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2008 EPL 84 30004

(http://iopscience.iop.org/0295-5075/84/3/30004)

A Letters Journal Exploring the Frontiers of Physics

November 2008

EPL, **84** (2008) 30004
doi: 10.1209/0295-5075/84/30004

www.epljournal.org

The Peierls distorted chain as a quantum data bus for quantum state transfer

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received 3 May 2008; accepted in final form 22 September 2008
published online 17 October 2008

PACS 03.65.Ud – Entanglement and quantum nonlocality (*e.g.* EPR paradox, Bell's inequalities, GHZ states, etc.)
PACS 75.10.Jm – Quantized spin models
PACS 03.67.Lx – Quantum computation architectures and implementations

Abstract – We study the quantum state transfer (QST) of an electron spin along the half-filled Peierls distorted chain (PDC). As has been proved, this solvable and feasible gapped model can accomplish a high-fidelity and long-distance QST. Moreover, numerical simulations are performed in and near the non-distortion point which is beyond the range of perturbation. The result shows that the efficiency of the QST is sensitive to the uniform-to-distorted transition of the PDC, which is related to the transition between the conductor and the insulator. Then this scheme can also be employed to probe the conductor-to-insulator transition of the PDC.

Introduction. – In many protocols of quantum information processing, it is crucial to transmit a quantum state with high fidelity [1]. While various schemes of quantum state transfer (QST) were proposed and demonstrated experimentally for optical systems [2] even with the atom ensemble [3,4], other groups have tried to implement such a task based on solid-state systems [5,6]. It was recognized that a quantum spin chain [7,8] or a Bloch electron system [9] with artificial nearest-neighbor (NN) couplings can be used as a data bus. In [8], it was shown that the spectrum-parity matching is responsible for the perfectness of most protocols for QST [10]. However, these schemes based on solid-state systems are too artificial with very specially designed NN couplings. Moreover, only the single-particle cases were considered.

In this paper, we use a Peierls distorted chain (PDC) [11], which is a tight-binding chain with the staggered NN hopping integrals, to act as a quantum data bus. For a QST scheme based on gapped systems, the key issue is the period which usually grows up exponentially as the transfer distance increases. Since the PDC is solvable, we can study a larger size system, where it is found that this chain can transfer a quantum state for electrons, which is similar to the spin ladder for spins [12], but with higher fidelity and longer distance. It should be noticed that, it is not easy to find such a good data bus, since the energy gap between the ground and first excited states is crucial: if the gap is too large, the QST period increases exponentially with the transfer distance; while if it is too small, the fidelity of QST becomes lower. Furthermore, since the PDC can be solved exactly, we can perform analytical investigations for the sender and receiver with any arbitrary distance between them. Besides these advantages, the PDC is a more natural material, which is originated from the Su-Schrieffer-Heeger (SSH) model [13,14], which describes the polyacetylene. In the large atomic mass limit and the half-filled case, the SSH model reduces to the PDC, and the dimerization induces an energy gap for the half-filled PDC making the QST feasible.

This paper is organized as follows: In the second section, the model setup and the single-particle spectrum of the PDC are introduced. The big energy gap in the middle of the spectrum corresponds to the energy gap between the ground and first excited states of the PDC in the half-filled case. As an approximation up to the second order, an effective Hamiltonian H_{AB} with respect to AB is deduced by using the Fröhlich transformation [15,16] in the third section. Moreover, the fidelity of QST is defined and plotted. In the fourth section, to demonstrate that H_{AB} is a good approximation for QST, the reduced

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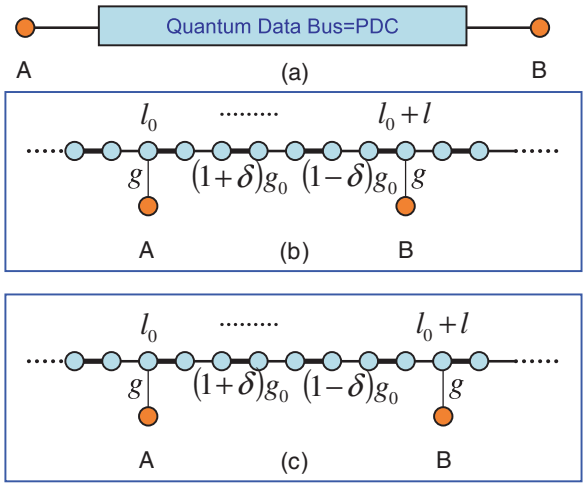

Fig. 1: (Color online) (a) The schematic illustration of the system. The sender A and receiver B are connected to two sites of the PDC with distance l being (b) odd and (c) even. Case (b) has the mirror inversion symmetry and is studied extensively in this paper since it has a high efficiency for QST, while case (c) has a lower efficiency.

density matrices of AB for the ground and first excited states of the whole system are calculated. Our scheme is also investigated numerically in a crossover region between two types of dimerization in the fifth section. It is shown that, at the uniform-to-distorted transition point, the QST becomes the fastest with a similar loss of the fidelity to what has been discussed in [17]. The conclusions are presented finally in the last section.

Model setup. – The sender A and receiver B are connected to two sites l_0 and l_0+l of the PDC (fig. 1(a)), and two connection ways are shown in fig. 1(b) and (c). Only the case that both l_0 and l are odd is discussed (fig. 1(b)). The obtained results can be applied to the case that l is odd but l_0 is even, since the whole chain is with the periodic boundary condition. A similar study can also be applied to the case in fig. 1(c). We do not discuss this case in detail but give a conclusion that such a setup has a lower efficiency for QST.

The tight-binding Hamiltonian of PDC reads

$$H_{PDC} = -\sum_{n,\sigma} g_0 \left[1-(-1)^n \delta\right] \left(c^\dagger_{n+1,\sigma} c_{n,\sigma} + \text{h.c.}\right), \quad (1)$$

where $c_{n,\sigma}$ ($c^\dagger_{n,\sigma}$) is annihilation (creation) operator of electrons on site n with spin σ, and $g_0[1-(-1)^n\delta]$ is the hopping integral, where δ denotes the distortion of the hopping integral, which realizes the staggered NN hopping integrals of the PDC. We consider the QST between A and B via the PDC between them. The purpose is through the hopping of a polarized electron to transfer a qubit state, which is a superposition state of the spin up and down. Since (1) does not contain any spin-dependent interaction, the electron spin polarization is conserved. Then the spatial motion of the electron along the PDC carries a spin state from one location to another. Generally, the coupling constant κ of moving particles is much larger than that of spin-spin interactions, since the latter is usually a second-order perturbation term. For example, the Heisenberg model is originated from the half-filled Hubbard model with the on-site repulsion U in the limit $\kappa \ll U$ [18], where the spin-spin coupling strength $J \sim \kappa^2/U$. Since the period of QST is inversely proportional to κ, J, and the period of QST is required to be much less than the decoherence time, the efficiency of QST by using moving particles should be better than that by using spin interactions. The connection of AB to the chain is described by

$$H_I = -g \sum_{\sigma} \left(c^\dagger_{A,\sigma} c_{l_0,\sigma} + c^\dagger_{B,\sigma} c_{l_0+l,\sigma} + \text{h.c.}\right), \quad (2)$$

where $c_{A,\sigma}$ and $c_{B,\sigma}$ ($c^\dagger_{A,\sigma}$ and $c^\dagger_{B,\sigma}$) are annihilation (creation) operators of the electron on A and B with spin σ. l_0 and l_0+l denote the connecting sites of the chain.

Since the transfer distance l is short compared to the whole size N of the chain, the boundary effect on the QST is neglectable. Then we can take the periodic boundary condition to diagonalize the Hamiltonian (1) as

$$H_{PDC} = \sum_{k,\sigma} \epsilon_k \left(\alpha^\dagger_{k,\sigma}\alpha_{k,\sigma} - \beta^\dagger_{k,\sigma}\beta_{k,\sigma}\right), \quad (3)$$

where the dispersion relation is

$$\epsilon_k = 2g_0\sqrt{\cos^2\frac{k}{2} + \delta^2 \sin^2\frac{k}{2}}. \quad (4)$$

The excitations are described by fermion operators

$$\alpha_{k,\sigma} = \frac{1}{\sqrt{N}} \sum_{j=1}^{N/2} e^{-ikj} \left(c_{2j-1,\sigma} - e^{i\theta_k} c_{2j,\sigma}\right) \quad (5)$$

and

$$\beta_{k,\sigma} = \frac{1}{\sqrt{N}} \sum_{j=1}^{N/2} e^{-ikj} \left(c_{2j-1,\sigma} + e^{i\theta_k} c_{2j,\sigma}\right), \quad (6)$$

where $k = 4\pi m/N$, $m = 0, 1, 2, \ldots, N/2-1$, and

$$e^{i\theta_k} = \frac{g_0}{\epsilon_k}\left[(1+\delta) + (1-\delta)\, e^{-ik}\right]. \quad (7)$$

The single-particle spectrum (4) is illustrated in fig. 2. There is a big energy gap between two bands

$$2\Delta = 2\min\{\epsilon_k\} = 4|g_0\delta|, \quad (8)$$

which approximately corresponds to the energy gap between the ground and first excited states of the PDC in the half-filled case.

In the following sections, it will be shown that such an energy gap (8) in the half-filled case leads to a desirable robust quantum data bus, and for the subsystem AB, the original energy degeneracy will be removed by switching on g, and then split into two sub-levels with the level spacing $\Delta E = 2|g_{eff}|$, as illustrated in fig. 2. Here, g_{eff} is the effective hopping integral which can be obtained analytically in the next section.

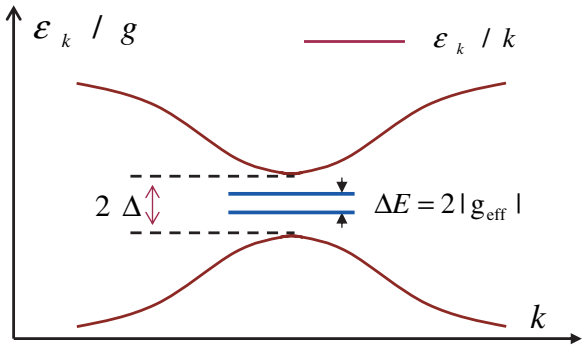

Fig. 2: (Color online) The single-particle spectrum of the PDC. In the half-filled case, the gap 2Δ approximately corresponds to the energy gap between the ground and first excited states of the PDC. For the subsystem AB, by switching on g, the degeneracy will be removed and split into two sub-levels with the level spacing $\Delta E = 2|g_{eff}|$ as shown in eq. (20).

Effective long-range hopping. – To deduce an effective Hamiltonian about the indirect coupling between A and B, the Fröhlich transformation is utilized [15,16], whose original approach was used successfully for the BCS theory of superconductivity. In detail, for the Hamiltonian of the total system

$$H = H_{PDC} + H_I \tag{9}$$

in our scheme, through finding an anti-Hermitian operator S which obeys

$$[S, H_{PDC}] = H_I, \tag{10}$$

the effective Hamiltonian H_{eff} of H can be obtained as

$$\begin{aligned} H_{\mathit{eff}} &= e^{-S} H e^{S} \\ &= H_{PDC} + (H_I - [S, H_{PDC}]) \\ &\quad + \frac{1}{2}\left[(H_I - [S, H_{PDC}]), S\right] \\ &\quad + \frac{1}{2}\left[H_I, S\right] + \cdots \\ &\cong H_{PDC} + \frac{1}{2}\left[H_I, S\right]. \end{aligned} \tag{11}$$

Substitute $c_{l_0,\sigma}$ and $c_{l_0+l,\sigma}$ in eq. (2) with $\alpha_{k,\sigma}$ and $\beta_{k,\sigma}$ according to (5) and (6), we have

$$\begin{aligned} H_I = -\frac{g}{\sqrt{N}} \sum_k \Big[& e^{ik\frac{l_0+1}{2}} c^\dagger_{A,\sigma} \left(\alpha_{k,\sigma} + \beta_{k,\sigma}\right) \\ & + e^{ik\frac{l_0+l}{2} - i\theta_k} c^\dagger_{B,\sigma} \left(\alpha_{k,\sigma} - \beta_{k,\sigma}\right) + \text{h.c.} \Big]. \end{aligned} \tag{12}$$

According to the expressions (3) and (12), if we set S as

$$\begin{aligned} S = -\frac{g}{\sqrt{N}} \sum_{k,\sigma} \frac{1}{\epsilon_k} \Big[& e^{ik\frac{l_0+1}{2}} c^\dagger_{A,\sigma} \left(\alpha_{k,\sigma} - \beta_{k,\sigma}\right) \\ & - e^{ik\frac{l_0+l}{2} - i\theta_k} c^\dagger_{B,\sigma} \left(\alpha_{k,\sigma} + \beta_{k,\sigma}\right) - \text{h.c.} \Big], \end{aligned} \tag{13}$$

it will satisfy condition (10). Then this expression about S is right.

Substituting (3), (12) and (13) into (11), the effective Hamiltonian becomes

$$H_{\mathit{eff}} = H_{AB} + H_0, \tag{14}$$

where

$$H_{AB} = \sum_\sigma g_{\mathit{eff}} \left(c^\dagger_{A,\sigma} c_{B,\sigma} + c^\dagger_{B,\sigma} c_{A,\sigma}\right) \tag{15}$$

denotes an effective hopping of an electron between A and B with

$$g_{\mathit{eff}} = \frac{2g^2}{N} \sum_k \frac{e^{-ik\frac{l-1}{2} + i\theta_k}}{\epsilon_k}, \tag{16}$$

and

$$\begin{aligned} H_0 = & \sum_{k,\sigma} \epsilon_k \left(\alpha^\dagger_{k,\sigma}\alpha_{k,\sigma} - \beta^\dagger_{k,\sigma}\beta_{k,\sigma}\right) \\ & + \frac{g^2}{2N} \sum_{k,\sigma,k',\sigma'} \frac{1}{\epsilon_{k'}} \Big[A(k,k') \left(\alpha^\dagger_{k,\sigma}\alpha_{k',\sigma'} - \beta^\dagger_{k,\sigma}\beta_{k',\sigma'}\right) \\ & + B(k,k') \left(\beta^\dagger_{k,\sigma}\alpha_{k',\sigma'} - \alpha^\dagger_{k,\sigma}\beta_{k',\sigma'}\right) + \text{h.c.} \Big] \end{aligned} \tag{17}$$

describes the dynamics of the data bus with

$$\begin{aligned} A(k,k') &= e^{-i(k-k')\frac{l_0+1}{2}} [1 + e^{i(\theta_k - \theta_{k'})}], \\ B(k,k') &= e^{-i(k-k')\frac{l_0+1}{2}} [1 - e^{i(\theta_k - \theta_{k'})}]. \end{aligned} \tag{18}$$

Straightforward calculations show that, in the thermodynamic limit, $N \longrightarrow \infty$, the hopping constant (16) becomes

$$g_{\mathit{eff}} = \frac{g^2}{g_0} \frac{(-1)^{\frac{l-1}{2}}}{1+\delta} \left(\frac{1-\delta}{1+\delta}\right)^{\frac{l-1}{2}}. \tag{19}$$

On the other hand, eq. (15) is a model consisting of two sites in the half-filled case, where two eigenvalues of the ground and first excited states are $-|g_{\mathit{eff}}|$ and $|g_{\mathit{eff}}|$, respectively. Then the corresponding energy difference between two states is

$$\Delta E = 2|g_{\mathit{eff}}|. \tag{20}$$

This energy difference is depicted in fig. 2. When the total system consisting of A, B and the PDC is half-filled, ΔE represents the energy difference between the ground and first excited states of the whole system.

It needs to be pointed out that, only under some specific conditions, the effective Hamiltonian H_{AB} can work well. These conditions include some restrictions about parameters δ, g, and so on, which are considered in the next section. Moreover, to obtain a perfect QST, the energy gap between the ground and first excited states of the data bus should be large enough to protect the QST from the thermal fluctuation. Then in our scheme, the PDC should be half-filled, since, as shown in fig. 2, when the PDC is half-filled, its energy gap between the ground and first excited states is 2Δ, which can be large enough.

Now let us consider the QST scheme via our system. Assume Alice is at the sender site A, and Bob is at the

receiver site B. The medium PDC is half-filled and in its ground state. Let Alice hold an electron with a spin state she wants to communicate with Bob as

$$|\varphi\rangle = \cos\frac{\theta}{2}|\downarrow\rangle + e^{i\phi}\sin\frac{\theta}{2}|\uparrow\rangle, \tag{21}$$

where $|\uparrow\rangle$ ($|\downarrow\rangle$) denotes the spin-up (-down) state. Thus, the initial state $|\psi(0)\rangle_{AB} = |\varphi\rangle_A \otimes |0\rangle_B$ about AB is

$$|\psi(0)\rangle_{AB} = \left(\cos\frac{\theta}{2}c^{\dagger}_{A,\downarrow} + e^{i\phi}\sin\frac{\theta}{2}c^{\dagger}_{A,\uparrow}\right)|0\rangle_{AB}, \tag{22}$$

where $|0\rangle_A$ ($|0\rangle_{AB}$) denotes the empty state, *i.e.*, there is no electron at A (both A and B). At the instant

$$t = \tau = \frac{\pi}{\Delta E} = \frac{\pi}{2|g_{eff}|}, \tag{23}$$

the system evolves into a new factorized state $|\psi(\tau)\rangle_{AB} = |0\rangle_A \otimes |\varphi\rangle_B$. Here,

$$|\psi(\tau)\rangle_{AB} = \left(\cos\frac{\theta}{2}c^{\dagger}_{B,\downarrow} + e^{i\phi}\sin\frac{\theta}{2}c^{\dagger}_{B,\uparrow}\right)|0\rangle_{AB} \tag{24}$$

realizes a perfect quantum swapping. Then at this time, Bob at the receiver site B receives an electron with the spin state $|\varphi\rangle$, *i.e.*, he receives the state Alice sends to him. In this process, the fidelity of QST can be defined as

$$F(t) = \mathrm{Tr}\left[\rho_B(t)|\varphi\rangle_B\langle\varphi|\right], \tag{25}$$

where

$$\rho_B(t) = \mathrm{Tr}_{A,PDC}|\Psi(t)\rangle\langle\Psi(t)|. \tag{26}$$

In the above formula, $\Psi(t)$ is the state of the whole system consisting of A, B and the PDC at time t, and $\mathrm{Tr}_{A,PDC}$ means tracing off the states of A and PDC.

It is desirable that the above scheme can work well over a longer distance. In the inset of fig. 3, the fidelity (25) obtained in our scheme is plotted with $t=\tau$, $N=500$, $l=10n+9$, $n\in[0,7]$, $\delta=0.01$, $g=0.01$, and $g_0=1$. It is very high, larger than 0.99, even for a long-distance transfer. On the other hand, for a desirable QST scheme, the QST time should not be too long. In our scheme, since τ is the characteristic time, from (23), the effective hopping integral $|g_{eff}|$ should not decay too fast as l increases. According to (19), $|g_{eff}|$ decays exponentially when l increases. However, $\sqrt{(1-\delta)/(1+\delta)}$ is close to 1 for small δ, where $|g_{eff}|$ will not decay rapidly for any finite l. To demonstrate this, $|g_{eff}|$ is plotted in fig. 3 according to the analytical formula (19) and numerical result ΔE (20). Two results agree with each other well, and τ is proportional to l, which is crucial for scalable quantum information processing.

Reduced density matrices. – In the above section, $|g_{eff}|$ has been studied as a function of l in a specific range of parameters. It is shown that our scheme can realize a high-fidelity and long-distance QST. However, the obtained conclusion should be based on the fact that H_{AB} given by (15) is a valid approximation in the studied range. In this section, such a validity of H_{AB} is investigated by comparing the eigenstates of H_{AB} with the density matrices reduced from the ground and first excited states of the total system (9). Note that (9) does not contain any spin-spin interaction term. Then this system can be regarded as a spinless fermion system, and the feasibly obtained results can be applied to the original system. Therefore, in the following discussion, the spin degree of freedom is ignored for simplicity.

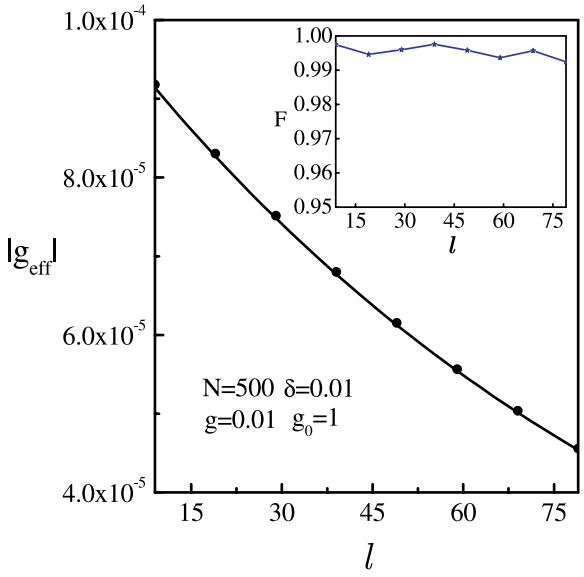

Fig. 3: (Color online) $|g_{eff}|$ obtained from the approximate analytical result (19) (line) and numerical result ΔE (20) (solid dot) with $l=10n+9$ and $n\in[0,7]$. Two results agree with each other well, and $|g_{eff}|$ does not decay very fast as l increases. The inset is about the fidelity of QST at the instant τ. It is very high, larger than 0.99, even for a long-distance transfer.

Define the states $|n_-,n_+\rangle_{AB}$ as

$$|1,0\rangle_{AB} = \frac{1}{\sqrt{2}}\left(a^{\dagger}_A - a^{\dagger}_B\right)|0\rangle_{AB}, \tag{27}$$

$$|0,1\rangle_{AB} = \frac{1}{\sqrt{2}}\left(a^{\dagger}_A + a^{\dagger}_B\right)|0\rangle_{AB}, \tag{28}$$

where $a^{\dagger}_A$ and $a^{\dagger}_B$ are spinless fermion operators on A and B. These two states are the eigenstates of H_{AB} in the half-filled subspace, and n_- (n_+) is the particle number in the anti-bonding (bonding) state. Moreover, we define the states $|n_-,n_+;\eta\rangle$ and $|n_-,n_+\rangle$ for the total system (9). $|n_-,n_+;\eta\rangle$ is the eigenstate of the total system with $g=0$, which is denoted as $|n_-,n_+;\eta\rangle = |n_-,n_+\rangle_{AB}\otimes|\eta\rangle_{PDC}$, where $|\eta\rangle_{PDC}$ is the ground state ($\eta=0$) and the excited states ($\eta=1,2,\ldots$) of the half-filled PDC. $|n_-,n_+\rangle$ is the ground or first excited state of the total system with $g\neq 0$, which is spanned by the state possessing the same parity as $|n_-,n_+;\eta\rangle$, and corresponds to the eigenvalue $E_{n_-n_+}$. Next the calculation task is the quantity defined as

$$P_{n_-n_+} = \mathrm{Tr}\left(\rho_R\rho_{n_-n_+}\right), \tag{29}$$

where

$$\rho_R = \mathrm{Tr}_{PDC}(|n_-, n_+\rangle\langle n_-, n_+|) \tag{30}$$

and

$$\rho_{n_-n_+} = |n_-, n_+\rangle_{AB}\langle n_-, n_+|. \tag{31}$$

Substituting (30) and (31) into (29), $P_{n_-n_+}$ becomes

$$P_{n_-n_+} = \sum_{\eta} |\langle n_-, n_+|n_-, n_+; \eta\rangle|^2. \tag{32}$$

In fact, $P_{n_-n_+}$ is the overlap between the eigenstates of H_{AB} and the reduced states from the exact eigenstates of the total system. Then if $P_{n_-n_+}$ is near unity, it can be concluded that H_{AB} is a good approximation for QST.

In the following discussion, the efforts are made to explicitly express $|n_-, n_+\rangle$ with the help of the Gellmann-Low theorem [19], where $|n_-, n_+\rangle$ can be expressed with $|n_-, n_+; 0\rangle$ as

$$|n_-, n_+\rangle = \frac{U(0, -T)|n_-, n_+; 0\rangle}{e^{-iE_{n_-n_+}T}\langle n_-, n_+|n_-, n_+; 0\rangle}. \tag{33}$$

Here $T \to \infty(1 + i\epsilon)$, and

$$U(t, t_0) = \mathcal{T}\exp\left\{-i\int_{t_0}^{t} \mathrm{d}t' H'_I(t')\right\} \tag{34}$$

is the time-evolution operator in the interaction picture, where $\mathcal{T}$ is the time-ordering symbol, and $H'_I(t') = \exp(iH_{PDC}t')H_I\exp(-iH_{PDC}t')$. Similarly,

$$\langle n_-, n_+| = \frac{\langle n_-, n_+; 0|U(T, 0)}{e^{-iE_{n_-n_+}T}\langle n_-, n_+; 0|n_-, n_+\rangle}. \tag{35}$$

With these expressions, $P_{n_-n_+}$ in (32) can be rewritten as

$$P_{n_-n_+} = \frac{\langle n_-, n_+; 0|Q|n_-, n_+; 0\rangle}{e^{-iE_{n_-n_+}(2T)}|\langle n_-, n_+; 0|n_-, n_+\rangle|^2}, \tag{36}$$

where

$$Q = \mathcal{T}\left\{Q_{n_-n_+}\exp\left[-i\int_{-T}^{T} \mathrm{d}t' H'_I(t')\right]\right\}, \tag{37}$$

and

$$Q_{n_-n_+} = |n_-, n_+\rangle_{AB}\langle n_-, n_+| \otimes \mathbf{1}. \tag{38}$$

Note that for further calculations, (36) cannot be calculated directly, because there is no explicit expression for $|n_-, n_+\rangle$. To get rid of this difficulty, we use the normalization condition $\langle n_-, n_+|n_-, n_+\rangle = 1$ as

$$\frac{\langle n_-, n_+; 0|U(T, -T)|n_-, n_+; 0\rangle}{e^{-iE_{n_-n_+}(2T)}|\langle n_-, n_+; 0|n_-, n_+\rangle|^2} = 1. \tag{39}$$

Then $P_{n_-n_+}$ (36) becomes

$$P_{n_-n_+} = \frac{\langle n_-, n_+; 0|Q|n_-, n_+; 0\rangle}{\langle n_-, n_+; 0|U(T, -T)|n_-, n_+; 0\rangle}. \tag{40}$$

Table 1: $1 - P_{n_-n_+}$ and $-P^{(2)}_{n_-n_+}$ with $N = 500$, $l = 10n + 9$, $n \in [0, 7]$, $g = 0.01$, $g_0 = 1$, and $\delta = 0.01$. $1 - P_{10}$ $(1 - P_{01})$ is close to $-P^{(2)}_{10}$ $(-P^{(2)}_{01})$. All of the values are very small, which indicates that $P_{n_-n_+}$ is close to 1, and H_{AB} is a good approximation for QST when the parameters satisfy the condition (44).

l	$1 - P_{10}$ $(\times 10^{-3})$	$-P^{(2)}_{10}$ $(\times 10^{-3})$	$1 - P_{01}$ $(\times 10^{-3})$	$-P^{(2)}_{01}$ $(\times 10^{-3})$
9	4.48	4.53	0.54	0.54
19	0.41	0.41	4.62	4.66
29	4.66	4.70	0.37	0.37
39	0.38	0.38	4.64	4.68
49	4.60	4.64	0.43	0.43
59	0.50	0.50	4.53	4.57
69	4.45	4.49	0.58	0.58
79	0.67	0.67	4.37	4.40

Next we expand $P_{n_-n_+}$ (40) as the series of g. Calculations show that for small g, the zeroth order $P^{(0)}_{n_-n_+}$ is unity, the first order $P^{(1)}_{n_-n_+}$ is zero, and the second order $P^{(2)}_{n_-n_+}$ is non-zero and can be expressed as

$$P^{(2)}_{n_-n_+} = -\sum_k \frac{2g^2}{\epsilon_k^2 N}\left[1 + (n_- - n_+)\cos\left(k\frac{l-1}{2} - \theta_k\right)\right]. \tag{41}$$

It approximately describes the difference between the eigenstates of H_{AB} and the reduced states from the exact eigenstates of the total system. In order to study it, the upper bound of $P^{(2)}_{n_-n_+}$ is obtained as

$$|P^{(2)}_{n_-n_+}| \leqslant \frac{4}{N}\sum_k \frac{g^2}{\epsilon_k^2}. \tag{42}$$

In the thermodynamic limit, it becomes

$$|P^{(2)}_{n_-n_+}| \leqslant \frac{g^2}{2g_0^2\delta}. \tag{43}$$

When $|P^{(2)}_{n_-n_+}| \ll 1$, $P_{n_-n_+}$ is close to 1. That is to say, when $g^2/(2g_0^2\delta) \ll 1$, or

$$g/(\sqrt{2}g_0) \ll \sqrt{\delta}, \tag{44}$$

H_{AB} can work well.

As an example, $1 - P_{n_-n_+}$ and $-P^{(2)}_{n_-n_+}$ with $N = 500$, $l = 10n + 9$, $n \in [0, 7]$, $g = 0.01$, $g_0 = 1$, and $\delta = 0.01$ are listed in table 1, where the parameters satisfy the condition (44). $1 - P_{n_-n_+}$ is calculated from the exact diagonalization, and $-P^{(2)}_{n_-n_+}$ is obtained from the analytical expression (41). No matter n_- $(n+)$ equals 1 (0) or 0 (1), two results are in agreement with each other approximately. Moreover, they are very small, which indicates that $P_{n_-n_+}$ is close to 1, and the eigenstates of H_{AB} can well describe the states of AB in the ground and first excited states of the total system. In the next section, the QST is investigated in a more extensive range about δ, where (44) may be violated.

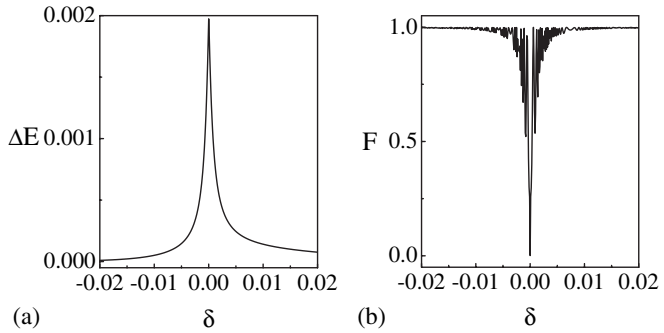

Fig. 4: (a) The energy difference ΔE between the ground and first excited states and (b) the fidelity F with $N = 200$, $l = 49$, $g = 0.01$, and $g_0 = 1$. When δ approaches to zero, ΔE has a sharp peak, while F drops rapidly. That is to say, the critical properties affect the dynamics of the QST. Around the point $\delta = 0$, although the QST becomes fast, the fidelity decreases rapidly, which indicates that the features of AB can be used to detect the transition of the PDC.

Uniform-to-distorted transition of the PDC. – The magnitude of δ is crucial for the QST since the speed and fidelity of QST is sensitive to the dimerization. There are two types of dimerization corresponding to $\pm|\delta|$. So it is interesting to investigate what happens near the transition point $\delta = 0$. In this region, the above analytical condition (44) is violated, which is due to the vanishing δ. Then for this case, we should do numerical simulations to study the energy difference ΔE between the ground and first excited states of the half-filled total system (9), and the fidelity F defined as (25) with $t = \pi/\Delta E$. Since $\pi/\Delta E$ is the recurrent time, both of them can characterize the properties of QST. Figure 4 is plotted with $N = 200$, $l = 49$, $g = 0.01$, and $g_0 = 1$. When the system approaches to $\delta = 0$, which corresponds to a uniform chain, ΔE has a sharp peak, while F drops rapidly. It indicates that the critical properties affect the dynamics of the QST. Although the QST becomes fast around the critical point, the fidelity decreases rapidly. This phenomenon is very similar to that discovered by [17], and it indicates that the features of two probing sites AB, such as the recurrent time, or the fidelity, can be used to detect the transition of the PDC from uniform to distorted, which relates to the transition between the conductor and insulator.

Conclusions. – A proper gapped system can be used as a data bus to accomplish the QST. The challenge comes from two aspects: a long-distance QST should be fast and robust, and the scheme should be feasible in the experiments. We find that the PDC originating from the well-known SSH model and existing in conducting polymers (polyacetylene) has the potential to overcome these two challenges. Moreover, the PDC has an exact solution, which makes it possible to obtain analytical results.

In addition, for the vanishing distortion limit of the PDC, which is beyond the region of perturbation, we employ numerical simulations to investigate the QST. It is found that the fidelity of QST strongly depends on the distortion δ of the PDC. Then, from a measurement perspective, the features of sites A and B, such as the fidelity or the recurrent time of the QST, can be used to detect the uniform-to-distorted transition of the PDC, which relates to a transition between the conductor and insulator. These observations have universality, which may motivate us to investigate the function of other natural materials.

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We acknowledge the support of the NSFC (grant No. 90203018, 10474104, 10447133, 10874091), the Knowledge Innovation Program (KIP) of Chinese Academy of Sciences, the National Fundamental Research Program of China (No. 2001CB309310).

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