

Factorization of the dephasing process in a quantum open system

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The fluctuation-dissipation relation is well known for a quantum open system with energy dissipation. In this paper a similar underlying relation is found between the bath fluctuation and the dephasing of the quantum open system, for which energy is conserved, but the information is leaking into the bath. To obtain this relation we revisit the universal, but simple dephasing model with a quantum nondemolition interaction between the bath and the open system. Then we show that the decoherence factor describing the dephasing process is factorized into two parts, to indicate the two sources of dephasing, the vacuum quantum fluctuation, and the thermal excitations defined in the initial state of finite temperature.

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I. INTRODUCTION

A realistic quantum system can rarely be isolated from its surrounding environment (or called the “bath”) completely [1,2]. When it is coupled to the bath with a large number of degrees of freedom, decoherence happens. There are usually two distinct decoherence effects of the bath on the quantum system: when the energy exchange is allowed by the interaction between the open system and the bath, the system energy usually dissipates into the environment irreversibly and we name this effect quantum dissipation [3–5]; another effect is called quantum dephasing, which occurs with no energy exchange, but an irreversible process of information loss happens in the considered open system [6–8].

As to quantum dissipation, the well-known fluctuation-dissipation relation [4,9–11] reveals to what extent the bath fluctuation depends on the quantum dissipation process of the open system, or how the phenomenological damping rate is determined microscopically by the random couplings to the degrees of freedom of the bath. As for quantum dephasing, however, a similar statement for the quantum decoherence only with dephasing and without loss of energy does not exist.

This paper is devoted to finding the intrinsic relation between the pure dephasing and some random nature of the bath. We begin with a general dephasing model for an open system interacting with a bath of many harmonic oscillators via a coupling of quantum non-demolition [12]. This model is known to be universal in weak coupling limit [3]. Here, we characterize the dynamic process of dephasing with the so-called decoherence factor, which linearly accompanies the off-diagonal elements of time evolution of the reduced density matrix for the system obtained by tracing over the variables of the bath. We find that the decoherence factor is a product of two factors, one of which is determined by the excitations of the bath while another does not vanish for the

bath initially in vacuum state. This observation clearly indicates that there exist two sources of quantum dephasing, which originate from both the vacuum quantum fluctuation and the thermal excitations of the heat bath, respectively.

The paper is organized as follows. In Sec. II, we describe a universal model for quantum open system interacting with a bath of many bosons through nondemolition couplings. In Sec. III, we show that a factorization structure appears in the dynamic dephasing process of the quantum open system. In Sec. IV, we present the central result of this paper, i.e., the relation between dephasing and the fluctuation of the bath. In Sec. V, we study the dephasing of the quantum open system while the bath is in the thermal equilibrium state. Finally, in the last section we conclude this paper with the remarks about the relationship between the quantum dephasing and the generalized thermalization due to the entanglements of the system with the bath.

II. BOSON BATH FOR DEPHASING

In general, the quantum dephasing process is microscopically considered as the vanishing of off-diagonal elements of the time evolution of reduced density matrix of an open system interacting with its surrounding environment or bath of infinite degrees of freedom. The simplest model is the composite system consisting of a system interacting with the bath of infinite bosons [3].

The model Hamiltonian

$$H = H_S + H_I + H_B$$

is decomposed into three parts, the system part H_S , the bath part $H_B = \sum_j \hbar \omega_j a_j^\dagger a_j$ with the bosonic creation (annihilation) operators a_j^\dagger (a_j) for the mode of frequencies ω_j ($j=1,2,\dots$) and the interaction between the quantum open system and the bath [13]

$$H_I = \hbar G \sum_j (\xi_j a_j + \text{H.c.}), \quad (1)$$

where we assume that $\hbar \xi_j$ has the dimension of energy. The operator-volumed coupling G depends on the system

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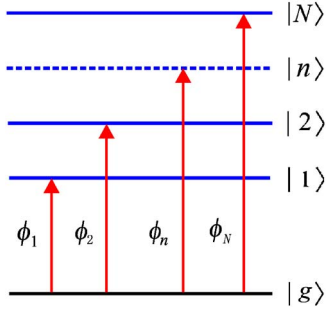


FIG. 1. (Color online) The energy level configuration for the quantum open system defined within the subspace of excited states $|n\rangle$ spanned by the transition operators $\phi_n=|g\rangle\langle n|$ from the ground state $|g\rangle$.

variables. To conserve the energy of the open system, if dephasing can happen, the coupling is required to be of quantum nondemolition, i.e., $[H_S, H_I]=0$ and $[H_B, H_I] \neq 0$, or $[H_S, G]=0$. In general the considered system has N energy levels with eigenstates $|n\rangle$ and corresponding eigenvalues Ω_n ($n=1, 2, \dots, N$) [14], i.e.,

$$H_S = \sum_{n=1}^N \hbar \Omega_n |n\rangle\langle n|, \quad (1)$$

$$G = \sum_{n=1}^N g_n |n\rangle\langle n|, \quad (2)$$

where g_n is dimensionless since $\hbar G \xi_j$ has the same dimension of energy as that of $\hbar \xi_j$. The mode number N may be infinite for a macroscopic heat bath. We assume that the system can be embedded into a $(N+1)$ -dimensional space with an extended basis vector $|g\rangle$ such that $|n\rangle = \phi_n^\dagger |g\rangle$ (see Fig. 1), where the transition operators $\phi_n = |g\rangle\langle n|$ has commutation relations with the projection operators $p_n = |n\rangle\langle n|$

$$[\phi_n, p_m] = \delta_{mn} \phi_n. \quad (3)$$

A typical example of this model is that the open system is a single mode boson with Hamiltonian $H_S = \hbar \omega_0 b^\dagger b$ defined by the bosonic creation (annihilation) operator b^\dagger (b) for the mode of frequency ω_0 . The interaction $H_I = \hbar b^\dagger b \sum_j (\xi_j a_j + \text{H.c.})$ between the boson and the bath [15] was frequently used to give a quantum approach for the measurement of the boson state. It originates from the generic oscillator coupling of the system coordinate q to the bath variables x_j , i.e.,

$$\sum_j \xi_j q x_j \sim \sum_j [\xi_j b^\dagger a_j + \xi_j b a_j^\dagger + \text{H.c.}] \quad (4)$$

in a large detuning limit $|\omega_j - \omega_0| \gg \xi_j$.

We can exactly solve the Heisenberg equation for the operators ϕ_n and a_j

$$\dot{\phi}_n = -i\Omega_n \phi_n - iX g_n \phi_n, \quad (5)$$

$$\dot{a}_j = -i\omega_j a_j - i\xi_j^* G$$

with the quantum noise operator

$$X = \sum_j (\xi_j a_j + \text{H.c.}).$$

Here, the noise operator satisfies the Brownian conditions in $\langle X(t) \rangle = 0$ and $\langle X(t')X(t) \rangle \neq 0$ in an equilibrium state ρ and the thermodynamic average $\langle \dots \rangle$ is defined as $\langle A \rangle = \text{Tr}(\rho A)$.

Since G is conservable or $G(t) = G(0) = G_0$, we explicitly obtain

$$\phi_n(t) = \phi_n(0) e^{-i\Omega_n Z(t) + i g_n F(t) G_0} e^{-i\Omega_n t},$$

$$a_j(t) = e^{-i\omega_j t} a_j(0) - i \xi_j^* \eta_j(t) G_0, \quad (6)$$

where the phase operator in $b(t)$ reads

$$Z(t) = \sum_j [\xi_j \eta_j(t) a_j(0) + \text{H.c.}]$$

and $\eta_j(t) = i(e^{-i\omega_j t} - 1)/\omega_j$. Then the noise operator can be expressed as a linear combination of initial operators $a_j(0)$, $a_j^\dagger(0)$ and G_0 , i.e.,

$$X(t) = Y(t) - G_0 \dot{F}(t), \quad (7)$$

where the “ q -number” term

$$Y(t) = \sum_j [\xi_j e^{-i\omega_j t} a_j(0) + \text{H.c.}] \quad (8)$$

is due to the free evolution of the reservoir modes, and the c -number term

$$F(t) = 2 \int \frac{d\omega}{\omega} J(\omega) \left(t - \frac{\sin \omega t}{\omega} \right) \quad (9)$$

can be regarded as a time dependent “force” and defined by the spectral density function of the bath

$$J(\omega) = \sum_j |\xi_j|^2 \delta(\omega - \omega_j). \quad (10)$$

This arises from the back action of the system on the bath.

III. FACTORIZING DEPHASING PROCESS

To demonstrate the dynamic process of quantum dephasing of the open system, we now calculate the reduced density matrix for the time evolution of the open system. A pure dephasing process means that the off-diagonal elements of the reduced density matrix of the open system vanish, while the diagonal elements remain unchanged in such an ideal case. Usually the off-diagonal elements depend on both the initial state and the dynamic variables of the bath and can vanish in the thermodynamic limits.

We assume that the initial state of the composite system is of the factorization form

$$|\psi(0)\rangle = \sum_n c_n |n\rangle \otimes |\{m_j\}\rangle.$$

This means that the initial state of the open system is a superposition of the eigenstates $|n\rangle = \phi_n^\dagger |g\rangle$ while the bath is initially in Fock state $|\{m_j\}\rangle = \prod_j |m_j\rangle$. Here m_j stands for the

excitation number of Fock state $|m_j\rangle$. Let $|0\rangle = |g\rangle \otimes |\{0_j\}\rangle$, which is invariant under the operation of the evolution operator $U(t) = \exp(-iHt/\hbar)$, i.e., $U(t)|0\rangle = |0\rangle$.

Then, according to the explicit expressions (6) for operators ϕ_n and a_j , the time evolution of the composite system is calculated with a similar method in Ref. [16] as

$$\begin{aligned} |\psi(t)\rangle &= U(t)|\psi(0)\rangle = \sum_n c_n [U(t)\phi_n^\dagger U^\dagger(t)]U(t)|g\rangle \otimes |\{m_j\}\rangle \\ &= \sum_n c_n \phi_n^\dagger(-t)|g\rangle \otimes |\{m_j\}\rangle e^{-i\sum_j m_j \omega_j t} \\ &= \sum_n c_n(t)|n\rangle \otimes e^{ig_n Z(-t)} |\{m_j\}\rangle \end{aligned}$$

or

$$|\psi(t)\rangle = \sum_n c_n(t)|n\rangle \otimes |\beta_n\rangle. \quad (11)$$

Here,

$$c_n(t) = c_n e^{-i\Omega_n t} e^{ig_n^2 F(t)} e^{-i\sum_j m_j \omega_j t}$$

and the coherent state

$$|\beta_n\rangle = \prod_j D_j(g_n \alpha_j) |m_j\rangle \quad (12)$$

is defined by the displacement operators $D_j(\alpha) = \exp(\alpha a_j^\dagger - \alpha^* a_j)$ with

$$\alpha_j = -i\xi_j^* \eta_j(t) = \frac{\xi_j^*}{\omega_j} (e^{-i\omega_j t} - 1).$$

From the time evolution of density matrix $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$ for the composite system, we calculate the reduced density matrix of the open system

$$\rho_s(t) = \sum_n c_n c_n^* |n\rangle\langle n| + \sum_{n \neq m} c_n c_m^* e^{i\theta_{mn}(t)} \langle\beta_m|\beta_n\rangle |n\rangle\langle m| \quad (13)$$

by tracing over the variables of the bath. Here

$$\theta_{mn}(t) = (\Omega_m - \Omega_n)t + (g_n^2 - g_m^2)F(t)$$

is the time dependent real number.

To characterize the coherence of the quantum open system, we use the decoherence factor [13]

$$D_{n,m}(t) = \langle\beta_m|\beta_n\rangle \equiv \prod_j D_{mn}[m_j]. \quad (14)$$

Initially, the decoherence factor $D_{n,m}(0) = 1$, i.e., the system owns a completely ideal quantum coherence. Here, each of two factors in the decoherence factor

$$D_{mn}[m_j] = \langle m_j | D_j((g_n - g_m)\alpha_j) | m_j \rangle$$

is calculated as

$$D_{mn}[m_j] = e^{(-1/2)z_{nm;j}(t)} L_{m_j}[z_{nm;j}(t)] \quad (15)$$

in terms of the Laguerre polynomial $L_{m_j}[z_{nm;j}(t)]$ of variable

$$z_{nm;j}(t) = (g_n - g_m)^2 |\xi_j \eta_j(t)|^2.$$

With the above results, we can observe that the decoherence factor can be decomposed into two parts, i.e.,

$$D_{n,m}(t) = D_{n,m}^{(0)}(t) D_{n,m}^{\{\{m_j\}\}}(t), \quad (16)$$

where

$$D_{n,m}^{(0)}(t) = \prod_j e^{(-1/2)z_{nm;j}(t)}$$

originates from the vacuum quantum fluctuation of the bath and

$$D_{n,m}^{\{\{m_j\}\}}(t) = \prod_j L_{m_j}[z_{nm;j}(t)] \quad (17)$$

means the bath excitation. Actually, when the bath is initially in the vacuum state $|\{m_j=0\}\rangle$, the second factor $D_{n,m}^{\{\{m_j\}\}}(t)$ becomes a unity operator since $L_{m_j=0}(x) = 1$. The decoherence factor $D_{n,m}(t) = D_{n,m}^{(0)}(t)$ means that the dephasing only results from the vacuum fluctuation of the bath without thermal excitation. When the initial state of the bath ($|\{m_j\}\rangle$) is occupied by a large amount of excitation, the macroscopic feature of the bath (high excitations) can induce the dephasing of the open system. This is because $L_{m_j}(x)$ approaches the zero-order Bessel function $J_0(x)$ when $m_j \rightarrow \infty$ [17]. Then

$$D_{mn}[m_j] = e^{(-1/2)z_{nm;j}(t)} J_0[z_{nm;j}(t)]. \quad (18)$$

Since the Bessel function with real variables is a decaying oscillating function, the decoherence factor $D_{n,m}(t)$ (16) approaches zero when t tends to infinity. We will discuss how a thermal equilibrium state can induce the fast dephasing. The above arguments show that, even though the system energy is conserved, there still exist the two sources of the pure quantum dephasing induced by the bath, i.e., the vacuum quantum fluctuation and the thermal excitations, which leak the information of the system into the bath.

We can explicitly demonstrate these results in the continuous limit. As an illustration we take Ohmic spectral density of the bath [3,19]

$$J(\omega) = \gamma \omega e^{-\omega\Gamma} \quad (19)$$

to make the sum in

$$D_{n,m}^{(0)}(t) = e^{-(g_n - g_m)^2 \sum_j |\xi_j \eta_j(t)|^2 / 2} \quad (20)$$

as an integral

$$\sum_j |\xi_j \eta_j(t)|^2 = \int d\omega J(\omega) \frac{4}{\omega^2} \sin^2 \frac{\omega t}{2}. \quad (21)$$

Here, γ is a dimensionless coupling constant, and Γ the bath's response frequency. When the initial state of the bath is in vacuum state, the decoherence factor in Eq. (16) becomes

$$D_{n,m}(t) = (1 + \Gamma^2 t^2)^{-1/2} (g_n - g_m)^2 \gamma. \quad (22)$$

Figure 2 shows that the decoherence factor (22), the decay of which is similar to an exponential decay. Actually, at short time limit, $\Gamma^2 t^2 \ll 1$, the decoherence factor (22) becomes Gaussian decaying function as

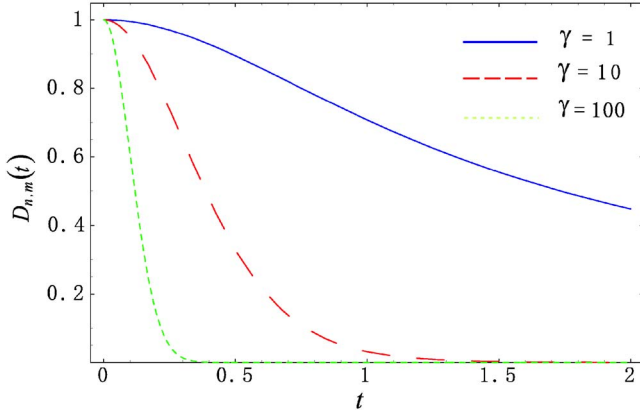


FIG. 2. (Color online) Plot of $D_{n,m}(t)$ (decoherence factor) (22) vs t (time) for various value of the coupling parameter γ . $|g_m - g_n| = 1$ and $\Gamma = 1$.

$$D_{n,m}(t) = e^{-(t/\tau)^2}. \quad (23)$$

Here the characteristic time τ of the dephasing of the open system is defined by

$$\tau^{-1} = (g_n - g_m)\Gamma \sqrt{\frac{\gamma}{2}}.$$

IV. DEPHASING-FLUCTUATION RELATION

Now we take another approach to calculate the decoherence factor, which will offer us a new angle to understand the source of dephasing. Here, we adopt the notation of the standard deviation $\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$ for a given operator A . We write down the decoherence factor

$$D_{n,m}(t) = \langle \{I_j\} | e^{i\phi(t)} | \{I_j\} \rangle \quad (24)$$

in terms of the phase difference operator

$$\phi(t) = (g_n - g_m)Z(-t)$$

for the bath initially with a factorized state $|\{I_j\}\rangle = \prod_j |I_j\rangle$. We denote the average $\langle \{I_j\} | A | \{I_j\} \rangle$ by $\langle A \rangle$.

For the small variation $|\phi(t) - \langle \phi(t) \rangle|$ of the phase $\phi(t)$ around its average $\langle \phi(t) \rangle$ over any state we have approximately [18]

$$D_{n,m}(t) = \langle e^{i\phi(t)} \rangle \approx e^{i\langle \phi(t) \rangle} e^{(-1/2)[\Delta\phi(t)]^2}. \quad (25)$$

It shows that the phase fluctuation $\Delta\phi(t)$ results in the loss of quantum coherence or quantum dephasing characterized by the decaying of the decoherence factor. For the initial Fock state with $|I_j\rangle = |m_j\rangle$ we discussed in the last section, $\langle \phi(t) \rangle = 0$ and the phase fluctuation due to the bath fluctuation can be separated explicitly into two parts, i.e.,

$$[\Delta\phi(t)]^2 = [\Delta\phi(t)]_0^2 + [\Delta\phi(t)]_f^2, \quad (26)$$

where the vacuum fluctuation part is

$$[\Delta\phi(t)]_0^2 = \sum_j z_{nm;j}(t) \quad (27)$$

and the bath excitation part

$$[\Delta\phi(t)]_f^2 = \sum_j 2m_j z_{nm;j}(t). \quad (28)$$

Actually the result is the same as what we have obtained above in some cases. This is because the couplings are linear with respect to the operators a_j and a_j^\dagger . Through some simple calculations, we obtain the decoherence factor as

$$D_{n,m}(t) = e^{(-1/2)[\Delta\phi(t)]_0^2} e^{(-1/2)[\Delta\phi(t)]_f^2} \equiv D_{n,m}^{(0)}(t) D_{n,m}^{(f)}(t), \quad (29)$$

where the thermal excitation part can be rewritten as

$$D_{n,m}^{(f)}(t) = \prod_j e^{-f_j(m_j)} = \prod_j e^{-m_j \tilde{r}_{nm;j}(t)}. \quad (30)$$

When the bath is initially in vacuum state $|\{m_j\}\rangle = |\{m_j=0\}\rangle$, $f_j(m_j) = 0$. Then the decoherence factor becomes $D_{n,m}(t) = D_{n,m}^{(0)}(t)$. Otherwise we have $f_j(m_j) > 0$, it means that the decoherence factor in Eq. (29) will decay with time.

Next we consider the difference of the decoherence factors obtained through different approaches in Eqs. (16) and (29), respectively. When m_j and $|(g_n - g_m)\xi_j|/\omega_j$ are all small, i.e., the bath is setup in a low-excitation state and coupling is weak, the Laguerre polynomial in Eq. (17) can be approximately as

$$L_{m_j}(x) \approx e^{-m_j x}. \quad (31)$$

Then the two decoherence factor in Eqs. (16) and (29) has the same form. We consider the time evolution of the average excitation number of the bath, i.e.,

$$N_B(t) = \sum_j \langle a_j^\dagger(t) a_j(t) \rangle = N_B(0) + \delta N_B(t) \quad (32)$$

where the initial average excitation number of the bath is $N_B(0) = \sum_j m_j$ and

$$\delta N_B(t) = \langle G_0^2 \rangle \sum_j |\xi_j \eta_j(t)|^2 = \langle G_0^2 \rangle \int d\omega J(\omega) \frac{4}{\omega^2} \sin^2 \frac{\omega t}{2} \quad (33)$$

is the quantum fluctuation of the bath excitation numbers, which is independent of the initial state of the bath. In the above equation, the term $\langle G_0^2 \rangle$ indicates the average of the square of the population number of the open system. This means

$$\sum_j |\xi_j \eta_j(t)|^2 = \frac{\delta N_B(t)}{\langle G_0^2 \rangle}. \quad (34)$$

Then the decoherence factor (16) can be rewritten in terms of the fluctuation of excitation number

$$D_{n,m}(t) \propto \exp \left[-\frac{1}{2} (g_n - g_m)^2 \frac{\delta N_B(t)}{\langle G_0^2 \rangle} \right]. \quad (35)$$

This equation shows that the decoherence factor is determined by the quantum fluctuation of bath $\delta N_B(t)$ and $\langle G_0^2 \rangle$. When the initial state of the open system is given, the value of $\langle G_0^2 \rangle$ is fixed in the mean while. That is to say, $\langle G_0^2 \rangle$ is not relevant to the source of dephasing of the open system, but

the energy of the open system will contribute to the dephasing. Thus we conclude that the quantum fluctuation of the bath excitation characterized by $\delta N_B(t)$ induces the dephasing of the open system.

V. DEPHASING IN THERMAL EQUILIBRIUM STATE

When the bath is prepared in a pure state we have found that the dephasing-fluctuation relation for quantum dephasing is similar to the well-known fluctuation-dissipation relation for quantum dissipation. Correspondingly the dephasing process can be understood separately according to the quantum fluctuation and the pure state excitation of the bath. In this section we will show that these observations also hold exactly for the case that the bath is initially in a thermal equilibrium state.

For the dephasing problem of the open system at finite temperature, the thermal equilibrium bath is described by the density matrix

$$\rho_B = \prod_j \frac{e^{-\beta \hbar \omega_j a_j^\dagger a_j}}{\text{Tr}(e^{-\beta \hbar \omega_j a_j^\dagger a_j})}. \quad (36)$$

Initially, the density operator of the composite system is a direct product

$$\rho(0) = |\psi(0)\rangle\langle\psi(0)| \otimes \rho_B$$

with the initial state $|\psi(0)\rangle = \sum_n c_n |n\rangle$ of the open system. Using the coherent-state representation, the density operator can be rewritten as

$$\rho_B = \prod_j \int d^2 \lambda_j \rho(\lambda_j) |\lambda_j\rangle\langle\lambda_j| \quad (37)$$

with the P representation for diagonal elements

$$\rho(\lambda_j) = \frac{1}{\pi \langle m_j \rangle} e^{-|\lambda_j|^2 / \langle m_j \rangle}, \quad (38)$$

where $\langle m_j \rangle = (e^{\beta \hbar m_j \omega_j} - 1)^{-1}$ is the average excitation number in the mode of the frequency ω_j .

By applying the results obtained in the previous sections, we can calculate the reduced density matrix

$$\rho_s(t) = \sum_n c_n c_n^* |n\rangle\langle n| + \sum_{n \neq m} c_n(t) c_m^*(t) |n\rangle\langle m| D_{n,m}^{[T]}(t) \quad (39)$$

to characterize the quantum coherence of the open system. Then the decoherence factor in the above equation is factorized as [17]

$$\begin{aligned} D_{n,m}^{[T]}(t) &= \prod_j \int d^2 \lambda_j \frac{e^{-|\lambda_j|^2 / \langle m_j \rangle}}{\pi \langle m_j \rangle} \langle \lambda_j | D_j(\alpha_{jmn}) | \lambda_j \rangle \\ &= D_{n,m}^{(0)}(t) \prod_j e^{-g_j(T)}, \end{aligned} \quad (40)$$

where $\alpha_{jmn} = (g_n - g_m) \alpha_j$ and

$$g_j(T) = \frac{z_{nm;j}}{e^{\beta \hbar \omega_j} - 1}. \quad (41)$$

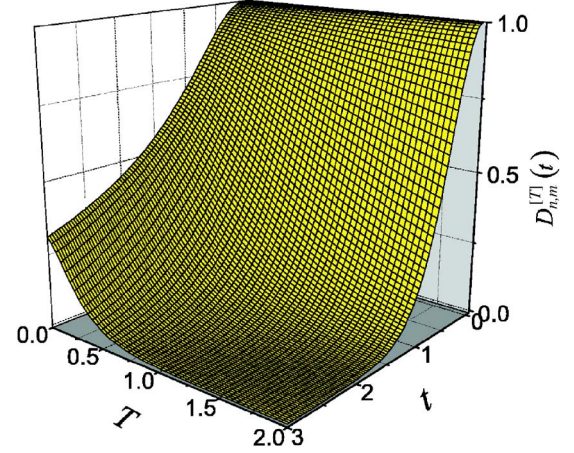


FIG. 3. (Color online) Plot of $D_{n,m}^{[T]}(t)$ (decoherence factor) (40) vs t (time) and T (temperature). $|g_m - g_n| = 1$, $\gamma = 1$, and $\Gamma = 1$.

Figure 3 shows that the decoherence factor $D_{n,m}^{[T]}(t)$ depends on the time t and the temperature T of the bath. When the temperature T increases, the decaying decoherence factor will be enhanced, i.e., the open system loses its quantum coherence.

When the temperature approaches the absolute zero degree ($\beta \rightarrow \infty$), we approximately have $(e^{\beta \hbar \omega_j} - 1)^{-1} \approx e^{-\beta \hbar \omega_j}$. Then the decoherence factor (40) becomes

$$D_{n,m}^{[T]}(t) = D_{n,m}^{(0)}(t) \exp\left(-\frac{(g_n - g_m)^2 \gamma t^2}{\hbar^2} k_B^2 T^2\right). \quad (42)$$

It shows that, in the low-temperature limit, the above decoherence factor exponentially decays as the square of temperature T^2 increases, which is called Gaussian decay. When the bath is prepared in the high temperature ($\beta \hbar \omega_j \rightarrow 0$), we have $e^{\beta \hbar \omega_j} - 1 \approx \beta \hbar \omega_j$, and then

$$g_j(T) = (g_n - g_m)^2 \frac{|\xi_j \eta_j(t)|^2}{\beta \hbar \omega_j} \propto T. \quad (43)$$

It means that, when the bath approaches the high-temperature limit, the decoherence factor (40) exponentially decays as the temperature T increases.

In addition, Fig. 4 shows that the rise of temperature will accelerate dephasing described by Eq. (40).

VI. CONCLUSION WITH REMARKS

In summary, through a universal model for a quantum open system with nondemolition coupling to the bath, we find the intrinsic relation between the dephasing of the open system and the quantum and thermal fluctuation of its bath. Usually, the coupling of a quantum open system to its environment is very complicated, and intuitively the dephasing process should depend on the details of system-bath couplings. So generally the present model used in this paper seems to be oversimplified. However, we would like to mention the study of Caldeira and Leggett about quantum dissipation [3], which proves that any bath with weak couplings to the open system can be modeled as a collection

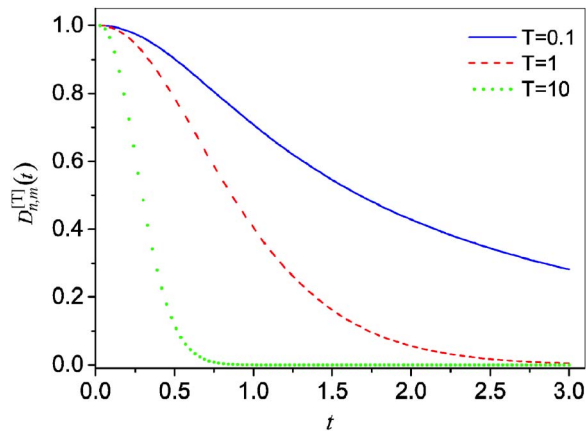


FIG. 4. (Color online) Plot of $D_{n,m}^{[T]}(t)$ (decoherence factor) (40) vs t (time) for various values of temperature T . $|g_m - g_n| = 1$, $\gamma = 1$ and $\Gamma = 1$.

of noninteracting bosons. As for the quantum dephasing, we even found a similar conclusion to universally consider the quantum dephasing problem in quantum computing [20]. Thus the present investigation can also be considered to be universal, and the dephasing process is generally clarified as the two kinds of origins, the quantum fluctuation in the vacuum and the thermal excitation in finite temperature.

Before concluding this paper, it is necessary to say something about the repletion and difference between the dynamic dephasing process studied in this paper and the thermalization to the thermal equilibrium induced by the bath [21–25]. First, we have to note that the dephasing process in this paper can describe the thermal equilibrium with the canonical state in a straightforward way. Most recently people have

revisited the investigation to explore the possibility of replacing the equal *a priori* probability postulate in statistical mechanics by a general canonical principle [21]: an arbitrary entangling pure state of the total system consisting of the “small” system plus “large bath” can be traced over the variables of the bath to give a generalized canonical state due to the “large number law” or overwhelming majority of wave functions in the subspace by obeying some global constraint for the “universe.” Actually, if this global constraint is particularly determined by the energy interval encompassed by the microcanonical ensemble, the above mentioned “tracing” operation results in the thermal equilibrium distribution. Such a thermalization process can also be described as a typical quantum dissipation phenomenon, a dynamically quantum process with energy exchange between the system and the bath [2,26]. However, there is no energy exchange in our present dephasing model and thus it gives rise to a mixture rather than the thermal equilibrium distribution. Maybe another type of global constraint exists other than the energy interval. We believe that it is an open question, which could be solved in a general framework. We will continue the exploration along this line in future studies.

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