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Objectivity in Quantum Measurement

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Abstract The objectivity is a basic requirement for the measurements in the classical world, namely, different observers must reach a consensus on their measurement results, so that they believe that the object exists "objectively" since whoever measures it obtains the same result. We find that this simple requirement of objectivity indeed imposes an important constraint upon quantum measurements, i.e., if two or more observers could reach a consensus on their quantum measurement results, their measurement basis must be orthogonal vector sets. This naturally explains why quantum measurements are based on orthogonal vector basis, which is proposed as one of the axioms in textbooks of quantum mechanics. The role of the macroscopicality of the observers in an objective measurement is discussed, which supports the belief that macroscopicality is a characteristic of classicality.

Keywords Quantum measurement · Many-world interpretation · Objectivity

1 Introduction

In the classical world, the *objectivity* is a basic requirement for measurements, that is, the different observers measuring the same object must reach a consensus on their

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results, so that they can be convinced that the object exists "objectively" since whoever measures it obtains the same result independent of the observers.

But in the Copenhagen version of quantum mechanics interpretation (QMI), this objectivity is not guaranteed since the measurement by an observer could cause a dramatical and stochastic change in the quantum state, namely, the "wave-function collapse" (WFC), and the WFC is inevitable in the Copenhagen interpretation, because the measuring apparatus (or observer) is treated as a purely classical term [1].

In the Copenhagen version of QMI, the classical apparatus is indispensable in the constitution of quantum theory as it should be, but at the same time it is not governed by quantum law. From the logical point of view, this is clearly unsatisfactory [2–6]. To get rid of this inconsistent point in the quantum theory, various "built-in" interpretations have been proposed without postulating the pure classicality of measuring apparatus, which leads to the WFC. The decoherence approach [1,7–10], the consistent history theory [11,12], and the many-worlds interpretation (MWI) [13,14] are well known representatives of these tentative solutions. Besides, there are more drastic solutions: Bohm's hidden variable approach [15,16], 't Hooft's deterministic and dissipative theory [17–20], and Adler's trace dynamics theory [21,22], in which quantum mechanics is interpreted as an effective theory emerging from some underlying structure.

The objectivity in quantum measurements has been discussed in the studies of quantum Darwinism [10,23–27]. In the theory of quantum Darwinism, it is noticed that environments consist of many subsystems, and observers acquire information about a system by intercepting copies of its pointer states deposited in fragments of the environment. In this sense, the objectivity of quantum measurements naturally emerges. The number of copies of the data in the environment about pointer states is the measure of objectivity.

In this paper, we ask a question: if we require a quantum measurement be "objective", what constraint would be imposed by this requirement of objectivity?

Here we should first describe the "objective quantum measurement" with mathematical clarity. We understand the quantum measurement as the establishment of the one-to-one correlation between the system S and the observer D, and this is encoded in the joint density matrix ρ_{SD}. The objectivity requires, (1) the correlation between the system and any observer must be the same; (2) The correlation between any two observers should be the same as that between the observer and the system.

These two conditions summarize the key requirement of the objectivity, namely, all the observers could obtain the same measurement result and verify their result with each other [6]. With this definition, we can treat the objectivity of quantum measurement by comparing the bipartite reductions (ρ_{SD}, $\rho_{SD'}$ and $\rho_{DD'}$) of the total density matrix $\rho_{SDD'}$. It turns out that, to satisfy the above simple objectivity conditions, any correlations obtained in the quantum measurement must be based on orthogonal vector basis. Moreover, two observers are enough to ensure this constraint.

It has been accepted as a basic axiom in quantum mechanics that the states we obtain after quantum measurements are orthogonal ones. Here our study shows this could be a natural constraint imposed by the objectivity requirement. If some observation is not based on orthogonal basis, its objective existence cannot be confirmed.

In Refs. [23,25], it was noticed that, by considering a faithful information transfer in the idealistic pre-measurement, namely, $|u\rangle|A_0\rangle \rightarrow |u\rangle|A_u\rangle$, $|v\rangle|A_0\rangle \rightarrow |v\rangle|A_v\rangle$, automatically the unitarity of the evolution guarantees that only orthogonal basis of the system ($\langle u|v\rangle = 0$) can be well distinguished in quantum measurements. In our study, the process how the correlations (SD, SD' or DD') are established is not concerned. By checking whether the correlations in the total density matrix $\rho_{SDD'}$ satisfy the objectivity conditions, the orthogonality of the measurement basis is also obtained directly. Moreover, for not only the system S but also the observers D/D', the measurement basis must be orthogonal basis. Namely, once the measurement result of a quantum system is objectively obtained, it must have been measured in orthogonal basis, and the measurement devices also must be working in orthogonal basis.

2 Quantum Measurements with Two or More Observers

In the QMI based on decoherence, a quantum measurement or observation is completed in two steps:

Step 1, the pre-measurement. A non-demolition coupling of the system S to the apparatus (observer) D is established and unitarily leads to a quantum entanglement between S and D.

Step 2, the decoherence. The environment E surrounding S selects the preferred basis $\{|\mathsf{s}_n\rangle\}$, and a classical correlation is created from the quantum entanglement developed in the pre-measurement [8,28,29].

Suppose that a system S initially prepared in a pure state is to be measured. The states of D and E are denote by $|\mathsf{d}_n\rangle$ and $|\mathsf{E}_n\rangle$ respectively. Then in the above mentioned Step 1 of the quantum measurement (pre-measurement), the total system (universe) $S + D + E$ will evolve into a partially entangled state

$$|\Phi_1\rangle = \left[\sum_n c_n |\mathsf{s}_n\rangle \otimes |\mathsf{d}_n\rangle\right] \otimes |\mathsf{E}\rangle, \tag{1}$$

from an initial product state $|\Phi_0\rangle = |\psi_S(0)\rangle \otimes |\mathsf{d}\rangle \otimes |\mathsf{E}\rangle$. Here, $|\mathsf{d}_n\rangle = U_n(D)|\mathsf{d}\rangle$ is a state of D correlated to the system state $|\mathsf{s}_n\rangle$ and $U_n(D)$ is the S-state dependent evolution matrix. In Step 2, the environment will become entangled with the system so that the total system reaches a GHZ type state

$$|\Phi_2\rangle = \sum_n c_n |\mathsf{s}_n\rangle \otimes |\mathsf{d}_n\rangle \otimes |\mathsf{E}_n\rangle, \tag{2}$$

where the environment states $|\mathsf{E}_n\rangle = U_n(E)|\mathsf{E}\rangle$ are orthogonal to one another, i.e., $\langle \mathsf{E}_m|\mathsf{E}_n\rangle = \delta_{mn}$. By tracing over the variables of E, one reaches then a correlation between S and D represented by the reduced density matrix $\rho_{SD} = \mathrm{tr}_E|\Psi\rangle\langle\Psi|$, that is,

$$\rho_{SD} = \sum_n |c_n|^2 |\mathsf{s}_n, \mathsf{d}_n\rangle\langle \mathsf{s}_n, \mathsf{d}_n|, \tag{3}$$

where $|\mathsf{s}_n, \mathsf{d}_n\rangle = |\mathsf{s}_n\rangle \otimes |\mathsf{d}_n\rangle$.

The above is a sketchy description of the implementation of quantum measurement with the help of environment E. It is pointed out that one does not need to require the orthogonality among the device states $\{|\mathsf{d}_n\rangle\}$ to distinguish the system states $\{|\mathsf{s}_n\rangle\}$. But an ideal quantum measurement will require the orthogonality among the device states. We will return to this topic later.

It is noticed from Eq. (2) that when $\{|\mathsf{d}_n\rangle\}$ and $\{|\mathsf{E}_n\rangle\}$ are both orthogonal vector sets, from mathematical point of view, the distinction between the observer and the environment is just nominal. Indeed, as far as the measurement of the system state is concerned, here the state $|\Phi_2\rangle$ enjoys a symmetry with respect to the exchange between $|\mathsf{d}_n\rangle$ and $|\mathsf{E}_n\rangle$. Thus boundary between the observer and the environment is not inherent in the current decoherence approach. It has been stressed by Zurek that in the decoherence approach the environment has been recognized as a witness of the measurement, it essentially plays the role of another measuring device or observer, and a large environment with redundancy of degrees of freedom can be divided into several portions, which could be regarded as observers [30].

It is thus not unnatural if we replace the environment with another observer and consider a scheme of quantum measurement with two observers. In this scheme, the total system is made up of a system S and two observers D and D'. These two observers can be also regarded as two fractions in the frame of quantum Darwinism. Let $\{|\mathsf{d}_n\rangle\}$ and $\{|\mathsf{d}'_n\rangle\}$ be two bases of the state spaces of D and D' respectively. The quantum measurement is then implemented through a tripartite decomposition

$$|\Psi\rangle = \sum_n c_n|\mathsf{s}_n\rangle \otimes |\mathsf{d}_n\rangle \otimes |\mathsf{d}'_n\rangle. \tag{4}$$

In this case, both the reduced density matrices $\rho_{SD} = \mathrm{tr}_{D'}|\Psi\rangle\langle\Psi|$ and $\rho_{SD'} = \mathrm{tr}_D|\Psi\rangle\langle\Psi|$ characterize a correlation between the system S and an observer D/D'. And $\rho_{DD'} = \mathrm{tr}_S|\Psi\rangle\langle\Psi|$ gives the correlation between the two observers, which makes it possible to compare their results. If $|\mathsf{s}_n\rangle$ are orthogonal states of S, the correlation

$$\rho_{DD'} = \mathrm{tr}_S|\Psi\rangle\langle\Psi| = \sum_n |c_n|^2|\mathsf{d}_n, \mathsf{d}'_n\rangle\langle\mathsf{d}_n, \mathsf{d}'_n| \tag{5}$$

has a classical form. If $|\mathsf{s}_n\rangle$ are not orthogonal states of S, there will not be a perfect classical correlation between the two observers as above. Instead, it reads

$$\tilde{\rho}_{DD'} = \rho_{DD'} + \sum_{m\neq n} c^*_m c_n|\mathsf{d}_n,\ \mathsf{d}'_n\rangle\langle\mathsf{d}_m,\ \mathsf{d}'_m| \cdot \langle\mathsf{s}_m|\mathsf{s}_n\rangle. \tag{6}$$

Here the term $\tilde{\rho}_{DD'} - \rho_{DD'}$ will negatively influence the comparison between the results of the two observers. This is a hint that non-orthogonal states can not be distinguished objectively. This point will be made clear later after a definition of measurement related objectivity is proposed mathematically.

We observe that partially tracing is omnipresent in the domain of quantum measurement. Physically it should imply doing some average or coarse-graining by the Born rule. With this remark we end this section.

3 Objectivity of Quantum Measurement

Now we see that the quantum measurement is understood as the establishment process of the system-observer correlation, which is encoded in the bipartite density matrices. With this consideration, we can discuss the objectivity requirement for quantum measurements with mathematical clarity.

As we mentioned before, the objectivity is a basic requirement for measurements in the classical world. It at least has two basic requirements, i.e., the different observers should obtain the same result, and they can check their result with each other. Since the quantum measurement is understood as the establishing process of correlations, we can verify whether this objectivity requirement is satisfied by checking the bipartite density matrices ρ_{SD}, $\rho_{SD'}$ and $\rho_{DD'}$. These three density matrices must have the same form to guarantee they encode the same correlation.

Therefore, the above requirements can be summarized in to the following three *objectivity conditions*:

$$\rho_{SD} = \sum_n p_n |\mathsf{s}_n, \mathsf{d}_n\rangle\langle \mathsf{s}_n, \mathsf{d}_n|, \tag{7a}$$

$$\rho_{SD'} = \sum_n p_n |\mathsf{s}_n, \mathsf{d}'_n\rangle\langle \mathsf{s}_n, \mathsf{d}'_n|, \tag{7b}$$

$$\rho_{DD'} = \sum_n p_n |\mathsf{d}_n, \mathsf{d}'_n\rangle\langle \mathsf{d}_n, \mathsf{d}'_n|. \tag{7c}$$

These three density matrices have the same form. The first two equations mean the observers D and D' establish the same correlation with the system S. The third equations means D and D' compare their result and reach a consensus.

Notice that the correlations are established based on the basis $\{|\mathsf{s}_n\rangle\}$, $\{|\mathsf{d}_n\rangle\}$ and $\{|\mathsf{d}'_n\rangle\}$, but here we do not require them to be orthogonal vector sets. Usually this orthogonality of measurement basis is presumed as a basis principle in priori. Now, through the following two propositions, we are going to show that the orthogonality of the basis $\{|\mathsf{s}_n\rangle\}$, $\{|\mathsf{d}_n\rangle\}$ and $\{|\mathsf{d}'_n\rangle\}$ is a natural result, if we require the quantum measurement must satisfy the above three objectivity conditions (7a–7c).

Proposition 1 *For a tripartite density matrix $\rho_{SDD'}$, if its reduced matrices $\rho_{SD} = \mathrm{tr}_D[\rho_{SDD'}]$ and $\rho_{SD'} = \mathrm{tr}_D[\rho_{SDD'}]$ have the forms of*

$$\rho_{SD} = \sum_n p_n |\mathsf{s}_n, \mathsf{d}_n\rangle\langle \mathsf{s}_n, \mathsf{d}_n|, \tag{8}$$

$$\rho_{SD'} = \sum_n p_n |\mathsf{s}_n, \mathsf{d}'_n\rangle\langle \mathsf{s}_n, \mathsf{d}'_n|, \tag{9}$$

then there exists an orthonormal vector set $\{|\Phi_i\rangle\}$, such that the tripartite $\rho_{SDD'}$ can be written as

$$\rho_{SDD'} = \sum_i \lambda_i |\Phi_i\rangle\langle\Phi_i|, \qquad \lambda_i > 0$$

$$|\Phi_i\rangle = \sum_n \mathsf{C}_n^{(i)} |\mathsf{s}_n,\, \mathsf{d}_n,\, \mathsf{d}'_n\rangle. \tag{10}$$

Here $\{|\mathsf{s}_n\rangle\}$, $\{|\mathsf{d}_n\rangle\}$ *and* $\{|\mathsf{d}'_n\rangle\}$ *are complete basis sets for the Hilbert space* $\mathcal{H}_S$, $\mathcal{H}_D$ *and* $\mathcal{H}_{D'}$ *respectively, but not necessarily orthogonal ones.*

We leave the proof in the appendix. Any density matrix like $\rho_{SDD'}$ can be diagonalized, but it is worth noticing that this proposition implies a strong constraint on the eigen basis $\{|\Phi_i\rangle\}$, namely, they must have a GHZ-like form [Eq. (10)] (here $\{|\mathsf{s}_n\rangle\}$, $\{|\mathsf{d}_n\rangle\}$, $\{|\mathsf{d}'_n\rangle\}$ may not be orthogonal basis).

Indeed the conditions in the above Proposition 1 can be replaced by any two of the three objectivity conditions (7a–7c), and the conclusion is the same. If we consider some more properties of quantum measurements, we will find that the constrained form of $\rho_{SDD'}$ [Eq. (10)] imposed by Proposition 1 can be further strengthened.

As we mentioned before, the quantum measurement is understood as the correlation establishing process by the unitary transformation. In the idealistic case, the initial state of the observer D/D' is prepared in a pure state. The initial state of the system S to be measured is arbitrary, namely, it can be an either pure or mixed state. But the unitary transformation to establish the pre-measurement should be the same for any initial state of S in a specific quantum measurement process, $\rho_t = U\rho_0 U^\dagger$.

Therefore, for the same pre-measurement process, pure and mixed initial states should have equal position in the constraint imposed by the objectivity requirement, since indeed the observers have no way to tell the difference whether the initial state is pure or mixed in this measurement process. Here we consider the initial state of S is a pure state, the state $\rho_{SDD'}$ after pre-measurement should also be a pure state, namely, the above Eq. (10) should be written as $\rho_{SDD'} = |\Phi\rangle\langle\Phi|$, and $|\Phi\rangle = \sum_n c_n |\mathsf{s}_n,\, \mathsf{d}_n,\, \mathsf{d}'_n\rangle$.

With this in mind, now we are going to prove the following proposition:

Proposition 2 *We consider that the state* $\rho_{SDD'}$ *after pre-measurement is prepared from a pure initial state* $|\psi_S\rangle \otimes |\mathsf{d}\rangle \otimes |\mathsf{d}'\rangle$ *by a unitary transformation, in this case:*

1. *if the objectivity conditions* (7a, 7b) *hold, then* $\{|\mathsf{d}_n\rangle\}$ *and* $\{|\mathsf{d}'_n\rangle\}$ *must be orthonormal vector sets;*
2. *if all the three objectivity conditions* (7a–7c) *hold, then* $\{|\mathsf{s}_n\rangle\}$ *must also be an orthonormal vector set.*

Proof As we discussed above, since $\rho_{SDD'}$ is prepared from a pure initial state by a unitary transformation, it also must be a pure state. According to Proposition 1, it must have the form of $\rho_{SDD'} = |\Phi\rangle\langle\Phi|$, and $|\Phi\rangle = \sum_n c_n |\mathsf{s}_n,\, \mathsf{d}_n,\, \mathsf{d}'_n\rangle$ (this summation only encloses terms of $c_n \neq 0$). Thus, its reduced density matrices are

$$\rho_{SD} = \mathrm{tr}_{D'} |\Phi\rangle\langle\Phi| = \sum_{m,n} c_m^* c_n \langle \mathsf{d}'_m|\mathsf{d}'_n\rangle \cdot |\mathsf{s}_n, \mathsf{d}_n\rangle\langle \mathsf{s}_m, \mathsf{d}_m|,$$

$$\rho_{SD'} = \mathrm{tr}_{D} |\Phi\rangle\langle\Phi| = \sum_{m,n} c_m^* c_n \langle \mathsf{d}_m|\mathsf{d}_n\rangle \cdot |\mathsf{s}_n, \mathsf{d}'_n\rangle\langle \mathsf{s}_m, \mathsf{d}'_m|.$$

Comparing these expressions with the objectivity conditions (7a, 7b) immediately leads to the conclusion that $c_m^* c_n \langle \mathsf{d}'_m | \mathsf{d}'_n \rangle = c_m^* c_n \langle \mathsf{d}_m | \mathsf{d}_n \rangle = 0$ when $m \neq n$. This implies $\langle \mathsf{d}_m | \mathsf{d}_n \rangle = \langle \mathsf{d}'_m | \mathsf{d}'_n \rangle = \delta_{mn}$, i.e., both $\{|\mathsf{d}_n\rangle\}$ and $\{|\mathsf{d}'_n\rangle\}$ are orthonormal vector sets, thanks to the fact that c_n are nonzero complex numbers. The first part of the proposition is thus proved, and the proof for the second part follows the same reason. □

From Proposition 2 we see that if we require the two observers D/D' could obtain the same measurement result, namely, they establish the same correlation with the system S [objectivity conditions (7a, 7b)], their measurement basis $\{|\mathsf{d}_n\rangle\}$ and $\{|\mathsf{d}'_n\rangle\}$ must be orthonormal vector sets. Further, if the two observers D/D' could verify that they obtain the same result by checking their own correlation $\rho_{DD'}$ [objectivity condition (7c)], then the basis $\{|\mathsf{s}_n\rangle\}$ of the system S, which is what they measured, also must be an orthonormal set.

Therefore, all the basis $\{|\mathsf{s}_n\rangle\}$, $\{|\mathsf{d}_n\rangle\}$ and $\{|\mathsf{d}'_n\rangle\}$ in the quantum measurement are orthonormal set, and the state $|\Phi\rangle$ is strictly a GHZ state. It is worth noticing that this is a natural constraint imposed by the requirement of objectivity, and we no more need to presume in priori as a basic principle that the measurement basis must be orthogonal sets. Once the measurement result of a quantum system is objectively obtained, it must have been measured in orthogonal basis, and the measurement devices also must be working in orthogonal basis. Otherwise, the objectivity of the quantum system cannot be confirmed, namely, non-orthogonal basis cannot be objectively measured.

It should be clear that all the results in this section can be generalized to multi-observer cases without difficulty. We would rather not go into the details.

4 Ideal Measurement from Macroscopicality: Central Spin Model

To achieve the above objective measurement, we need a unitary evolution satisfying $U\big(|\mathsf{s}_n\rangle \otimes |\mathsf{d}, \mathsf{d}'\rangle\big) = |\mathsf{s}_n\rangle \otimes |\mathsf{d}_n, \mathsf{d}'_n\rangle$. This can be completed by a Hamiltonian of the non-demolition type: $[\hat{H}_S, \hat{H}_{SD}] = 0$, $[\hat{H}_S, \hat{H}_{SD'}] = 0$ and $[\hat{H}_{SD}, \hat{H}_{SD'}] = 0$, where $\hat{H}_{SD}$ and $\hat{H}_{SD'}$ are the interaction between S and D/D'. And such a unitary transformation could be achieved by a dedicate control of the interaction time.

Besides, there is another more natural way to realize this unitary transformation by noticing that each macroscopic observer is usually composed of infinitely many degrees of freedom, and the orthogonality $\langle \mathsf{d}_m | \mathsf{d}_n \rangle = \langle \mathsf{d}'_m | \mathsf{d}'_n \rangle = \delta_{mn}$ can be achieved asymptotically in the thermodynamical limit.

Consider a composite system $S + D^{(1)} + D^{(2)} + \cdots + D^{(N)}$, where S is meant to be a quantum system to be measured and $D^{(1)}, D^{(2)}, \ldots, D^{(N)}$ stand for "elementary" observers. Choose a non-demolition type Hamiltonian $\hat{\mathcal{H}}$, such that an correlated state is prepared as

$$|\Psi\rangle = \sum_n c_n |\mathsf{s}_n\rangle \bigotimes_{i=1}^{N} |d_n^{(i)}\rangle. \tag{11}$$

Generally speaking, one cannot expect $\{|d_n^{(j)}\rangle\}$ to be orthogonal vector sets without a dedicate control of the interaction and the evolution time, and thus it is not easy to satisfy the above objectivity requirement. But if we divide the N "elementary" observers into two parts, i.e., by defining $|\mathsf{D}_n\rangle := \bigotimes_{i=1}^{M} |d_n^{(i)}\rangle$ and $|\mathsf{D}'_n\rangle := \bigotimes_{i=M+1}^{N} |d_n^{(i)}\rangle$, then in the macroscopical limit $M \to \infty$ and $N \to \infty$ we could have

$$\langle \mathsf{D}_m|\mathsf{D}_n\rangle = \prod_{i=1}^{M} \langle d_m^{(i)}|d_n^{(i)}\rangle \to 0,$$
$$\langle \mathsf{D}'_m|\mathsf{D}'_n\rangle = \prod_{i=M+1}^{N} \langle d_m^{(i)}|d_n^{(i)}\rangle \to 0, \tag{12}$$

for $m \neq n$ if only $|\langle d_m^{(i)}|d_n^{(i)}\rangle| < 1$ for $m \neq n$, which are easy to satisfy. This means that the "elementary" observers $\{D^{(1)}, D^{(2)}, \ldots, D^{(M)}\}$ and $\{D^{(M+1)}, D^{(M+2)}, \ldots, D^{(N)}\}$ can be coarse-grained into two macroscopic observers D and D' effectively. This observation convinces us that macroscopicality may well be regarded as a characteristic of a quantum observer.

To illustrate the above argument, let us present a concrete example [8,31,32]. In this example, the quantum system S to be measured is a central spin with two states $|\mathsf{e}\rangle$ and $|\mathsf{g}\rangle$, and the central spin is surrounded by another N spin-$\frac{1}{2}$ particles, which serve as the above mentioned "elementary" observers $D^{(1)} + D^{(2)} + \cdots + D^{(N)}$. The Hamiltonian of the total system $S + D^{(1)} + D^{(2)} + \cdots + D^{(N)}$ reads

$$\hat{\mathcal{H}} = \mathcal{E}|\mathsf{e}\rangle\langle\mathsf{e}| + \sum_{i=1}^{N} \left(\omega_i \hat{\sigma}_i^z + g_i \hat{\sigma}_i^x\right) + |\mathsf{e}\rangle\langle\mathsf{e}| \cdot \left[\sum_{i=1}^{N} \eta_i \hat{\sigma}_i^z\right], \tag{13}$$

where $\hat{\sigma}_i^z = |\uparrow\rangle_i\langle\uparrow| - |\downarrow\rangle_i\langle\downarrow|$ and $\hat{\sigma}_i^x = |\uparrow\rangle_i\langle\downarrow| + |\downarrow\rangle_i\langle\uparrow|$ are the Pauli matrices for the i-th spin.

In the spirit of the preceding discussion, we coarse-grain the N "elementary" observers into two macroscopic observers D and D' which contains N_1 and N_2 spins respectively ($N_1 + N_2 = N$), namely, we take $D = D^{(1)} + D^{(2)} + \cdots + D^{(N_1)}$ and $D' = D^{(N_1+1)} + D^{(N_1+2)} + \cdots + D^{(N_1+N_2)}$. For clarity, we rewrite the Hamiltonians of the systems D and D' as

$$\hat{H}_D = \sum_{i=1}^{N_1} \left(\omega_{1,i} \hat{\sigma}_{1,i}^z + g_{1,i} \hat{\sigma}_{1,i}^x\right),$$
$$\hat{H}_{D'} = \sum_{i=1}^{N_2} \left(\omega_{2,j} \hat{\sigma}_{2,j}^z + g_{2,j} \hat{\sigma}_{2,j}^x\right). \tag{14}$$

It is then a routine work to check that

$$
\begin{aligned}
e^{-i\hat{\mathcal{H}}T}\left(|\mathsf{g}\rangle\bigotimes_{i=1}^{N}|\uparrow\rangle\right) &= \qquad |\mathsf{g}\rangle\otimes|\mathsf{D}_{\mathsf{g}}\rangle\otimes|\mathsf{D}'_{\mathsf{g}}\rangle,\\
e^{-i\hat{\mathcal{H}}T}\left(|\mathsf{e}\rangle\bigotimes_{i=1}^{N}|\uparrow\rangle\right) &= e^{-i\mathcal{E}T}\ |\mathsf{e}\rangle\otimes|\mathsf{D}_{\mathsf{e}}\rangle\otimes|\mathsf{D}'_{\mathsf{e}}\rangle,
\end{aligned}
\tag{15}
$$

where

$$
\begin{aligned}
|\mathsf{D}_{\mathsf{g}}\rangle &= \bigotimes_{i=1}^{N_1} R_{1,i}^{(\mathsf{g})}(T)|\uparrow\rangle, \qquad |\mathsf{D}'_{\mathsf{g}}\rangle = \bigotimes_{i=1}^{N_2} R_{2,i}^{(\mathsf{g})}(T)|\uparrow\rangle,\\
|\mathsf{D}_{\mathsf{e}}\rangle &= \bigotimes_{i=1}^{N_1} R_{1,i}^{(\mathsf{e})}(T)|\uparrow\rangle, |\mathsf{D}'_{\mathsf{e}}\rangle = \bigotimes_{i=1}^{N_2} R_{2,i}^{(\mathsf{e})}(T)|\uparrow\rangle,
\end{aligned}
\tag{16}
$$

with $R_{n,i}^{(\alpha)}(T) = \exp[-iH_{n,i}^{(\alpha)}T]$ for $\alpha = \mathsf{g}, \mathsf{e}$ and $n = 1, 2$. Here $H_{n,i}^{(\alpha)}$ are single effective Hamiltonians defined as follows:

$$
\begin{aligned}
H_{n,i}^{(\mathsf{g})} &= \omega_{n,i}\hat{\sigma}_{n,i}^{z} + g_{n,i}\hat{\sigma}_{n,i}^{x},\\
H_{n,i}^{(\mathsf{e})} &= (\omega_{n,i}+\eta_{n,i})\hat{\sigma}_{n,i}^{z} + g_{n,i}\hat{\sigma}_{n,i}^{x}.
\end{aligned}
\tag{17}
$$

By straightforward calculation, we obtain

$$
\begin{aligned}
\left|\langle\mathsf{D}_{\mathsf{g}}|\mathsf{D}_{\mathsf{e}}\rangle\right| &= \prod_{i=1}^{N_1}\langle\uparrow|\left[R_{1,i}^{(\mathsf{g})}(T)\right]^{\dagger}\cdot R_{1,i}^{(\mathsf{e})}(T)|\uparrow\rangle\\
&= \prod_{i=1}^{N_1}\left(1-\sin^2\mu_{1,i}^{(\mathsf{e})}T\cdot\sin^2\phi_{1,i}^{(\mathsf{e})}\right)\left(1-\sin^2\mu_{1,i}^{(\mathsf{g})}T\cdot\sin^2\phi_{1,i}^{(\mathsf{g})}\right),\\
\left|\langle\mathsf{D}'_{\mathsf{g}}|\mathsf{D}'_{\mathsf{e}}\rangle\right| &= \prod_{i=1}^{N_2}\langle\uparrow|\left[R_{2,i}^{(\mathsf{g})}(T)\right]^{\dagger}\cdot R_{2,i}^{(\mathsf{e})}(T)|\uparrow\rangle\\
&= \prod_{i=1}^{N_2}\left(1-\sin^2\mu_{2,i}^{(\mathsf{e})}T\cdot\sin^2\phi_{2,i}^{(\mathsf{e})}\right)\left(1-\sin^2\mu_{2,i}^{(\mathsf{g})}T\cdot\sin^2\phi_{2,i}^{(\mathsf{g})}\right),
\end{aligned}
\tag{18}
$$

where

$$
\begin{aligned}
\mu_{n,i}^{(\mathsf{e})} &= \left[(\omega_{n,i}+\eta_{n,i})^2+g_{n,i}^2\right]^{\frac{1}{2}}, \quad \sin\phi_{n,i}^{(\mathsf{e})} = \frac{g_{n,i}}{\mu_{n,i}^{(\mathsf{e})}},\\
\mu_{n,i}^{(\mathsf{g})} &= \left[\omega_{n,i}^2+g_{n,i}^2\right]^{\frac{1}{2}}, \quad \sin\phi_{n,i}^{(\mathsf{g})} = \frac{g_{n,i}}{\mu_{n,i}^{(\mathsf{g})}},
\end{aligned}
\tag{19}
$$

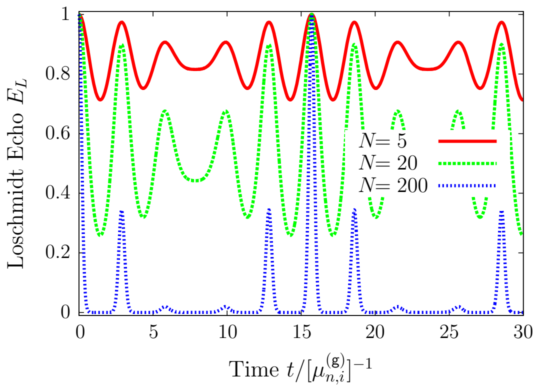

Fig. 1 When the spin number $N_{1,2} \to \infty$, the Loschmidt echo $E_L^{1,2}$ [Eq. (18)] approaches zero. We set $\mu_{n,i}^{(\mathrm{g})} = 1$ as the energy unit, and $\mu_{n,i}^{(\mathrm{e})} = 1.2$, $g_{n,i} = 0.2$

for $n = 1, 2$. It is noticed that $\left|\langle \mathsf{D_g}|\mathsf{D_e}\rangle\right|$ and $\left|\langle \mathsf{D'_g}|\mathsf{D'_e}\rangle\right|$ are none other than the so called Loschmidt echoes. Let us denote them by E_L^1 and E_L^2 respectively. From the expressions of the Loschmidt echoes it should be clear that each product factor is a non-negative number and smaller than 1, thus in the thermodynamic limit $N_{1,2} \to \infty$, for a generic T, we have $\left|\langle \mathsf{D_g}|\mathsf{D_e}\rangle\right| \simeq 0$ and $\left|\langle \mathsf{D'_g}|\mathsf{D'_e}\rangle\right| \simeq 0$ (see Fig. 1).

5 Conclusions

In this paper, we show that the requirement of objectivity indeed could impose an important constraint on quantum measurements, namely, if we require the quantum measurement to be objective, then the measurement basis must be orthogonal vector sets. Usually this is presumed as a basic principle in priori, but here we show that this can be a natural constraint imposed by the requirement of objectivity.

The quantum measurement is understood as the establishing process of correlations. And the objectivity requires that different observers could obtain the same result, and they can verify with each other. This is a very natural requirement in our classical world. Our result implies if the quantum measurement is not based on orthogonal basis, its objective existence cannot be confirmed, in another word, non-orthogonal basis cannot be objectively measured.

The emergence of classicality in quantum measurement is closely related to the objectivity condition. This point is illustrated with the central spin model, where the N "elementary" observers are coarse-grained into two macroscopic observers enjoying orthogonal pointer state sets for an ideal measurement. In this example, it is clearly seen how classical correlations result from the macroscopical observers and a support is provided for the belief that macroscopicality is a characteristic of classicality.

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A Proof for the Proposition 1

Proposition 1 *For a tripartite density matrix ρ_{ABC}, if its reduced matrices $\rho_{AB} = \mathrm{tr}_C[\rho_{ABC}]$ and $\rho_{AC} = \mathrm{tr}_B[\rho_{ABC}]$ have the forms of*

$$\rho_{AB} = \sum_n p_n |\mathsf{a}_n, \mathsf{b}_n\rangle\langle \mathsf{a}_n, \mathsf{b}_n|, \tag{20}$$

$$\rho_{AC} = \sum_n p_n |\mathsf{a}_n, \mathsf{c}_n\rangle\langle \mathsf{a}_n, \mathsf{c}_n|, \tag{21}$$

then there exists an orthonormal vector set $\{|\Phi_i\rangle\}$, such that the tripartite ρ_{ABC} can be written as

$$\rho_{ABC} = \sum_i \lambda_i |\Phi_i\rangle\langle\Phi_i|, \qquad \lambda_i \geq 0$$

$$|\Phi_i\rangle = \sum_n \mathsf{C}_n^{(i)} |\mathsf{a}_n, \mathsf{b}_n, \mathsf{c}_n\rangle. \tag{22}$$

Here $\{|\mathsf{a}_n\rangle\}$, $\{|\mathsf{b}_n\rangle\}$ and $\{|\mathsf{c}_n\rangle\}$ are complete basis sets for the Hilbert space $\mathcal{H}_A$, $\mathcal{H}_B$ and $\mathcal{H}_C$ respectively, but not necessarily orthogonal ones.

For clarity, we use A, B, C here to replace the S, D, D' in the main text. To prove this proposition, we need the following lemma:

Lemma *Let $\mathbf{P}$ be a positive definite matrix and $\mathbf{C}$ a semi-positive one. If $\mathrm{tr}[\mathbf{C}\cdot\mathbf{P}] = 0$, then $\mathbf{C}$ is a zero matrix.*

Proof We decompose the positive matrix $\mathbf{P}$ in its eigen basis as $\mathbf{P} = \sum_n \lambda_n |n\rangle\langle n|$, where all $\lambda_n > 0$. Then we have $\mathrm{tr}[\mathbf{C}\cdot\mathbf{P}] = \sum_n \lambda_n \langle n|\mathbf{C}|n\rangle = 0$. To make sure $\langle n|\mathbf{C}|n\rangle = 0$ for all the basis $\{|n\rangle\}$, $\mathbf{C}$ must be a zero matrix. □

With the help of the above lemma, the proof of Proposition 1 lies as follows.

Proof For the tripartite density matrix ρ_{ABC}, we can always write it as the eigen spectrum decomposition $\rho_{ABC} = \sum_i \lambda_i |\Phi_i\rangle\langle\Phi_i|$, where $|\Phi_i\rangle$ are orthonormal basis, and $\lambda_i > 0$ are the non-zero eigenvalues respectively. But now we could only write down $|\Phi_i\rangle$ in a general form

$$|\Phi_i\rangle = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{l=1}^{L} \mathsf{C}_{mnl}^{(i)} |\mathsf{a}_m, \mathsf{b}_n, \mathsf{c}_l\rangle, \tag{23}$$

where $\mathsf{C}_{nml}^{(i)}$ are complex numbers. It then follows that

$$\rho_{ABC} = \sum_{\substack{mnl\\ m'n'l'}} \varrho_{mnl,\, m'n'l'} |\mathsf{a}_m, \mathsf{b}_n, \mathsf{c}_l\rangle\langle \mathsf{a}_{m'}, \mathsf{b}_{n'}, \mathsf{c}_{l'}|,$$

$$\varrho_{mnl,\,m'n'l'} := \sum_i \lambda_i \cdot \mathsf{C}^{(i)}_{mnl}\overline{\mathsf{C}}^{(i)}_{m'n'l'}, \tag{24}$$

and the reduced density matrix ρ_{AB} becomes

$$\rho_{AB} = \mathrm{Tr}_C[\rho_{ABC}] = \sum_{\substack{m,n\\ m',n'}} \Big(\sum_{l,l'} \varrho_{mnl,\,m'n'l'} \langle \mathsf{c}_{l'}|\mathsf{c}_l\rangle\Big) |\mathsf{a}_m, \mathsf{b}_n\rangle\langle \mathsf{a}_{m'}, \mathsf{b}_{n'}|. \tag{25}$$

Comparing this with the required form of ρ_{AB} [Eq. (20)], we come to the following equation

$$\sum_{l,l'} \varrho_{mnl,\,m'n'l'} \langle \mathsf{c}_{l'}|\mathsf{c}_l\rangle = \delta_{mm'}\delta_{nn'} \cdot \delta_{mn} p_n. \tag{26}$$

Now we introduce two $L \times L$ matrices $\mathbf{C}^{(mn;m'n')}$ and $\mathbf{P}$, which are defined by

$$[\mathbf{C}^{(mn;m'n')}]_{l,l'} = \varrho_{mnl,\,m'n'l'}, \quad \mathbf{P}_{l',l} = \langle \mathsf{c}_{l'}|\mathsf{c}_l\rangle. \tag{27}$$

With their help, Eq. (26) can be written in a compact form

$$\mathrm{tr}[\mathbf{C}^{(mn;m'n')} \cdot \mathbf{P}] = \delta_{mm'}\delta_{nn'} \cdot \delta_{mn} p_n. \tag{28}$$

One notices that when $m = m'$, $n = n'$, $m \neq n$, we have

$$\mathrm{tr}[\mathbf{C}^{(mn;mn)} \cdot \mathbf{P}] = 0. \tag{29}$$

It is easy to verify that $\mathbf{C}^{(mn;mn)}$ is a semi-positive matrix,[1] and $\mathbf{P}$ is positive definite.[2] Therefore, according to above lemma, we know that $\mathbf{C}^{(mn;mn)}$ is a zero matrix when $m \neq n$. Thus we obtain

$$[\mathbf{C}^{(mn;mn)}]_{l,l} = \varrho_{mnl,\,mnl} = \sum_i \lambda_i \cdot |\mathsf{C}^{(i)}_{mnl}|^2 = 0. \tag{30}$$

Since all the $\lambda_i > 0$ in the above summation, that leads to

$$\mathsf{C}^{(i)}_{mnl} = 0, \quad \forall\, i,\, l,\, m \neq n. \tag{31}$$

In the same way, by comparing with ρ_{AC} [Eq. (21)], we can prove

$$\mathsf{C}^{(i)}_{mnl} = 0, \quad \forall\, i,\, n,\, m \neq l. \tag{32}$$

1 Obviously, the coefficient matrix $\varrho_{mnl,\,m'n'l'}$ of the density operator ρ_{ABC} is semi-positive. Notice that $[\varrho_{mnl,\,m'n'l'}]$ can be regarded as a block matrix, and $[\mathbf{C}^{(mn;mn)}]_{l,l'} = \varrho_{mnl,\,mnl'}$ is one of its principal blocks, thus $\mathbf{C}^{(mn;mn)}$ is semi-positive.

2 For any non-zero vector $\mathbf{v} := (v_1, v_2, \ldots, v_L)^T$, we have $\mathbf{v}^\dagger \cdot \mathbf{P} \cdot \mathbf{v} = \sum_{l,l'} v^*_{l'} \langle \mathsf{c}_{l'}|\mathsf{c}_l\rangle v_l = \langle\tilde{\psi}|\tilde{\psi}\rangle > 0$, where $|\tilde{\psi}\rangle := \sum_l v_l|\mathsf{c}_l\rangle$

Therefore, the only possible non-zero coefficients $\mathsf{C}_{mnl}^{(i)}$ are those satisfying $m = n = l$, thus, we write the coefficients as $\mathsf{C}_{mnl}^{(i)} = \delta_{mn}\delta_{ml} \cdot \mathsf{C}_n^{(i)}$, then we obtain the expression

$$|\Phi_i\rangle = \sum_n \mathsf{C}_n^{(i)} |\mathsf{a}_n, \mathsf{b}_n, \mathsf{c}_n\rangle \tag{33}$$

and complete the proof. □

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