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Dynamic generation of entangling wave packets in *XY* spin system with decaying long-range couplings

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The dynamic generation of spin entanglement between two distant sites in a *XY* model with $1/r^2$ decay long-range couplings was studied. Due to the linear dispersion relation $\varepsilon(k)\sim|k|$ of magnons in such a model, a well-located spin state can be dynamically split into two moving entangled local wave packets without changing their shapes. Interestingly, when such two wave packets meet at the diametrically opposite site after the fast period $\tau=N\pi/J$, the initial well-located state is completely recurrent. Numerical calculation was performed to confirm the analytical result even if the ring system of sizes *N* up to several thousands is considered. The truncation approximation for the coupling strengths was also studied. Numerical simulation shows that the above conclusions still hold even if the range of the coupling strength is truncated to a relatively short scale compared with the size of the spin system.

quantum information, entanglement, spin model

In quantum information processing, it is also crucial to generate entangled qubits, which can be used to perfectly transfer a quantum state over a long distance. For an optical system this task has been completed long time ago, but for a solid state system it remains a great challenge in both experiment and theory to create quantum entanglement by a solid state device. Recently many proposals to entangle distant spins have been proposed based on various physical mechanisms[1−5]. Most protocols for accomplishing quantum state transfer in a spin array are based on the fixed inter-qubit couplings[6−21]. The simplest coupled spin system with uniform nearest neighbor (NN) couplings has been studied by the pioneer work[7]. The paradigms to generate maximal entanglement and to perform perfect qubit-state transmission over an arbitrary distance are the protocols using the pre-engineered inhomogeneous NN couplings[8−10]. Besides, a practical scheme realizes

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the transmission of Gaussian wave packet as a flying qubit via the spin system with uniform NN couplings[16,17]. The advantage of such a scheme rests with its fast transfer, which means that the period of transfer time is proportional to the distance: the higher fidelity for longer distance. We also notice that there is current interest in studying the systems with long-range inter-qubit interactions[22−24].

In this paper, we revisit the issue of entanglement generation and quantum state transfer between two distant qubits in a qubit array with long-range inter-qubit interactions. An alternative way to construct a perfect medium for quantum state transfer may require the long-range interactions beyond NN interactions; but the interactions should be required to decay rapidly in order to avoid the direct connecting coupling between two distant qubits that trivially causes quantum entanglement. Now we propose a novel protocol based on a pre-engineered *XY*-model with long-range $1/r^2$-decay interactions. It is found that such a model has the same function as that of the modulated NN coupling spin model[8,9], offering significant advantages over the other protocols in the tasks of perfectly transferring quantum states and generating entanglement between two sites over longer distance.

This paper is organized as follows. In section 1, we consider a model with linear dispersion relation and show that it ensures the perfect state transfer and the creation of entanglement between the spatial separated qubits. In section 2, we propose a spin model with $1/r^2$- decay interactions that processes the desired dispersion relation. In section 3, we perform numerical calculations to confirm some of the obtained analytical results and the validity of the truncation approximation.

1 Formalism

1.1 Pre-engineered model with linear dispersion relation

Usually, we regard an $S = 1/2$ spin as a qubit, and a coupled spin system as a qubit array respectively. The simplest coupled qubit array is usually described by the spin-1/2 *XY* model. Our proposal makes the quantum spin array behave as a perfect spin-network, in which a spin flip at any site (or the superposition of local single-spin-flip states) can evolve into two entangled, local wave packets. Consider the prescribed spin-1/2 *XY* model on a ring system with *N* sites. The Hamiltonian reads

$$H = 2\sum_{i,r} J_r (S_i^x S_{i+r}^x + S_i^y S_{i+r}^y) \tag{1}$$

or

$$H = \sum_{i,r} J_r (S_i^+ S_{i+r}^- + S_i^- S_{i+r}^+), \tag{2}$$

where S_i^x, S_i^y and S_i^z are Pauli matrices for the *i*th site, and $S_i^{\pm} = S_i^x \pm iS_i^y$; J_r are the coupling strengths for the two spins separated by the distance *r*. Since the Hamiltonian conserves spin, i.e. $[\sum_i S_i^z, H] = 0$, the dynamics can be reduced to that in some invariant subspaces. Thus we can only concentrate on the single excitation subspace hereafter. If a single-site flipped state can be correctly transferred, a qubit state should also be transferred correspondingly because the saturate ferromagnetic state with all spins up $|0\rangle \equiv \prod_{i=1}^{N} |\uparrow\rangle_i$ is an eigenstate of the Hamiltonian. Actually, in the single-spin-flip subspace or the single excitation (magnon) subspace, the basis states are denoted by the single-site flipped state (or δ-pulse) $|i\rangle = S_i^- |0\rangle$, $i \in [1, N]$. Therefore, if state $|i\rangle$

can be evolved to state $|j\rangle$ after a period of time τ, $|j\rangle=\exp(-iH\tau)|i\rangle$, we have

$$(\alpha|\uparrow\rangle_j+\beta|\downarrow\rangle_j)\sum_{l\neq j}^{N}|\uparrow\rangle_l=\mathrm{e}^{-iH\tau}(\alpha|\uparrow\rangle_i+\beta|\downarrow\rangle_i)\sum_{l\neq i}^{N}|\uparrow\rangle_l, \tag{3}$$

i.e. a qubit state $\alpha|\uparrow\rangle+\beta|\downarrow\rangle$ can be transferred from i to j. Furthermore, any single-magnon state can be expressed as

$$|\psi\rangle=\sum_i A_i S_i^-|0\rangle\equiv\sum_i A_i|i\rangle \tag{4}$$

and

$$|\psi\rangle=\sum_k D_k S_k^-|0\rangle\equiv\sum_k D_k|k\rangle, \tag{5}$$

respectively in spatial $\{|i\rangle\}$ and momentum $\{|k\rangle\}$ spaces. Here, we have used the spin-wave operator $S_k^-=\sum_j \mathrm{e}^{-ikj}S_j^-/\sqrt{N}$ with discrete momentum $k=2\pi n/N$, $n\in[-N/2, N/2-1]$.

We begin with the assumption that there exists an optimal distribution of J_r, which ensures the single-magnon spectrum possessing a linear dispersion relation, i.e.

$$H_s=\sum_k \varepsilon_k|k\rangle\langle k|,\ \ \varepsilon_k=\frac{J}{2\pi}|k|, \tag{6}$$

as illustrated in Figure 1. Here J is a constant denoting the unit of the coupling strength between the qubits. In the following, we first show that such a kind of systems can perform perfect state transfer and long-range entanglement generation, and then we can provide a practical example that satisfies this linear dispersion relation.

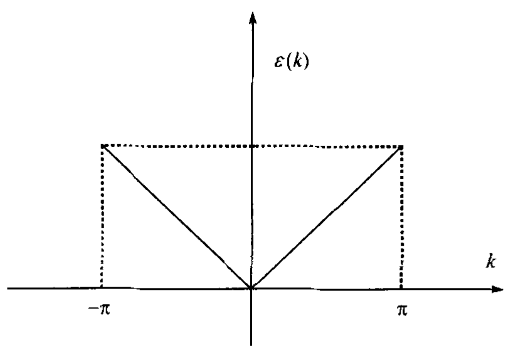

Figure 1 Schematic illustration for the ideal dispersion relation $\varepsilon(k)$~$|k|$ of a system which is shown to be a perfect entangler. Such a kind of system can be realized by the XY model with pre-engineered long-range couplings.

1.2 Time evolution of wave packets in the linear dispersion regime

In order to investigate the dynamics of generating entanglement in the spin system with linear dispersion relation mentioned above, we concentrate on the case where the initial state is a single-site flipped state $S_i^-|0\rangle$, $i\in[1,N]$. Intuitively, the well localized state has equal probability amplitudes with respect to momentum eigenstates $|\pm|k|\rangle$, due to the inverse Fourier transformation

$$S_j^-=\frac{1}{\sqrt{N}}\sum_k \mathrm{e}^{ikj}S_k^- \xrightarrow{N\to\infty} \frac{1}{\sqrt{N}}\frac{N}{2\pi}\int_{-\pi}^{\pi}\mathrm{e}^{ikj}S_k^-\mathrm{d}k=\frac{\sqrt{N}}{2\pi}\int_0^{\pi}(\mathrm{e}^{ikj}S_k^-+\mathrm{e}^{-ikj}S_{-k}^-)\mathrm{d}k. \tag{7}$$

Thus it should be split into two classes of waves with $\pm|k|$ driven by the Hamiltonian with linear single-magnon dispersion relation eq. (6). As illustrated in Figure 2, if the superposition of the two classes of waves forms two local and spatial separated wave packets in the real space, it will lead to the entanglement of the two wave packets.

Generally, we first consider a wave packet dynamics with an initial state

$$|\psi(N_{\rm A},0)\rangle = S^{-}_{N_{\rm A}}|0\rangle = \frac{1}{\sqrt{N}}\sum_k {\rm e}^{ikN_{\rm A}} S^{-}_{k}|0\rangle, \tag{8}$$

which is a single flip at site $N_{\rm A}$. The time evolution starting with this state can be calculated as

$$|\psi(N_{\rm A},t)\rangle = {\rm e}^{-iHt}|\psi(N_{\rm A},0)\rangle = |\phi_{-}(N_{\rm A},t)\rangle + |\phi_{+}(N_{\rm A},t)\rangle, \tag{9}$$

where

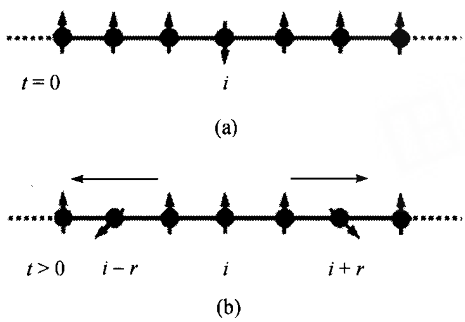

Figure 2 (color online) Schematic illustration of the generation process of two distant entangled qubits. At $t = 0$ the initial state is a single-spin flip on the saturated ferromagnet. At $t > 0$ the initial δ-pulse magnon is separated into two components which have maximal entanglement. At $t = \tau$, the recurrence time, the two sub-wave packets meet together and the initial state is recurrent.

$$|\phi_{\pm}(N_{\rm A},t)\rangle = \mp\frac{\sqrt{N}}{2\pi}\int_0^{\mp\pi} {\rm e}^{ik\left(N_{\rm A}\pm\frac{J}{2\pi}t\right)}{\rm d}k|k\rangle \tag{10}$$

denote the two wave packets with explicit forms in the real space

$$|\phi_{\pm}(N_{\rm A},t)\rangle = \mp\frac{\sqrt{N}}{2\pi}\int_0^{\pm\pi} {\rm e}^{ik\left(N_{\rm A}\pm\frac{J}{2\pi}t\right)}{\rm d}k\frac{1}{\sqrt{N}}\sum_j {\rm e}^{-ikj}|j\rangle = \frac{1}{2\pi}\sum_j\left[\frac{\mp{\rm e}^{\mp i\pi\left(N_{\rm A}-j\pm\frac{J}{2\pi}t\right)}\pm1}{i\left(N_{\rm A}-j\pm\frac{J}{2\pi}t\right)}\right]|j\rangle. \tag{11}$$

We will show that such two wave packets are localized and non-spreading, with velocity $v_{\pm} = \pm J/2\pi$, respectively. Actually, from eq. (11) one has

$$|\phi_{\pm}(N_{\rm A},t+t_0)\rangle = \left|\phi_{\pm}\left(N_{\rm A}\pm\frac{J}{2\pi}t,t_0\right)\right\rangle \tag{12}$$

or

$${\rm e}^{-iHt}|\phi_{\pm}(N_{\rm A},t_0)\rangle = T_{Jt/2\pi}|\phi_{\pm}(N_{\rm A},t_0)\rangle, \tag{13}$$

where T_a acts as a translational operator for the states $|\phi_{\pm}(l,t)\rangle$: $T_a|\phi_{\pm}(l,t)\rangle \equiv |\phi_{\pm}(l\pm a,t)\rangle$. In general, the spatial coordinate is treated as a discrete variable while the temporal coordinate is continuous. In order to make T_a operable, the translational spacing a should be an integer. In the following, we only consider the states at the discrete instants t, which ensure a to be integers, i.e. $Jt/2\pi = [Jt/2\pi]$ (the integer part of $Jt/2\pi$). In terms of

$$l_{\pm}(j,t) = N_{\rm A} - j \pm \left[\frac{J}{2\pi}t\right], \tag{14}$$

the wave packets eq. (11) can be rewritten as

$$\left|\phi_{\pm}(N_{\mathrm{A}},t)\right\rangle=\frac{1}{2\pi}\sum_{j}\left\{\frac{1}{il_{\pm}(j,t)}[\mp e^{\mp i\pi l_{\pm}(j,t)}\pm 1]\right\}\left|j\right\rangle. \tag{15}$$

Obviously, eq. (13) shows that state $\left|\phi_{\pm}(N_{\mathrm{A}},t)\right\rangle$ is shape-invariant with velocity $v_{\pm}=\pm J/2\pi$. Now we show the locality of these states. The projections

$$\left\langle j\middle|\phi_{\pm}(N_{\mathrm{A}},t)\right\rangle=\begin{cases}\frac{1}{2}, & l_{\pm}(j,t)=0,\\ \mp\frac{i}{\pi\, l_{\pm}(j,t)}, & \text{odd } l_{\pm}(j,t),\\ 0, & \text{even } l_{\pm}(j,t)\neq 0,\end{cases} \tag{16}$$

of $\left|\phi_{\pm}(N_{\mathrm{A}},t)\right\rangle$ onto the localized states $\left|j\right\rangle$ show that the shape-invariant states are well-localized. And each wave packet has the probability amplitude

$$\left|\phi_{\pm}(N_{\mathrm{A}},t)\right|^{2}\simeq\frac{1}{4}+\frac{2}{\pi^{2}}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^{2}}=\frac{1}{2}, \tag{17}$$

which indicates that the initial state eq. (8) is split into two wave packets completely. On the other hand, the dispersion relation eq. (6) requires the translational invariance of the Hamiltonian since the momentum is the conserved quantity, which leads to the periodicity of the states $\left|\phi_{\pm}\right\rangle$:

$$\left|\phi_{\pm}\left(N_{\mathrm{A}}\pm\frac{J}{2\pi}t,t\right)\right\rangle=\left|\phi_{\pm}\left(N_{\mathrm{A}}\pm\frac{J}{2\pi}t\mp m_{\pm}N,t\right)\right\rangle \tag{18}$$

where $m_{\pm}=1, 2, \cdots,$ are integers. Obviously, when $\tau=N(m_{+}+m_{-})\pi/J,$ the initial state $\left|\psi(N_{\mathrm{A}},0)\right\rangle=S_{N_{\mathrm{A}}}^{-}\left|0\right\rangle$ evolves to

$$\left|\psi(N_{\mathrm{A}},\tau)\right\rangle=S^{-}_{N_{\mathrm{A}}-\frac{N}{2}(m_{+}-m_{-})}\left|0\right\rangle, \tag{19}$$

which is just the recurrence of $\left|\psi(N_{\mathrm{A}},0)\right\rangle$ on the positions $N_{\mathrm{A}}-N(m_{+}-m_{-})/2$ at the instants $N(m_{+}+m_{-})\pi/J$. The physics of this phenomenon can be understood as the interference of two wave packets eq. (11). It also accords with the prediction for the system with spectrum-symmetry matching condition (SSMC)[10,20].

In the single-magnon invariant subspace, the single flip states $\{\left|j\right\rangle\}$ at site j constitute the complete basis. Then the above conclusion can be applied to any state in this subspace. Consider an arbitrary initial state $\left|\Phi(0)\right\rangle=\sum_{j}A_{j}\left|j\right\rangle=\sum_{j}A_{j}\left|\psi(j,0)\right\rangle$, which is a coherent superposition of single flip states. At time t, it evolves to be

$$\left|\Phi(t)\right\rangle=e^{-iHt}\left|\Phi(0)\right\rangle=\left|\Phi_{+}(t)\right\rangle+\left|\Phi_{-}(t)\right\rangle, \tag{20}$$

where

$$\left|\Phi_{\pm}(t)\right\rangle=\sum_{j}A_{j}\left|\phi_{\pm}(j,t)\right\rangle \tag{21}$$

represent two invariant-shape states. Accordingly, at the instants $\tau=N(m_{+}+m_{-})\pi/J,$ the final state is a translation of $\left|\Phi(0)\right\rangle$,

$$|\Phi(\tau)\rangle = T_{N(m_+-m_-)/2}|\Phi(0)\rangle. \tag{22}$$

Furthermore, during the period of time $t \neq \tau$, wave packets $|\Phi_\pm(t)\rangle$ are still well localized in space if the initial state $|\Phi(0)\rangle$ is local. Then the revival of a wave packet can be used to implement perfect quantum state transfer.

Considering an example with $A_j = u\delta_{j,N_\mathrm{D}} + v\delta_{j,N_\mathrm{C}}$, then the initial state $|\Phi(0)\rangle$ can be written in the form

$$|\mathrm{DC}\rangle = uS_{N_\mathrm{D}}^- \prod_{i=1}^{N} |\uparrow\rangle_i + vS_{N_\mathrm{C}}^- \prod_{i=1}^{N} |\uparrow\rangle_i = (u|\uparrow\rangle_{N_\mathrm{D}} |\downarrow\rangle_{N_\mathrm{C}} + v|\downarrow\rangle_{N_\mathrm{D}} |\uparrow\rangle_{N_\mathrm{C}} \prod_{i\neq N_\mathrm{D},N_\mathrm{C}} |\uparrow\rangle_i, \tag{23}$$

where $|u|^2 + |v|^2 = 1$. Obviously, it is an arbitrary pairwise entangled state between sites N_D and N_C. Applying the result eq. (22) to such an initial state $|\mathrm{DC}\rangle$, one can get the conclusion that this pairwise entanglement can be perfectly transferred to the location at the opposite diameter end in a ring.

1.3 Entanglement of two separated spins

Now we turn our attention to the entanglement of the two separated spins induced by the wave packets eq. (11). The reduced density matrix of a state $|\Phi(t)\rangle$ for two spins located at sites i and j has the form[25–29]

$$\rho_{ij} = \begin{pmatrix} v_{ij}^+ & 0 & 0 & 0 \\ 0 & w_{ij} & z_{ij} & 0 \\ 0 & z_{ij} & w_{ij} & 0 \\ 0 & 0 & 0 & v_{ij}^- \end{pmatrix}, \tag{24}$$

with respect to the standard basis vectors $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$. Here, the matrix elements are

$$\begin{aligned} v_{ij}^+ &= \frac{1}{4} + \frac{1}{2}\langle\Phi(t)|[2S_i^z S_j^z \pm (S_i^z + S_j^z)]|\Phi(t)\rangle, \\ w_{ij} &= \frac{1}{4} - \langle\Phi(t)|S_i^z S_j^z|\Phi(t)\rangle, \\ z_{ij} &= \frac{1}{2}\langle\Phi(t)|(S_i^+ S_j^- + S_i^- S_j^+)|\Phi(t)\rangle. \end{aligned} \tag{25}$$

Correspondingly, the concurrence of two spins located at sites i and j for a state $|\Phi(t)\rangle$ can be calculated by

$$C_{ij} = \max\{0,\ 2(|z_{ij}| - \sqrt{v_{ij}^+ v_{ij}^-})\}. \tag{26}$$

Since the state we are concerned about is in the invariant subspace with $S_z = N/2 - 1$, we have $v_{ij}^- = 0$ and then the concurrence reduces to

$$C_{ij} = |\langle\Phi(t)|(S_i^+ S_j^- + S_i^- S_j^+)|\Phi(t)\rangle|. \tag{27}$$

Consider a local initial state having reflectional symmetry with respect to a point N_A, which has the form

$$|\Phi(0)\rangle = f_{N_A}(0)|N_A\rangle + \sum_{j=1}^{N/2} f_j(0)[|N_A + j\rangle + (-1)^R |N_A - j\rangle], \tag{28}$$

where R = even (odd) represents the parity of the state under the reflection. Due to the linear dispersion relation eq. (6), the final state at some instant t actually describes two spatial separated local non-spreading wave packets. However, the reflectional symmetry with respect to N_A of the final state still holds, and $f_{N_A}(t) \approx 0$. The concurrence between two spins at $N_A + j$ and $N_A - j$ for the state $|\Phi(t)\rangle$ is

$$\begin{aligned} C(j,t) = C_{N_A+j,N_A-j} &= |\langle\Phi(t)|(S^+_{N_A+j}S^-_{N_A-j} + S^-_{N_A+j}S^+_{N_A-j})|\Phi(t)\rangle| \\ &= 2|(-1)^R f_j^*(t)f_j(t)| = 2|f_j(t)|^2. \end{aligned} \tag{29}$$

Obviously, the total concurrence $\sum_{j>0} C(j,t) = 1$ due to the normality of the wave function $|\Phi(t)\rangle$. Usually, the total concurrence is regarded as the concurrence of two wave packets. Taking the single flip at N_A in the form eq. (8) to be the initial state as an example, straightforward calculation shows that non-zero concurrences are

$$C(j,t_0) = \frac{1}{2}, \quad C(j,t_n) = \frac{2}{(2n-1)^2\pi^2}. \tag{30}$$

For this to occur, one requires that $t_0 = 2\pi j/J$, $t_n = 2\pi[j + (2n-1)]/J$, $n = 1, 2, \cdots$. It indicates that the concurrence with magnitudes eq. (30) for two spins at $N_A + j$ and $N_A - j$ can be generated at the moment $t_0, t_1, t_2, \cdots, t_n$.

2 Finite long-range interaction model

In this section, we consider the possibility to realize the above scheme in a system which possesses the linear dispersion relation based on a one-dimensional arrangement of spins (qubits) coupled by long-range interactions. Actually, using the identity

$$|2\xi| = \frac{N}{2} - \sum_{r=\text{odd}}^{\infty} \frac{4N}{r^2\pi^2}\cos\frac{r\pi(2\xi)}{N}, \tag{31}$$

we have

$$\frac{J}{2\pi}|k| \approx J_0 + \sum_{r=\text{odd}}^{N/2-1} 2J_r\cos(kr), \tag{32}$$

where

$$J_0 = \frac{J}{4}, \quad J_r = \frac{J}{r^2\pi^2}. \tag{33}$$

Then the Hamiltonian matching the case eq. (6) in the single-magnon invariant subspace can be rewritten as

$$H = J_0 + \sum_{r=\text{odd}}^{N/2-1} J_r h_r, \quad h_r = \sum_i (S_i^+ S_{i+r}^- + h.c.). \tag{34}$$

Notice that although such a model involves the long-range interaction, J_r decays rapidly as r increases. So we call this finite "long-range interactions". The above analytical analysis shows that such finite long-range interactions can lead to nontrivial long-range entanglement and QST.

3 Numerical results and truncation approximation

In the previous sections, it is found that the finite long-range interactions can lead to nontrivial long-range entanglement and QST for the large N limit system. In this section, numerical simulations are performed for finite systems to illustrate the result we obtained above and investigate its application. The numerical exact diagonalization method is employed to calculate the time evolution of the single flip state eq. (8) and the Gaussian wave packet in the following form

$$|\psi_G(N_A,\alpha)\rangle = \frac{1}{\sqrt{\Omega}}\sum_i e^{-\frac{\alpha^2}{2}(N_A-i)^2} S_i^- |0\rangle, \tag{35}$$

in the finite N system. Here, Ω is the normalized factor and N_A, α determine the center and the shape of the wave packet. In Figures 3 and 4, the time evolution of a single flip state eq. (8) and a Gaussian wave packet eq. (35) of $\alpha = 0.1$ in a $N = 100$ ring system are plotted. They show that the local initial state is split into two wave packets in both cases and the wave packets keep their shapes without spreading. Similarly, numerical simulations are also employed for the open chain systems. The similar conclusions are also obtained for open chain system which will be discussed in the following.

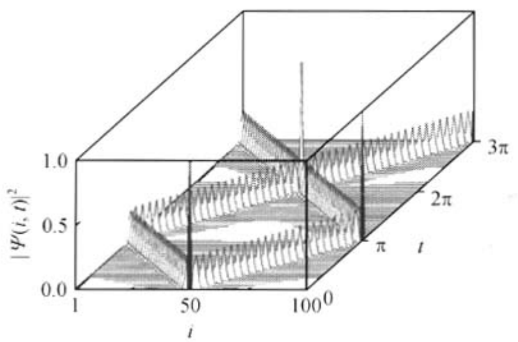

Figure 3 Time evolution of a δ-pulse in an $N = 100$ ring system obtained by numerical simulation. Here the time t is in the unit of N/J. It shows that the two sub-waves are local and keep their shapes.

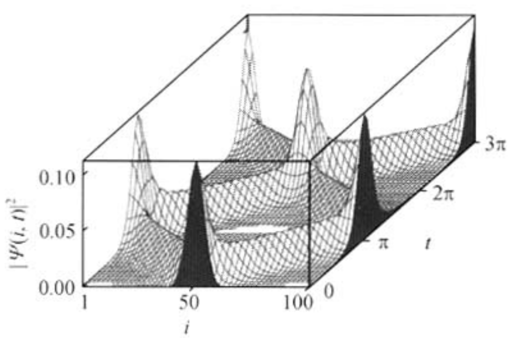

Figure 4 Time evolution of a Gaussian wave packet of $\alpha = 0.1$ in a $N = 100$ ring system obtained by numerical simulation. Here the time t is in the unit of N/J. It shows that the two sub-waves are local and keep their shapes.

A natural question is that what role the long-range couplings play for the peculiar behavior of the propagation of the local wave in such a system. As shown above, the long-range hopping couplings decay rapidly. Then if the size of the system is large enough, it is believed that the too long-range couplings can be neglected. Thus, the "long-range couplings" in the form eq. (33) can be regarded as the relatively local, or "finite long-range" couplings. In order to investigate this problem, or the boundary between the so-called "long-range" and "finite long-range" couplings, we consider the truncated Hamiltonian

$$H = J_0 + \sum_{r=\mathrm{odd},<r_0} J_r h_r, \tag{36}$$

where r_0 is the truncation distance. From the above analysis, the initial state $|\psi(N_A,0)\rangle$ should recur at the positions N_A at the instant $2\pi N/J$, if the system is perfect for the quantum state transfer. The autocorrelation $|A(t)|=|\langle\psi(N_A,0)|\psi(N_A,t)\rangle|$ is an appropriate quantity to investigate the role that r_0 plays. During the period of time $\sim[0,3\pi N/J]$, the maxima of autocorrelations $|A_{max}|=\max\{|A(t)|\}$ of the state eq. (8) in the systems with $N=500$, 1000, 1500, 2000 and $r_0 = 10, 20, \cdots, 100$ are calculated numerically by the exact diagonalization method. In Figure 5, the dependence of the quantity on the truncation distance is plotted. It shows that $|A_{max}|$ approaches 1 when r_0 is around 90, which is called the critical truncation distance or the boundary between the "long-range" and "finite long-range" couplings, for the systems with different N. It indicates that in the case of $N \gg r_0$, the wave packets still travel without spreading. Making use of this observation, we find that although the interactions between spins are "long-range", the $1/r^2$-distribution of coupling strength allows us to limit the maximal interaction range while minimizing the degradation of the quantum coherence obtained from the ideal model. Then we have the conclusion that the $1/r^2$-decay couplings can be regarded as local couplings, or finite long-range couplings. In other words, the robust long-range entanglement between the two distant qubits is not due to the direct long-range coupling interaction between them.

From the above observation, we find that, for the large size system, the long-range couplings ($r_0 \geqslant 90$) can be neglected. Then in the thermodynamic limit, a ring system is equivalent to a more practical system, i.e. an open chain system. To demonstrate this, the numerical simulation is employed to investigate the concurrence $C(l, r_0, t)$ between two far separated sites $N/2 \pm l$ ($l \sim N/2$) for the N-site system with different truncation r_0. In Figure 6, plots of $C(l, r_0, t)$ for the systems with $N = 1000$, $l = 400$ and different r_0 are presented. It shows that for an open chain system, long-range entanglement between two distant qubits can be achieved via the time evolution of a single flip state. The maximal entanglement created by such a system is 0.5, as measured by the two-point concurrence.

Finally, we would like to explain why we choose the linear dispersion relation. First and foremost, linear dispersion relation is the simplest case which satisfies SSMC[10,20]. Besides, our result shows that the linear dispersion relation can be accomplished by "microscopic long-range" but "macroscopic short-range" interaction, which is nontrivial in practice. In addition, one can also take other dispersion relations with SSMC by choosing other types of interaction. However, it is difficult to find an appropriate dispersion relation which can be realized by "macroscopic short-range interactions". So the linear dispersion relation is our best choice.

4 Summary

In summary, the system with long-range couplings is investigated analytically and numerically. It is found that the $1/r^2$-decay long-range coupling model can exhibit approximately linear dispersion $\varepsilon \sim |k|$. The dynamics of such a model possesses a novel feature that an initial local wave packet can be separated into two entangled local wave packets. Furthermore, during the traveling period each wave packet can keep their shape without spreading. Numerical simulation indicates that there exists a critical truncation distance r_0, which limits the range of the interaction but does not affect the generation of entanglement between the two distant qubits in the distance $l \gg r_0$. This model

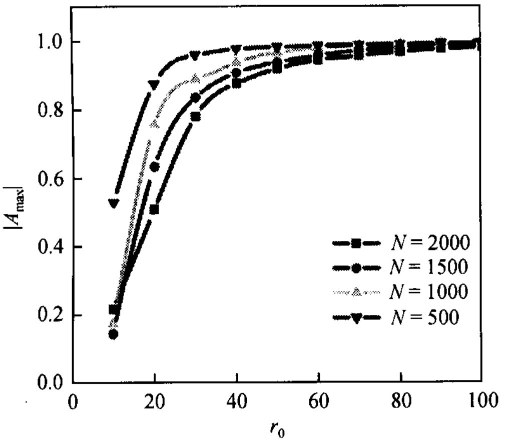

Figure 5 The maximal autocorrelation functions of the initial δ-pulse state in the N-site systems with the truncation distance r_0. It demonstrates that the critical r_0, at which $|A_{max}|$ start to approach 1, does not depend on N strongly.

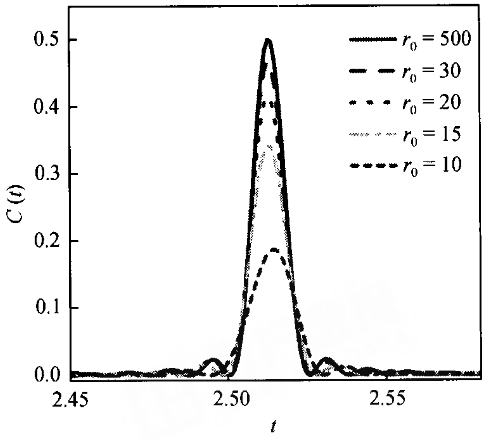

Figure 6 The time-dependent concurrences of two spins separated by the distance $2l = 800$ in the chain system with $N = 1000$, $r_0 = 10, 15, 20, 30$ and 500. Here the initial state is the single flip state, and time t is in the unit of N/J. The results for $r_0 = 500$ are in agreement with the analytical analysis. The results for different truncation approximations show that the range of couplings can be taken on a small scale due to the $1/r^2$- decay of coupling constants.

opens up the possibility to realize the solid state based entangler for creating two entangled but spatially separated qubits.

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Dynamic generation of entangling wave packets in XY spin system with decaying long-range couplings

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material presented in quant-ph/0403073, quant-ph/0502040, quant-ph/0504160, quant-ph/0510035, quant-ph/0512012 and quant-ph/0603283. It includes two large review chapters on entanglement and distillation.

4.外文期刊 ZHANG Zhan-Jun.LIU Yi-Min.MAN Zhong-Xiao Many-Agent Controlled Teleportation of Multi-qubit Quantum Information via Quantum Entanglement Swapping

We present a method to teleport multi-qubit quantum information in an easy way from a sender to a receiver via the control of many agents in a network. Only when all the agents collaborate with the quantum information receiver can the unknown states in the sender's qubits be fully reconstructed in the receiver's qubits. In our method, agents's control parameters are obtained via quantum entanglement swapping. As the realization of the many-agent controlled teleportation is concerned, compared to the recent method [C.P. Yang, et al., Phys. Rev. A 70 (2004) 022329], our present method considerably reduces the preparation difficulty of initial states and the identification difficulty of entangled states, moreover, it does not need local Hadamard operations and it is more feasible in technology.

5.外文期刊 Mio Murao Quantum information and entanglement

We present telecloning, which is an example of an alternative approach for quantum information processing using multiqubit entanglement as a resource. Telecloning simultaneously performs quantum optimal cloning and quantum communication using a particular entangled state. It performs remote quantum operation on spatially separated output qubits for an input qubit. It is also an example of the packaging and preservation of a quantum operation using entanglement.

6.OA论文 Ghojavand. Majid Targeted Efficient Entanglement Distribution through Un-modulated Spin Chains: A specific look to future Nanoscale quantum information systems

In this work, the optimization problem of entanglement provision of targeted site pairs is studied in connected spin graphs; however, as a special case, in previous studies mainly entanglement distribution between an isolated site and another site of a spin system has been under consideration. Here, it is found that in symmetric spin graphs evidently larger amount of entanglement could be achieved between counterpart sites. Moreover, it is shown, in certain symmetric spin rings only by excitation of one fixed site each site pair could be efficiently entangled, while in other structures optimization of generated entanglement necessitates change of encoded sites and their encoding in a complex manner. Hence, this special structures could be treated as an elegant solution for implementation feasibility of future nanoscale quantum information chips because, they could be simultaneously used as nodes, information transmission lines, and according to present work as entanglement distribution buses by only one contact point with external environment; whereas, in such dimensions, minimization of employed resources for fulfilling different necessary parts is a critical issue.

7.外文期刊 Walborn. SP.Almeida. MP.Ribeiro. PHS.Monken. CH Quantum information processing with hyperentangled photon states

We discuss quantum information processing with hyperentangled photon states - states entangled in multiple degrees of freedom. Using an additional entangled degree of freedom as an ancilla space, it has been shown that it is possible to perform efficient Bell-state measurements. We briefly review these results and present a novel deterministic quantum key distribution protocol based on Bell-state measurements of hyperentangled photons. In addition, we propose a scheme for a probabilistic controlled-not gate which operates with a 50 % success probability. We also show that despite its probabilistic nature, the controlled-not gate can be used for an efficient, nonlocal demonstration of the Deutsch algorithm using two separate photons.

8.外文期刊 Rigolin. G.de Oliveira. MC Quantification of continuous variable entanglement with only two types of simple measurements

Here we propose an experimental set-up in which it is possible to obtain the entanglement of a two-mode Gaussian state, be it pure or mixed, using only simple linear optical measurement devices. After a proper unitary manipulation of the two-mode Gaussian state only number and purity measurements of just one of the modes suffice to give us a complete and exact knowledge of the state's entanglement.

9.外文期刊 Wineland DJ..Barrett M..Britton J..Chiaverini J..DeMarco B..Itano WM. Jelenkovic B..Langer C..Leibfried D..Meyer V..Rosenband T..Schatz T. Quantum information processing with trapped ions

Experiments directed towards the development of a quantum computer based on trapped atomic ions are described briefly. We discuss the implementation of single-qubit operations and gates between qubits. A geometric phase gate between two ion qubits is described. Limitations of the trapped-ion method such as those caused by Stark shifts and spontaneous emission are addressed. Finally, we describe a strategy to realize a large-scale device. [References: 46]

10.OA论文 Soeda. Akihito.Murao. Mio Comparing globalness of bipartite unitary operations acting on quantum information: delocalization power, entanglement cost, and entangling power

We compare three different characterizations of the globalness of bipartite unitary operations based on different tasks, namely, delocalization power, entanglement cost for LOCC implementation, and entangling power. We present extended analysis on the globalness in terms of delocalization in two ways. First, we show that the delocalization power differs whether the global operation is applied on one piece of quantum information or two pieces. Second,

by introducing the concept of dislocation, we prove that the local unitary equivalents of controlled-unitary operations assisted by LOCC cannot dislocate one piece of quantum information when applied on two pieces of quantum information. This confirms that the local unitary equivalents of controlled-unitary operations, which are LOCC one-piece relocalizeable, belong to a class of global operations with relatively weak globalness in terms of dislocation of quantum information.