Born-Oppenheimer approximation of quantized cavity-atom system and localization control of atomic tunneling*

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Received August 8, 1994

Abstract The generalized Born-Oppenheimer approximation theory is applied to the localization control of state tunneling of a two-level atom in a cavity field with single mode. The nonadiabatic effect of tunneling of atomic chiral states in coherent cavity field is analyzed quantitatively and the condition for realizing localization is given strictly. Besides, the influence of variation in temperature on tunneling of atomic state is discussed.

Keywords: cavity quantum electrodynamics, Born-Oppenheimer approximation, tunneling, localization.

The development of techniques in quantum electrodynamics makes it possible to manufacture microwave cavities in a scale comparable with the atomic wavelength and to realize the artificial control of dynamical behavior of atoms in cavity[1]. For instance, by means of constraints on scale of cavity, spontaneous emission can be greatly enhanced[2], and making use of the adiabatic decay of cavity mode to cool atoms, the scales of time and spatial length can be precisely determined[3-6]. This new area in quantum optics and atom physics is called cavity quantum electrodynamics. Recently, through an appreciate preparation of multiple or single mode cavity field, people have tried to establish experimentally and theoretically the ways for artificially inhibiting or enhancing the tunneling rate of atomic chiral state so as to realize the localization of atomic chiral state[7-10]. However, most of the theoretical studies on the localization and tunneling of atomic state are semi-classical and phenomenological and short of systematical analytic treatment. One of the purposes of this paper is to build a systematical analytic theory for the control of atomic tunneling. Using the generalized Born-Oppenheimer (BO) approximation theory[11-13] suggested by the author several years ago, a systematic method is presented to analyze the adiabatic and non-adiabatic effects, and the influence of initial coherent and mixed states is discussed.

1 Description of single mode cavity-two-level atom system

To grasp the essence of the problem, let us consider the simplest case of cavity quantum

* Project supported by the National Natural Science Foundation of China.

electrodynamics for a cavity-atom system: a two-level atom is placed in a single mode cavity and the coupling between them is independent of the atomic mass-center. Let $h=1$ and write down the Hamiltonian for the cavity-atom system.

$$\hat{H}=-\frac{\Delta}{2}|1\rangle\langle 1|-|2\rangle\langle 2|+\omega a^{+}a+g(a+a^{+})(|1\rangle\langle 2|+|2\rangle\langle 1|), \tag{1.1}$$

where $|1\rangle$ and $|2\langle$ are the ground and excitation states with energy level difference Δ; a^{+} and a are the creation and annihilation operators for the cavity mode with the frequency ω; g denotes the coupling constant. Notice that in the case of strong coupling that is well away from resonance, the rotating wave approximation is not valid and the model (1) cannot make the exactly-solvable Jaynese-Cummings (JC) model[14-15]. For this reason, we must build an analytic method for approaching the resonance cavity-atom system. Refs. [8—10] even used the variational method to study the ground state with high-frequency cavity field. In this case ω is very large, and Δ can be regarded as a relatively small parameter, indicating that the eigenstates

$$|\pm\rangle=\frac{1}{\sqrt{2}}(|1\rangle\pm|2\rangle)$$

of $|1\rangle\langle 2|+|2\rangle\langle 1|$, as the chiral bare state after having dressed certain variables of cavity field, is expected to become the stable state of each order for the cavity-atom system. These stable state will correspond to the localizations problem of atomic chiral state. Obviously, without the dressing cavity, the off diagonal elements of $\hat{H}$ in the basis $|+\rangle$, $|-\rangle$ have indelible contributions that will cause the completely coherent tunneling from $|+\rangle$ to $|-\rangle$. Thus, only if certain cavity field is dressed, this tunneling phenomenon can be controlled. We can draw an analogy for double-well potentials to imagine this problem. In a double-well, the lowest two energy eigenstates possess odd and even parities, respectively. Their superpositions $|\varphi_{\pm}\rangle=\frac{1}{\sqrt{2}}(|\varphi_1\pm|\varphi_2\rangle)$ represent the localized states in right and left wells, respectively. Because $|\varphi_1\rangle$ and $|\varphi_2\rangle$ have different energies, the energy split will lead to the phase difference between $|\varphi_+\rangle$ and $|\varphi_-\rangle$, proportional to time . It then results in the changes of signal of $|\varphi_{12}\rangle$ in $|\varphi_{\pm}\rangle$ to realize the transitions between $|\varphi_{\pm}$ and causes the coherent tunneling. Notice that a random environment coupling may lead to dissipation and Brownian motion so that the control of system does not make sense. However, if the single mode cavity is used to replace the random environment, the effective control of tunneling may be realized.

2 Analytic treatment of dynamical process: BO approximation

According to the above analysis, the eigenstates of Hamiltonian (1.1) can be regarded as the superpositions of the chiral state

$$|\psi\rangle=\varphi_{+}|+\rangle+\varphi_{-}-\rangle, \tag{2.1}$$

where the dressing factor $\varphi_{\pm}\rangle$ satisfies the vector-valued motion equation in the generalized BO approximation approach[11-13].

$$H_0\varphi + V\varphi = E = \varphi, \tag{2.2}$$

where

$$H_0=\begin{pmatrix}\hat{H}_+ & 0\\ 0 & \hat{H}_-\end{pmatrix},\quad V=-\frac{\Delta}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\quad \varphi=\begin{pmatrix}\varphi_+\\ \varphi_-\end{pmatrix} \tag{2.3}$$

and

$$\hat{H}_{\pm} = \omega a^+ a \pm g(a^+ + a) \tag{2.4}$$

just denote two forced oscillators of external forces with opposite directions, respectively. By drawing analogy to the generalized BO approximation theory in refs. [11—13], the above approach regarded eigenstates of $|1\rangle\langle 2|+|2\rangle\langle 1|$ as quantized variables, but other variables in the model as collective variables. The central idea in this approach is to transform the function equation with multiple components depending on perturbation parameters in Fock space into the matrix equation with operator-valued elements and then to utilize the ordinary perturbation theory to obtain the solutions for the dressing factors. In fact, eqs. (2.2) have two sets of zeroth order solutions,

$$\varphi_+^{[0]}(n)=\begin{pmatrix}\varphi_+(n)\\ 0\end{pmatrix},\quad \varphi_+^{[0]}(n)=\begin{pmatrix}0\\ \varphi_-(n)\end{pmatrix}, \tag{2.5}$$

which are obtained from eigen-equation

$$\hat{H}_{\pm}\,\varphi_{\pm}(n) = E_n^{[0]}\varphi_{\pm}(n)\,. \tag{2.6}$$

Starting from eq. (2.3), one may get the first-order correction to eq. (2.2),

$$\varphi_{\pm}^{[1]}(n)=\sum_{m\neq n}\frac{\Delta\langle\varphi_{\pm}(m)|\varphi_{\pm}(n)\rangle}{2(E_m^{[0]}-E_n^{[0]})}\,\varphi_{\pm}^{[0]}(m). \tag{2.7}$$

Eq. (2.7) gives the solution $\varphi_{\pm}^{[1]}(n)$ to the dressing factors with first order corrections,

$$|\psi_{\pm}(n)\rangle=\varphi_{\pm}(n)|\pm\rangle|\frac{\Delta}{2\omega}\sum_{m\neq n}\frac{\langle\varphi_{\pm}(m)|\varphi_{\pm}(n)\rangle}{m-n}\,\varphi_{\pm}(m)|\pm\rangle, \tag{2.8}$$

$$n=0,\ 1,\ 2,\cdots$$

where we have used the explicit form of $E_n = E_n^{(0)}$

$$E_n=(N-\alpha^2)\omega, \alpha=g/\omega. \tag{2.9}$$

Corresponding to E_n, $\varphi_{\pm}(n)$ is given in terms of the coherent state operator

$$D(z)=e^{+za^+-z^*a} \tag{2.10}$$

as

$$\varphi_{\pm}(n)=D(\pm\alpha)|n\rangle=\frac{1}{\sqrt{n!}}D(\pm\alpha)a^{+n}|0\rangle. \tag{2.11}$$

Based on the above discussion we calculate the correction factors

$$\Delta^{\pm}_{nm} = \frac{\Delta\langle\varphi_{\pm}(m)\varphi_{\pm}(n)\rangle}{2\omega(m-n)}$$

$$= \sum_{l=0}^{\min(m,n)} \frac{\Delta\sqrt{m!\,n!}\,(\pm 2\alpha)^{m+n-2l}}{2\omega(m-n)(m-l)!\,(n-l)!}\,\mathrm{e}^{-2|\alpha|^2}. \tag{2.12}$$

From the properties of coherent state where min(m,n) denotes the minimum m and n, one has

$$|\psi_{\pm}(n)\rangle \cong \varphi_{\pm}(n)|\pm\rangle + \sum_{m\neq n} \Delta^{\pm}_{nm}\,\varphi_{\mp}(m)|\pm\rangle. \tag{2.13}$$

In the viewpoint of BO approximation, the first term of eq. (2.13) represents the eigenstate of adiabatic approximation, while the last term refers to the non-adiabatic correlations. The adiabatic condition in $|\Delta^{\pm}_{mn}| \ll 1$, or

$$\frac{\sqrt{m!\,n!}\cdot(2g)^{m-n}\cdot\Delta}{2\omega^{m-n-1}} \ll 1. \tag{2.14}$$

Condition (2.14) means the ligh frequency cavity field, i.e. $\Delta/\omega \ll 1$, and the not too strong coupling $g/\omega \ll 1$.

3 Control of tunneling and localization

We first consider the case of zero temperature. If the system is initially prepared in a singled model field of coherent state $|z\rangle = D(z)|0\rangle$ with strength $|z|^2$, and a chiral bare state $|+\rangle$ for the atom, then the initial state of cavity-atom is

$$|\psi_{\uparrow}(0)\rangle = |z\rangle \otimes |+\rangle \cong \sum_{n=0}^{\infty} f_n(z)[|\psi_+(n)\rangle - \sum_{m\neq n} \Delta^{+}_{nm}|\psi_-(m)\rangle], \tag{3.1}$$

where

$$f_n(z) = \frac{(\alpha+z)^n}{\sqrt{n!}}\,\mathrm{e}^{-\frac{1}{2}|\alpha+z|^2-\alpha(z-z^*)}.$$

Using expression (2.13) of the first-order approximate eigenstates $|\varphi_{\pm}(n)\rangle$, we obtain the evolution state at t,

$$|\psi_{\uparrow}(t)\rangle = \mathrm{e}^{\mathrm{i}\omega\alpha^2 t}\sum_{n=0}^{\infty} f_n(z)(\mathrm{e}^{-\mathrm{i}n\omega t}|\psi_+(n)\rangle - \sum_{m\neq n}\Delta^{+}_{nm}\,\mathrm{e}^{-\mathrm{i}m\omega t}|\psi_-(m)\rangle)$$

$$= \mathrm{e}^{\mathrm{i}\omega\alpha^2 t}\sum_{n=0}^{\infty} f_n(z)[(\mathrm{e}^{-\mathrm{i}n\omega t}\varphi_+(n)|+\rangle + \sum_{m\neq n}(\mathrm{e}^{\mathrm{i}n\omega t} - \mathrm{e}^{-\mathrm{i}m\omega t})\varphi_-(m)|-\rangle]. \tag{3.2}$$

Eq.(3.3) shows that in presence of light field, the tunneling rate from chiral state $|+\rangle$ to $|-\rangle$

$$P = \sum_{nn'=0}^{\infty}\sum_{m\neq n'n} \Delta^{+}_{nm}\Delta^{+*}_{n'm} f_n(z) f^*_{n'}(z)\,\chi_{nm}(t)\chi^*_{n'm}(t)$$

$$\chi_{nm}(t) = \mathrm{e}^{-\mathrm{i}n\omega t} - \mathrm{e}^{-\mathrm{i}n\omega t}. \tag{3.3}$$

The probability ramming atom in $|+\rangle$ is 1-P. It is observed from eqs. (3.3) and (3.1) that when adiabatic condition (2.14) holds, $P\to 0$ and the tunneling effect is inhibited and then the atom is localized in the chiral state $|+\rangle$. If $z=-\alpha$, then $f_n(z)=\delta_{n0}$. At this time, the cavity is in the state $\varphi_+(0)|+\rangle$. The above formula of tunneling rate is simplified as

$$P=\sum_{m\neq 0}4|\Delta_{0m}^{+}|^2\sin^2\left(\frac{1}{2}m\omega t\right). \tag{3.4}$$

This means that the tunneling rate is completely controlled by the parameters ω and Δ of the cavity and the coupling constant g. Refs. [7—10] pointed out this fact qualitatively, but without giving the quantitativel expression (3.4). Expression (3.3) is a complete new result.

Finally, we analyse the influence of the cavity temperatures on the control of tunneling-localization. For convenience, we suppose that the cavity is initially in a thermal equilibrium state of temperature T. In this sense, the initial state is described by a density matrix

$$\rho(t)=\sum_{n=0}^{\infty}\frac{1}{\Omega}e^{-n\beta\omega}|n\rangle\langle n|\otimes|+\rangle\langle+|,$$

$$\Omega=(1-e^{\beta\omega})^{-1},\ \beta=\frac{1}{\kappa T}. \tag{3.5}$$

From it we obtain the state density matrix at time t

$$\rho(t)=\sum_{n=0}^{\infty}\frac{1}{\Omega}|\psi_{n\uparrow}(t)\rangle\langle\psi_{n\uparrow}(t)|, \tag{3.6}$$

where $|\psi_{n\uparrow}(t)\rangle$ is the solution to the Schrödinger equation with initial state $-n+\rangle$, which is determined by BO approximation solution (2.13) as

$$|\psi_{n\uparrow}(t)>=\sum<m|D(\alpha)n>e^{i\alpha^2\omega t}$$

$$*(e^{-im\omega t}\varphi_+(m)|+\rangle+\sum_{k\neq m}^{m}\Delta_{m\kappa}^{+}x_{m\kappa}(t)\varphi_-(k)|-\rangle). \tag{3.7}$$

Therefore, the tunneling rate from $|+\rangle$ to $|-\rangle$ is

$$P=T_r(|-\rangle\langle-|\rho(t)=\sum_n\frac{1}{\Omega}e^{-n\beta\omega}S_n(t), \tag{3.8}$$

where factors

$$S_n(t)=T_r(|-\rangle\langle-|\psi_{n\uparrow}(t)\rangle\langle\psi_{n\uparrow}(t)|)$$

$$=\sum\sum_{k\neq m,m'}^{\infty}\langle m|D(\alpha)|n\rangle\langle m'|D(\alpha)|n\rangle\Delta_{m\kappa}^{+}\Delta_{m'\kappa}^{+*}\chi_{m\kappa}(t)\chi_{m'\kappa}(t)^{*} \tag{3.9}$$

are positive. Expression (3.8) shows that as the temperature rises, (β decreases), the tunneling rate increases so that the localization is broken. In order to localize the atom to a chiral state, we must decrease the temperature of the cavity.

4 Conclusion

By building a systematically analytic method, this paper has studied the tunneling and localization of atom state in ideal cavity of super-short microwave. Though our discussion is only constrained to the simplest case of two-level atom and single mode cavity, it is not difficult to generalize the method developed in this paper to complicated cases, e.g. the tunneling effect of many-level atoms. It is also expected that some of the physical conclusions are valid for more complicated cases. For example, in the case where the motion of atomic mass-center affects the cavity quantum mechanics, its effects on control of atomic tunneling and localization can be investigated.

Acknowledgement The author thanks Prof. C. N. Yang for instructing him to enter the new field of cavity quantum electrodynamics of atom-cavity system.

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