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Analytic treatment of high-order adiabatic approximations of two-neutrino oscillations in matter

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In this paper the high-order adiabatic approximation method proposed by the author is used to approach the nonadiabatic effects of two-neutrino oscillations in matter. We not only obtain the analytic expressions for high-order approximate solutions of the neutrino-oscillations equation but also give nonadiabatic corrections to the probability of the adiabatic transition between two neutrinos in matter with arbitrarily varying density.

About three years ago, the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism for the enhancement of neutrino oscillations in matter was suggested to solve the solar-neutrino puzzle.[1] In the MSW mechanism, there is an effective potential visible only to electron neutrinos, which is proportional to the electron density in matter and gives an additional contribution to the mass-squared value. It results in a resonance oscillation of electron neutrinos to muon neutrinos.

All the discussions about this mechanism working in practice depend on the solutions of the neutrino-oscillation equation[1-3]

$$i\frac{\partial}{\partial t}\begin{bmatrix}\nu_e(t)\\ \nu_\mu(t)\end{bmatrix}=\frac{1}{2E}\begin{bmatrix} m_1^2\cos^2\theta+m_2^2\sin^2\theta+2\sqrt{2}GEN(t) & \frac{1}{2}(m_2^2-m_1^2)\sin2\theta \\ \frac{1}{2}(m_2^2-m_1^2)\sin2\theta & m_1^2\sin^2\theta+m_2^2\cos^2\theta\end{bmatrix}\begin{bmatrix}\nu_e(t)\\ \nu_\mu(t)\end{bmatrix} \tag{1}$$

in the charge-current basis $|\nu_e\rangle$, $|\nu_\mu\rangle$, where G is the weak coupling constant, $N(t)$ the electron density in matter, E the energy of a neutrino, and θ the vacuum mixing angle.

Under the adiabatic condition that the variation of $N(t)$ over an oscillation length is very small, a number of authors have performed analytic treatments of Eq. (1) (Refs. 4–8). In particular, the exact solutions of (1) were obtained (i) in the case of linearly varying density by Haxton[9] and Petcov[10] and (ii) in the case of exponentially varying density by Kaneko,[11] Pizzochero,[6] Toshev,[12] and Petcov.[7]

However, we will pay attention to the nonadiabatic case with arbitrarily varying density. Because (1) is a Schrödinger-type equation, the high-order adiabatic approximation method[13,14] suggested in studying Berry's phase by the author is naturally used to approach it.

Following the discussion in Refs. 13 and 14 we first solve the eigenequation of the matrix in (1), obtaining the eigenvalues

$$\lambda_{1,2}(t)=\frac{1}{4E}(2\sqrt{2}GEN(T)+m_1^2+m_2^2 \mp\{[2\sqrt{2}GN(t)E-(m_2^2-m_1^2)\cos2\theta]^2 +(m_2^2-m_1^2)^2\sin^2\theta\}^{1/2}) \tag{2}$$

and the eigenstates

$$|\nu_1(t)\rangle=\begin{bmatrix}\cos\theta(t)\\ -\sin\theta(t)\end{bmatrix},\quad |\nu_2(t)\rangle=\begin{bmatrix}\sin\theta(t)\\ \cos\theta(t)\end{bmatrix}, \tag{3}$$

respectively, where $\theta(t)$ is determined by

$$\sin^2 2\theta(t)=(m_2^2-m_1^2)^2\sin^2 2\theta/\{[2\sqrt{2}GEN(t)-(m_2^2-m_1^2)\cos2\theta]^2+(m_2^2-m_1^2)^2\sin^2\theta\}\ . \tag{4}$$

Letting

$$|\psi(t)\rangle=\sum_{i=1}^{2}C_i(t)\exp\left[-i\int_0^t\lambda_i(t')dt'\right]|\mu_i(t)\rangle \tag{5}$$

and substituting it into (1), we obtain

$$\dot C_k(t)=(-1)^k C_j(t)\dot\theta(t)\exp[(-1)^k 2i\alpha(t)]\ ,\quad j,k=1,2;\ j\neq k\ , \tag{6}$$

where

$$\alpha(t)=\tfrac{1}{2}\int_0^t[\lambda_2(t')-\lambda_1(t')]dt'\equiv\alpha(N(t))\ .$$

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Integrating (6) by parts, we have

$$C_k(t)-C_k(0)=R_k(t)-R_k(0)\ ,$$
$$R_k(t)=(-1)^{k+1}\exp[(-1)^k 2i\alpha(t)] \times \sum_{n=0}^{\infty}\left[(-1)^k\frac{i}{2\Delta(t)}\right]^{n+1}\left[\frac{\partial}{\partial t}-\frac{\dot\Delta(t)}{\Delta(t)}\right]^n \times[\dot\theta(t)C_j(t)]\ , \tag{7}$$

where $\Delta(t)=\frac{1}{2}[\lambda_2(t)-\lambda_1(t)]$. By introducing a continuous real parameter λ for each term $\sim\dot\theta(t)/\Delta(t)$, Eq. (7) can be rewritten as

$$C_k(t)-C_k(0)=\sum_{n=0}^{\infty}\left[(-1)^k\frac{i\lambda}{2}\right]^{n+1} \times\{\exp[(-1)^k 2i\alpha(t)]g_n(t)-g_n(0)\}\ ,$$
$$g_n(t)=\frac{1}{\Delta(t)^{n+1}}\left[\frac{\partial}{\partial t}-\frac{\dot\Delta(t)}{\Delta(t)}\right]^n[C_i(t)\dot\theta(t)]\ . \tag{8}$$

At the end of the calculation, we will take $\lambda=1$. This approach is similar to that in time-dependent perturbation theory of quantum mechanics. We express $C_k(t)$ as a power series in λ,

$$C_k(t)=\sum_{l=0}^{\infty}\left[(-1)^k\frac{i\lambda}{2}\right]^l C_k^{[l]}(t)\ , \tag{9}$$

and substitute it into (8), obtaining an equality between two power series in λ. For this equality to be satisfied, we have

$$C_k^{[0]}(0)=C_k(0)\ ,\quad C_k^{[m]}(t)=f_k^{[m]}(t)-f_k^{[m]}(0)\ ,$$
$$f_k^{[m]}(t)=(-1)^{k+1}\{\exp[(-1)^k 2i\alpha(t)]/\Delta(t)\} \times\sum_{l=0}^{m-1}\left[\frac{1}{\Delta(t)}\left[\frac{\partial}{\partial t}-\frac{\dot\Delta(t)}{\Delta(t)}\right]\right]^{m-l-1} \times[C_k^{[l]}(t)\dot\theta(t)]\ . \tag{10}$$

Then, we successively obtain each order adiabatic approximate solution of the neutrino-oscillation equation (1). In practice, if the conditions

$$\prod_{l=1}\left[\frac{d^l}{dt^l}N(t)\right]^{n_l}\Big/[\Delta(t)]^P\ll 1\quad\left[\sum_l ln_{l=P}\right] \tag{11}$$

are satisfied, then we can neglect all the terms $C_k^{[p']}$ $(P'\geq P)$ and the p-order adiabatic approximations of the solution of (1) are valid for the problem.

Because of (5), the probability for observing an electron neutrino after traveling a distance L in matter is

$$P(\nu_e\to\nu_e)=|\langle\nu_e|\psi(L)\rangle|^2 =|C_1|^2\cos^2\theta(L)+|C_2|^2\sin^2\theta(L) +\mathrm{Re}[C_1^*(L)C_2(L)\sin 2\theta(L)e^{-2i\alpha(t)}]\ . \tag{12}$$

Under the adiabatic condition $\dot\theta(t)/\Delta(t)\ll 1$, the first-order approximate solution $C_i(t)=C_i(0)$ $(i=1,2)$ obtained from (10) gives

$$P_{\mathrm{ad}}(\nu_e\to\nu_e)=\cos^2[\theta(0)+\theta(L)]\sin^2\alpha(L) +\cos^2[\theta(0)-\theta(L)]\cos^2\alpha(L)\ , \tag{13}$$

which was also obtained in Ref. 2.

For the electron neutrino starting at very high electron density $[\theta(0)=\pi/2$ because of $N(t=0)\to\infty]$ in the Sun, the initial conditions for (1) are

$$C_1^{[0]}(0)=0\ ,\quad C_2^{[0]}(0)=1\ ,\quad C_i^{[m]}(0)=0 \quad (i=1,2\ ;\ m=1,2,3,\ldots)\ . \tag{14}$$

According to (10), we obtain

$$C_1^{(0)}(t)=0\ ,\quad C_2^{[0]}(t)=1\ ,$$
$$C_1^{[1]}(t)=\exp[-2i\alpha(t)]\dot\theta(t)/\dot\alpha(t)\ ,\quad C_2^{[1]}(t)=0\ ,$$
$$C_1^{[2]}(t)=\exp[-2i\alpha(t)][\dot\theta(t)/\dot\alpha(t)^2-\ddot\alpha(t)\dot\theta(t)/\dot\alpha(t)^3]\ ,$$
$$C_2^{[2]}(t)=-\dot\theta(t)^2/\dot\alpha(t)^2\ ,\ \ldots\ . \tag{15}$$

Under the conditions

$$\left[\frac{d^3}{dt^3}N(t)\right]\Big/[\Delta(t)]^3\ ,\quad\left[\frac{d^2}{dt^2}N(t)\right]\dot N(t)\Big/[\Delta(t)]^3\ ,\quad[\dot N(t)]^3/[\Delta(t)]^3\ll 1\ , \tag{16}$$

the probability of seeing an electron neutrino after traveling a distance L in matter of the Sun is

$$P^{[3]}(\nu_e\to\nu_e)=\sin^2\theta(L)+\{\dot\theta^2(L)[1+\sin^2\theta(l)]+[\dot\Delta(L)\dot\theta(L)/\Delta(l)-\ddot\theta(L)]\sin^2\theta(L)\}/[4\Delta(t)^2]\ . \tag{17}$$

The last term of (14) shows a nonadiabatic effect on two-neutrino oscillations, which comes from the third-order approximate solution of (1).

In order to compare the above result with the results that have been obtained by others, we obviously calculate in two cases $N_1(t)=N_0+kt$ and $N_2(t)=N_0\exp(-t/T_0)$ by making use of (4) and (14), obtaining

$$P_{1,2}^{[3]}(\nu_e\to\nu_e)=\sin^2\theta_{1,2}(L)+\frac{1}{8\alpha_{1,2}^2(L)}\tan^2 2\theta_{1,2}(L)\{1+\sin^2\theta_{1,2}(L)[1-\cot 2\theta_{1,2}(L)+\tan 2\theta_{1,2}(L)]\}\ , \tag{18}$$

where $\theta_{1,2}(t)=\theta(N_{1,2}(t))$ and $\alpha_{1,2}(t)=\alpha(N_{1,2}(t))$, respectively.

The method used in this paper will be used to study the nonadiabatic corrections to the adiabatic transition of three-neutrino oscillations in matter.[15]

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