big]. \end{aligned} \tag{9}$$

The atom-cavity system dressed by the spatial degree of freedom is in general initial states at $t=0$:

$$|\psi(0)\rangle = \left(\sum_{n=0}^{\infty} B_2(n)|n, 2\rangle + B_1(n)|n+1, 1\rangle \right) \otimes \int dp C(p)|p\rangle, \quad (10)$$

where $C(p)$ is the probability amplitude for the centre of mass in the momentum representation. As follows, we let $C_2(n, p) = B_2(n)C(p)$, $C_1(n, p) = B_1(n)C(p)$. Note that the differences between the functions f, f' and f'' ($f = \alpha_n, A_n, \Omega_n$ and so on) are the momentum shifts, that is $f' = f(p - \hbar k/2)$, $f'' = f(p + \hbar k/2)$, and there are momentum shifts $\pm \hbar k$ for the ground and excited states. Physically, a mixture of ground and excited states should be split by the inhomogeneous cavity field into three beams when the beam enters the field along the direction perpendicular to x or into three purses in momentum space when along the x direction.

Knowing the above exact solution, we are in the position to study the dynamical effect of spatial motion of mass centre. One is the population inversion of the two levels and the other the transition probability. In principle, to complete the definite description of dynamics of a generalized J-C model, one must specify various initial conditions, which correspond to different statistical properties of the single-mode field and different initial preparations of the atom. It is naturally expected that these differences in initial states can result in marked differences in the dynamical behaviours of the cavity-atom system.

Let us first consider the case with an initial state in which there are exactly m photons and the momentum of mass centre for the atom in excited state obeys the Gaussian distribution

$$|C(p)|^2 = \left(\frac{2a^2}{\pi} \right)^{1/2} \exp \left(\frac{-2a(p-p_0)^2}{\hbar^2} \right). \quad (11)$$

In this case, the initial conditions are

$$\begin{aligned} C_2(p, m) &= \delta_{mn} C(p), \\ C_1(p, m) &= 0, \end{aligned} \quad (12)$$

and the exact solution (9) immediately gives the probability as

$$\begin{aligned} P(|m, 2\rangle \rightarrow |m+1, 1\rangle) &= \int dp |C(p)|^2 \left\{ \frac{g^2(m+1)}{\Delta^2/4 + g^2(m+1)} \sin^2 \right. \\ &\quad \left. \times \left[\left(g^2(m+1) + \frac{\Delta^2}{4} \right)^{1/2} t \right] \right\}, \end{aligned} \quad (13)$$

with the atom in the ground state dressed by $m+1$ quanta. Here

$$\Delta = \Delta(p) = \omega_0 - \omega + \frac{pk}{m} - \frac{\hbar k^2}{2m}. \quad (14)$$

Note that, if the initial momentum is a definite p , then the ground-state component of the atomic evolution state has a momentum shift $-\hbar k$. This shift may lead to splitting of the atomic beam or purse. This exact result shows that, except for the original detuning $\omega_0 - \omega$, two additional terms are added, one of which is the

Doppler shift pk/m and the other the frequency shift $-\hbar k^2/2m$ associated with the recoil of the atom with mass m .

Because the Doppler effect modifies the effective oscillation frequency, the transition between the ground and the excited state is very sensitive to the initial distribution of momentum. Figure 1 shows the behaviour of an initially excited two-state atom subsequent to interacting with a single-mode cavity field with definite photon number under the influence of the motion of the atomic mass centre. In this and all subsequent graphs, we assume that $\omega - \omega_0 - \hbar k^2/2m = 0$ (near-resonant case), $k/m = 1$. So, from equation (14), we have $\Delta = p$, $g = 1$, and $p_0 = 2$. Note that the infinitely large a means a zero width of the wave packet in momentum space and that the atom has a definite momentum. In this case, the probability shows a complete oscillation without explicit appearance of the Doppler effects, such as that shown in figure 1 (a) (with large a). As the breadth

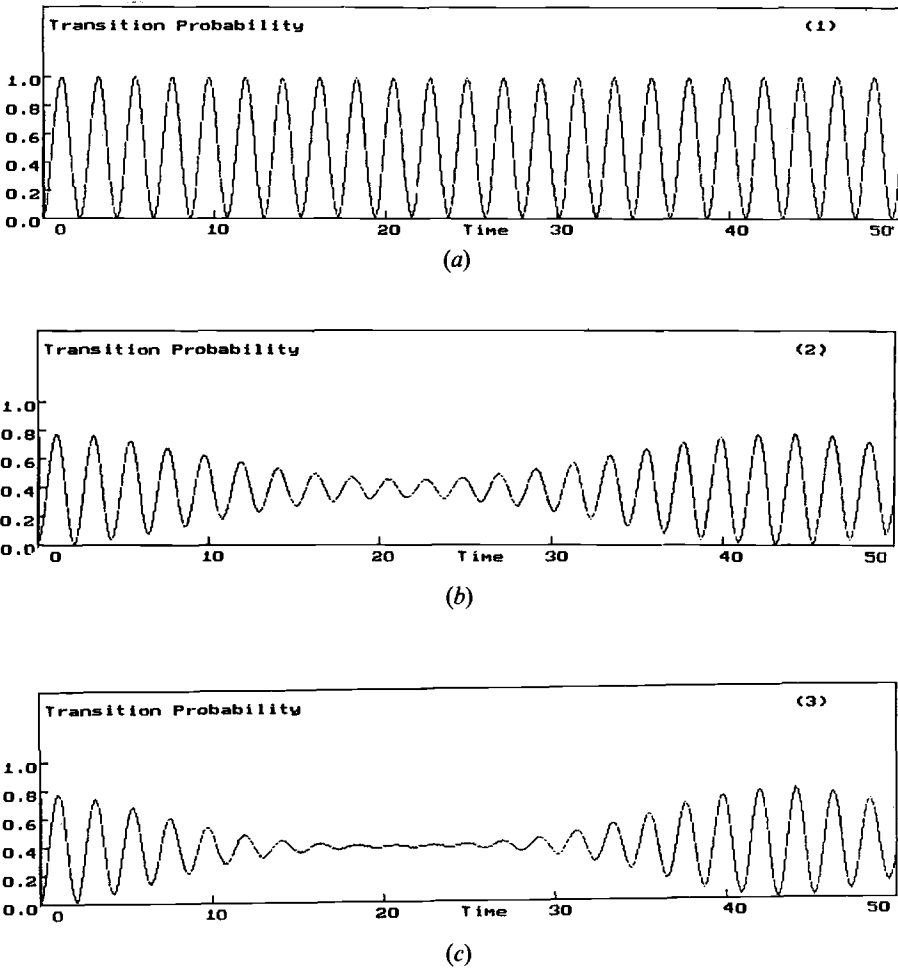


Figure 1. Transition probabilities for a cavity field which has a definite photon number and an initial momentum of atomic mass centre which has a Gaussian distribution (we assume that the photon number $m = 0$; here $\Delta = p$, $g = 1$ and $p_0 = 2$): (a) $a = 8$; (b) $a = 3.5$; (c) $a = 1.5$.

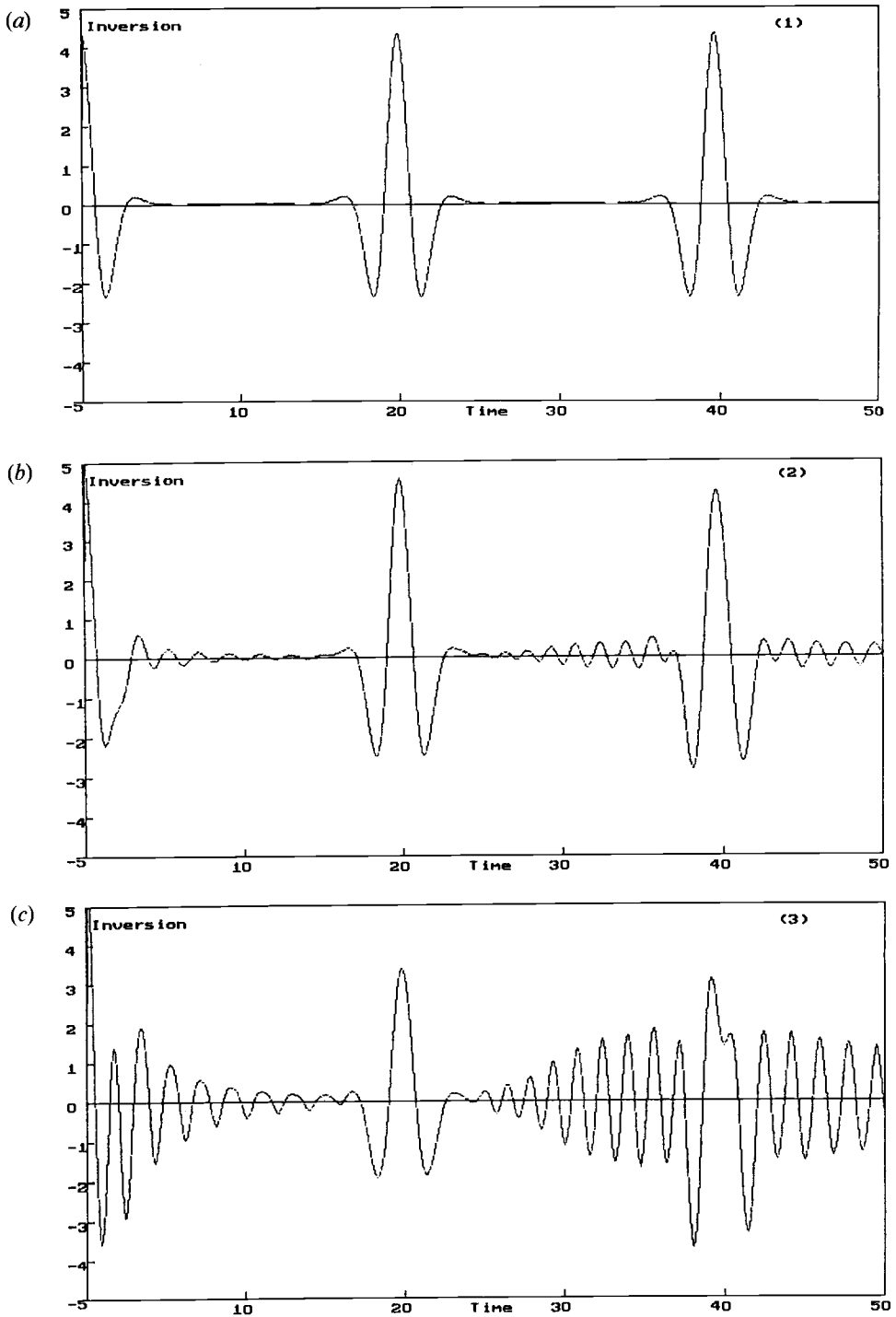


Figure 2. The population inversion for a cavity field which initially is in a coherent state and with an initial momentum of mass centre which has a Gaussian distribution (the mean photon number is 5, and the other parameters are the same as in figure 1): (a) $a=0.1$; (b) $a=0.06$; (c) $a=0.03$.

increases, the different oscillation components corresponding to frequencies determined by the different p are superposed, causing an oscillation collapse and revival through the Doppler effect (e.g. figures 1 (b) and (c)). It should be noted that the above case is quite similar to that for the original J-C model with a coherent-state cavity field as the mean photon number increases [9].

In order to see the effect of the moving mass centre on the population inversion more clearly, we modify the Hamiltonian (1) to the intensity-dependent case [13], that is let $a \rightarrow a(a^+a)^{1/2}$, $a^+ \rightarrow (a^+a)^{1/2}a^+$. The above-mentioned Doppler and recoil shifts of the frequency must have an influence on the evolution of expectation value of atomic internal energy. This value is calculated analytically:

$$E(t) = \langle \psi(t) | \hbar\omega_0 (|2\rangle\langle 2| - |1\rangle\langle 1|) | \psi(t) \rangle = \frac{\hbar\omega_0}{2} \exp(-|\alpha|^2) \int dp |C(p)|^2 \times \left(\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} \frac{\Delta^2(p)/4 + g^2(n+1)^2 \cos\{[g^2(n+1)^2 + (\Delta(p)^2/4)]^{1/2} 2t\}}{\Delta^2(p)/4 + g^2(n+1)^2} \right) \quad (15)$$

when the cavity is initially in a coherent state $|\alpha\rangle$. The frequency shift appearing in $\Delta(p)$ shows the effect of motion of the atomic mass centre on the atomic internal energy. Figure 2 shows $\langle E(t) \rangle$ calculated from the analytical result (9) for the non-resonance cases with different $\Delta(p)$. It is worth noting that, even though there is a cavity field, at the resonance case $\Delta(p) = 0$ the motion of the mass centre does not have any influence on the population inversion of levels and the evolution of the excited energy. As the wave packet width increases, that is a decreases, the population inversion will oscillate in the collapse region, and finally the collapse and revival phenomenon will disappear owing to the Doppler effect.

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