Angular Momentum of a Magnetically Trapped Atomic Condensate

P. Zhang,[1] H. H. Jen,[1] C. P. Sun,[2] and L. You[1,3]

[1]School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA
[2]Institute of Theoretical Physics, Chinese Academy of Science, Beijing 100080, China
[3]Center for Advanced Study, Tsinghua University, Beijing 100084, People's Republic of China

(Received 27 December 2005; revised manuscript received 12 September 2006; published 17 January 2007)

For an atomic condensate in an axially symmetric magnetic trap, the sum of the axial components of the orbital angular momentum and the hyperfine spin is conserved. Inside an Ioffe-Pritchard trap (IPT) whose magnetic field (B field) is not axially symmetric, the difference of the two becomes surprisingly conserved. In this Letter we investigate the relationship between the values of the sum or difference angular momentums for an atomic condensate inside a magnetic trap and the associated gauge potential induced by the adiabatic approximation. Our result provides significant new insight into the vorticity of magnetically trapped atomic quantum gases.

DOI: 10.1103/PhysRevLett.98.030403 PACS numbers: 03.75.Mn, 03.65.Vf, 03.75.Lm, 67.57.Fg

A magnetic trap constitutes one of the key enabling technologies for the recent successes in atomic quantum gases [1]. The most commonly employed magnetic traps includes the quadrupole trap (QT) as in the magneto-optical-trap (MOT) configuration [2] and the Ioffe-Pritchard trap (IPT) [3]. The direction of the B field forming the magnetic trap is generally a function of the spatial position. For a trapped atom, its hyperfine spin adiabatically follows the changing B-field direction, and the atom remains aligned (or antialigned) with respect to the local B field. As a result of the adiabatic approximation, the center of mass motion of a magnetically trapped atom experiences an induced gauge potential from the changing B field [4,5].

In a variety of magnetic traps, e.g., in a QT, the B field is invariant with respect to rotations along a fixed z axis. As the case of single particle dynamics [6], such a SO(2) symmetry leads to the conservation of the z component J_z ($=L_z + F_z$) of the total angular momentum or the sum of the z components of the atomic spatial angular momentum $\vec{L}$ and the hyperfine spin $\vec{F}$. A different symmetry exists for an IPT giving rise to a corresponding conserved quantity D_z ($=L_z - F_z$), the difference of L_z and F_z. To our knowledge, this surprising property has never before been identified explicitly. We feel obliged to present this Letter because it significantly affects the vortical properties of a global condensate ground state in a magnetic trap.

Our work is focused on a detailed investigation of the relationship between the gauge potential and the associated values of $J_z(D_z)$ in a magnetic trap. For a spin-F condensate, due to the appearance of the adiabatic gauge potential, the possible values of J_z or D_z are restricted to a definite region $[-F, F]$. The gauge potential of our formulation is directly related to the effective trap rotation studied earlier in Ref. [5]. While Ho and Shenoy mainly studied the orbital angular momentum component in an IPT [5], instead we concentrate on the conserved quantity, the sum (J_z) or difference (D_z) for a QT or a IPT.

This Letter is organized as follows. We first consider a spin-1 condensate in a QT. Making use of an effective energy functional appropriate for the adiabatic approximation [5], we prove $J_z \in [-1, 1]$ with the actual value determined by the angle between the z axis and the direction of the B field. We then generalize to the spin-F case. Finally, our result is extended to an IPT.

The Hamiltonian of a spin-1 atomic condensate with N atoms in a magnetic trap ($\hbar = 1$) is $H = H_S + H_I$ with the single atom part

$$H_S = \int \hat{\psi}^\dagger(\vec{r})\left[-\frac{\nabla^2}{2M} + \mu_B g_F B(\vec{r})\vec{F}\cdot\hat{n}(\vec{r})\right]\hat{\psi}(\vec{r})d\vec{r},$$

and the atom-atom interaction Hamiltonian

$$H_I = \sum_{m,n,p,q}\int \hat{\psi}_m^{(z)\dagger}(\vec{r})\hat{\psi}_n^{(z)\dagger}(\vec{r}')V_{pq}^{mn}(\vec{r},\vec{r}')\hat{\psi}_p^{(z)}(\vec{r})\hat{\psi}_q^{(z)}(\vec{r}')d\vec{r}d\vec{r}'.$$

$\hat{\psi}(\vec{r}) = [\hat{\psi}_{-1}^{(z)}(\vec{r}), \hat{\psi}_0^{(z)}(\vec{r}), \hat{\psi}_1^{(z)}(\vec{r})]^T$ denotes the annihilation field operator for the z-quantized F_z component of m, n, p, $q = 0, \pm 1$. M is the atomic mass, and μ_B the Bohr magneton. $B(\vec{r})$ and $\hat{n}(\vec{r})$ denote the strength and direction of the local B field. The *Lande* g factor is $g_{F=1} = -1/2$.

Within the mean field approximation, the field operator $\hat{\psi}(\vec{r})$ is replaced by its average $\langle\hat{\psi}(\vec{r})\rangle$. To introduce the adiabatic approximation, we define a group of normalized scalar wave functions $\varphi_u(\vec{r})$:

$$\langle\hat{\psi}(\vec{r})\rangle = \sum_{b=0,\pm 1}\sqrt{N}\,\xi^B(b,\vec{r})\varphi_b(\vec{r}), \tag{1}$$

where $\xi^B(b,\vec{r})$ is the eigenstate of the B-quantized spin component $\vec{F}\cdot\hat{n}(\vec{r})$ with eigenvalue b, satisfying $\vec{F}\cdot\hat{n}(\vec{r})\xi^B(b,\vec{r}) = b\xi^B(b,\vec{r})$ and $\xi^{B\dagger}(b,\vec{r})\xi^B(b',\vec{r}) = \delta_{b,b'}$. In the z-quantized representation, $\xi^B(b,\vec{r}) = [\xi_{-1}^B(b,\vec{r}), \xi_0^B(b,\vec{r}), \xi_1^B(b,\vec{r})]^T$. In this study, it is important to distinguish $\xi^B(b,\vec{r})$ from the eigenstates $\xi_z(0,\pm 1)$ of F_z with eigenvalues $0, \pm 1$. In explicit form, we have $\xi_z(-1) = [1,0,0]^T$, $\xi_z(0) = [0,1,0]^T$, and $\xi_z(1) = [0,0,1]^T$.

A magnetic dipole precesses around the direction of a B field. Majorona transitions between different $\xi^B(b,\vec{r})$ states can be neglected when $B(\vec{r})$ is large enough. Thus the atomic hyperfine spin adiabatically freezes in the low-field seeking state $\xi^B(-1, \vec{r})$ during the trapped center of mass motion, and $\varphi_{-1}(\vec{r}) = \varphi(\vec{r})$ and $\varphi_{0,+1}(\vec{r}) = 0$. Similarly the z-quantized mean field becomes

$$\hat{\psi}(\vec{r}) \approx \langle\hat{\psi}(\vec{r})\rangle = \sqrt{N}\xi^B(-1, \vec{r})\varphi(\vec{r}), \tag{2}$$

with $\varphi(\vec{r})$ a B-quantized scalar function.

Substituting Eq. (2) into the expression of H, we can obtain the expression of the condensate energy $E_{\rm ad}$ as a functional of the scalar wave function $\varphi(\vec{r})$: $E_{\rm ad}[\varphi] = N\int \varphi^*(\vec{r})\mathcal{E}_{\rm ad}\varphi(\vec{r})d\vec{r}$. Here $\mathcal{E}_{\rm ad}$ is defined as [4,5]:

$$\mathcal{E}_{\rm ad} = \frac{[-i\vec{\nabla} + \vec{A}(\vec{r})]^2}{2M} + \mu_B B(\vec{r}) + W + V_o + \frac{g_2}{2}|\varphi|^2. \tag{3}$$

The gauge potential induced by the adiabatic approximation is $\vec{A}(\vec{r}) = -i\xi^{B\dagger}(-1, \vec{r})\nabla\xi^B(-1, \vec{r})$ and $W = [|\nabla\xi^{B\dagger}(-1, \vec{r})\nabla\xi^B(-1, \vec{r})| - \vec{A}\cdot\vec{A}]/(2M)$. The trap potential $\mu_B B(\vec{r})$ is augmented by $V_o(\rho, z)$ from other sources, e.g., an optical potential with rotational symmetry along the z axis. Under the approximation of contact pseudopotentials, trapped atoms collide in the same spin aligned state. Thus the g_2 term is proportional to the scattering length a_2 in the total spin channel $F_{\rm tot} = 2$.

We show below that the gauge potential $\vec{A}(\vec{r})$ constrains the values of J_z for a condensate ground state. In cylindrical coordinate (ρ, ϕ, z), the B field of a QT is expressed as $\vec{B}(\vec{r}) = B'(\rho\hat{e}_\rho - 2z\hat{e}_z)$. An optical potential $V_o(\rho, z)$ is introduced to push atoms away from the region of small $B(\vec{r})$, eliminating the deadly Majorona transitions [7,8]. Because this B field is cylindrically symmetric, $\vec{F}\cdot\hat{n}(\vec{r})$ commutes with the z component of the total atomic angular momentum $J_z = L_z + F_z$. This allows us to choose $\xi^B(-1, \vec{r})$ as the common eigenstate of $\vec{F}\cdot\hat{n}(\vec{r})$ and J_z with any possible eigenvalues. In this study we choose $\xi^B(-1, \vec{r})$ for convenience to satisfy

$$J_z\xi^B(-1, \vec{r}) = 0. \tag{4}$$

This constraint on J_z limits the respective values for L_z or F_z, in a sense equivalent to a gauge choice. The actual value of J_z for the ground state of a condensate is determined by appropriate system parameters. Explicitly, a simple rotation gives $\xi^B(-1, \vec{r}) = \exp[i\phi]\times\exp[-i\vec{F}\cdot\hat{e}_\phi\beta(\rho, z)]\xi_z(-1)$. The angle between the z axis and $\hat{n}(\vec{r})$ is $\beta(\rho, z) = \arccos[-2z/\sqrt{\rho^2 + 4z^2}]$.

The B-quantized ground state scalar mean field wave function is denoted as $\varphi_g(\vec{r})$, determined from a minimization of $E_{\rm ad}[\varphi]$. The cylindrical symmetry of both $\vec{B}(\vec{r})$ and V_o assures $E_{\rm ad}[\varphi(\rho, \phi, z)] = E_{\rm ad}[\varphi(\rho, \phi + \theta, z)]$ for any θ. Because of this SO(2) symmetry, if there exists only one normalized ground state (up to global phase factors) as in the situation considered here for a condensate, it has to be a common eigenstate for all rotation operators $\exp[\theta\partial/(\partial\phi)]$, i.e., an eigenstate of $i\partial/(\partial\phi)$. Thus, we take $\varphi_g(\vec{r}) = \tilde{\varphi}_g(\rho, z)\exp(is\phi)$. On substituting into Eq. (2), we obtain the z-quantized mean field $\langle\hat{\psi}(\vec{r})\rangle_g$ for the ground state

$$\langle\hat{\psi}(\vec{r})\rangle_g = \sqrt{N}\xi^B(-1, \vec{r})\tilde{\varphi}_g(\rho, z)\exp(is\phi), \tag{5}$$

which is an eigenstate of J_z with an eigenvalue s, i.e.,

$$J_z\langle\hat{\psi}(\vec{r})\rangle_g = s\langle\hat{\psi}(\vec{r})\rangle_g, \tag{6}$$

because of Eq. (4).

Equation (5) and the expansion $\langle\hat{\psi}(\vec{r})\rangle_g = \sum_{m=0,\pm1}\langle\hat{\psi}_m^{(z)}(\vec{r})\rangle_g\xi_z(m)$ gives

$$\langle\hat{\psi}_m^{(z)}\rangle_g = \sqrt{N}\xi_m^B(-1, \vec{r})\varphi_g(\vec{r}) = b_m(\rho, z)\tilde{\varphi}_g(\rho, z)e^{i(s-m)\phi}, \tag{7}$$

with $b_m = \xi_z^\dagger(m)e^{-iF_y\beta(\rho,z)}\xi_z(-1)$. The individual spin components of state Eq. (7) naturally carry topological windings as a direct result of the conservation of J_z.

To determine the value of s or J_z for the ground state, we first compute the gauge potential $\vec{A}(\vec{r})$ for a QT. With the expression of $\xi^B(-1, \vec{r})$, we find $\vec{A}(\vec{r}) = \cos\beta(\rho, z)\hat{e}_\phi/\rho$. Using the expressions for $\vec{A}(\vec{r})$, $\mathcal{E}_{\rm ad}$, and $E_{\rm ad}$, it is easy to show that for a scalar wave function $\tilde{\varphi}_g(\rho, z)\exp(im\phi)$ with any integer m, the energy functional $E_{\rm ad}$ satisfies

$$E_{\rm ad}[\tilde{\varphi}_g(\rho, z)e^{im\phi}] = E_{\rm ad}[\tilde{\varphi}_g] + \Delta E_m[\tilde{\varphi}_g], \tag{8}$$

with $E_m[\tilde{\varphi}_g] = \int d\vec{r}|\tilde{\varphi}_g^*(\rho, z)|^2[(m^2 + 4m\cos\beta)/(2M\rho^2)]$. In addition to the centrifugal term proportional to m^2, a term linear in m appears due to the $\vec{A}\cdot\nabla$ term in $\mathcal{E}_{\rm ad}$.

In the following we show that the above linear term is important for the value s, which we determine with a variational approach. Because $E_{\rm ad}$ takes its minimal value in the state $\tilde{\varphi}_g\exp[is\phi]$, we have $E_{\rm ad}[\tilde{\varphi}_g e^{is\phi}] \le E_{\rm ad}[\tilde{\varphi}_g e^{i(s\pm1)\phi}]$. Together with Eq. (8), we find the necessary condition satisfied by s: $\Delta E_s[\tilde{\varphi}_g] \le \Delta E_{s\pm1}[\tilde{\varphi}_g]$ or $|s + C| \le 1/2$, where the coefficient C is defined as

$$C = \frac{\int d\vec{r}|\tilde{\varphi}_g^*(\rho, z)|^2[\cos\beta(\rho, z)/\rho^2]}{\int d\vec{r}|\tilde{\varphi}_g^*(\rho, z)|^2[1/\rho^2]}.$$

When the correlation between $\cos\beta(\rho, z)$ and ρ^{-2} is neglected, the factor is approximated by $C \approx \int d\vec{r}|\tilde{\varphi}_g^*(\rho, z)|^2\cos\beta(\rho, z)$, i.e., the expectation value of $\cos\beta(\rho, z)$ for state $\tilde{\varphi}_g$. Since $|C| < 1$, we have $|s + C| \ge |s| - 1$, which is the same as $s \in [-1, 1]$. Without the gauge potential $\vec{A}(\vec{r})$, we would have $\Delta E_m[\tilde{\varphi}_g] = m^2\int d\vec{r}|\tilde{\varphi}_g^*(\rho, z)|^2(2M\rho^2)^{-1} \ge 0$. Thus the value of s definitely would be zero. Then the appearance of a nonzero valued s arises due to the induced gauge potential.

We now generalize our result to atoms with an arbitrary F and inside any axially symmetric B fields. Analogously we can prove that the value s of J_z in the ground state satisfies the necessary condition

$$|s - \eta_F FC| \le 1/2, \tag{9}$$

and $s \in [-F, F]$ with $\eta_F = \mathrm{sign}(g_F)$. The result of $s \in [-F, F]$ and the conservation of J_z is independent of the form of the atomic interaction potential. Although its strength g_2 does affect the wave function shape, thus can influence the value of s through the factor C.

The condition Eq. (9) also allows for a rough estimate of L_z. A straightforward calculation gives $\langle L_z \rangle = s - \eta_F F \int d\vec{r} |\tilde{\varphi}_g^*(\rho, z)|^2 \cos\beta(\rho, z)$ for the spinor mean field $\langle \hat{\psi}(\vec{r}) \rangle_g$. Neglecting the correlation between ρ^{-2} and $\cos\beta(\rho, z)$ as before, the value of $\langle L_z \rangle$ becomes approximately $s - \eta_F FC$, which lies always in the region $[-1/2, 1/2]$ according to Eq. (9). Therefore, the value $\langle L_z \rangle$, or the weighted average of the winding numbers, is generally a small number, despite the winding number $s - m$ itself, for the component $\langle \hat{\psi}_m^{(z)}(\vec{r}) \rangle_g$, may take any integer in the region $[-2F, 2F]$. We find $\langle F_z \rangle = \eta_F F \int d\vec{r} |\tilde{\varphi}_g^*(\rho, z)|^2 \cos\beta(\rho, z)$ from the expression of $\langle L_z \rangle$, a qualitative reflection that atomic hyperfine spin is aligned ($g_F > 0$) or antialigned ($g_F < 0$) with respect to the local B field.

Our result above allows for the direct creation of vortex states in a quadrupole trapped atomic condensate. For example, assume a spin-1 condensate in a QT plus an "optical plug" [8] satisfies $V_o(\rho, z) = V_o(\rho, -z)$, then we find $C = 0$ and $s = 0$ due to the spatial reflection symmetry about the $x - y$ plane. The ground state components $\langle \hat{\psi}_{\pm 1}^{(z)}(\vec{r}) \rangle_g$ then automatically carry persistent currents with winding numbers ∓ 1 according to Eq. (7). In addition, the low-field seeking atoms are trapped near the x-y plane at $z = 0$ because $|B(\vec{r})|$ is an increasing function of z. The populations for the three z-quantized states, determined by $\xi_0^B(-1, \vec{r})$ and $\xi_{\pm 1}^B(-1, \vec{r})$, are of the same order of magnitudes. Therefore, when a ground state condensate in the "plugged" QT is created, its ± 1 components $\langle \hat{\psi}_{\pm 1}^{(z)}(\vec{r}) \rangle_g$ are single quantized vortex states and can be directly resolved with a B field as used in Ref. [9].

The qualitative example above is confirmed by the numerical solution for a condensate of 5×10^6 ^{23}Na atoms in a quadrupole plus a plug trap. We take $B' = 22$ Gauss/cm and $V_o = U_o \exp[-\rho^2/\sigma^2]$ with $U_o = (2\pi) 8 \times 10^4$ Hz and $\sigma = 7.4$ μm. The ground state distribution $p_i = \int d\vec{r} |\langle \hat{\psi}_i^{(z)}(\vec{r}) \rangle|^2$ is found to be $p_{\pm 1} = 27.2\%$ and $p_0 = 45.6\%$. The phase and density distributions for the three components $\langle \hat{\psi}_{0,\pm 1}^{(z)}(\vec{r}) \rangle_g$ are shown in Fig. 1(a) and 1(b).

We also can expand the ground state $\langle \hat{\psi}(\vec{r}) \rangle_g$ in terms of the eigenstates $\xi_x(m)$ of F_x with eigenvalues m: $\langle \hat{\psi}(\vec{r}) \rangle_g = \sum_{m=0,\pm 1} \sqrt{N} \langle \hat{\psi}_m^{(x)}(\vec{r}) \rangle_g \xi_x(m)$. We then immediately note that $\langle \hat{\psi}_m^{(x)}(\vec{r}) \rangle_g$ is a superposition of vortex states with definite winding numbers 0 or ± 1, e.g., $\langle \hat{\psi}_0^{(x)}(\vec{r}) \rangle_g = (\sqrt{N}/\sqrt{2})[b_{-1}(\rho, z) e^{i\phi} - b_1(\rho, z) e^{-i\phi}]$. The density distribution of $\langle \hat{\psi}_{0,\pm 1}^{(x)}(\vec{r}) \rangle_g$ as shown in Fig. 1(c) clearly illustrates the interference pattern along the $\hat{e}_\phi$ direction. As is demonstrated in Fig. 1(c), the middle panel for $|\langle \hat{\psi}_0^{(x)}(\vec{r}) \rangle|^2$ clearly displays the double peak structure along the azimuthal direction, arising from the interference of the terms proportional to $e^{i \pm \phi}$. Thus, if a B field is used to separate the components $\langle \hat{\psi}_m^{(x)}(\vec{r}) \rangle_g$, a superposition of vortices with different winding numbers would be obtained.

We now extend our result for an axially symmetric magnetic trap to the widely used IPT whose B field possesses a different symmetry. In the region near the z axis, $\vec{B}(\vec{r}) = B'[\cos(2\phi)\hat{e}_\rho - \sin(2\phi)\hat{e}_\phi + h\hat{e}_z]$, the angle $\beta(\rho, z)$ between the local B field and the z axis satisfies $\cos\beta(\rho, z) = h/\sqrt{\rho^2 + h^2}$. In this case J_z is no longer conserved due to the lack of the SO(2) symmetry. However, we find that D_z is now conserved because it commutes with $\vec{F} \cdot \vec{B}(\vec{r})$. Therefore, we can select the low-field seeking hyperfine spin state $\xi^B(\eta_F F, \vec{r})$ as the eigenstate of D_z with an eigenvalue $-\eta_F F$, the same spin state as used in [5], again defined through a rotation $\xi^B(\eta_F F, \vec{r}) = \exp[-i\vec{F} \cdot \hat{n}_\perp \beta] \xi_z(\eta_F F)$. For $h > 0$, we find the induced gauge potential becomes $\vec{A}(\vec{r}) = -\eta_F F(1 - \cos\beta(\rho, z))\hat{e}_\varphi/\rho$.

Adopting the same notation as before, we denote the ground state spinor mean field wave function as $\langle \hat{\psi}(\vec{r}) \rangle_g = \sqrt{N} \varphi_g(\vec{r}) \xi^B(\eta_F F, \vec{r})$. Interestingly, we find $E_{\mathrm{ad}}[\varphi(\rho, \phi, z)] = E_{\mathrm{ad}}[\varphi(\rho, \phi + \theta, z)]$ remains satisfied, and the ground state condensate scalar wave function takes the form $\varphi_g = \tilde{\varphi}_g(\rho, z) \exp(iu\phi)$. Therefore, $\langle \hat{\psi}(\vec{r}) \rangle_g$ is the eigenstate of D_z with an eigenvalue $d = u - \eta_F F$, and its

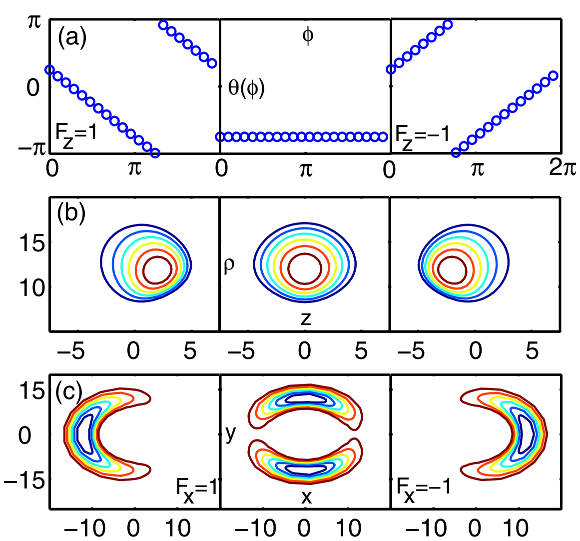

FIG. 1 (color online). (a) The phases of the z-quantized components $\langle \hat{\psi}_1^{(z)} \rangle$ (left), $\langle \hat{\psi}_0^{(z)} \rangle$ (middle), and $\langle \hat{\psi}_{-1}^{(z)} \rangle$ (right panel), as functions of the azimuth angle ϕ; (b) the density distributions $|\langle \hat{\psi}_1^{(z)} \rangle|^2$ (left), $|\langle \hat{\psi}_0^{(z)} \rangle|^2$ (middle), and $|\langle \hat{\psi}_{-1}^{(z)} \rangle|^2$ (right panel) of the z-quantized components as functions of ρ and z; (c) the integrated density distributions $\int |\langle \hat{\psi}_1^{(x)} \rangle|^2 dz$ (left), $\int |\langle \hat{\psi}_0^{(x)} \rangle|^2 dz$ (middle), and $\int |\langle \hat{\psi}_{-1}^{(x)} \rangle|^2 dz$ (right panel) of the x-quantized components as functions of x and y. The units for x, y, ρ, and z in (b) and (c) are all arbitrary.

components $\langle \hat{\psi}_m^{(z)}(\vec{r}) \rangle_g = \sqrt{N} b'_m(\rho, z) e^{i(m+u-\eta_F F)\phi}$ analogously carry a persistent current with a winding number $m + u - \eta_F F$. Here $b'_m = \xi_z^\dagger(m) e^{-iF_y \beta} \xi_z(\eta_F F)$. This result is consistent with the ground state vortex phase diagram for an $F = 1$ condensate found numerically in the $z = 0$ plane of an IPT [10]. The conservation of D_z as found by us, however, calls for a simpler labeling of each vortex phase because only one of three integers (m_1, m_0, m_{-1}) is independent, as with Eq. (15) of Ref. [10].

Following the same reasoning as before, we find

$$|d + \eta_F F(1 - C)| \leq 1/2, \tag{10}$$

for $d \neq \eta_F F$, and $d \in [-F, F]$ or the value of D_z in the ground state lies in the region $[-F, F]$.

In an IPT, atoms are trapped near the z axis where the B field is essentially along the z-axis direction and $\xi^B(\eta_F F, \vec{r})$ is approximately the eigenstate $\xi^z(\eta_F F)$. L_z then is essentially always zero corresponding to a ground state without a vortex. The angular momentum difference D_z then becomes $d = -\eta_F F$.

Several previous proposals [11] and experiments [12] on creating vortex states unknowingly have used the fact that $\langle \hat{\psi}(\vec{r}) \rangle_g$ in an IPT is an eigenstate of D_z. For example, in the experiment of Ref. [12], spin-1 atoms initially were prepared in the internal state $\xi^z(-1)$ ($F_z = -1$) with no vorticity in its spatial mode. This corresponds to $D_z = 1$. To create a vortex state, the bias field along the z axis was adiabatically inverted from the $+z$ to the $-z$ direction. In this process the atomic internal state was changed from $\xi^z(-1)$ to $\xi^z(1)$. If the whole operation is adiabatic, the commutator $[\vec{F} \cdot \vec{B}(\vec{r}, t), D_z] = 0$ will be maintained. Consequently, D_z is conserved. In the end, when the internal state was changed to $\xi^z(1)$ ($F_z = 1$), its orbital angular momentum became $L_z = D_z + F_z = 2$ or a double vortex spatial mode as was observed [12].

In Ref. [9] when the bias field is adiabatically switched off, the direction of the B field adiabatically changes to lie in the x-y plane. D_z is again conserved during this process ($=1$). When complete, the atomic state is changed from $\xi^z(-1)$ to a superposition of all three components $\xi^z(0, \pm 1)$. Then different F_z components are associated with vortex states with corresponding $L_z = D_z + F_z$. When the three internal states are separated, two of them are observed to contain vortices with nonzero winding numbers.

Creating a ground state condensate with $D_z \neq -\eta_F F$ is quite challenging inside an IPT. This was considered quantitatively by Ho and Shenoy [5], who obtained an approximate gauge potential resembling an effective trap rotation when expanded to the first order of ρ. However, their expansion easily fails away from the z axis. A numerical discussion on this challenge is provided in [10]. The more general constraint of $D_z \neq -\eta_F F$, i.e., the necessary condition Eq. (10) of the angular momentum difference D_z, was not obtained in [5,10]. From a direct calculation, it can be proved that if C is approximated by $\int d\vec{r} |\tilde{\varphi}_g^*(\rho, z)|^2 \cos\beta(\rho, z)$ and $d \neq \eta_F F$, we have $\langle L_z \rangle = d + \eta_F F \int d\vec{r} |\tilde{\varphi}_g^*(\rho, z)|^2 [1 - \cos\beta(\rho, z)]$, which can never be greater than $1/2$ according to Eq. (10).

In summary, we have investigated the angular momentum of a magnetically trapped condensate. Inside an axially symmetric trap such as a QT, the total angular momentum J_z along the symmetric z axis is found to be conserved, while the angular momentum difference D_z is conserved in an IPT. Both conservation laws reflect the underlying symmetries of the traps' magnetic fields, and the values of J_z or D_z in the ground states are determined by the gauge potential $\vec{A}(\vec{r})$. In the global ground state, the corresponding eigenvalues of J_z and D_z are limited to $\in [-F, F]$ with the precise values directly related to the angle between the local B field and the z axis.

Our results provide significant insights into the study of magnetically trapped condensates. The conservation laws we discuss reveal an important observational consequence: in the QT or IPT [10], the components $\langle \hat{\psi}_m^{(z)}(\vec{r}) \rangle_g$ of a condensate ground state automatically carry persistent currents with different winding numbers. Furthermore, according to the conditions Eqs. (9) and (10), the values of J_z or D_z, or the winding numbers of the spatial wave functions, can be controlled through the angle β between the local B field and the z axis. We have shown for a condensate in a QT with an optical plug, where the atomic populations for the $2F + 1$ components have approximately the same order of magnitude. Therefore, vortex states can be present already in a ground state condensate without requiring adiabatic operations as in [9,12].

We thank Dr. D. L. Zhou, Dr. Bo Sun, and Professors C. Raman and W. Ketterle for helpful discussions. This work is supported by NSF, NASA, and NSFC.

[1] T. Bergeman, G. Erez, and H. J. Metcalf, Phys. Rev. A **35**, 1535 (1987).
[2] E. L. Raab *et al.*, Phys. Rev. Lett. **59**, 2631 (1987).
[3] D. E. Pritchard, Phys. Rev. Lett. **51**, 1336 (1983).
[4] C. A. Mead and D. G. Truhlar, J. Chem. Phys. **70**, 2284 (1979); C. A. Mead, Phys. Rev. Lett. **59**, 161 (1987); C. P. Sun and M. L. Ge, Phys. Rev. D **41**, 1349 (1990).
[5] T. L. Ho and V. B. Shenoy, Phys. Rev. Lett. **77**, 2595 (1996).
[6] T. H. Bergeman *et al.*, J. Opt. Soc. Am. B **6**, 2249 (1989).
[7] K. B. Davis *et al.*, Phys. Rev. Lett. **75**, 3969 (1995).
[8] D. S. Naik and C. Raman, Phys. Rev. A **71**, 033617 (2005).
[9] A. E. Leanhardt *et al.*, Phys. Rev. Lett. **90**, 140403 (2003).
[10] E. N. Bulgakov and A. F. Sadreev, Phys. Rev. Lett. **90**, 200401 (2003).
[11] T. Isoshima *et al.*, Phys. Rev. A **61**, 063610 (2000).
[12] A. E. Leanhardt *et al.*, Phys. Rev. Lett. **89**, 190403 (2002).