FORTSCHRITTE DER PHYSIK

Quantum Dynamical Approach of Wavefunction Collapse in Measurement Process and Its Application to Quantum Zeno Effect

CHANG-PU SUN[1, 2], XUE-XI YI[1] and XIA-JI LIU[1]

[1] Institute of Theoretical Physics, Northeast Normal University,
   Changchun 130024, P. R. China
[2] Institute for Theoretical Physics, State University of New York,
   Stony BrooK, New York 11794, USA

Abstract

The systematical studies on the dynamical approach of wavefunction collapse in quantum measurement are report in this paper based on the Hepp-Coleman's model and its generalizations. Under certain physically reasonable conditions, which are easily satisfied by the practical problems, it is shown that the off-diagonal elements of the reduced density matrix for the measured system vanish in quantum mechanical evolution process of the universe formed by the measured system plus the measuring instrument-detector at the macroscopic limit with a very large particle number N. Various examples with detector made up of oscillators of different spectrum distribution are used to illustrate this observations. With the two-level system as an explicit illustration, the quantum information entropy is exactly obtained to quantitatively describe the degree of decoherence for the so-called partial coherence caused by detector. The entropy for the case with many levels is computed based on perturbation method in the limits with very large and very small N. As an application of this general approach for quantum measurement, a dynamical realization of the quantum Zeno effect are present to analyse its recent testing experiment in connection with a description of transition in quantum information entropy. Finally, the Cini's model for the correlation between the states of the measured system and the detector is generalized for the case with many energy-level. It is shown that this generalization can also be invoked to give the dynamical realization of wavefunction collapse.

1.   Introduction

Though quantum mechanics has been experimentally proved to be a quite successful theory, its interpretation is still an important problem that physicists should face [1-4]. To interpret the physical meaning of its mathematical formalism, one has to invoke the wave packet collapse (WPC) (or wavefunction collapse) postulate as an extra assumption added to the closed system of laws in quantum mechanics. This postulate is also called von Neumann's projection rule or wavefunction reduction process. Let us now describe it briefly. It is well known in quantum physics that, if measured quantum system S is in a state $|\phi\rangle$ that is a linear superposition of the eigenstates $|k\rangle$ of the operator $\hat{A}$ of an observable A just before a measurement, ie.,

$$|\phi\rangle = \sum c_k |k\rangle, \quad (c_k's \text{ are complex numbers}) \tag{1.1}$$

then a result of the measurement of A is one a_k of the eigenvalues of $\hat{A}$ corresponding to $|k\rangle$ with the probability $|c_k|^2$. The von Neumann's postulate tell us that, once a well-determined result a_n about A has been obtained, the state of S is no longer $|\phi\rangle$ and it must collapses into $|n\rangle$. This is because a immediately-successive measurement of A after the first one should repeats the same result. In terms of the density matrix

$$\varrho = |\phi\rangle \langle\phi| = \sum_{k, k'} c_k c_{k'}^* |k\rangle \langle k'|, \tag{1.2}$$

for the state $|\phi\rangle$, the above WPC process can be mathematically expressed as a projection or reduction

$$\varrho \rightarrow \hat{\varrho} = \sum |c_n|^2 |k\rangle \langle k|. \tag{1.3}$$

Because the off-diagonal elements represent coherence, through which the density matrix describes a non-classical probability, the WPC characterized by vanishing of off-diagonal elements means the loss of quantum coherence or called quantum decoherence [5].

The recent studies on the quantum decoherence in an open system S surrounded by an environment E was mainly motivated by the interests in the macroscopically-quantum effects such as dissipation in the quantum tunnelling and the semiclassical gravity theory for particle creation in quantum cosmology [6–10]. In quantum dissipation theory, an important treatment for quantum decoherence is invoking quantum Brownian motion through its master equation to describe the vanishing of the off-diagonal elements of reduced density matrix of S [1]. The recent investibations in this context were carried out with the Feynman-Vernons integral method for the Ohmic, sub-Ohmic or super-Ohmic environment (e. g., see ref. [8]). Without the use of path-integral [9, 10], an exactly-solvable dynamical model of quantum dissipation was presented by YU and this author to deal with the similar phenomenon. Notice that ZUREK and his collaborators especially emphasized the role of environment surrounding the open system, which monitors the observables of the system so that their eigenstates continuously decohere and then approach classical states [4, 8].

It should be noticed that there exists another dynamical theory based on HEPP-COLEMEN's (HCs) investigation to dynamically realize quantum decoherence characterized by the WPC [11, 12]. This theory and its generalization [13–15] were proceeded in a purely-quantum mechanical framework. In conventional theory of quantum mechanics, to realize the WPC, the external classical measuring apparatus detector must be used to detect the result. Then, someone thinks the WPC postulate to be not quite satisfactory since quantum mechanics is expected to be an universal theory valid for whole "universe", but the detector, as a part of the universe, behaves classically in the von Neumann's postulate. A reasonable description of the detector should be quantum essentially and it exhibits the classical or macroscopic features in certain limits. If one deal with the detector as a subsystem of the closed system – the universe C formed by the measured system S plus the detector D, it is possible that the quantum dynamics of the universe can result in the WPC through the interactions between S and D. Up to now, some exactly-solvable models along this line mentioned above have been presentd to analyse this problem. Among them, the Happ-Coleman (HC) model is very famous one and has been extensively studied in last twenty years [13–19]. In order to describe studies in this paper clearly, we need to see some details of this model.

In the original HC model, an ultrarelativistic particle with spin-1/2 is referred to the measured system S while a one-dimensional array of scatterers with spin-1/2 to the detector D. The interaction between S and D is represented by an coupling

$$H_I = \sum_{n=1}^{N} \frac{1}{2} \, V(x - a_n) \, \sigma_1^{(n)} (1 + \sigma_3^{(0)}) \tag{1.4}$$

where $\sigma_1^{(n)}$ is the first component of Pauli matrix; a_n is the position of the scatterer assigned to the n'th site in the array; the index 0 referes to S. The Hamiltonian for D is

$$H_s = c \hat{P} \tag{1.5}$$

where c, $\hat{P}$ and x are the light speed, the momentum and coordinate operators respectively for S. This model is quite simple, but it can be exactly solved to produce a deep insight on the dynamical description of the quantum measurement process. Starting with the initial state

$$|\psi(0)\rangle = \sum c_k \, |k\rangle \otimes |D\rangle \tag{1.6}$$

where $|D\rangle$ is pure state of D (it is usually taken to be ground state), the evolution state $|\psi(t)\rangle$ for the universe $C = S + D$ is defined by the exact solution to this model. Then, the reduced density matrix

$$\varrho_s(t) = \mathrm{Tr}_D(|\psi(t)\rangle \langle\psi(t)|) \tag{1.7}$$

of the measured system is obtained by taking the trace of the density matrix

$$\varrho(t) = |\psi(t)\rangle \langle\psi(t)| \tag{1.8}$$

of the universe to the variables of D. Obviously, $\varrho_s(t)$ depends on the particle number N of D. When $N \to \infty$, i.e., in the macroscopic limit, $\varrho_s(t) \to \hat{\varrho}$ after long enough time t. Namely, the Schrodinger evolution of the universe C leads to the WPC for the measured system. More recently, the original CH model was improved to describe the energy exchange between S and D by adding a free energy Hamiltonian

$$H_0 = \hbar\omega \sum_{n=1}^{N} \sigma_3^{(n)} \tag{1.9}$$

and correspondingly improving the interaction slightly. Notice that the improved model remains exactly-solvable [13].

Because the spin quantum number is fixed to be 1/2 in the original HC model or its improved versions, they can not describe the *classical characters* of the measurement. Usually, the classical feature of a quantum object is determined by taking certain value for some internal quantum numbers of the detector D or $\hbar = 0$. In the case of the angular momentum, this classical limit corresponds to infinite spin. The generalized dynamical approach, which can work effectively for both the classical and macroscopic limits, was successfully built by this author in 1993 [14, 15]. The first step of this approach is to establish such a generalization of the HC model manifesting the WPC as the dynamical process in the classical limit as well as in the the macroscopic limit simultaneously. Then, the essence for this model substantially resulting in the realization of the WPC as well as for those well-established was found to be the factorization of the evolution matrix of the universe in the interaction picture with help of a detailed study on the dynamics of the generalized HC model in both the exactly-solvable case and the non-solvable case. For the latter, the high-order adiabatic approximation (HOAA) method [20–23] is applied to its special case that the coupling parameter depends on the position of the measured ultrarelativistic particle quite slightly. Finally, we point out that this possible essence in the dynamical realization of the WPC, is largely independent of the concrete forms of model Hamiltonians. Notice, that in the dynamical models of the WPC for quantum

measurement, both the macroscopic and classical measuring apparatus can be regarded as the environment in the certain model of the quantum decoherence for quantum dissipation [4, 8]. This is because both they act as the classical or macroscopic monitors seeing the system.

However, because all of the previous dynamical models of decoherence for quantum measurement depend on the specific forms of Hamiltonians of D and S, it is necessary to present a model-independent dynamical approach for decoherence in quantum measurement process based on the HC model. It is expected that such an approach does not depend on the detailed structures of the Hamiltonians of S and D as fully as possible, but can be invoked to deal with the practical problems of quantum measurement such as quantum Zeno effect (QZE). This universal approach should also be used to describe the role of environment in decoherence for quantum dissipation. It is shown in present study that, through a suitable choice of the interaction between S and D, the Schrödinger evolution of the universe C formed may result in the phenomenon of decoherence in the reduced density matrix $\hat{\varrho}$ of S at the macroscopic limit of D with very large N. Mathematically, the mechanism of this phenomenon is that the accompanying factors in the off-diagonal elements of reduced density matrix $\hat{\varrho}_s$ caused by the dynamical evolution of C will vanish as N approaches infinity. It even was described in concrete examples [14, 15]. Notice again that in the previous models for the quantum measurement and decoherence, the considered systems S usually are specified as an ultrarelativistic particle or a two-level system while the detector D as a spin array. Here, what we require is only that the system is of the non-degenerate discrete spectrum and the interaction between D and S is chosen to result in a factorizable evolution matrix. However, D is required to satisfy a condition that any row or column of each factor corresponding to the factorizable evolution matrix at least has one non-vanishing off-diagonal element. This means that the back action of the measured system can effectively act on the detector so that the microscopic states can be read out from the macroscopic counting numbers contributed by all particles in detector. This condition is physically reasonable and can be satisfied in widespread circumstances. Some examples are invoked to illustrate that this condition can be realized by choosing suitable spectrum distributions of oscillators making up the detector.

To quantitatively describe the intermediate state of decoherence between the pure state and the most-largely mixed state, we need to calculate the entropy of S

$$s = s[\hat{\varrho}] = -\frac{K}{2} \, \text{Tr}(\hat{\varrho} \ln \hat{\varrho}) \tag{10}$$

for two cases: (I) For the two-level system and any finite N, an exact solution for s is obtained as the functional of $\hat{\varrho}$. (II) For the limits with very large and very small N, the approximate solutions of s by certain perturbation methods. These calculations show that the entropies indeed decrease as N increases and they will take the maximum values at the infinite N. This means that an ideal macroscopic detector or environment must cause the increment of entropy of the its monitored system to maximum extent.

The above general approach is used to built an exactly-solvable dynamical model for the quantum Zeno effect [25, 26] in connection with the recent experiment by ITANO et al. [26] about the inhibition of quantum transition between the atomic energy levels. The present investigation compromises the different points of view about this experiment testing quantum Zeno effect [27–37]. In thos model the detector is simplified as a system of N oscillators with a suitable interaction with the measured system – a two-level atom, we show that, due to gradually-vanishing of the off-diagonal elements in ϱ_S, the two-level system will be frozen in its initial level as the times of measurement in a given time interval becomes infinite. This is just the quantum dynamical realization of the QZE through a dynamical approach of the WPC. The information entropy for the process of quantum Zeno effect of two-level system is calculated to manifest an interesting behavior of

transition from random to regularity: for a given time interval, when the times L of measurement is less than a critical value L_c, the entropy changes at quite random as L changes; when L is larger than L_c, the entropy decreases monotonically as L becomes larger.

Finally, it has to be pointed out that the correlation between the states of the measured system and that of the detector has not been emphasized well in the original HC model and its generalization. But this problem was well analysed by CINI in his beautiful model [39]. The present investigation is also to emphasize on both the wavefunction collapse and the state correlation in the Cini model. In fact, the correlation between the states of measured system and the detector is crucial for a realistic process of measurement, which enjoys a scheme using the macroscopic counting number of the measuring instrument-detector to manifest the microscopic state of the measured system. The original Cini model for the system S and the measuring instrument-detector D is build only for a two-level system interacting with the detector D, which consists of N indistinguishable particle with two possible modes ω_0 and ω_1. For the two states u_+ and u_- of S, the detector has different strengths of interaction with them. Then, the large number N of "ionized" particle in the ionized state ω_1 transiting from the un-ionized state ω_0 shows this correlations. In this paper, we wish to generalize the Cini's model for the M-level system.

2.   Dynamical Description of the General Model

In this section we wish to describe a general dynamical model for quantum decoherence caused by a suitable interaction between the measured system S and a measuring instrument-detector (or an environment) D, which sometimes can be regarded as a reservoir at temperature T. The considered system is only required to be of the non-degenerate discrete spectrum. Let $|n\rangle$ $(n = 1, 2, ... M)$ be the discrete eigenstates of S corresponding to N energy levels E_n $(n = 1, 2, ... M)$. Therefore, the Hamiltonian is formally expressed as

$$\hat{H}_s = \sum_{n=1}^{M} E_n |n\rangle \langle n|. \tag{2.1}$$

D is made up of N particles with the single particle Hamiltonian $\hat{H}_k(x_k)$ for dynamical variables x_k (such as canonical coordinate, momentum and spin) of the k'th particle. Its Hamiltonian

$$\hat{H}_D = \sum_{k}^{N} \hat{H}_k(x_k) = \sum_{k, n_k} \langle n_k| \hat{H}_k(x_k) |n_k\rangle |n_k\rangle \langle n_k| \tag{2.2}$$

can be written in terms of $\hat{H}_k(x_k)$ or its eigenstates $|n_k\rangle$. Here, it has been assumed that there are not mutual interactions among detector's particles.

Physically, the interaction between S and D can be chosen to have the different strengths for the different states of S. This is because so-called measurement is just a schem to read out the states of S from the counting number of D, and different counting numbers should correspond to different states of S. Thus, one can write the interacting Hamiltonian as

$$H_I = \sum_{n} \sum_{k} g(t) V_n(x_k) |n\rangle \langle n| \tag{2.3}$$

where $g(t)$ $(= 1$ for $0 \leq t \leq \tau$; $= 0$ for $t < 0$ or $t > \tau)$ is a switching function; $V_n \neq V_m$ for $m \neq n$ characterizing the time of measurement. Besides the above-mentioned basic requirements, their is only a few of constrains on the model and one even need not to

know the specific forms of both the interaction and the Hamiltonian. Therefore, it is reasonable to say our model is quite universal in comparison with the previous models for quantum measurement.

The system S plus the detector D forms a composite system C, the "universe". In this sense, C is closed and thus its evolution can be described by an unitary operator $U(t)$ governed by the total Hamiltonian

$$H = H_s + H_D + H_I \equiv H_0 + H_I.$$

By changing it into the interaction picture and then backing to the original Schrödinger picture, a direct calculation gives the evolution operator

$$U(t) = U_0(t) \, U_I(t);$$
$$U_I(t) = \sum_n U_n(t) \, |n\rangle \, \langle n| \tag{2.4}$$

where

$$U_0(t) = e^{\frac{1}{i\hbar}\hat{H}_0 t} = \sum_{n=1}^{M} e^{\frac{-iE_n t}{\hbar}} \, |n\rangle \, \langle n| \otimes \prod_{k=1}^{N} e^{\frac{-i\hat{H}_k(x_k)t}{\hbar}};$$

$$U_n^{[k]}(t) = \begin{cases} \mathscr{P} \exp\left[\frac{1}{i\hbar} \int_0^t V_n[x_k(t')]\, dt'\right], & \text{for} \quad 0 \leqq t \leqq \tau \\ U_n^{[k]}(\tau), & \text{for} \quad t > \tau \end{cases} \tag{2.5}$$

Notice that $\mathscr{P}$ is the time-order operator and

$$x_k(t) = e^{\frac{-iH_k t}{\hbar}} x_k e^{\frac{iH_k t}{\hbar}} \tag{2.6}$$

is the representative of the variables x_k of D in the interaction picture.

It should be pointed out that the above mentioned evolution operator $U_I(t)$ in the interaction picture just possesses the factorizable structure found in ref. [14, 15] by one (CPS) of the authors, which is the key to result in von Neumann's wave packet collapse in the quantum measurement.

3.   Quantum Decoherence at Macroscopic Limit

If the system S were closed, the Schrödinger time evolution should not lead to the phenomenon of decoherence – the WPC defined by eq. (1.1), that is to say, a pure state as a coherent superposition of some eigenstates of an observable of S can not evolve into a mixed state. This is because the unitary time evolution operator $U(t)$ preserves the rank of density matrix for S, which, however, has rank 1 for a pure state and has rank larger than 1 for a mixed state. We can also describe this impossibility in terms of the definition of quantum entropy in section 5.

However, in our present model, S is considered as an open system interacting with D. Though the unitary evolution of the universe C formed by S plus D cannot change C to a mixed state from a pure state, its induced effective evolution of S given by removing the variables of D may be non-unitary and thus the rank of the reduced density matrix can be changed in the evolution. In this sense, it is possible to realize the decoherence for quantum measurement in a quantum dynamical process.

Assume that D is initially in a mixed state, e.g., an thermal equilibrium state at temperature T, when the interaction between S and D switches on. This initial state of D is denoted by the density matrix

$$\varrho_D(0) = \prod_{k=1}^{N} \otimes \sum_{n_k} P_{n_k} |n_k\rangle \langle n_k| \tag{3.1}$$

where P_{n_k} is the classical probability of k'th particle of D in the basis states $|n_k\rangle$

$$\sum_{n_k} P_{n_k} = 1 .$$

Let the initial state of S is a pure state, which is a coherent superposition

$$|\phi\rangle = \sum_{n=1}^{N} C_n |n\rangle \tag{3.2}$$

of the energy eigenstates of S without the interaction with D. Then, we write down the density operator of the initial state for the universe $C = S + D$:

$$\varrho(0) = |\phi\rangle \langle\phi| \otimes \varrho_D(0) = \sum_{m,n} C_m C_n^* |m\rangle \langle n| \otimes \varrho_D(0) . \tag{3.3}$$

Solving the von Neumann equation

$$i\hbar \frac{\partial}{\partial t} \varrho(t) = [\varrho(t), \hat{H}] \tag{3.4}$$

we can obtain the density operator of C at time t as

$$\varrho(t) = U(t)^{\dagger} \varrho(0) U(t) .$$

Taking the partial trace of $\varrho(t)$ over the variables of D, we have the reduced density matrix

$$\varrho_s(t) = \mathrm{Tr}_D(\varrho(t)) = \sum_{n=1}^{M} |C_n|^2 |n\rangle \langle n| + \sum_{n>n'}^{M} (C_n C_{n'}^* F_{n,n'}(N,t) |n\rangle \langle n'| + h \cdot c) \tag{3.5}$$

where the off-diagonal terms are accompanied by factors

$$F_{n,n'}(N,t) = e^{\frac{-i(E_n - E_{n'})t}{\hbar}} \mathrm{Tr}(U_{n'} \varphi(t) U_n(t) \varrho_D(0))$$

$$= e^{\frac{-i(E_n - E_{n'})t}{\hbar}} \prod_{k=1}^{N} \sum_{n_k} \langle n_k| (U_n^{[k]\dagger}(t) U_n^{[k]}(t)) |n_k\rangle P_{n_k}$$

$$= e^{\frac{-i(E_n - E_{n'})t}{\hbar}} \prod_{k=1}^{N} F_{n,n'}^{[k]}(t) . \tag{3.6}$$

In the above expression, Tr_k means taking partial trace over the k'th variable of D. Now, we are trying to find a central condition, under which as the particle number N of D approaches infinity, $F_{m,n}(N,T,t)$ approaches zero for $m \neq n$, i.e.,

$$|F_{m,n}(N,t)| \to 0 \quad \text{as} \quad N \to \infty . \tag{3.7}$$

Eq. (3.7) determines the vanishing of the off-diagonal elements of $\varrho_s(t)$, that is to say, it will approximate the classical behavior of the open system S in the macroscopic limit with very large N.

To this end, we rewrite the accompanying factor $F_{n,n'}(N, t) \equiv F$. Then,

$$|F_{n,n'}^{[k]}(t)| = \left| \sum_{n_k} \langle n_k| U_{n'}^{[k]\dagger}(t)\, U_n^{[k]}(t)|n_k\rangle\, P_{n_k} \right| \leqq \sum_{n_k} |\langle n_k| U_{n'}^{[k]\dagger}(t)\, U_n^{[k]}(t)|n_k\rangle|\, P_{n_k}$$

$$\leqq \sum_{n_k} P_{n_k} = 1. \tag{3.8}$$

Here, it has been taken into account that

$$|\langle n_k| U_{n'}^{[k]\dagger}(t)\, U_n^{[k]}(t)|n_k\rangle|^2 = 1 - \sum_{m_k \neq n_k} |\langle m_k| U_{n'}^{[k]\dagger}(t)\, U_n^{[k]}(t)|m_k\rangle|^2 \leqq 1, \tag{3.9}$$

for an unitary operator $U_{n'}^{[k]\dagger}(t)\, U_n^{[k]}(t)$ and $n' \neq n$. In terms of the non-zero positive real number

$$\Delta_k^{n,n'}(t) = -\ln |F_{n,n'}^{[k]}(t)| \tag{3.10}$$

the norm of accompanying factor $F_{n,n'}(N, t)$ is expressed as

$$|F_{n,n'}(N, t)| = e^{-\sum_{k=1}^{N} \Delta_k^{n,n'}}. \tag{3.11}$$

Due to eq. (3.8), $\Delta_k^{m,n} \leqq 0$; In general, under a resonable condition that there is at least one non-vanishing off-diagonal element in a given row or column of each factor $U_{n'}^{[k]\dagger}(t)\, U_n^{[k]}(t)$, $\Delta_k^{m,n}(t)$'s do not approach zero as $k \to \infty$ at $t = \tau$ according to eq. (3.9). In this sense, $\sum_{k=1}^{\infty} \Delta_k^{m,n}(t)$ is a diverging series with the limit of infinity. In next section, some examples are present to obey the above mentioned condition explicitly.

The above discussion shows the possibility of realizing quantum decoherence in a quantum dynamical process at the macroscopic limit with very large N. The above discussion does not depend on the specific forms of both the single particle Hamiltonian $H_k(x_k)$ and the interaction $V_n(x_k)$. It can be invoked to find a vast class of quite general dynamical models for quantum measurement to describe quantum decoherence – the WPC as the result of the dynamical evolution at macroscopic limit!

4.   Concrete Examples with D Made up of Oscillators

In this section, we will use a typical model to explicitly illustrate how the above approach works effectively for the dynamically-vanishing of the off-diagonal elements of the evolving density matrix, concretely speaking, under what kind of circumstance, the series $SR = \sum_{k=1}^{\infty} \Delta_k^{mn}(t)$ in the exponential accompanying factor diverges into infinity. Let D be made up of N oscillators with Hamiltonian

$$H = \sum_{i=1}^{N} \hbar \omega_k a_k^+ a_k \tag{4.1}$$

and the interaction between D and S be

$$H_I = \sum_{n=1}^{M} \sum_{i=1}^{N} \mu_n g_i (a_i^+ + a_i)\, |n\rangle \langle n| \tag{4.2}$$

where the requirement $\mu_m \neq \mu_n$ (for $m \neq n$) means that the coupling of D with S has different strengths for the different states $|n\rangle$ of S.

In this sense, according to WEI-NORMAN's algebraic method [38], a factor $U_n^{[k]}(t)$ of the evolution matrix can be assumed as

$$U_n^{[k]}(t) = e^{f_k^n(t)} \, e^{A_k^n(t) a_k^\dagger} \, e^{B_k^n(t) a_k}. \tag{4.3}$$

Here, the coefficients $f_k^n(t)$, $A_k^n(t)$, and $B_k^n(t)$ to be determined satisfy a system of equations

$$\dot{A}_k^n = - \dot{B}_k^n(t)^* = \frac{g_k \mu_n}{i \hbar \omega} e^{i\omega t},$$

$$\dot{f}_k^n(t) = B_k^n{}^*(t) \, \dot{A}_k^n(t). \tag{4.4}$$

It leads to

$$A^{nk}(t) = - B_k^n{}^*(t) = \frac{\mu_n g_k}{\hbar \omega_k} \left[1 - e^{i\omega_k t} \right] \tag{4.5}$$

and the real part of $f_k^n(t)$

$$\mathrm{Re}(f_k^n(t)) = - \frac{1}{2} |A_k^n(t)|^2 = - \frac{\mu_n g_k}{\hbar \omega_k} (1 - \cos \omega_k t) \tag{4.6}$$

is negative for $\omega_k \neq n\pi/t$.

To master the kernel of the problem, we consider an simple case with zero temperature. The corresponding density matrix is $\varrho_D(0) = |0\rangle \langle 0|$ where the ground state of D

$$|0\rangle = |0_1\rangle \otimes |0_2\rangle \otimes \cdots \otimes |0_N\rangle$$

is a direct product of the vacuum states $|0_i\rangle$ $(i = 1, 2, \ldots, N)$ of N oscillators. In this sense,

$$|F_{mn}^{[k]}(t)| = \left| \langle 0| \, U_n^{[k]}(t)^\dagger \, U_m^{[k]}(t) \, |0\rangle \right| = e^{-\varDelta_k^{mn}(t)} \tag{4.7}$$

where

$$\varDelta_k^{mn}(t) = 2(\mu_m - \mu_n)^2 \, \frac{g_k^2 \sin^2 \frac{\omega_k t}{2}}{(\hbar \omega_k)^2} \equiv \eta_k(t). \tag{4.8}$$

In the following discussion, we will detail the analysis about diverging of the series $\sum_k \varDelta_k^{mn}$ for various spectrum distributions of D. The most simple case is that D has a constant discrete spectrum, i.e., $\omega_k = \omega = \text{constant}$, $g_k = g = \text{constant}$. In this case

$$|F_{mn}(N, t)| = \exp \left\{ -2N(\mu_m - \mu_n)^2 \, \frac{g}{(\hbar \omega)^2} \sin^2 \frac{\omega t}{2} \right\} \equiv |f(N, t)| \tag{4.9}$$

approaches zero as $N \to \infty$ except for the period points $\omega t = 2k\pi$ $(k = 0, 1, 2 \ldots)$. Generally, the series $SR = \sum_k \varDelta_k^{mn}$ can be reexpressed in terms of a unspecific spectrum distribution $\varrho(\omega_k)$ as

$$SR = 2(\mu_m - \mu_n)^2 \sum_k \frac{\varrho(\omega_k) \, g_k^2 \sin^2 \frac{\omega_k t}{2}}{(\hbar \omega_k)^2}. \tag{4.10}$$
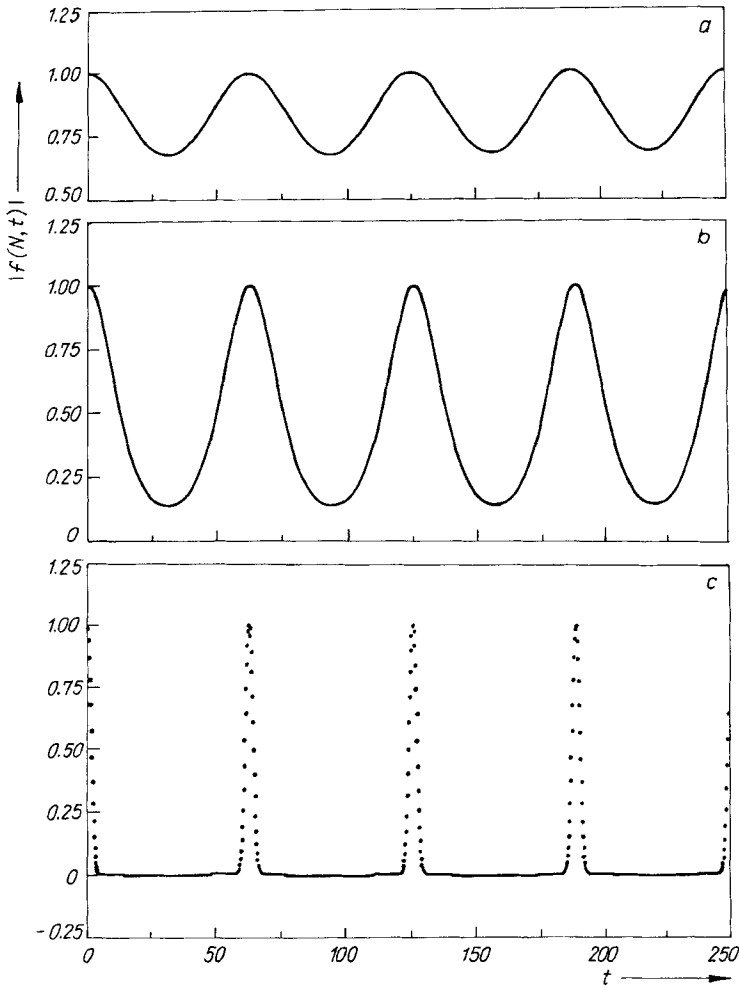
Figure 1: The norm $|f(N, t)|$ of coherence factor as the function of time t for the case with spectral distribution $\omega_k = \omega$. Figures 1-a, 1-b, and 1-c correspond to the times of measurement equal to 5, 10 and 500 respectively. Here, $\omega_k = \omega = 0.1$, $\frac{g}{(\hbar\omega_k)^2} = 0.00001$. For larger N (e.g. $N = 500$), the coherence enjoyed by $|f(N, t)|$ almost disappear for $\bar{t}_k < t < \bar{t}_{k+1}$. However, at the common period points $t_k = \frac{2\pi k}{\omega}$, $(k = 1, 2, ..., N)$ the coherence is resumed in a very short time.

In fact, in the case of discrete spectrum, the spectrum distribution means a degeneracy that there are $\varrho(\omega_k)$ oscillators possessing the same frequency ω_k. So long as the term

$$\varrho(\omega_k)\, g_k^2 \sin^2 \frac{\omega_k t}{2} / (\hbar\omega_k)^2$$

does not approach zero as $k \to \infty$, the above series SR must diverge into infinity. For example, if $\omega_k = k\,\omega$, $\varrho(\omega_k) \propto k^{\eta+2}/g_k^2$, $(\eta > 0)$, then this series must diverge into infinity for each term $\propto k^\eta \sin^2 \frac{k\omega t}{2} > 0$ except for the period points $\omega t = 2k\pi$ $(k = 0, 1, 2, ...)$.

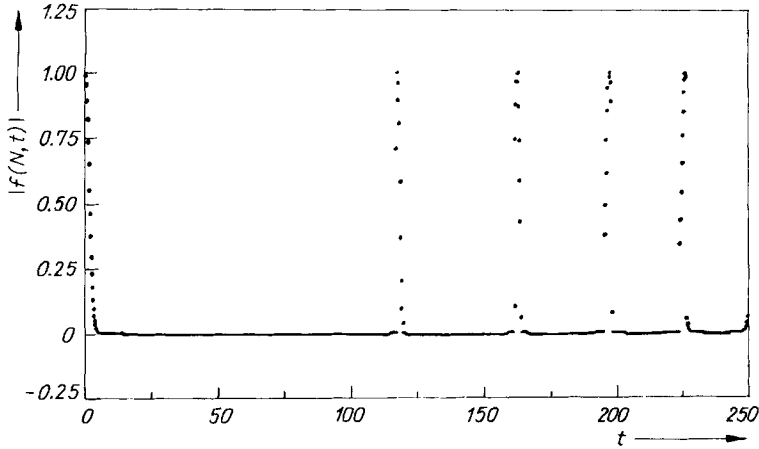Figure 2: The normal of coherence factor for the spectral distribution, the ω_k is random with a cut-off frequence $\omega_c = 500$. The time for resuming the coherence is $t_c(k) = \frac{2\pi k}{\omega_c}$.

Notice that $f(N, t) \equiv F_{m, n}(N, t) = 1$ and 0 means the complete coherence and the complete decoherence respectively. Because of the cut-off of frequency, the $\eta_k(t)$ do not approach zero except for the period points $t = \frac{(2n + 1)\pi}{\omega_k}$ as $k \to \infty$. Also due to $\eta_k > 0$, the series $\sum_{k=1}^{\infty} \eta_k(t)$ must diverge into infinity, that is to say, $f(N, t) \to 0$ according to eq. (3.4). Therefore, when the detector is in the macroscopic limit ($N \to \infty$), the off-diagonal elements in $\varrho_S(t)$ vanishes and the WPC appears as the result of the dynamical evolution. It is recognized from eq. (4.8) that, to realized the WPC, we must constrict the interaction for each time of the measurement only to take place in a time interval τ less than the oscillation period $\tau_c = \frac{2\pi}{\omega k}$. Otherwise, the coherence terms suppressed by the factor $f_k(t) \equiv F_{m, n}^k(t)$ will be resumed at the common period points $t = t_c$ at which $\eta_k(t_c) = 0$ and $|f_k(t)| = 1$. The figure 1 with a constant spectrum $\omega_k = \omega$ shows how the decoherence appears in accompany with: $|f(N, t)| \to 0$ as N increases for $t \neq t_c$, and how the coherence is resumed at $t = t_c$. Figure 2 displays the same problems for the random spectrum distribution that the frequencies ω_k take random value with a cut-off. According the above analysis, one should let the interaction switch off before the coherence restores in the wavefunction collapse quantum dynamically. For this reason, the random spectral distribution and the constant spectral distribution are much appreciated as the most simple case for our present study for the QZE.

For the case with continuous spectrum, some interesting circumstances can result from the concrete spectrum distributions. In the first example with

$$\varrho(\omega_k) = \frac{1}{g_k^2}$$

the series is convergence to a positive number proportional to time t

$$SR = \sum_k \Delta_k^{mn}(t) = \frac{2(\mu_m - \mu_n)}{\hbar^2} \int_0^{\infty} \frac{\sin^2 \frac{\omega_k t}{2}}{\omega_k^2} d\omega_k = \frac{\pi(\mu_m - \mu_n)^2 t}{2\hbar^2}. \tag{4.11}$$

This shows that the norm of the accompanying factor $F_{m, n}(\infty, t)$ is an exponential decaying factor, i.e.,

$$|F_{m,n}(\infty, t)| = e^{-\frac{\pi(\mu_m - \mu_n)^2}{2\hbar^2} t}. \tag{4.12}$$

This quite interesting result is much similar to that in the quantum dissipation [7, 9, 10]. As $t \to \infty$ the off-diagonal elements of density matrix vanish simultaneously!

In another example for continuous spectrum, the spectrum distribution is Ohmic type (e.g., see ref. [7, 9, 10]), i.e.,

$$J(\omega) = \frac{\pi}{2} \sum_{j=1}^{N} \frac{g_j^2}{\omega_j} \delta(\omega - \omega_j) = \eta \omega_j. \tag{4.13}$$

Its alternative reformulation is

$$\varrho(\omega_k) = \frac{2\eta \omega_k^2}{\pi g_k^2} \equiv \varepsilon \frac{\omega_k^2}{g_k^2} \tag{4.14}$$

and then

$$\sum_k \Delta_k^{mn}(t) \to \frac{2(\mu_m - \mu_n)^2 \varepsilon}{\hbar^2} \int_0^\infty \sin^2 \frac{\omega_k t}{2} \, d\omega_k \to \infty, \quad \text{for} \quad m \neq n. \tag{4.15}$$

In summary, so long as we choose a suitable spectrum distribution of oscillators in the detector, the series $\sum_k \Delta_k^{mn}$ can diverge into infinity, that is to say, the dynamical evolution of S plus D can result in the WPC or quantum decoherence in the reduced density matrix of S, though the discussion in this section was proceeded with the oscillator detector, any detector (or environment) weakly coupling to system may be equivalent to a system of oscillators according to the proof given by CALDEIRA and LEGGETT [4]. Therefore, the discussion in this section do not lose the generality of the problem.

5.   Entropy Increment in Decoherence Process

Since quantum decoherence decreases the information available to the observation about the quantum open system S, the quantum entropy

$$s[\varrho] = -\frac{K}{2} \operatorname{Tr}(\varrho \ln \varrho) \tag{5.1}$$

as a functional of the density matrix ϱ can be used to characterize the degree of decreasing information quantitatively. In comparison with the statistical thermodynamics, the decoherence process can be understood as an irreversible process in terms of the concept of entropy increment. In fact, if $|\lambda\rangle$ is the eigenstate of ϱ with eigenvalue λ, then eq. (5.1) is re-expressed as

$$S[\varrho] = -\frac{K}{2} \sum_\lambda \lambda \ln \lambda. \tag{5.2}$$

Obviously, the entropy is invariant under an unitary transformation and the time evolution of a closed system must not change its entropy. In our problem, the entropies of the initial and final states are zero and

$$s[\varrho_f] = -\frac{K}{2} \sum_{n=1}^{M} |C_n|^2 \ln |C_n|^2 \tag{5.3}$$

respectively. This manifests that the decoherence process is certainly a process of increasing entropy. Notice that eq. (5.2) defines the maximum entropy of the system S for a given initial state (3.2) of S, which corresponds to complete decoherence. In a well-established theory for quantum decoherence, it is expected that the partially- and completely-decohering states are also described very well. For this reason, we calculate the corresponding entropy of the intermediate state characterized by the reduced density matrix $\varrho_s(t)$ for finite N.

For the seek of simplicity, we first consider the two-state system with $M = 2$ physically. This case may be understood as spin $\frac{1}{2}$ procession interacting with a reservoir. In this sence, the reduced density matrix with finite N is explicitly written down

$$\varrho_s = \begin{bmatrix} |C_1|^2 & C_1 C_2^* F \\ C_2 C_1^* F^* & |C_2|^2 \end{bmatrix} \tag{5.4}$$

where

$$F = F_{12}(N, T) = e^{-\sum_{i=1}^{N} \Delta_k^{1,2}(t)}.$$

In terms of its eigenvalues $\lambda_1 = \frac{1}{2}(1 + x)$ and $\lambda_2 = \frac{1}{2}(1 - x)$ where,

$$x = x(N) = \sqrt{1 - 4(1 - |F|^2) |C_1|^2 |C_2|^2}$$
$$= \sqrt{1 - 4(1 - e^{-2 \sum_{i=1}^{N} \Delta_k(t)}) |C_1|^2 |C_2|^2}, \tag{5.5}$$

the explicit expression of entropy is

$$s(N) = -\frac{K}{2} [(1 + x) \ln(1 + x) + (1 - x) \ln(1 - x) - 2 \ln 2] \tag{5.6}$$

where

$$\| C_1| - |C_2 \| \leqq x \leqq 1.$$

Obviously,

$$\frac{ds(N)}{dx} = -\frac{K}{2} \ln \frac{(1 + x)}{(1 - x)} < 0$$

and x decrease as N increases. Then, $s = s(N)$ is a monotonically-increasing function in the above domain of x. When $N = \infty$, x takes its maximum value so that there appears maximum entropy

$$s_m = -\frac{K}{2} (|C_1|^2 \ln |C_1|^2 + |C_2|^2 \ln |C_2|^2). \tag{5.7}$$

The above quantitative analysis shows us that the complete decoherence means the maximum value of entropy. For the sufficiently large N, the average value $\Delta = N^{-1} \sum_{k=1}^{N} \Delta_k^{1,2}(t)$ is roughly independents for N

$$|F| = e^{-2N\Delta}$$

is approximated by exponential function of N.

The explicit entropy functions for the partial decoherence in mesoscopic case with a finitely large N can not be easily solved for a general system of $M (> 2)$ energy levels.

However, for few cases with very large N or very small N, some approximation methods can be developed to calculate the entropy analytically.

For the case with small N, we define

$$\Delta_{mn}(N) = F_{mn}(N) - 1. \tag{5.8}$$

Since the function $F_{n,n}(N, t) = 1$ and $F_{m,n}(N, t)$ approaches the unity for small N, the norm of $\Delta_{m,n}(N)$ is quite small in this case. Thus, the reduced density (3.5) can be decomposed into the unperturbed part

$$\varrho_0 = \varrho(0) = \sum C_m C_n^* |m\rangle \langle n| = |\psi\rangle \langle \psi|$$

and the perturbed part

$$\Delta \varrho = \sum_{m,n=1}^{M} \Delta_{m,n}(N) C_m C_n^* |m\rangle \langle n|. \tag{5.9}$$

The unperturbed part ϱ_0 denotes a pure state and then has a non-degenerate eigenstates $|\psi\rangle$ with eigenvalue 1. Choose $|\psi_0\rangle \equiv |\psi\rangle$ to be the first one of basis vectors for the Hilbert space. Then, the other M-1 basis vectors $|\psi_k\rangle = P|k\rangle$ $(k = 1, 2, ..., N-1)$ for the M-1 dimensional degenerate subspace orthonormal to $|\psi_0\rangle$ may be constructed in terms of the complementary projection operator

$$P = 1 - |\Psi\rangle \langle \Psi| = \sum_{m=1}^{N} (\delta_{mn} - C_m^* C_n) |n\rangle \langle m|. \tag{5.10}$$

It is not difficult to prove that such $N - 1$ vectors are linearly-independent. It follows from the Schmite rule that $N - 1$ vectors

$$|u_k\rangle = \sum_{k'} S_{k'k} |\psi_{k'}\rangle, \quad k = 1, 2, ..., N - 1$$

are built as the orthogonomal basis for the whole space corresponding to the zero eigenvalues of ϱ_0. Therefore, the time independent perturbation theory determines the approximate eigenvalues of $\varrho = \varrho_0 + \Delta \varrho$ up to second order

$$\lambda_0 \simeq 1 - \sum_{k=1}^{N-1} |\langle \Psi| \Delta \varrho |u_k\rangle|^2 + \cdots \equiv 1 - \delta + \cdots \tag{5.11}$$

$$\lambda_k = |\langle \Psi| \Delta \varrho |U_k\rangle|^2 + \cdots \equiv \delta_k + \cdots, \quad k = 1, 2, ... M - 1.$$

Obviously, the normalization of density matrix still holds as $\sum_{k=0}^{N-1} \lambda_k = 1$ under this perturbation. A direct calculation from eq. (5.11) results in the entropy function in small N limit as

$$S(N) = -\frac{k}{2} \left[(1 - \delta) \ln(1 - \delta) + \sum_k \delta_k \ln \sum_k \delta_k \right] \tag{5.12}$$

$$= -\frac{K}{2} \sum_{k=1}^{M-1} |\langle \Psi| \Delta \varrho |u_k\rangle|^2.$$

Another case that can be handled analytically based on approximation method is that with large N. In this sense, $|F_{mn}(N, t)|$ is so small that the off-diagonal parts of $\hat{\varrho}$ (ex. (3.5)) can be regarded as a perturbation. The unperturbed part

$$\varrho_M = \sum_{m=1}^{M} |C_m|^2 \, |m\rangle \, \langle m| \tag{5.13}$$

represents the mixed state with the maximum entropy and $|k\rangle$ and $|C_k|^2$ are just its eigenstates and the corresponding eigenvalues. Invoking the time-independent perturbation theory and regarding

$$\Delta \varrho = \hat{\varrho} - \varrho_M \tag{5.14}$$

as the perturbation, we write down the second's corrections for the eigenvalue

$$\Delta \lambda_n^{[2]} = \sum_{m \neq n} \frac{|F_{mn}|^2 \, |C_m|^2 \, |C_n|^2}{|C_m|^2 - |C_n|^2} \tag{5.15}$$

for the first order solution $\lambda_n^{(0)} = |C_N|^2$. Then, the approximate entropy is obtained up to of second order

$$s(N) = -\frac{K}{2} \sum_{k} [|C_k|^2 \, (\ln |C_k|^2 + 2 \, \Delta \lambda_k^{(2)})] . \tag{5.16}$$

6.   Application to Quantum Zeno Effect

The quantum Zeno effect (QZE) was theoretically proposed by MISRA and SUDARSHAN [25] based on the postulate of the WPC. It is argued that an unstable particle will never be found to decay when it is continuously observed, more generally speaking, a frequent measurement inhibits the transitions between quantum states. Recently, ITANO, HEINZEN, BOLLINGER and WINELAND (IHBW) [26] reported that they have observed the QZE in an experiment about atomic transition based on Cook's proposal. They claimed that the freezing of stimulated transition probability appeared when the two-level atomic system is subjected to frequent measurements of the level population. Then, the QZE havs attracted much interest in its theoretical studies last years [27–37].

Among these discussions, PETROSKY, TUSAKI and PRIGOGINE's (PTP's) work [27–28] shows that the result in IHBW's experiment can be recovered through conventional quantum mechanics without invoking a repeated WPC. This is quite similar to PERES's observation [37] that the a modified Hamiltonian may mimic the wavefunction collapse to slow down the quantum transition and then realized the QZE in a pure framework of quantum mechanics. It seems that the conclusion of IHBW's experiment is challenged by the theoretical analysis in PTP's work, and the same result can be obtained in term of either the use of the WPC by frequent measurement, or the application of the Schrödinger's evolution as a pure quantum dynamical process [29, 30]. Thus, It is difficult to say whether the IHBW's experiment has proved the existence of the QZE or not.

To compromise the above mentioned different standpoints in a reasonable framework, we will reconsider the QZE and its corresponding IHBW's experiment from a distinct point of view based on the dynamical approach for quantum measurement in this paper. Since the WPC for quantum measurement in present approach can be regarded as a quantum dynamical process in certain circumstance, it is natural to understand the QZE and IHBW's experiment as the results of a measurement monitoring the system continuously, but the WPC characterizing measurement can be realized here quantum dynamically.

6.1.   Model Hamiltonian for QZE and its Evolution Matrix

Now, we present a dynamical model for the QZE. Let the measured object be a two-level systems S with Hamiltonian

$$H_0 = \frac{\Omega}{2} (|1\rangle \langle 2| + |2\rangle \langle 1|) \tag{6.1}$$

where $|1\rangle$ and $|2\rangle$ are the ground and excited states respectively. Ω is Rabi frequency of the external field coupling with atom. The continuous measurement for a given time interval T is imaged as a limit of the L times successive measurements at different times $t = kT/L$ $(0 \leq k \leq L)$ with $L \to \infty$. For k'th measurement, the interaction of measuring instrument-detector D on the system turns on at time $t_k = kT/L$ and then turns off at at time $t_k + \tau$ for $k = 0, 1, 2, \ldots$. Here, the τ is very short in comparison with T/L. This case is quite similar to the realistic experiment by ITANO et al. [26]. In their experiment involving three levels of ${}^9\text{Be}^+$ ions, a on-resonance radiation frequency field is applied to ${}^9\text{Be}^+$ with L shot on-resonance optical pulses-measurement pulses to perform a quantum measurement. Each optical pulse results in the off-diagonal elements of the density matrix to zero with the postulate of the WPC.

In our model, the instrument D consists of N harmonic oscillators with free Hamiltonian

$$H_D = \sum_{i=1}^{N} \hbar \omega_i a_i^+ a_i \tag{6.2}$$

where a_i^+ and a_i are creation and annihilation operators for i's mode of boson states respectively. To realize the WPC, some additional requirements such as the frequency cut-off mentioned in sections must be introduced for the spectral distribution. The model-interaction

$$H_I = \frac{1}{2} \sum_{i=1}^{N} g(t)(a_i + a_i^+) \left[(|1\rangle \langle 1| - |2\rangle \langle 2|) \cos \frac{\Omega}{2} t - \sin \frac{\Omega}{2} t (|2\rangle \langle 1| - |1\rangle \langle 2|) + 1 \right] \tag{6.3}$$

is turned-on and turned-off by a switching function

$$g(t) = \begin{cases} g, & \text{if } t_k \leq t \leq t_k + \tau \\ 0, & \text{otherwhere.} \end{cases}$$

Specifying the general approach in section 3 to present model, we obtain the exact solution $U(t)$ for the Schrödinger equation of the total system C in terms of the free evolution matrix $U(t) = U_0(t) U_e(t)$:

$$U_0(t) = U_D(t) \cdot U_S(t):$$

$$U_D(t) = e^{-i \sum a_k^+ a_k \omega_k t} = \prod_{k=1}^{N} e^{-i a_k^+ a_k \omega_k t};$$

$$U_S(t) = \begin{bmatrix} \cos \frac{\Omega}{2} t & -i \sin \frac{\Omega}{2} t \\ -i \sin \frac{\Omega}{2} t & \cos \frac{\Omega}{2} t \end{bmatrix}, \tag{6.4}$$

and the evolution matrix in the interaction picture is

$$U_e(t) = U_e(t) \, |1\rangle \, \langle 1| + |2\rangle \, \langle 2|:$$

$$U_e(t) = \prod_{k=1}^{M} e^{f_k(t)} \, e^{A_k(t) \, a^+} \, e^{B_k(t) \, a} \equiv \prod_{k=1}^{M} U^{[k]}(t) \tag{6.5}$$

where

$$A_k(t) = -B_k^*(t) = \int_0^t \frac{g(t)}{i\hbar} \, e^{i\omega_k t} \, dt = \frac{g}{i\hbar} \left[\sum_{j=0}^{l-1} \int_{t_j}^{t_j+\tau} e^{i\omega_k t} \, dt + \int_{t_l}^{t_l+t} e^{i\omega_k t} \, dt \right]$$

$$= \frac{g}{\hbar\omega} \left[(1 - e^{i\omega_k \tau}) \, e^{\frac{i\omega_k l T}{N}} + (1 - e^{i\omega_k \tau}) \frac{1 - e^{\frac{i\omega T l}{L}}}{1 - e^{\frac{i\omega T}{L}}} \right], \tag{6.6}$$

$$\dot{f}_k(t) = B_k^*(t) \, \dot{A}_k(t) \tag{6.7}$$

for $t_l \leq t \leq t_l + \tau$. Here, we have used the properties of the switching function $g(t)$. If $\tau \ll T$, then the approximate function $A_k(t)$ is

$$A_k(t) = \frac{g}{\hbar\omega} \, (1 - e^{i\omega_k \tau}) \, e^{\frac{i\omega l T}{L}}. \tag{6.8}$$

It follows from the above equations that

$$\eta_k(t) = \mathrm{Re} \, [f_k(t)] = -\frac{1}{2} \int_0^t (\dot{A}_k(t') \, A_k^{*\prime}(t') + \dot{A}_k(t')^* \, A_k(t')) \, dt = -\frac{1}{2} \, |A_k(t)|^2. \tag{6.9}$$

Notice that real part of $f_k(t)$ is negative and $e^{\mathrm{Re}\,(f_k(t))}$ is not larger than unity.

6.2.   QZE Based on Dynamical Realization of QZE

For the measured system S with an initial pure state

$$|\psi\rangle = c_1 \, |1\rangle + c_2 \, |2\rangle$$

and the measuring instrument D with initial mixture state described by a density matrix

$$\varrho_D = \prod_{k=1}^{N} \varrho_D(k),$$

the initial state of the composite system C formed by S plus D is a mixture state with density matrix

$$\varrho(0) = |\psi\rangle \, \langle\psi| \otimes \varrho_D.$$

Then, the state density matrix of C at $t (< t_1)$ is

$$\varrho(t) = U_0(t) \, U_e(t) \, \varrho(0) \, U_e^+(t) \, U_0^+(t).$$

Now, we understand the process of measurement as a procedure to determine the reduced density matrix, which contains the total information of the measured system S. The first measurement results in a reduced density matrix for S

$$\varrho_s = Tr_D \varrho(t) = U_S(t)(|c_1|^2 |1\rangle \langle 1| + |c_2|^2 |2\rangle \langle 2| + f(N, t)|1\rangle \langle 2|$$
$$+ f(N, t)^* |2\rangle \langle 1|) U_S^+(t) \qquad (6.10)$$

by taking the trace for the variables of the detector D where the coherence factor

$$f(N, t) = Tr_D(U_e(t) \varrho_D) = \prod_{k=1}^{N} f_k(N, t) \equiv \prod_{k=1}^{N} Tr_k[U^{[k]}(t) \varrho_D(k)] \qquad (6.11)$$

is factorizable. This factorization is crucial for the appearance of the WPC. Similar to that in section 3,

$$|f_k(N, t)| = |Tr_k(U^{[k]}(t) \varrho_D(k))| \leq 1.$$

Thus, the N-multiple product of all $|f_k(N, t)|$'s must approach zero as $N \to \infty$ unless most of $|f_k(N, t)|$'s are unity simultaneously. It has been illustrated that the case that all or most of $|f_k(N, t)|$'s are unity can be eliminated by choosing a suitable spectrum distribution with cut-off of frequency. For example, we take $\varrho_D(k)$ to a pure state $|0\rangle \langle 0|$, obtaining

$$f(N, t) = \langle 0| U_I(t) |0\rangle = e^{-\sum_{k=1}^{N} \eta_k(t)} \qquad (6.12)$$

where $|0\rangle = |0_1\rangle \otimes |0_2\rangle \otimes \cdots \otimes |0_M\rangle$ is the vacuum state of the detector D consisting of M oscillators.

$$\eta_k(t) = -\text{Re}[f_k(t)] = \frac{1}{2} |A_k|^2 = \frac{g^2}{(\hbar \omega_k)^2} (1 - \cos \omega_k t). \qquad (6.13)$$

Notice that eq. (6.13) is quite similar to eq. (4.8). When the detector is macroscopic $(N \to \infty)$, the off-diagonal elements in $\varrho_S(t)$ vanishes and the WPC appears as the result of the dynamical evolution, that is

$$\varrho(t) \to \varrho_S = U_S(t)(|c_1|^2 |1\rangle \langle 1| + |c_2|^2 |2\rangle \langle 2|) U_S^+(t). \qquad (6.14)$$

According to the above discussion, after the first time of measurement, the density matrix is

$$\varrho_s\left(t_1 = \frac{T}{L}\right) = (c_1^{[1]} |1\rangle \langle 1| + c_2^{[2]} |2\rangle \langle 2| + A^{[1]} |1\rangle \langle 2| + A^{[1]*} |2\rangle \langle 1|) + O^{[1]}\left(\frac{1}{N}\right) \qquad (6.15)$$

where

$$c_1^{[1]} = |c_1|^2 \alpha^2 + |c_2|^2 \beta^2, \quad c_2^{[1]} = |c_2|^2 \alpha^2 + |c_1|^2 \beta^2$$

$$A^{[1]} = \frac{i}{2} |c_1|^2 \beta - \frac{i}{2} |c_2|^2 \qquad (6.16)$$

$$\alpha = \alpha(L) = \cos \frac{\Omega T}{2L}, \quad \beta = \beta(L) = \sin \frac{\Omega T}{2L}.$$

Notice that the last term in eq. (4.1) is

$$O^{[1]}\left(\frac{1}{N}\right) = U_S^{(t)}[f(N, t)|1\rangle \langle 2| + f^*(N, t)|2\rangle \langle 1|] U_S^+(t) \qquad (6.17)$$
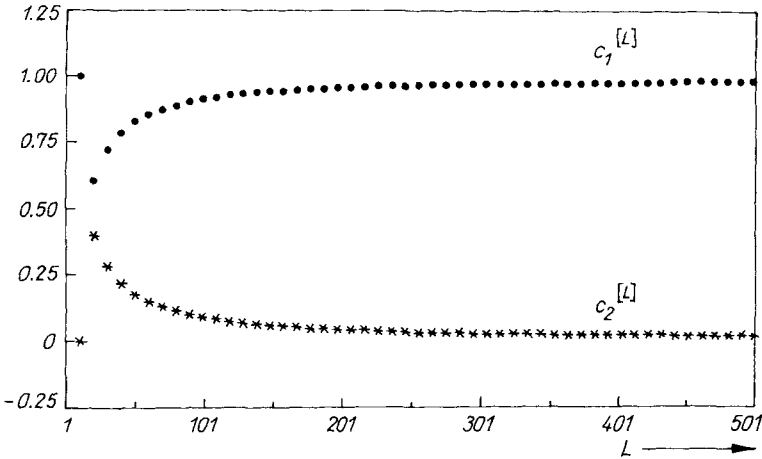
Figure 3: The probabilities $c_1^{[L]}$ and $c_2^{[L]}$ finding the system at states $|1\rangle$ and $|2\rangle$ respectively.

which vanishes as $N \to \infty$, the first term in eq. (3.1)

$$U_S(t_1)(|c_1|^2 |1\rangle \langle 1| + |c_2|^2 |1\rangle \langle 1|) U_S^+(t_1)$$

represents the mixed state with complete decoherence. Subsequently, the measurement of first time cancels the off-diagonal term of $\varrho_s(0) = |\psi\rangle \langle\psi|$ as $N \to \infty$. The measurement of second time will cancel the off-diagonal terms $A^{[1]}|1\rangle \langle 2|$ and $A^{[1]*}|2\rangle \langle 1|$ in eq. (6.15). Similarly, after the measurement of k'th time

$$\varrho_s[k] = c_1^{[k]}|1\rangle \langle 1| + c_2^{[k]}|2\rangle \langle 2| + A^{[k]}|1\rangle \langle 2| + A^{[k]*}|2\rangle \langle 1| + O_k^{[k]}\left(\frac{1}{N}\right)$$

$$= U_S(t_k)(c_1^{[k-1]}|1\rangle \langle 1| + c_2^{[k-1]}|2\rangle \langle 2|) U_S(t_k)^+ + O_k^{[1]}\left(\frac{1}{N}\right) \qquad (6.18)$$

where $c_1^{[k]}$ and $c_2^{[k]}$ are determined by the recurrent relations

$$c_1^{[k]} = c_1^{[k-1]}\alpha^2 + c_2^{[k-1]}\beta^2, \quad c_2^{[k]} = c_2^{[k-1]}\alpha^2 + c_1^{[k-1]}\beta^2 \qquad (6.19)$$

where

$$O_k^{[1]}\left(\frac{1}{N}\right) = U_s(t_{k-1})[A^{[k-1]} f(N, t_k) |1\rangle \langle 2| + A^{[k-1]*} f(N, t_k)^* |2\rangle \langle 1|] U_s^+(t_{k-1})$$

also disappear as $N \to \infty$.

It is not difficult to solve $c_1^{[k]}$ and $c_2^{[k]}$ explicitly from eq. (6.19), but we only invoke the computer simulation to evaluate the variations of $c_1^{[L]}$ and $c_2^{[L]}$ as the measurement times L increase. If the system is initially in pure state $|1\rangle$, the initial conditions are $c_1 = 1$ and $c_2 = 0$. It is illustrated in Figure 3 that, as $L \to \infty$, the distributions $c_1^{[L]} \to 1$ while $c_2^{[L]} \to 0$. Notice that $c_1^{[L]}$ first decrease and then increase to 1. This means that, for a macroscopic instrument ($N \to \infty$), the system will be forced back to the state $|1\rangle$ as the successive measurements become continuous for $L \to \infty$. This just the QZE!

41*

6.3.   "Transition" of Quantum Entropy for QZE

Now, we consider the entropy of the measured system S as a function of the times L of measurement. Considering that the entropy is invariant under an unitary transformation due to the original definition, one can obtain the entropy for the QZE after L'th measurement

$$s(L) = s[\varrho_S(L)] = -\frac{K}{2}(c_1^{[L]} \ln c_1^{[L]} + c_2^{[L]} \ln c_2^{[L]}).$$ (6.20)

The entropy $s = s[\varrho_s(L+1)]$ takes its maximum value

$$S_{\max}(L+1) = \frac{K}{2} \ln 2$$ (6.21)

only when

$$c_1^{[L]} = c_2^{[L]}.$$

According to the recurrent relations eq. (6.19), the above equation leads to

$$c_1^{[L]} - c_2^{[L]} = (\alpha^2 - \beta^2)(c_1^{[L-1]} - c_2^{[L-1]})$$

$$= (\alpha^2 - \beta^2)^L (c_1^2 - c_2^2).$$

Then, it is easily observed that the above equation holds only when

$$\alpha^2(L) = \beta(L)^2$$

or

$$\frac{\omega T}{L} = \frac{\pi}{2}(2l+1), l = 0, \pm 1, \pm 2, \dots.$$ (6.22)

The entropy s takes several maximum values

$$s_{\max}(L) = -\frac{K}{2}[c_1^{[\bar{L}]} \ln c_1^{[\bar{L}]} + c_2^{[\bar{L}]} \ln c_2^{[\bar{L}]}]$$ (6.23)

where

$$\bar{L} = \left[\frac{2\omega T}{(2l+1)\pi}\right], \quad l = 0, \pm 1, \pm 2, \dots$$ (6.24)

Here, $[x]$ denotes the integer part of the real number for $x \geq 0$, if $x \leq 0$ then $[x] = 0$.

Let us consider some special examples for the above discussions. If $\omega T = \pi$, $\bar{L} = 2$ (for $l = 0$), and there only one point for the maximum entropy

$$s_{L+1} = -\frac{K}{2}(c_1^{[2]} \ln c_1^{[2]} + c_2^{[L]} \ln c_2^{[L]}).$$ (6.25)

Generally, if $\omega T = k\pi$ for positives integer k,

$$\bar{L} = \left[\frac{2k}{2l+1}\right] = 2k, \left[\frac{2k}{3}\right], \left[\frac{2k}{5}\right], \dots, \left[\frac{2k}{2X+1}\right]$$ (6.26)

where $X \leq k-1$, that is to say, there are k points $2k$, $[\frac{2k}{3}]$, $[\frac{2k}{5}]$, ..., $[\frac{2k}{2k+1}]$ for the maximum values of entropy.
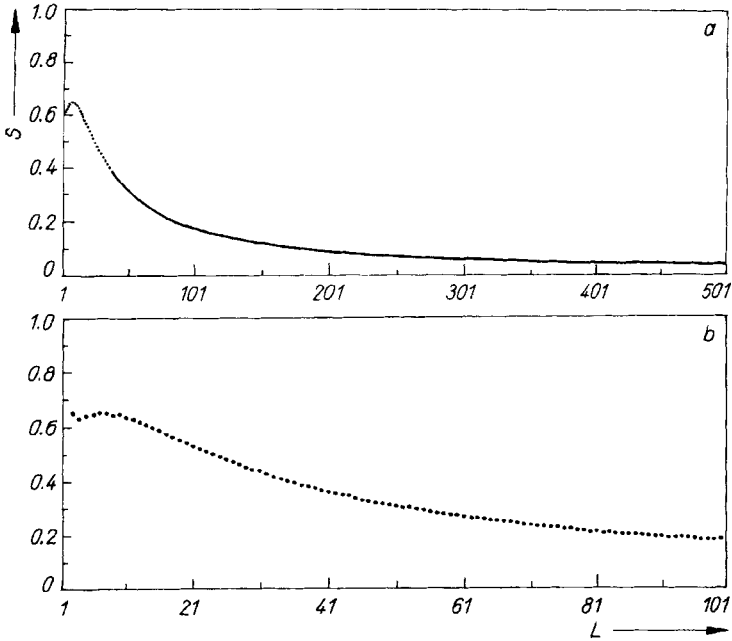
Figure 4: The entropy as the function of times L of measurement for $\omega T = 3\pi$, it represents a "transition" of the information entropy from the random to regularity. Figure 4-b is part of Figure 4-a.
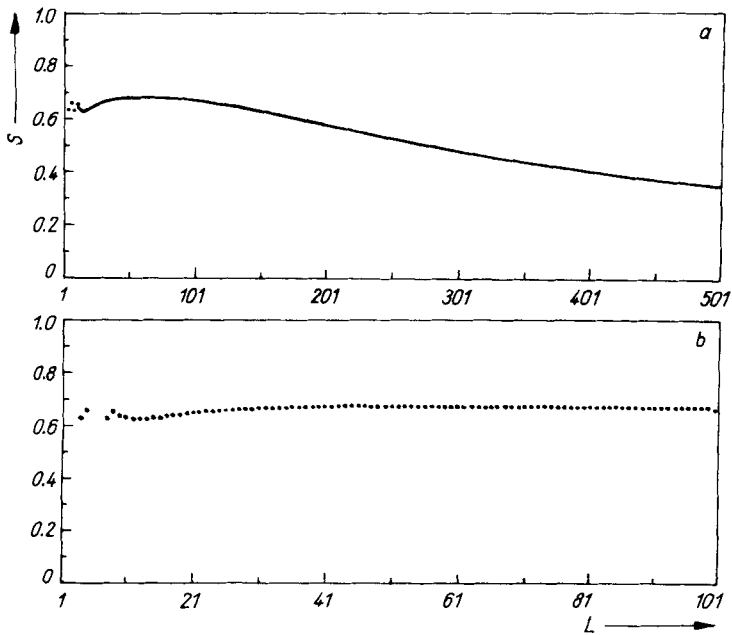

Figure 5: The entropy as the function of times L of measurement for $\omega T = 10\pi$, it represents a "transition" of the information entropy from the random to regularity. Figure 5-b is part of Figure 5-a. $\omega T = 10\pi$, and Figure 5-c,d with $\omega T = 10\pi$.

For the general case with $\omega T = kT$, the above analysis shows us that, if L is less than the critical value $L_c = 2k$, the variation of $S(L)$ is a "random function" of L, which is not monotonic; however, when L is larger than the critical value $2k$, $s(L)$ is a monotonically-decreasing function of L. When ωT is not an integer times of π, the critical point for L is $\frac{2\,\omega\,T}{(2\,l+1)}\pi$. Such a feature of "transition" from random to regularity for quantum entropy in the QZE is illustrated in Figures 4 and 5. Physically, the measurement process with the QZE defines a transition of the information entropy from random to regularity.

7.   Generalization of Cini's Model for Quantum Measurement

It has to be pointed out that the correlation between the states (CBS) of the measured system and that of the detector has not been emphasized well in the original HC model and its generalizations. This problem was alternatively analysed by CINI in his beautiful dynamical model [39]. The present investigation is to emphasize on both the WPC and CBS for a generalization of the Cini model. In fact, the correlation between the states of measured system and the detector is crucial for a realistic process of measurement, which enjoys a scheme using the macroscopic counting number of the measuring instrument-detector to manifest the microscopic of the measured system. The original Cini model for the correlation between the states of measured system S and the measuring instrument-detector D is only build for a two-level system interacting with the detector D, which consists of indistinguishable bosons with two possible modes ω_0 and ω_1. For the two states u_+ and u_- of S, the detector has different strengths of interaction with them. Then, the large number N of "ionized" particle in the ionized mode ω_1 transiting from the un-ionized mode ω_0 shows this correlations.

7.1.   Generalized Cini Model

Now, we wish to generalize the Cini's model for the M-level system. The measured system S with M-levels has the model Hamiltonian

$$\hat{H}_S = \sum_{n=1}^{M} E_n |\Phi_n\rangle \langle \Phi_n| \tag{7.1}$$

where $|\Phi_n\rangle$ are the eigenstates corresponding to the eigenvalues E_n ($n = 1, 2, ..., M$). The detector D is a two-boson system with the free Hamiltonian

$$\hat{H}_D = \hbar \omega_1 a_1^+ a_1 + \hbar \omega_2 a_2^+ a_2 \tag{7.2}$$

where a_i and a_i^+ are the creation and annihilation operators and they satisfy

$$[a_i, a_j^+] = \delta_{ij}, \ [a_i, a_j] = [a_i^+, a_j^+] = 0. \tag{7.3}$$

In the *Schrödinger* representation, the interaction is described by

$$\hat{H}_I(t) = \sum_n g e^{-\eta t} (w_n |\Phi_n\rangle \langle \Phi_n|) (e^{i(\omega_2 - \omega_1)t} a_1^+ a_2 + e^{i(\omega_1 - \omega_2)t} a_2^+ a_1) \tag{7.4}$$

where the non-degenerate weights w_n represent the different strengths for the different states $|\Phi_n\rangle$ of the system. The exponential decay factor $e^{-\eta t}$ ($\eta > 0$) is here introduced to

turn off the interaction after suitable time so that the coherence can not restore in the evolution process. This point can be explicitly seen in the following discussion. The introduction of time-dependent factors $e^{\pm i(\omega_1 - \omega_2)t}$ is quite similar to that in ref. [13] where these factors are used to describe the energy exchange due to the presence of the free Hamiltonian H_D. Notice that there was not the free Hamiltonian for detector in the original Hepp-Coleman model such as H_D in our present model. Let $H = H_S + H_D + H_I$ be the total Hamiltonian in the *Schrödinger* representation for the composite system C formed by S plus D.

Transforming the problem into the interaction representation with the evolution operator

$$U_0(t) = \exp\left[\frac{1}{i\hbar}(H_S + H_D)\,t\right]$$

one has the interaction potential

$$V_I(t) = e^{-\eta t}\, g \sum_{n=1}^{M} W_n\, |\Phi_n\rangle\, \langle\Phi_n|\, (a_1^+ a_2 + a_1 a_2^+). \tag{7.5}$$

In order to diagonalize $V_I(t)$, we invoke the canonical transformation as in ref. [39]

$$a_1 = \frac{1}{\sqrt{2}}\,(b_1 - b_2), \quad a_2 = \frac{1}{\sqrt{2}}\,(b_1 + b_2) \tag{7.6}$$

where the new boson operators b_i and b_i^+ still satisfy the same bosonic commutation relation. In terms of these operators, $V_I(t)$ is rewritten as the diagonal form

$$V_I(t) = g e^{-\eta t} \sum_{n=1}^{M} W_n\, |\Phi_n\rangle\, \langle\Phi_n|\, (b_1^+ b_1 - b_2^+ b_2). \tag{7.7}$$

Then, considering that the interaction part $V_I(t)$ commutes with each other at different time, ie.

$$[V_I(t),\, V_I(t')] = 0$$

one can express the evolution operators as

$$U_I(t) = \exp\left[\frac{1}{i\hbar}\int_0^t V_I(t')\,dt'\right] \equiv \sum_{n=1}^{M} U_n(t)\, |\Phi_n\rangle\, \langle\Phi_n|$$

$$= \sum_{n=1}^{M} \exp\left[-it_\eta g W_n(b_1^+ b_1 - b_2^+ b_2)\right] |\Phi_n\rangle\, \langle\Phi_n| \tag{7.8}$$

where

$$t_\eta = \frac{1 - e^{-\eta t}}{\eta}$$

takes the real time as $\eta \to 0$. Thus, t can be regarded as the η-deformation of time t. When $t \to \infty$, $t_\eta \to \frac{1}{\eta}$.

7.2.   Correlation of States from Evolution of State

Now, we consider the evolution of the total system starting with an initial state at $t = 0$

$$|\Psi(0)\rangle = \sum_{k=1}^{M} C_k |\Phi_k\rangle \otimes |N, 0\rangle \tag{7.9}$$

where

$$|m, n\rangle = \frac{a_1^{+m} a_2^{+n}}{\sqrt{m! \, n!}} |0\rangle \tag{7.10}$$

denotes a Fock state of two-boson system with n particles in "ionized" state and m particles in un-ionized state. It is hoped to manifest a correlation between the states $|m, n\rangle$ of detector and the state $|\Phi_l\rangle$ of the system in a dynamical evolution of the state $|\Psi(t)\rangle$ in some limiting case so that one can read out the state $|\Phi_l\rangle$ from the manifestation of the state $|m, n\rangle$ of the detector. Notice that the eigenstates of the operator

$$\hat{O} = b_1^+ b_1 - b_2^+ b_2 \tag{7.11}$$

are

$$|\lambda, N - \lambda\} = \frac{1}{\sqrt{\lambda! \, (N - \lambda)!}} \, b_1^{+\lambda} b_2^{+(N-\lambda)} |0\rangle$$

$$= \left(\frac{1}{\sqrt{2}}\right)^N \sum_{m=0}^{\lambda} \sum_{n=0}^{N-\lambda} \frac{\sqrt{\lambda! \, (N-\lambda)!}}{\sqrt{(\lambda-m)! \, (m!)(N-\lambda-n)! \, (n!)}} |m, n\rangle \tag{7.12}$$

with the eigenvalues

$$\varepsilon_N(\lambda) = 2\lambda - N \tag{7.13}$$

where $\lambda = 0, 1, 2, \ldots, N$ for a given integer N. The original Fock state can be expended in terms of $|\lambda, N - \lambda\}$ as

$$|N, 0\rangle = \left(\frac{1}{\sqrt{2}}\right)^N \sum_{\lambda=0}^{N} \frac{\sqrt{N!}(-1)^{N-\lambda}}{\sqrt{\lambda!(N-\lambda)!}} |\lambda, N - \lambda\}. \tag{7.14}$$

Then, one obtains the wavefunction of thew total system at time t

$$|\Psi_I(t)\rangle = U_I(t) |\Psi(0)\rangle \tag{7.15}$$

$$= \sum_{\lambda=0}^{N} \sum_{k=1}^{M} C_k \left(\frac{1}{\sqrt{2}}\right)^N \frac{\sqrt{N!}(-1)^{N-\lambda}}{\sqrt{\lambda!(N-\lambda)!}} \, e^{-igt_\eta W_k(2\lambda-N)} |\Phi_k\rangle \otimes |\lambda, N - \lambda\}$$

namely,

$$|\Psi_I(t)\rangle = \sum_{k=1}^{M} C_k |\Phi_k\rangle \otimes \sum_{n=0}^{N} a_n(t, k) |n, N - n\rangle \tag{7.16}$$

where

$$a_n(t, k) = \frac{(-1)^{N-n} \sqrt{N!}}{\sqrt{n! \, (N-n)!}} \cos^n(g W_k t_\eta) \sin^{N-n}(g W_k t_\eta). \tag{7.17}$$

Obviously, the probability of finding n "ionized" particles in second bosonic mode is

$$P_n = |a_n|^2 = \frac{N!}{n! \, (N-n)!} \cos^{2n}(g \, W_k \, t_\eta) \sin^{2 \, (N-n)}(g \, W_k \, t_\eta) \qquad (7.18)$$

or

$$P_n = C_n^N \, p^n (1-p)^{N-n} \qquad (7.19)$$

where

$$C_n^N = \frac{N!}{(N-n)! \, n!}, \quad p_k(t) = \cos^2(g \, W_k \, t_\eta).$$

When N is very large so that the Stirling formula is valid, it can be proved that when $n_k = \bar{n}_k = N p_k(t)$, the probability p_n has its maximum.

$$P_{\bar{n}_k} = C_{\bar{n}_k}^N \left(\frac{\bar{n}_k}{N} \right) \left(\frac{N - \bar{n}_k}{N} \right)^{N - \bar{n}_k}. \qquad (7.20)$$

Notice that the derivation is the same as that in ref. [39], but $\bar{n}_k$ depends on the index k. As proved in ref. [39], P_{n_k} is very strongly peaked around its maximum $P_{\bar{n}_k}$, which becomes unity when $N \to \infty$. Therefore, if the detector is very macroscopic ($N \to \infty$), then $P_{n_k}(N \to \infty) = \delta_{n_k \bar{n}_k}$ that leads to

$$|\Psi_I(t)\rangle_{N \to \infty} = \sum_{k=1}^{M} C_k |\Phi_k\rangle \otimes |\bar{n}_k(t), \, N - \bar{n}_k(t)\rangle. \qquad (7.21)$$

When $\bar{n}_k(t) \neq \bar{n}_{k'}(t)$ for $k \neq k'$, a one-to-one correlation between the states $|\Phi_k\rangle$ of S and the states of D. In this sense, if the detector is found in the state $|\bar{n}_k(t), \, N - \bar{n}_k(t)\rangle$, it can be concluded that the system is in this state $|\Phi_k\rangle$. A realistic detector must have a good fringe visibility, which can manifests the macroscopic differences between any two states of $|\bar{n}_k, \, N - \bar{n}_k\rangle$ for different k. It required that there is not the considerable overlap between $|\bar{n}_k, \, N - \bar{n}_k\rangle$ and $|\bar{n}_{k'}, \, N - \bar{n}_{k'}\rangle$ for $k \neq k'$. In fact, if $\bar{n}_k = \bar{n}_{k'}$, then

$$g \, W_k \, t_\eta = g \, W_{k'} \, t_\eta + n \pi, \, n = 0, 1, 2, \ldots$$

or

$$g \, t_\eta (W_k - W_{k'}) = n \pi.$$

Otherwise, if one lets the interaction between S and D decay very fast so that the limiting time

$$\frac{1}{\eta} \lesssim \frac{\pi}{2 \, g (W_k - W_{k'})}$$

for any $k \neq k'$, then $\bar{n}_k(t) \neq \bar{n}_{k'}(t)$ for any long time evolution.

Notice that the above mentioned problem of overlap of correlation states is a disadvantage in the original Cini's model, where there is not the decay factor of interaction. So, in the time $t_n = \frac{n \pi}{g W}$ the states $|\bar{n}, \, N - \bar{n}\rangle$ and $|N, 0\rangle$ are completely overlap. Here, it is the case of two levels with $W_2 = W = \frac{1}{\sqrt{N}}$, $W_1 = 0$, thus, at $t = t_n$ the correlations vanish for the original Cini's model. Introducing the decay factor $e^{-\eta t}$ in the interaction is the key point to avoid vanishing of correlation in our model. In fact, such decay of interaction can appear in realistic physics. For example, an atom is prepared in a microwave cavity loaded with an electromagnetic field which can decay at a suitable rate.

In this example, the atom and the cavity are regarded as the system and the detector respectively. Notice that this example is quite useful for the studies of atomic cooling [40].

7.3.   Wavefunction Collapse in Cini Model

We can also use the above generalized Cini's model to describe the WPC for the M-level system quantum mechanically. Let the system D be initially prepared in a coherent superposition of M-level

$$|\Psi\rangle = \sum_{k=1}^{M} C_k |\Phi_k\rangle \tag{7.22}$$

and the detector be adjusted in the initial state $|N, 0\rangle$, then the density matrix for the initial state of total system is expressed as

$$\hat{\varrho} = \sum_{k, k'} C_k C_{k'}^* |\Phi_k\rangle \langle \Phi_{k'}| \otimes |N, 0\rangle \langle N, 0|. \tag{7.23}$$

Using the evolution operator $U_I(t)$ in the interaction representation, one formally write down the density matrix for the total system at t

$$\hat{\varrho}(t) = U_I(t) \varrho(0) U_I^+(t).$$

Because we are only interested in the final state of the system other than that of the detector for the consideration of WPC, so we must take trace over the variable of detector in the total density matrix to obtain a reduced density matrix for S

$$\hat{\varrho}_S(t) = \mathrm{Tr}_D \varrho(t) = \sum_{m, m'} \langle m, m' | \varrho(t) | m, m' \rangle = \sum_k |C_k|^2 |\Phi_k\rangle \langle \Phi_{k'}|$$
$$+ \sum_{k \neq k'} e^{-it_\eta (E_k - E_{k'})/\hbar} \cos^N [g(W_k - W_{k'})t_\eta] C_k C_{k'}^* |\Phi_k\rangle \langle \Phi_{k'}| \tag{7.24}$$

where we have used

$$\sum_{n=0}^{N} |a_n(t, k)|^2 = 1 \tag{7.25}$$

and

$$\sum_{n=0}^{N} a_n(t, k) a_n^*(t, k') = \cos^N [g(W_k - W_{k'})t_\eta]. \tag{7.26}$$

Notice that each off-diagonal element in the density matrix is accompanied by a time-dependent factor

$$F^N(k, k') = \cos^N [g(W_k - W_{k'})t_\eta] \tag{7.27}$$

which is a N-multiple product of factors $\cos[g(W_k - W_{k'})t_\eta]$. Recalling that due to the existence of the strong decay factor $e^{-\eta t}$ so that

$$\eta \geq \frac{g(W_k - W_{k'})}{\pi}$$

holds for any $k \neq k'$, we observed that the deformed time t_η changes from $t_\eta = 0$ to $t_\eta = \frac{1}{\eta}$ as the real time changes from $t = 0$ to $t \to \infty$ respectively. In this sense,

$$g(W_k - W_{k'})t_\eta < \pi \tag{7.28}$$

and then

$$0 \leq |\cos[g(W_k - W_{k'})t_\eta]| < 1$$

or

$$|\cos[g(W_k - W_{k'})t_\eta]| = e^{-f_k(t_\eta)}, \ f_k(t_\eta) > 0. \tag{7.29}$$

This observation leads to

$$F^N(k, k') = e^{-N f_k(t_\eta)}$$

which obviously approach zero as $N \to \infty$. Therefore, when the detector is macroscopic, $(N \to \infty)$ the off-diagonal terms of the density matrix vanish and the wavefunction collapse is realized quantum dynamically.

To end this paper, we present some comments on the above discussions. Though this paper provides one with an extensive generalization and the unified description for a number of dynamical models (e. g., the HC model) for the quantum decoherence, we have to say that a disadvantage in the original model still exists in the present models. This is the oscillation of $U_n^{[k]}(t)^\dagger U_{n'}^{[k]}(t)$ may enable most of the factors $F_{n,n'}^{[k]}(T, t)$ in eq. (3.6) to become unity at a specific time $t = \tau_c$ and thus the whole accompanying factor can not approach zero at this time. To suppress such kind of oscillation so that the decoherence appears in dynamical evolution, a phenomenological method is to use the switching function $g(t)$ in the interaction (2.3). However, the microscopic mechanism of this switching is not clear for us. We believe that the quantum dissipation caused by the detector or environment is a possible way to introduce such a switching mechanism microscopically. For the concrete example that D is made up of harmonic oscillators, the discussion in section 4 showed that this kind of dissipation may result from the specific distribution of a detector with *infinite* particles. How to realize the quantum decoherence directly through quantum dissipation is still an open question.

Acknowledgements

References

[1] J. von Neumann, Mathematische Grundlagen der Quantenmechanik, Berlin: Springer, 1993.
[2] V. B. Braginski and F. Y. Khalili, *Quantum Measurement*, Cambridge University Presses, 1992.
[3] Tso-hsiu Ho, Physics (in Chinese), **22** (1992) 419.
[4] W. H. Zurek, Phys. Today, 1991, No. 10, p. 36.
[5] B. Kramer, *Quantum Coherence in Mesoscopic Systems*, New York: Plenum, 1991.
[6] M. Gell-Mann and J. B. Hartle, Phys. Rev. D, **47** (1993), 3345.
[7] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Gary and W. Zwerger, Rev. Mod. Phys., **59** (1987), 1.

[8] J. P. PAZ, S. HABIB and W. H. ZUREK, Phys. Rev. D., **47** (1993), 488.
[9] L. H. YU and C. P. SUN, Phys. Rev. A, **49** (1994), 592.
[10] C. P. SUN and L. H. YU, Phys. Rev. A. **51** (1995) 1845.
[11] K. HEPP, Helv. Phys. Acta, **45** (1972), 237.
[12] J. S. BELL, Helv. Phys. Acta, **48** (1975), 93.
[13] H. NAKAZATO and S. PASCAZO, Phys. Rev. Lett., **70** (1993), 1.
[14] C. P. SUN, Phys. Rev. A, **48** (1993), 878.
[15] C. P. SUN, Chin. J. Phys., **32** (1994), 7.
[16] N. G. VAN KAMPEN, Physica A, **153** (1988), 97.
[17] M. NAMIK, S. PASCAZIO, Phys. Rev. A, **44** (1991), 39.
[18] M. Namik, Found. Phys. Lett, **5** (1992) 265.
[19] B. GAVEAU and L. S. SCHULMAN, J. Stat. Phys. **58** (1990), 1209.
[20] C. P. SUN, J. Phys. A, **21** (1988), 1595.
[21] C. P. SUN, Phys. Rev. D, **38** (1988), 2908.
[22] C. P. SUN, High Energy Phys. Nucl. Phys., **12** (1988), 252.
[23] C. P. SUN, Phys. Rev. D, **41** (1990), 1318.
[24] C. P. SUN, Physica Scripta, **42** (1993), 393.
[25] B. MISRA and E. SUDARSHAN, J. Math. Phys., **18** (1977), 756.
[26] W. M. ITANO, O. J. HEINZEN, J. J. BOLLINGER and D. J. WINELAND, Phys. Rev. A, **41** (1990), 2295.
[27] T. PETROVSKY, S. TASAKI and I. PRIGOGINE, Phys. Lett A, **151** (1990), 109.
[28] T. PETROVSKY, S. TASAKI and I. PRIGOGINE, Physica A, **170** (1991), 306.
[29] S. INAGAKI, M. NAMIKI and T. TAJIN, Phys. Lett. A, **166** (1992), 5–12.
[30] S. PASCAZIO, M. NAMIKI, G. BADAREK, H. RANCH, Phys. Lett. A, **179** (1993), 155.
[31] D. HOME, M. WHITAKE, Phys. Lett. A, **173** (1993), 327.
[32] R. ONOFRIO, C. PRESILLA and U. TAMBINI, Phys. Lett. A, **183** (1993), 135.
[33] T. P. ALTENMÜLLER and A. SCHENZLE, Phys. Rev. A, **48** (1993), 70.
[34] M. J. GAGEN, H. WISEMEN, G. J. MILBARN, Phys. Rev. A, **48** (1993), 132.
[35] T. P. ALTENMÜLLER and A. SCHENZLE, Phys. Rev. A, **49** (1994), 2016.
[36] G. S. AGARWAL and S. P. TEWARI, Phys. Lett. A, **185** (1994), 139.
[37] A. PERES, Amer. J. Phys. **48** (1980), 931.
[38] J. WEI and E. NORMAN, J. Math. Phys., **4A** (1963), 5.
[39] M. CINI, Nuovo Cimento, **73 B** (1983), 27.
[40] T. ZAUGG, M. WIKENS, P. MEYSTRE and G. LENZ, Optics Commun. **97** (1993), 189.