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RESEARCH ARTICLE

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Noise suppression for micromechanical resonator via intrinsic dynamic feedback

Abstract We study a dynamic mechanism to passively suppress the thermal noise of a micromechanical resonator through an intrinsic self-feedback that is genuinely non-Markovian. We use two coupled resonators, one as the target resonator and the other as an ancillary resonator, to illustrate the mechanism and its noise reduction effect. The intrinsic feedback is realized through the dynamics of coupling between the two resonators: the motions of the target resonator and the ancillary resonator mutually influence each other in a cyclic fashion. Specifically, the states that the target resonator has attained earlier will affect the state it attains later due to the presence of the ancillary resonator. We show that the feedback mechanism will bring forth the effect of noise suppression in the spectrum of displacement, but not in the spectrum of momentum.

Keywords fluctuations, cooling, intrinsic feedback

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1 Introduction

Recently, interest has been generated on cooling techniques for mechanical systems at nano- and micron-scales [1]. Among them, the typically employed is the feedback cooling technique where an external feedback circuit is responsible for detecting the motion of the target and feeding a counteracting force against this motion; through a general decrease of magnitude in the density noise spectrum, it was shown that the feedback can effectively reduce the fluctuation of the target and provide a cooling mechanism [3, 4]. Some experiments based on the models that contain the feedback loops have been carried out in the past few years. A few are directed towards the cooling of micron- to nanometer-size mechanical resonators, aiming to reach a macroscopic quantum mechanical ground state and serving as a powerful manifestation of quantum mechanical effects [5−8]. Other experiments succeed in slowing down the motion of micron-size mirrors through the radiation pressure of an optical field in a Fabry-Pérot (FP) cavity, aiming to reach a noise level and equivalently an effective temperature that are pertinent to the employment of high-precision detection of gravity waves [9−12].

The aforementioned implementations of feedback cooling through reduction of noise fluctuations invariably rely on an electrical circuitry external to the target system to be cooled. The controller here is usually fixed and attracts or repels the resonator through either electrostatic Coulomb force or Lorentzian force. If such an external detection-control unit could be eliminated in favor of a mechanism with self-detection of and self-adjustment to the target's thermal motion, we call the mechanism "self-cooling" [15, 16]. Devices implementing this self-cooling use less components and are free from the reliance on an external circuitry and hence prone to less noise sources.

In one case [13], an augmented cavity along with an extra optical cavity field is established on the other side of the mirror, in addition to the regular FP cavity, so as to counteract the radiation pressure from the original cavity field. This extra field cushions the motion of the pressure mirror and plays the role of feedback. In another case [14−16], a set of Josephson junctions behaving

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as a qubit, serially connected to a mechanical resonating beam, serves delayed supercurrent into the circuit according to the magnetic flux through the circuit loop. The magnetic flux is controlled by the vibrating motion of the beam, which in turn is controlled by the magnetic field generated by the current feed. Such a mutual dependence furnishes a self-feedback mechanism. It should be pointed out that both of the self-feedback setups require delayed feedback, which assumed *a priori* a non-Markovian approximation that explicitly depends on the history of the target's motion. In these phenomenological treatments, the cooling target either couples itself to a static controller and makes itself prone to the noise stemmed from the feedback, or couples to a mechanically static detection construct and receives manually delayed feedback.

Hereby, we present a dynamic model based on an intrinsic mechanism with non-Markovian feedback, which is obviously free from an external feedback loop and does not rely on a presupposition of historical dependence. This mechanism is illustrated by a simple mechanical system in which the target is modeled by a harmonic oscillator and attached to a dynamic controller, which is a relatively heavier resonator, through a spring. The target is controlled by an intrinsic feedback through the dynamics of coupling: earlier positions and velocities of the target affects the motion of the controller and this influence is subsequently fed back to the target. Consequently, the accumulation of earlier states of the target will affect the state of itself later. With proper parameter setup, the target essentially experiences a resistance and decelerates its motion; its displacement variance is shrunk, noise suppressed and effective temperature cooled down. The lack of a specific detection device for the motion of the target resonator and an external feedback circuit characterizes the intrinsic nature of the mechanical feedback. Our numerical analysis shows the existence of a noise suppression capability of our scheme, e.g. the variance of displacement can be reduced to $0.04 \times 10^{-21}\mathrm{m}^2$, and therewith a cooling capability under a practical setting accessible in current experiments. We note that the scheme is theoretically illustrative through its simple model setup yet widely applicable because the general oscillator systems can be extended to quantum bosonic systems and other cases. In fact, a similar model and mechanism has been proposed to actively cool down the torsional vibration of a nanomechanical resonator through spin-orbit interactions [17].

The model will be explained in Section 2 and its delay function then derived *a posteriori* to examine its non-Markovian dependence. The complete solution of the system dynamics is given in Section 3, with which we will derive the density noise spectrum and calculate the theoretical noise suppression rate. The associated numerical results will be presented in Section 4, given various parameter setups. The analysis is extended to the domain of momentum noise in Section 5.

2 Intrinsic feedback by coupling dynamics

2.1 The model

Our model setup (see Fig. 1) comprises two masses and three springs. The two masses are denoted by m and M, respectively. The mass m is the target and typically lighter whereas the mass M serves as an ancillary controller and is relatively heavier. The three springs are denoted by their Hooke's constants k, g and K, respectively. The spring of constant k attaches the lighter mass m to the fixed wall on the left and the spring of constant K attaches the heavier mass M to the fixed wall on the right. The spring of constant g strings the two masses together. Such a setup, intuitively, grants the heavier mass M the function of a suspension system and a medium for the feedback. The symbol $G(t)$ represents an external driving force which is necessary for the discussion of cooling but can be deemed zero for the present.

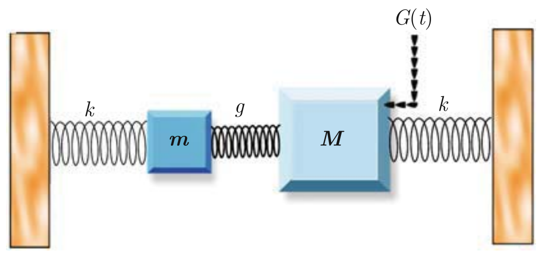

Fig. 1 The diagrammatic figure shows the arrangement of the three springs and the two masses. From left to right, they are: the spring of constant k, the target mass m, the spring of constant g, the ancillary mass M, and the spring of constant K. $G(t)$ is the harmonic driving force.

The connected springs will give rise to mechanical vibrations of the masses. We let $\bar{\omega} = \sqrt{(k+g)/m}$ denote the effective mechanical resonance frequency for the mass m, assuming the other mass M is fixed, and $\bar{\Omega} = \sqrt{(K+g)/M}$ the equivalent for the mass M, assuming the mass m is fixed. Besides these mechanical vibrations, we assume each of the masses experience a frictional damping and we let γ denote the damping coefficient for the mass m and Γ that for mass M.

Then according to the setup above, the coordinates of the two masses obey a coupled system of classical