Analogue of cavity quantum electrodynamics for coupling between spin and a nanomechanical resonator: Dynamic squeezing and coherent manipulations

Fei Xue,[1,*] Ling Zhong,[1] Yong Li,[2,1] and C. P. Sun[1,†]
[1]*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing, 100080, China*
[2]*Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, 4056 Basel, Switzerland*
(Received 27 September 2005; revised manuscript received 7 November 2006; published 11 January 2007)

In the cavity QED analogue for the coupling system of a spin and a nanomechanical resonator, we show that, when the nanomechanical resonator is quantized, a spin-boson model for this coupling system can refer to a Jaynes-Cummings (JC) or an anti-JC model. These observations predict some quantum optical phenomena, such as dynamic squeezing in the single oscillation mode of the nanomechanical resonator. By modulating the phase of rf magnetic field, one can switch the system between the JC and anti-JC model for the detection of the single spin.

DOI: 10.1103/PhysRevB.75.033407 PACS number(s): 03.65.Ta, 76.60.−k, 74.50.+r

INTRODUCTION

Many recent experiments have exhibited the possibilities to reach GHz mechanical oscillations by a nanomechanical resonator (NAMR).[1–4] With such a high oscillating frequency at mK temperature, the NAMR can be modeled as a quantized harmonic oscillator. Then it can be used to explore the new phenomena in the quantum-classical crossover, to serve as a mechanical analogue of quantum optical devices,[5,6] and as a quantum probe that can be viewed as a quantum version of magnetic-resonance force microscopy (MRFM) for single electron spin detection.[7,8]

For conventional MRFM when the number of spins in the sample tends to approach 1 and the NAMR may reach quantum regime, a fully quantum model is expected to describe this coupling system correctly. In this paper, to treat the coherent coupling between NAMR and spin in a fully quantum way, we make use of an analogue of the Jaynes-Cummings (JC) model in cavity QED. Such an artificially engineered cavity QED system is used to demonstrate the feature of the quantized NAMR through the physical effects similar to various quantum optical phenomena. For example, in the large detuning limit that the NAMR does not exactly resonate with the spin, the virtual transition of the spin will result in the interesting squeezing effect for the NAMR. In the resonant case, the quantum dynamics of the coupling system is described by the JC or anti-JC model and thus a typical collapse-revival phenomenon can be exhibited by the mode of the NAMR.

MODEL SETUP

As illustrated in Fig. 1, a ferromagnetic particle is glued on the middle of the NAMR and exerts a gradient magnetic field on the spin in a static magnetic field B_0 pointing in the z direction. The spin is also exposed to a rotating rf magnetic field $\vec{B}_1(t)=B_1(\cos(\omega_r t+\varphi), B_1\sin(\omega_r t+\varphi), 0)$ in the x-y plane. The magnetic tip produces a dipolar magnetic field $\vec{B}_{tip}=\mu_0[3(\vec{n}\cdot\vec{m})\ \vec{n}-\vec{m})]/(4\pi r^3)$ at the position of the spin.[9] Here, μ_0 is the vacuum magnetic conductance, $\vec{n}$ is the unit vector pointing in the direction from the tip to the spin, $\vec{m}$ is the magnetic moment of the ferromagnetic particle pointing in the z direction, and r is the distance between the tip and the spin. The coordinate of the spin is set as $(0,0,d)$. For a small vibration of tip, the magnetic field at the position of the spin produced by the magnetic tip is approximately $B_{tip}(z)=(A-Gz)$ along the z direction for the small deviation z of the tip. Here, $A=\mu_0 m/(2\pi d^3)$, $G=3\mu_0 m_z/(2\pi d^4)$, and $m_z=|\vec{m}|$.

The NAMR with the magnetic tip is modeled as a harmonic oscillator with effective mass m_{eff} and frequency ω_c. We introduce creation and annihilation operators a and $a^\dagger$ by $z=\lambda(a^\dagger+a)$, where $\lambda=\sqrt{\hbar/(2m_{eff}\omega_c)}$. In the rotating reference frame where the time dependence due to $\vec{B}_1(t)$ is eliminated, the Hamiltonian of the coupling system can finally be written as

$$H=\hbar\omega_c a^\dagger a+\gamma\hbar B_1(\cos\varphi S_z+\sin\varphi S_y) + \hbar g(a^\dagger S_- + aS_+ + aS_- + a^\dagger S_+), \quad (1)$$

where γ is the gyromagnetic ratio and $g\equiv\gamma\lambda G/2$. $S_\pm\equiv S_x\pm iS_y$ are defined by Pauli matrices $(1/2)\sigma_x\to S_z$, $(1/2)\sigma_y\to S_y$, and $(1/2)\sigma_z\to -S_x$ for the spin. We have also set $B_0+A+\omega_r/\gamma=0$ with special parameters so that the model can be simplified. This is a typical spin-boson model, an analogue of the JC model in cavity QED. It can be used to

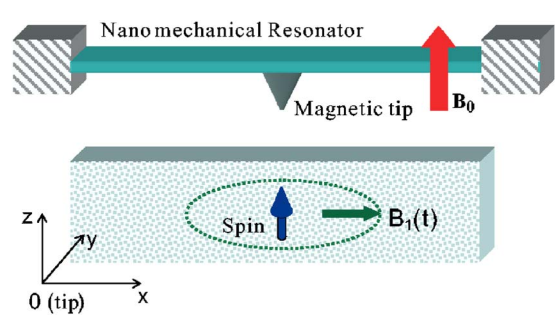

FIG. 1. (Color online) The setup of the device. A spin and an NAMR are coupled with each other by a tiny ferromagnetic particle attached to the NAMR.

demonstrate rich quantum optical phenomena on-resonance and off-resonance.

DYNAMIC SQUEEZING

We first consider the case of the off-resonance situation in which the squeezing effects of oscillations of the NAMR appear because of the interference between the two oscillation modes with respect to the two eigenstates of the spin. This may reduces the uncertainty of position or momentum below the standard quantum limit.[10,11] A similar phenomenon has been investigated for a NAMR that capacitively couples to its substrate,[12] and an NAMR that couples to a Cooper pair box.[13]

We suppose $\omega_c \ll \omega_s = \gamma B_1$, as ω_c is about 10^2 MHz with a quality factor of 10^4 in current experiments.[3,20] In the following discussion we have put $\varphi=0$ and $\hbar=1$ to display squeezing effects. In large detuning limit $g \ll \Delta \equiv |\omega_s - \omega_c|$, the Hamiltonian can be approximately written in a diagonal form $H_{diag} = H_1 \otimes |1\rangle\langle 1| + H_0 \otimes |0\rangle\langle 0|$ by adiabatically eliminating coherent effect between the spin states $|0\rangle$ and $|1\rangle$. H_k, $(k=0,1)$ reads

$$H_k \approx \omega_c a^\dagger a - g_k(a^\dagger a^\dagger + 2a^\dagger a + aa), \tag{2}$$

where $g_k = (-1)^k g^2/(4\Delta)$ and the constant terms $(-1)^k \omega_s/2 - g_k$ have been omitted.

Diagonalization of H_k gives $H_k = \Omega_k b_k^\dagger b_k + \varepsilon_k$. The Bogliubov excitation by $b_k = \mu_k a - \nu_k a^\dagger$ is defined by squeezing transformation with $\mu_k = (\sqrt{N_k} + 1/\sqrt{N_k})/2$, $\nu_k = (\sqrt{N_k} - 1/\sqrt{N_k})/2$, and $N_k = \sqrt{\omega_c \Delta / [\omega_c \Delta - (-1)^k g^2]}$. The eigenfrequency $\Omega_k \approx \omega_c[1 - (-1)^k g^2/(2\omega_c \Delta)]$ contains dispersive shift $\delta = g^2/(2\omega_c \Delta)$ due to the Lamb effect, while the corresponding excitation is shifted by $\varepsilon_k = (-1)^k[\omega_s/2 - g^2/(4\Delta)]$.

Starting from a quasiclassical state, the coherent state $|\alpha\rangle$ of mode a, at time t the NAMR will evolve into the partial wave function $|\Psi(t)\rangle_k = \exp(-i\Omega_k b_k^\dagger b_k t)|\alpha\rangle$ with respect to each spin state $|k\rangle$. To show the dynamic squeezing, $|\Psi(t)\rangle_k$ is explicitly calculated by considering the fact $B_k(t)|\Psi(t)\rangle_k = \alpha|\Psi(t)\rangle_k$ from the Heisenberg operators $B_k(t) = \mu_k(t) b_k + \nu_k(t) b_k^\dagger$ for $B_k(0) = a$, where $\mu_k(t) = \mu_k \exp(i\Omega_k t)$ and $\nu_k(t) = \nu_k \exp(i\Omega_k t)$. We further explicitly write quasiexcitation operator $B_k(t)$ in terms of the elementary photon excitation

$$B_k(t) = u_1(t) a + u_2(t) a^\dagger, \tag{3}$$

where $u_1(t) = \cos \Omega_k t + i(\mu_k^2 + \nu_k^2)\sin \Omega_k t$ and $u_2(t) = 2i\mu_k \nu_k \sin \Omega_k t$. Thus in the case of off-resonance, the time evolution of the NAMR follows the eigenstate of the quasiexcitation operator $B_k(t)$. And the eigenstate of $B_k(t)$ is found to be a squeezed state with respect to the elementary excitation of the NAMR of a and $a^\dagger$. The squeezing factor is calculated as

$$r_k(t) = \ln(2\mu_k \nu_k \sin \Omega_k t + \sqrt{1 + 4\mu_k^2 \nu_k^2 \sin^2 \Omega_k t}). \tag{4}$$

The dynamic squeezings of oscillations in the NAMR are illustrated by plotting the curves of γ_0 and γ_1 in Fig. 2(a). Depending on the spin state $|k\rangle$, the NAMR is dynamically squeezed with different frequencies Ω_k and different maximum squeezing factor $\ln N_k$. The maximum squeezing factor of the NAMR oscillation depends on g/Δ and the strength of coupling g/ω_c, which is shown in Fig. 2(b). In the plot we have assumed $\omega_c < \omega_s$. The greater squeezing occurs with the smaller detuning and stronger coupling. To guarantee the large detuning assumption, the detuning cannot be too small. However, large squeezing can still be attained with optimal coupling strengths.

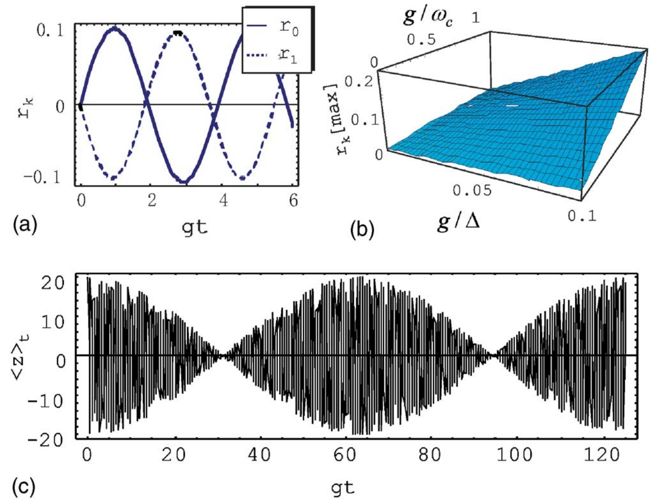

FIG. 2. (Color online) (a) The squeezing factor r_k of the oscillation of the NAMR initially in the coherent state $|\alpha\rangle$ for the different spin state $|k\rangle$. (b) The maximum squeezing factor $\ln N_k$. The detuning $g/\Delta \in [0.01, 0.1]$ and coupling strength $g/\omega_c \in [0.01, 0.99]$. (c) Oscillating of the NAMR at off-resonance situation.

In large detuning situation, this spin-boson system also exhibits "collapse-revival" phenomenon. For the NAMR initially prepared in the coherent state $|\alpha\rangle$, its position $\langle z\rangle_t$ can be calculated according to the fact that the time evolution of NAMR always follows the eigenstate of $B_k(t)$. Therefore we expand $a + a^\dagger$ in terms of $B_k(t)$ and $B_k^\dagger(t)$, and then $\langle z\rangle_t \propto \langle a + a^\dagger\rangle_t$ is obtained as

$$\langle z\rangle_t \propto \sum_k 2c_k c_k^* \operatorname{Re}\left(\alpha\left[\frac{1}{N_k}\cos \Omega_k t - i \sin \Omega_k t\right]\right). \tag{5}$$

Here, the spin is initially prepared in the state $c_0|0\rangle + c_1|1\rangle$. The oscillation of the NAMR collapses and revives, as illustrated in Fig. 2(c), is the result of the interference effect of the two oscillating mode Ω_k labeled by the spin state $|k\rangle$, which is a witness of the quantization of the NAMR.

JC AND ANTI-JC PHASES

Next we consider the situation that the spin resonates with the NAMR under the rotating-wave approximation. We show that, by modulating the phase φ of the rf magnetic field $B_1(t)$, the device can be switched between the JC model and the anti-JC model. Such a structure offers a new type of mechanical analogue of cavity QED among various solid-state cavity QED systems.[14–19]

In the following we assume $B_1 > 0$. When the phase of rf field $\varphi=0$, the Hamiltonian (1) becomes of JC type,

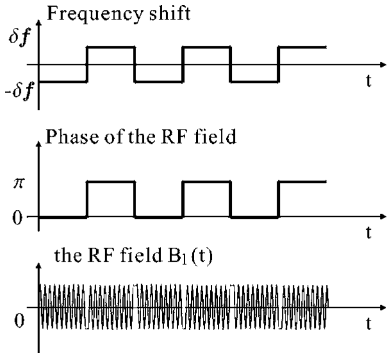

FIG. 3. Timing diagram for switching the coupling system between the JC and the anti-JC Hamiltonian. The initial state of the spin is supposed to be the ground state of the JC Hamiltonian. The frequency shift $\delta f = g^2/\Delta$ of the NAMR changes its sign when the system switches between the two Hamiltonians, while the state of the spin evolves little.

$$H_{JC} = \hbar\omega_c a^\dagger a + \hbar\omega_s S_z + \hbar g(a^\dagger S_- + aS_+), \tag{6}$$

in the rotating-wave approximation, where $\omega_s = \gamma B_1 > 0$ and the constant term $\hbar\omega_c/2$ is dropped. This can be understood in the interaction picture, in which the terms concerning aS_- and $a^\dagger S_+$ with frequencies $\pm(\omega_c+\omega_s)$ oscillates fast with higher frequency, and thus can be neglected. For this JC Hamiltonian in the large detuning limit $g/\Delta \ll 1$, where $\Delta = |\omega_c - \omega_s|$, the dressed eigenfrequency of the NAMR is shifted as $\omega_c + 2g^2\langle S\rangle/\Delta$. When the phase of the rf magnetic field $\varphi = \pi$, the Hamiltonian (1) becomes an anti-JC Hamiltonian

$$H_{AJC} = \hbar\omega_c a^\dagger a + \hbar\omega_s S_z + \hbar g(aS_- + a^\dagger S_+), \tag{7}$$

under the rotating-wave approximation. Here $\omega_s = -\gamma B_1 < 0$. This is understood in the interaction picture, in which the terms concerning aS_+ and $a^\dagger S_-$ with frequencies $\pm(\omega_c - \omega_s)$ rather than aS_- and $a^\dagger S_+$, oscillates fast with higher frequency, thus should be neglected. For this anti-JC Hamiltonian at the large detuning situation, the dressed energy level of the NAMR reads $\omega_c - 2g^2\langle S\rangle/\Delta$.

In the two situations of the JC and anti-JC phases discussed above, the ac Stark shifts have opposite signs. This motivates us to present a scheme for the detection of single spin. The trick is similar to the interrupted oscillating the cantilever-driven adiabatic reversals (iOSCAR) protocol,[7] and provides a convenient frequency shift detection without requesting the difficult measurement of the absolute frequency of the NAMR. The rf magnetic field is interrupted for half a cycle periodically as illustrated in Fig. 3(c). Then the phase of the rf magnetic field φ will swing between 0 and π correspondingly, which is illustrated in Fig. 3(b). Therefore the NAMR-spin system can switch between the JC and anti-JC models. When the switching is fast enough so that the spin only evolves little and is kept in its original state, i.e., the expectation of the spin $\langle S\rangle$ keeps its value, the frequency shift of the NAMR swings between $2g^2\langle S\rangle/\Delta$ and $-2g^2\langle S\rangle/\Delta$ periodically as the consequence of this Hamiltonian switching, which is illustrated in Fig. 3(a).

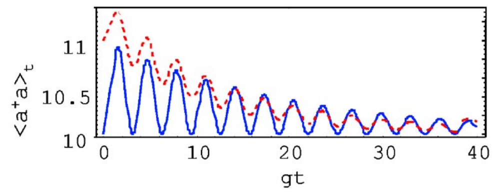

FIG. 4. (Color online) The damping of the NAMR. The energy of the NAMR $\langle a^\dagger a\rangle_t$ is plotted with the NAMR or spin in excited state under weak-coupling environment situation $\kappa/g = 0.2$. The red (dashed) line is for the NAMR initially in the excited state, and the blue (solid) line is for the spin initially in the excited state.

DAMPING OF THE NAMR

Various quantum optical phenomena due to the quantization of the NAMR can be demonstrated in principle, even for a novel scheme of single spin detection. But due to the coupling to complex environments, these phenomena may be washed out in practice. Therefore it is necessary to study the influence of environments. For a NAMR with hundreds MHz frequency and tens mK temperature feasible in current experimental technique,[20] the quality factor of the mechanical resonator is not high.[3,7] So there are many efforts to understand the relaxation of the NAMR in low frequency of the NAMR,[21–23] while the high-frequency relaxation mechanism is still not well understood. In this paper we finally focus on some aspects of this problem.

We model the environment of the NAMR as a multimode boson bath to study its decoherence problem. The Heisenberg equation about operators $(a^\dagger)^m a^n O$ $(O = S_z, S_\pm)$ leads to the Heisenberg-Langevin equations about their expectations,[24]

$$\frac{d}{dt}\langle a^\dagger a\rangle = -\kappa\langle a^\dagger a\rangle + gX + \kappa n_{th},$$

$$\frac{d}{dt}\langle S_z\rangle = -2gX,$$

$$\frac{d}{dt}X = g\langle S_z\rangle - \frac{1}{2}\kappa X + 2g\langle a^\dagger S_z a\rangle + g$$

$$\frac{d}{dt}\langle a^\dagger S_z a\rangle = -gX - \kappa\langle a^\dagger S_z a\rangle \tag{8}$$

which is closed for the "quadratic" operators such as $X = i\langle S^+ a + S^- a^\dagger\rangle$. Here the role of the bath is reflected by the average thermal photons n_{th} and the average $\langle\cdots\rangle_E$ over the environment. The above close system of equations can be solved by the Laplace transform method numerically. In Fig. 4, for weak coupling to the bath, we plot the time evolution of the expectation of the energy of the NAMR $\langle a^\dagger a\rangle$ with the initial condition $n_{th} = 10$, $\langle a^\dagger a\rangle + \langle S_z\rangle = 11$, $X = \langle a^\dagger S_z a\rangle = 0$. The case of stronger coupling to the environment, i.e., larger κ/g, is also studied. It is not surprising that the oscillation of the energy of the NAMR dies out faster with larger κ/g. With sufficient strong coupling to environment, no oscillation of the energy of the NAMR could be observed. This simple consideration suggests that, even for nonzero temperature environment, it is still possible to observe the energy oscillation between the NAMR and the spin as long as the coupling to the environment is weak enough.

EXPERIMENTAL FEASIBILITIES

Finally let us address some issues in the experiments. The frequency shifts of the NAMR depends on the coupling constant $g \sim 10^{-7} G\sqrt{\omega_c / k_{eff}}$ with $\gamma \approx 10^{10}$ and k_{eff} the effective spring constant of NAMR.[8] Using the experimental parameters,[7] $G \sim 10^5$ T/m, k_{eff}=0.11 mN/m, and ω_c=5.5 kHz, we obtain a frequency shift of about 7 Hz for the NAMR where $g/\Delta = 0.1$. The frequency shift is larger than that in the current OSCAR technique (about 4 mHz) by three orders of magnitude. Therefore when the temperature is reduced and the mechanical oscillation in the MRFM is quantized, the mechanical QED model provides an efficient protocol of single electron-spin detection. To reach such a scale of low temperature, there are some protocols[25–27] that promise to cool the temperature of the NAMR down to mK or even lower. However, reducing the temperature of the NAMR still remains a great challenge in the experiments. If one can set the frequency of the NAMR at tens MHz, then tens mK temperature is sufficient to prepare the NAMR in the ground state. And the frequency shift to be detected is about tens Hz. The required resolving power is about 10^{-6}, which is already achievable in the current experimental conditions.[15] Therefore our scheme is realizable with optimal combination of current experimental techniques.

In summary, this paper studied a mechanical analog of cavity QED structure with a quantized NAMR. We theoretically predict some analogues of quantum optical phenomena, such as the dynamic squeezing. For practices in the experiments, by considering the influence of thermal decoherence, we suggest that coherent manipulation of JC and anti-JC phases might provide a protocol for the detection of single spin.

ACKNOWLEDGMENTS

This work was supported by the NSFC with Grants No. 90203018, No. 10474104, No. 60433050, and No. 10574133 and NFRPC with Grants No. 2001CB309310 and No. 2005CB724508. We also thank Y. D. Wang for helpful discussions.

---

*Present address: Frontier Research System, The Institute of Physical and Chemical Research (RIKEN), Japan. Electronic address: Echoxue@riken.jp

†Electronic address: suncp@itp.ac.cn; URL: http://www.itp.ac.cn/~suncp

[1] X. M. H. Huang, C. A. Zorman, M. Mehregany, and M. L. Roukes, Nature (London) **421**, 496 (2003).

[2] A. N. Cleland and M. R. Geller, Phys. Rev. Lett. **93**, 070501 (2004).

[3] A. Gaidarzhy, G. Zolfagharkhani, R. L. Badzey, and P. Mohanty, Phys. Rev. Lett. **94**, 030402 (2005).

[4] A. Gaidarzhy, G. Zolfagharkhani, R. L. Badzey, and P. Mohanty, Appl. Phys. Lett. **86**, 254103 (2005).

[5] I. Bargatin and M. L. Roukes, Phys. Rev. Lett. **91**, 138302 (2003).

[6] C. P. Sun, L. F. Wei, Y. X. Liu, and F. Nori, Phys. Rev. A **73**, 022318 (2006).

[7] D. Rugar, R. Budakian, H. J. Mamin, and B. W. Chui, Nature (London) **430**, 329 (2004).

[8] G. P. Berman, F. Borgonovi, V. N. Gorshkov, and V. I. Tsifrinovich, IEEE Trans. Nanotechnol. **4**, 14 (2005).

[9] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (John Wiley & Sons, New York, 1999), p. 186.

[10] V. B. Braginsky and F. Y. Khalili, *Quantum Measurement* (Cambridge University Press, Cambridge England, 1995).

[11] M. D. LaHaye, O. Buu, B. Camarota, and K. C. Schwab, Science **304**, 74 (2004).

[12] M. P. Blencowe and M. N. Mybourne, LT 22 Proceedings, Physica B **280**, 555 (2000).

[13] Y. D. Wang, Y. B. Gao, and C. P. Sun, Eur. Phys. J. B **40**, 321 (2004).

[14] A. D. Armour, M. P. Blencowe, and K. C. Schwab, Phys. Rev. Lett. **88**, 148301 (2002).

[15] E. K. Irish and K. Schwab, Phys. Rev. B **68**, 155311 (2003).

[16] A. Blais, R. S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A **69**, 062320 (2004).

[17] A. Wallraff, D. L. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **431**, 162 (2004).

[18] I. Chiorescu, P. Bertet1, K. Semba, Y. Nakamura, C. J. P. M. Harmans, and J. E. Mooij, Nature (London) **431**, 159 (2004).

[19] Y. D. Wang, P. Zhang, D. L. Zhou, and C. P. Sun, Phys. Rev. B **70**, 224515 (2004).

[20] R. G. Knobel and A. N. Cleland, Nature (London) **424**, 291 (2003).

[21] P. Mohanty, D. A. Harrington, K. L. Ekinci, Y. T. Yang, M. J. Murphy, and M. L. Roukes, Phys. Rev. B **66**, 085416 (2002).

[22] G. P. Berman, F. Borgonovi, Hsi-Sheng Goan, S. A. Gurvitz, and V. I. Tsifrinovich, Phys. Rev. B **67**, 094425 (2003).

[23] H. Gassmann, M-S. Choi, H. Yi, and C. Bruder, Phys. Rev. B **69**, 115419 (2004).

[24] M. O. Scully and M. S Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, England, 1997).

[25] A. Hopkins, K. Jacobs, S. Habib, and K. Schwab, Phys. Rev. B **68**, 235328 (2003).

[26] C. H. Metzger and K. Karral, Nature (London) **432**, 1002 (2004).

[27] P. Zhang, Y. D. Wang, and C. P. Sun, Phys. Rev. Lett. **95**, 097204 (2005).