Quantum leakage of collective excitations of atomic ensemble induced by spatial motion

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Abstract We generalize the conception of quantum leakage for the atomic collective excitation states. By making use of the atomic coherence state approach, we study the influence of the atomic spatial motion on the symmetric collective states of 2-level atomic ensemble due to inhomogeneous coupling. In the macroscopic limit, we analyze the quantum decoherence of the collective atomic state by calculating the quantum leakage for a very large ensemble at a finite temperature. Our investigations show that the fidelity of the atomic system will not be good in the case of atom number $N \to \infty$. Therefore, quantum leakage is an inevitable problem in using the atomic ensemble as a quantum information memory. The detailed calculations shed theoretical light on quantum processing using atomic ensemble collective qubit.

Keywords: **quantum memory, atomic ensemble, qubit, quantum leakage.**

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Quantum memory[1,2] is an important element in quantum communication[3−6] and quantum computation[7−9] (or quantum information processing). It is crucial to choose the real quantum storage systems, which are able to be manipulated easily and immune from rapid quantum decoherence. In the past several years, many quantum systems are investigated as the carrier of quantum information. These systems include the entangled two-photon pair[10] in optical parametric down conversion, the array of trapped ions[11], the atomic system[1] in the cavity quantum electrodynamics, the Josephson Junction[12] controlled by the external field, the quantum dots[13], the semiconductor excitons[14] and NMR system[15] . Since there is some deficiency to some extent in these systems, people are trying to study better and more reliable memory systems. Recently the light group velocity can be slowed dramatically[1,2,17−20] in the atomic ensemble with the electromagnetic induced transparency (EIT)[16]. So people began to believe that this kind of system can store quantum information.

Now some research groups have investigated theoretically many ways to store quantum information using collective excited states of atomic ensemble. The collective effect of atomic internal states is very important to form atomic ensemble qubit. Recently we have studied the effect brought by the inhomogeneous coupling between atoms and external light field[1)] . Our results

1) Sun, C. P., Yi, S., You, L., Decoherence of collective atomic spin states due to inhomogeneous coupling (quant-ph/0203072).

offered theoretical bases for practically-useful atomic memory design.

It was found in our investigation that the formation of atomic collective exciton depends on the homogeneousness of coupling coefficient of each atom to external field[1)]. Then the symmetrical sub-space[21] of the atomic ensemble is invariant under action of the Hamiltonian with homogeneous coupling. So this sub-space can be used as a computation space (or called collective excitation space). In the case of coupling coefficient being inhomogeneous, the atomic symmetrical sub-space does not keep close and the state vector of the collective space will exceed the symmetrical sub-space in the evolution of atom system. This phenomenon is called quantum leakage, which will destroy the formation of the collective qubit of atomic ensemble.

In this paper we study another important influence of the motion of atomic center-of-mass on the atomic ensemble qubit. Actually, the essence of quantum leakage can sum up to the random motion of atoms, but in most cases, this kind of effect is mainly due to the inhomogeneity of the phase of complex coupling coefficients. This issue was not concerned in previous work. So in this paper we will discuss the quantum leakage of the atomic ensemble in a general way.

1 Quantum leakage of collective state of atomic ensemble

Generally we consider an atomic ensemble which consists of N two-level atoms ($m = 1$). For simplicity, the atomic ensemble is coupled to a one-dimension classic single-mode light with the frequency ω_L. By using the rotating wave approximation, the Hamiltonian can be written as

$$H_{ori} = \sum_{k=1}^{N} \left(\frac{1}{2} p_k^2 + \frac{1}{2} g^{(k)}(x_k) \sigma_+^{(k)} + h.c + \frac{1}{2} \omega^{(k)} \sigma_z^{(k)} \right), \tag{1}$$

where p_k is the momentum of the k-th atom, $g^{(k)}(x_k)$ the effective coupling coefficient between the k-th atom whose position is x_k and the light, $\omega^{(k)} = \epsilon^{(k)} - \omega_L$, and $\epsilon^{(k)}$ the transition frequency of the k-th atom.

For quantum memory using atomic ensemble, the atomic system is assumed to be homogeneous. That is, $g^{(k)} \equiv g$, $\omega^{(k)} \equiv \omega$, and $g^{(k)}$ is independent of the position x_k. Then let $S_\pm = \sum_{k=1}^{N} \sigma_\pm^{(k)}$ and $S_z = \frac{1}{2} \sum_{k=1}^{N} \sigma_z^{(k)}$ be the collective operators. Then the Hamiltonian is written as

$$H_c = \sum_{k=1}^{N} \frac{1}{2} p_k^2 + g S_+ + g S_- + \omega S_z. \tag{2}$$

So, from the state

$$|J, -J\rangle = |\downarrow\rangle_1 \otimes |\downarrow\rangle_2 \otimes \cdots |\downarrow\rangle_N \qquad (J = N/2), \tag{3}$$

with all the quasi-spin being down, a symmetrical sub-space V_c can be obtained by the action of S_+ on it as

$$\left\{ |J, M\rangle \propto (S_+)^{J+M} |J, -J\rangle | M = -J, \cdots, J \right\}. \tag{4}$$

This is an invariant sub-space under the action of H_c. When $N \to \infty$, the collective excitation operator

$$b^\dagger = \lim_{J \to \infty} \frac{S_-}{\sqrt{2J+1}} \tag{5}$$

1) See footnote on page 580.

satisfies the commutation relation $[b, b^\dagger] = 1$. So people can carry through quantum teleportation[3,4] experiment with the Einstein-Podysky-Rosen (EPR) state defined by collective conjugate operators $Q \propto b + b^\dagger$ and $P \propto b - b^\dagger$. This quantum state is called atomic ensemble collective qubit.

Recently we have studied the case that the coupling coefficient g_k does not depend on the position x_k directly, but it is still inhomogeneous ($g_k \neq g_j$)[1)]. Then V_c is not an invariant sub-space under the action of H_c. For a given initial state $|\phi(0)\rangle$, we define collective quantum leakage

$$L(t) = 1 - |\langle\phi(0)|U_c^\dagger(t)U(t)|\phi(0)\rangle|^2, \tag{6}$$

where $U(t)$ and $U_c(t)$ are the evolution matrices governed by the inhomogeneous Hamiltonian (1) and homogeneous Hamiltonian (2), respectively. This definition of $L(t)$ is consistent with that of footnote 2). In this work, to consider the atomic spatial motion at a certain temperature, we generalize the definition of quantum leakage as

$$L(t) = 1 - |F(t)|^2, \tag{7}$$

where

$$F(t) = Tr(U(t)\rho(0)U_c^\dagger(t)) \tag{8}$$

is the collective fidelity. It is obvious that the fidelity is 1 when the system is homogeneous.

2 Atomic ensemble in a running-wave light field

In this section we will discuss a practical example of the general model (1)— an atomic ensemble in a running-wave light field. Now we assume $g_k(x_k) = ge^{-iqx_k}$, and $\omega^{(k)} \equiv \omega = \text{const}$, for simplicity.

In order to solve this model, a time-independent unitary transformation is introduced as

$$W = \prod_{k=1}^{N} \exp\left[-\frac{i}{2}qx_k\sigma_z^{(k)}\right]. \tag{9}$$

Then the original Hamiltonian is transferred to

$$H = W^\dagger H_{ori} W = \sum_{k=1}^{N} H^{(k)}, \tag{10}$$

where

$$H^{(k)} = \frac{1}{2}{p_k}^2 + \frac{q^2}{8} + \frac{1}{2}g\sigma_+^{(k)} + h.c. + \frac{1}{2}(\omega - qp_k)\sigma_z^{(k)}. \tag{11}$$

Since the momentum operator p_k in $H^{(k)}$ commutes with the spin operators, the evolution operator can be written in a factorized form

$$U(t) = \prod_{k=1}^{N} \exp\left[-\frac{i}{2}qx_k\sigma_z^{(k)}\right] U^{(k)}(t) \exp\left[\frac{i}{2}qx_k\sigma_z^{(k)}\right], \tag{12}$$

where $U^{(k)}(t) = \exp[-it(H^{(k)})]$.

1) See footnote on page 580.

2) Prosen, T., Seligman, T. H., Decoherence of spin echoes (arXiv: nlin.CD/0201038).

Now we consider that the spatial state of atoms is in thermodynamic equilibrium. Then the density operator is

$$\rho_S = \prod_{k=1}^{N} \rho_S^{(k)} = \prod_{k=1}^{N} \int P_k(\beta, p_k)|p_k\rangle\langle p_k| dp_k, \tag{13}$$

where $P_k(\beta, p_k) = \frac{1}{z_k}\exp[-\beta\frac{1}{2}p_k^2]$, $z_k = \langle P_k|P_k\rangle\sqrt{2\pi/\beta}$, $\beta = 1/k_bT$, k_b is the Boltzmann constant and T is the atomic system temperature. Notice that N-atom state and the Hamiltonian are both factorized. It is convenient to introduce the N-atom quasi-spin coherent state. For the given atomic number $N = 2J$, the collective angular momentum state of the computation space is

$$|J, M\rangle = \frac{{S_+}^{J+M}}{\sqrt{(J+M)!(J-M)!}}|J, -J\rangle, \tag{14}$$

which can be written as the integral of atomic coherent state[1)][21]

$$|\theta\rangle = \prod_{k=1}^{2J} \frac{1}{\sqrt{2}} \exp\left(\frac{1}{2}e^{i\theta}\sigma_+^{(k)}\right)|\downarrow\rangle. \tag{15}$$

That is,

$$|J, M\rangle = n(J, M)\int_0^{2\pi} d\theta e^{-i(J+M)\theta}|\theta\rangle, \tag{16}$$

where

$$n(J, M) = \frac{1}{2\pi}\sqrt{\frac{(J+M)!(J-M)!2^N}{(2J)!}}. \tag{17}$$

Assume that the initial atomic ensemble state is $|J, M\rangle$. The initial total density operator

$$\begin{aligned}\rho(0) =& |J, M\rangle\langle J, M| \otimes \rho_S \\ =& [n(J, M)]^2 \int d\theta e^{-i(J+M)\theta} \int d\theta' e^{i(J+M)\theta'} \\ & \cdot \prod_{k=1}^{N} \int P_k(\beta, p_k)|p_k, e_k(\theta)\rangle\langle e_k(\theta'), p_k| dp_k\end{aligned} \tag{18}$$

is factorized. So

$$\begin{aligned}F(t) =& [n(J, M)]^2 \int d\theta e^{-i(J+M)\theta} \int d\theta' e^{i(J+M)\theta'} \prod_{k=1}^{N} \int P_k(\beta, p_k) dp_k \\ & \langle e_k(\theta'), p_k|U_0^{(k)\dagger} e^{-\frac{i}{2}qx_k\sigma_z^{(k)}} U^{(k)} e^{\frac{i}{2}qx_k\sigma_z^{(k)}}|p_k, e_k(\theta)\rangle,\end{aligned} \tag{19}$$

where

$$\begin{aligned}& \langle e_k(\theta'), p_k|U_0^{(k)\dagger} e^{-\frac{i}{2}qx_k\sigma_z^{(k)}} U^{(k)} e^{\frac{i}{2}qx_k\sigma_z^{(k)}}|p_k, e_k(\theta)\rangle \\ =& \Big\{\frac{1}{2}e^{-itqp_k/2}e^{i\theta}(\cos n^{(k)}t - i\sin n^{(k)}t\cos\alpha^{(k)}) \\ & [i\sin n_0^{(k)}t\sin\alpha_0^{(k)} + (\cos n_0^{(k)}t + i\sin n_0^{(k)}t\cos\alpha_0^{(k)})e^{-i\theta'}] \\ & + \frac{1}{2}e^{itqp_k/2}(\cos n'^{(k)}t + i\sin n'^{(k)}t\cos\alpha'^{(k)}) \\ & [\cos n_0^{(k)}t - i\sin n_0^{(k)}t\cos\alpha_0^{(k)} + i\sin n_0^{(k)}t\sin\alpha_0^{(k)}e^{-i\theta'}]\Big\}\langle p_k|p_k\rangle \\ =& \varphi(\theta, \theta', t)\langle p_k|p_k\rangle,\end{aligned} \tag{20}$$

1) See footnote on page 580.

and

$$n^{(k)} = \sqrt{g^2 + \frac{1}{4}\left[\omega - q\left(p_k + \frac{q}{2}\right)\right]^2}, \quad \alpha^{(k)} = arctg\frac{2g}{\omega - q(p_k + \frac{q}{2})}$$

$$n'^{(k)} = \sqrt{g^2 + \frac{1}{4}\left[\omega - q\left(p_k - \frac{q}{2}\right)\right]^2}, \quad \alpha'^{(k)} = arctg\frac{2g}{\omega - q(p_k - \frac{q}{2})}$$

$$n_0^{(k)} = \sqrt{g^2 + \frac{\omega^2}{4}}, \alpha_0^{(k)} = arctg\frac{2g}{\omega}. \tag{21}$$

From the above equations, we obtain the analytic result in detail:

$$\begin{aligned} F(t) \equiv & O_{M,M}(t) \\ = & [n(J,M)]^2 \int d\theta e^{-i(J+M)\theta} \int d\theta' e^{i(J+M)\theta'} \\ & \cdot \prod_{k=1}^{N} \int \sqrt{\frac{\beta}{2\pi}} \exp\left[-\beta\frac{1}{2}p_k^2\right] \cdot \varphi(\theta, \theta', t) dp_k. \end{aligned} \tag{22}$$

3 Results and discussions

In this section we investigate the above equation by numerical study. Given the values of g and ε, it is obvious that $\xi(t)$ $(= |O_{M,M}(t)|^2)$ is dependent on $J(= N/2), M, \omega_L(\propto q)$ and $\beta(= 1/k_bT)$. We will discuss on how the fidelity depends on these variables. First consider the N dependence of fidelity F. Assume the atomic collective state is $|J, M\rangle$, and the temperature is about the room temperature $(T = 300K)$, and the light frequency ω_L is given. Fig. 1(a) shows the time evolution of $|F|^2$. When $t > 0$, the phenomenon of collapse appears in the short term but in the long term the phenomenon of revival occurs. For the different case of N, the time period of revival peaks is the same but the height decreases more rapidly and the half-width becomes narrower when the atomic number N becomes bigger. $t_{1/2}$ is defined as the time in which the ξ is decreased to 0.5 for the first time. Fig. 2 shows the values of $t_{1/2}$ for different N. According to the numerical results, $t_{1/2}$ will decrease if the number of the atomic ensemble N increases, and asymptotically it is the exponential dependent decay of N. For a macroscopic N, the collapse should be very rapid and the revival will not occur any more.

Next we investigate the case of different M for a given N. Our numerical results show that $t_{1/2}$ is the longest when M equals $-J$, and $t_{1/2}$ will decrease if the value of M increases from $-J$ to J, as shown in fig. 2. It means that when only a few atoms are in the excited states, the evolution of $|F(t)|^2$ is relatively good.

We also have considered the case of different temperature for a given atomic number N with M being equal to J, as shown in fig. 3. It is seen that $t_{1/2}$ decreases to zero rapidly with the increasing temperature and approaches a certain value with $T \to 0k$. An intuitive illustration is that the more the temperature T increases, the more the movement of the atomic position x_k increases, that is, the influence of ignoring the term e^{-iqx_k} increases. When the temperature T becomes colder, $t_{1/2}$ does not increase but approach a certain value since even if the momentum P_k of an atom is almost zero at the limit of $T = 0$, the atom cannot stand at some site under the uncertainty relation $\Delta x_k \Delta P_k \geqslant \hbar/2$.

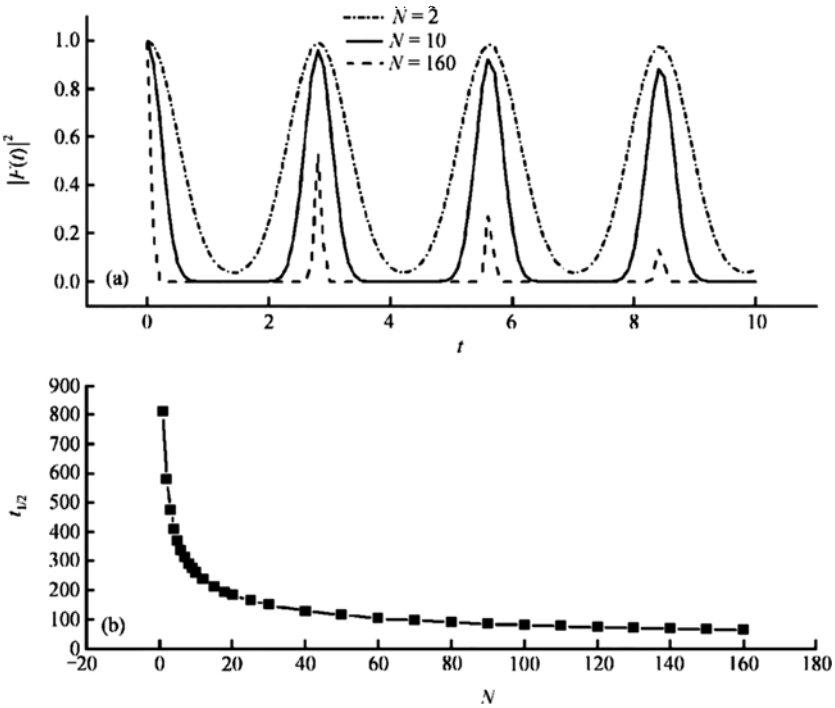

Fig. 1. (a) Comparison of time evolution of numerical $\xi(t)$ for different $N = 2, 10, 160$; (b) numerical results of $t_{1/2}$ according to a series of atomic number N.

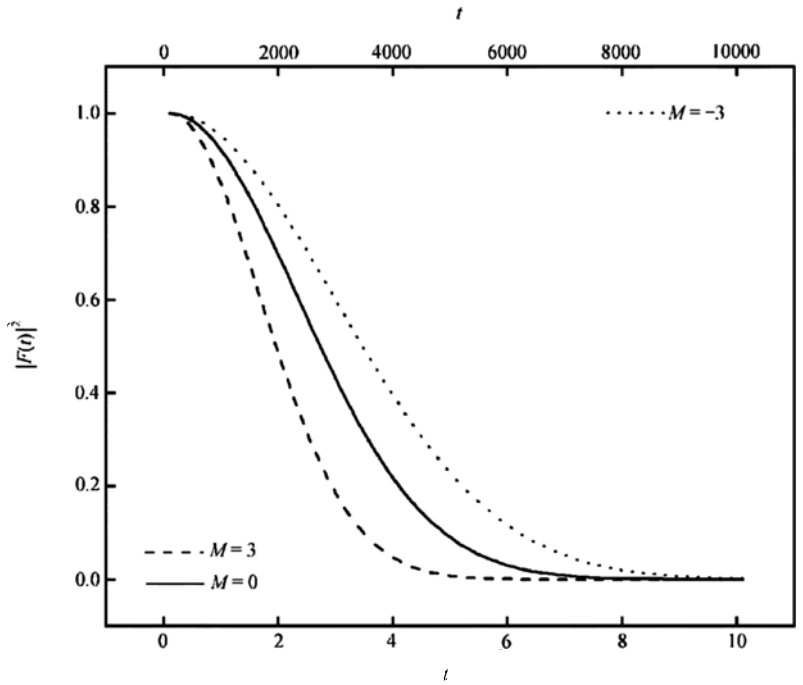

Fig. 2. The decay graph of time evolution of $\xi(t)$ for different $M = -3, 0, 3$ for the same atomic number $N = 6$. Notice that the line of the case of $M = -3$ is according to the top long time t -axis, the other lines of $M = 0, 3$ according to the bottom short time t-axis.

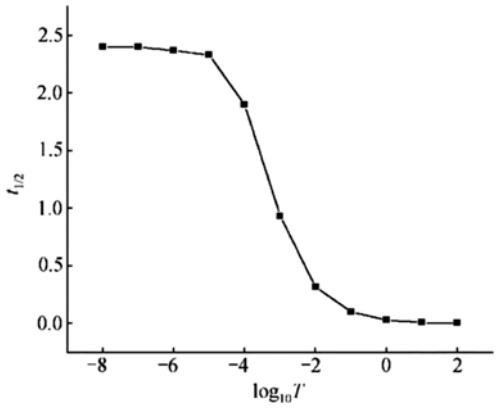

Fig. 3. The abscissa is the denarylogarithm of the temperature T. The ordinate is the values of $t_{1/2}$.

Then we consider the case of dependence on detuning ω (notice $\omega = \varepsilon - \omega_L$, $q \propto \omega_L$). It is seen in fig. 4 that when $\omega \gg g$, that is, the case of large detuning, ξ will increase evidently with an oscillation of high frequency. This is reasonable that the running-wave field will hardly influence the atoms in the case of large detuning and it brings a little effect by ignoring the term e^{-iqx_k}. When ω becomes large for a given P_k which is determined by the temperature T without changing q very much, the contribution of q and P_k become small and can be ignored for the case of very large value of ω. Then $|F(t)|^2$ is near 1 in this case as seen from the solid line C in fig. 4. It also can be seen from the fig. 4 that the evolution of ξ has the same decay envelope in the different case of the detuning ω with the same values of T, N and M.

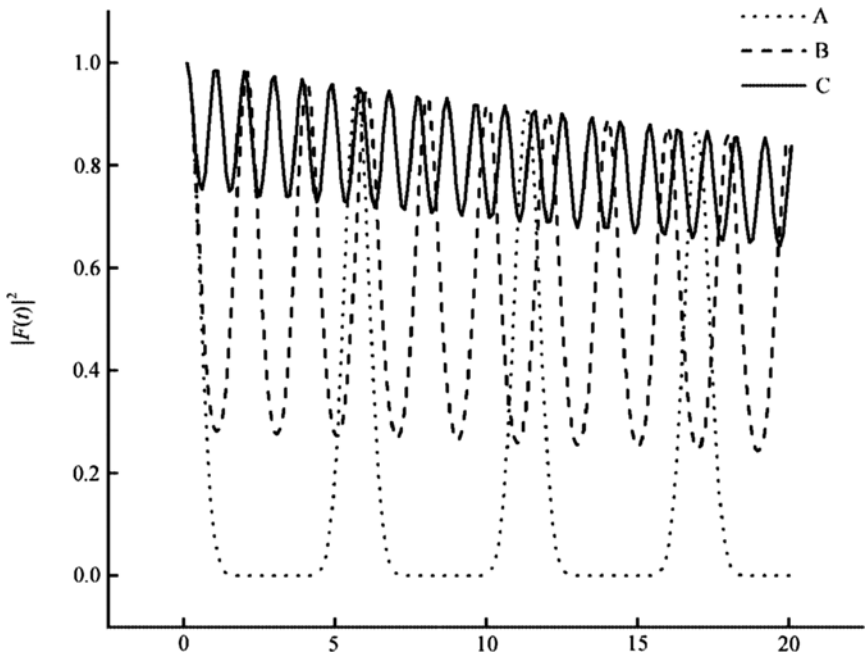

Fig. 4. Time evolution of $\xi(t)$ for different red detuning ω Line A according to $\omega = 1$, line B to $\omega = 3$ and line C to $\omega = 10$.

We discuss the quantum leakage for some cases when the initial inner state is a pure state $|J, M\rangle$ above. Factually it is easy to generalize the inner state $|J, M\rangle$ to a superposition state as

$$\Psi(0) = \sum_M C_M |J, M\rangle. \tag{23}$$

The quantum leakage is

$$L(t) = 1 - |\langle\phi_0(t)|\phi(t)\rangle|^2 = 1 - |\sum_M \sum_{M'} C^*_{M'} C_M O_{M,M'}(t)|^2, \tag{24}$$

where the non-diagonal term $O_{M,M'}(t)$ is like the diagonal term $O_{M,M}(t)$ as

$$\begin{aligned} & O_{M,M'}(t) \\ =& n(J,M') \cdot n(J,M) \int d\theta e^{-i(J+M)\theta} \int d\theta' e^{i(J+M')\theta'} \\ & \prod_{k=1}^{N} \int \sqrt{\frac{\beta}{2\pi}} \exp\left[-\beta \frac{1}{2} p_k^2 \right] \cdot \varphi(\theta, \theta', t) dp_k. \end{aligned} \tag{25}$$

Our numerical results show that the non-diagonal terms $O_{M,M'}(t)$ approach zero. So it only needs to calculate the diagonal terms and the results of ξ are like the case of $\Psi(0) = |J, M\rangle$ above.

4 Conclusion

In this paper we discuss the fidelity F of the 2-level atom ensemble coupled to a running-wave light. Our results show that $|F(t)|^2$ will collapse in a short time and will be exponential dependent decay of N asymptotically. Though $|F(t)|^2$ will revive in a relatively long time, the height of the revival peaks decrease sharply when N increases even if in the best case of initial state being $|J, M = -J\rangle$. Otherwise, the discussion of T dependence shows that though the fidelity is better at a lower temperature, it cannot eliminate effectively the quantum leakage by decreasing the temperature.

It seems that the fidelity can be good in the case of large detuning. But factually it is difficult to realize because of 2 reasons below. First, the envelope decay of $|F(t)|^2$ is independent of detuning and decreases rapidly when N increases. Secondly, in the case of large detuning, the light field has little action with the atomic ensemble, so it cannot eliminate effectively quantum leakage. All in all, our results show that when $N \to \infty$, the fidelity of atomic ensemble system will be bad in any case. Quantum leakage is an inevitable problem in using atomic ensemble as quantum information memory. This is consistent with our analysis about the quantum decoherence[22].

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