Quantum state swapping via a qubit network with Hubbard interactions

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We study the quantum state transfer (QST) in a class of qubit networks with on-site Coulomb interaction, which are described by the generalized Hubbard model with engineered hopping strengths. It is proved that the N-site system of two electrons with opposite spins can be rigorously reduced into N one dimensional engineered single Bloch electron models with central potential barrier. With this observation, we find that such system can perform a perfect QST, the quantum swapping between two distant electrons with opposite spins. Numerical results show that such QST and the resonant tunneling occur for the optimal on-site repulsive interaction.

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I. INTRODUCTION

Recently increasing investigations[1–9,11,12] have explored the possibilities to transfer quantum states through a class of solid-state data buses, such as the artificial spin chain with engineered nearest neighbor (NN) couplings.[1–3,5–9] Some interesting physical mechanisms have been discovered behind the protocols of quantum state transfer (QST) through the spin chain systems. For example, it is discovered that the gap structures of spectrum of the strongly correlated systems are responsible for the role of data bus;[7] and the spectrum marching parity symmetry is a sufficient condition for perfect QST.[3,6,9,10]

We also notice that most of the explorations for QST are carried out only for the noninteracting systems, and it seems that the on-site Coulomb interactions may destroy the QST protocols. In this paper, we study the influences of on-site interactions on the dynamic process of QST by making use of the generalized Hubbard model with engineered NN hopping integrals as same as that in the artificial Bloch electron model in Ref. 2.

To give prominence to our central context, we only consider the simplest interacting system with only two electrons of opposite spins. Here, the main result we achieved is the reduction of interacting model that the N-site two-electron engineered Hubbard model can be decomposed into N single-particle engineered models on l-sites chain $(l=1,3,\ldots,2N-1)$ with an additional central potential barriers (CPBs). This discovery enlightens us to explore the possibility of implementing the perfect QST, such as the quantum state swapping (QSS), since the reduced models still keep the mirror symmetry. The detailed numerical simulations demonstrate that there indeed exists such perfect QSS even certain on-site repulsion U is considered.

This paper is organized as follows. In Sec. II, we present the engineered spin model with on-site Coulomb interactions and point out that it is equivalent to a two-leg ladder of spinless Bloch electrons. Labeling sites in the two legs with the standard angular momentum bases, we then prove the two-leg network in Fig. 1(b) can be mapped into the product representations $SO(3)\otimes SO(3)$ as the direct sum of N central potential barrier model. With the reduction structure of the model in Sec. II, we show by numerical analysis in Sec. III that, when U takes an appropriate value, the effective levels can satisfy the spectrum-symmetry matching condition[9] (SSMC) approximately since the effective levels of such state was approximately shifted uniformly. Then a near-perfect QST can happen to the engineered spin model. Finally, in Sec. IV, we describe in detail a special near-perfect QST process, an approximate swap process of two electrons located on the two ends of the Hubbard chain. Based on the numerical results, we propose an experience formula for the fidelity as a function of time.

II. ENGINEERED HUBBARD MODEL AND ITS REDUCTION

Our model for QSS is the generalizations of the engineered spin model in Ref. 2 [see Fig. 1(a)] by adding the

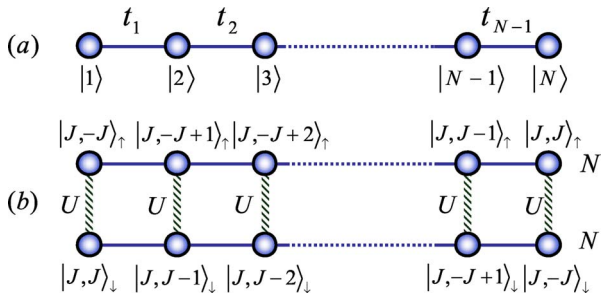

FIG. 1. (Color online) (a) The engineered Hubbard model of N sites with two electrons. (b) The two-leg ladder of spinless Bloch electrons, which is equivalent to the above Hubbard model. The rungs represent the on-site interactions. We label the sites in the two legs with the standard angular momentum bases $|JM\rangle$ in two opposite orders.

on-site Coulomb interactions. It can also be regarded as an engineered Hubbard model[13] with artificial hopping. The model Hamiltonian reads

$$H = -\sum_{j,\sigma} (t_j c^{\dagger}_{j,\sigma} c_{j+1,\sigma} + \text{H.c.}) + U \sum_{j} n_{j\uparrow} n_{j\downarrow}, \tag{1}$$

where $c^{\dagger}_{j,\sigma}$ is the creation operator of electron at site j with spin $\sigma = \uparrow, \downarrow$ and U is the on-site repulsion. The hopping integrals are engineered as[2]

$$t_j = \sqrt{j(N-j)}. \tag{2}$$

Similar type of NN coupling strength for coupled harmonic oscillator chain has also been investigated for the task of perfect entanglement transmission.[14] The usual Hubbard model has been extensively studied in connection with correlation effects in narrow-band solids and the concept of entanglement.[15] Such engineered NN couplings was first suggested in Ref. 2 for the task of QST through a quantum spin XX system with single excitation, which is equivalent to a tight binding model (TBM) with spinless Bloch electrons. If there were not the on-site repulsion, the engineered TBM with the Hamiltonian

$$H_{tbm} = -\sum_{j} (t_j c^{\dagger}_{j} c_{j+1} + \text{H.c.}), \tag{3}$$

where $c^{\dagger}_{j}(c_{j+1})$ is a creation (annihilation) operator, can become a collective spin

$$H_{tbm} = -2J_x \tag{4}$$

for such engineered NN couplings t_j. Here, the single electron state $|j\rangle = c^{\dagger}_{j}|0\rangle$ can be mapped to an angular momentum state by

$$|J,M\rangle = |j = J + M + 1\rangle \tag{5}$$

for $J=(N-1)/2$ and $M=-J,-J+1,\ldots,J-1,J$. Therefore, the time evolution operator

$$U(t) = e^{i2J_x t} \tag{6}$$

is a rotation operator, which easily rotates the state from $|J,-J\rangle = |1\rangle$ to $|J,J\rangle = |N\rangle$ at time $t=\pi/2$. This rotation completes a perfect QST. Actually, it has been shown in Refs. 2, 3, and 9 that an arbitrary many-particle state in such system can be transferred to its mirror counterpart perfectly. It is due to the fact that the energy-level structure and the parity of the corresponding eigenstate satisfy the spectrum-symmetry matching condition (SSMC) introduced in Ref. 9: let ϕ_n be the common eigenfunction of H and symmetry operator R with the eigenvalues ε_n and p_n, respectively. It is easy to find that any state ψ at time τ can evolve into its symmetrical counterpart $R\psi$ if the eigenvalues ε_n and p_n match with each other and satisfy the SSMC

$$e^{-i\varepsilon_n \tau} = p_n. \tag{7}$$

It can be imagined that the energy levels should be shifted by certain deviations from the original spectrum when the on-site repulsion is switched on. Nevertheless, there still exists the possibility that the new set of shifted energy levels satisfy the SSMC for an appropriate U since the certain symmetry remains as will be shown as follows. We first illustrate our discovery for this issue schematically in Fig. 1. Suppose that there are only two electrons with opposite spins in the engineered Hubbard model [see Fig. 1(a)]. The on-site interaction occurs only when the two electrons occupy a same site. Alternatively, the Hubbard chain is equivalent to the two-leg spinless Bloch electron ladder with N rungs [see Fig. 1(b)]. The rungs denote the on-site interaction. Each site in the leg corresponds to a single electron Bloch state $|j\rangle_{\sigma} = c^{\dagger}_{j\sigma}|0\rangle$, where $\sigma = \uparrow, \downarrow$; $j = 1,2,\ldots,N$.

According to Ref. 2, we can associate these states to the angular momentum states

$$|J,M\rangle_{\uparrow} = |J + M + 1\rangle_{\uparrow},$$

$$|J,-M\rangle_{\downarrow} = |J + M + 1\rangle_{\downarrow}, \tag{8}$$

where $J=(N-1)/2$ and $M=-J,-J+1,\ldots,J-1,J$. Then it is easy to check that for the engineered hopping integrals t_j, the lowing operator of angular momentum can be realized in terms of the fermion operators as

$$J^{(\uparrow)}_{-} = J^{(\uparrow)}_{x} - iJ^{(\uparrow)}_{y} = \sum_{j} t_j c^{\dagger}_{j,\uparrow} c_{j+1,\uparrow},$$

$$J^{(\downarrow)}_{-} = J^{(\downarrow)}_{x} - iJ^{(\downarrow)}_{y} = \sum_{j} t_j c^{\dagger}_{j+1,\downarrow} c_{j,\downarrow}, \tag{9}$$

for $\sigma = \uparrow, \downarrow$, which generates the group $SO(3)$ together with

$$J^{(\sigma)}_{+} = (J^{(\sigma)}_{-})^{\dagger},$$

$$J^{(\uparrow)}_{z} = \sum_{j} (j - 1 - J) c^{\dagger}_{j,\uparrow} c_{j,\uparrow},$$

$$J^{(\downarrow)}_{z} = \sum_{j} (J + 1 - j) c^{\dagger}_{j,\downarrow} c_{j,\downarrow}. \tag{10}$$

Then we can rewrite the Hamiltonian

$$H = -2J^{(\uparrow)}_{x} - 2J^{(\downarrow)}_{x} + V \tag{11}$$

in terms of the projection of on-site interaction

$$V = U \sum_{M} |J,M;J,-M\rangle\langle J,M;J,-M|, \tag{12}$$

where the two-particle associated state $|J,M;J,-M\rangle$ is defined by $|J,M;J,M'\rangle = |J,M\rangle_{\uparrow} \otimes |J,M'\rangle_{\downarrow}$.

The intrinsic dynamic symmetry of the above generalized model is described as $SO(3) \otimes SO(3)$. Thus the addition theorem for two angular momenta[16] can be employed to reduce the representation of this generalized Hubbard model according to the decomposition of the product representation

$$D^{[J]} \otimes D^{[J]} = \sum_{L=0}^{2J} \oplus D^{[L]}, \tag{13}$$

where $D^{[J]}$ is an irreducible representation of $SO(3)$. The key point in our treatment is to express the on-site interaction term V as the sum of irreducible tensor operators. To this end, we use the Clebsch-Gordan (CG) coefficients

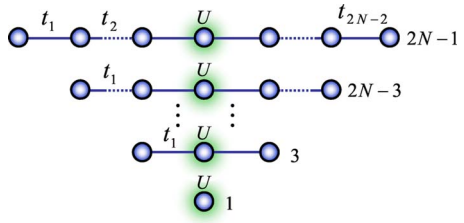

FIG. 2. (Color online) The equivalent two-leg network in Fig. 1(b) can be further reduced according to the product representations $SO(3)\otimes SO(3)$, as the direct sum of N central potential barrier model.

$$C^{LM}_{J,M_1;J,M_2} = \langle (JJ)L,M|J,M_1;J,M_2\rangle \tag{14}$$

to write the eigenvector $|L,M\rangle \equiv |(JJ)L,M\rangle$ of the total angular momentums as

$$|L,M\rangle = \sum_{M_1+M_2=M} C^{LM*}_{J,M_1;J,M_2}|J,M_1;J,M_2\rangle. \tag{15}$$

From the corresponding inverse transformation, the interaction term can be decomposed as

$$V = U\sum_{LL'}\sum_{M} C^{L0}_{J,M;J,-M}C^{L'0}_{J,M;J,-M}|L,0\rangle\langle L',0|. \tag{16}$$

By using the orthogonal relation of CG coefficients

$$\sum_{M} C^{L0}_{J,M;J,-M}C^{L'0}_{J,M;J,-M} = \delta_{LL'}, \tag{17}$$

the on-site interaction is further reduced as the sum of the irreducible tensors, i.e.,

$$V = \sum_{L} W^{[L]} = U\sum_{L} |(JJ)L,0\rangle\langle (JJ)L,0|. \tag{18}$$

Therefore, we have proved that the engineered Hubbard Hamiltonian for two electrons with opposite spins can be written as the direct sum of N irreducible sub-Hamiltonians

$$H^{(L)} = H_0^{(L)} + W^{[L]} = -2J_x + W^{[L]}. \tag{19}$$

The model described by each $H^{(L)}$ can be inversely mapped into a single spinless fermion model with a CPB, whose Hamiltonian is

$$H^{(L)} = -\sum_{j=-L}^{L-1}(t_j a_j^\dagger a_{j+1} + \text{H.c.}) + Ua_0^\dagger a_0, \tag{20}$$

where $a_j^\dagger$ is the creation operator of new fermion and

$$t_j = \sqrt{(L+j+1)(L-j)}. \tag{21}$$

As illustrated in Fig. 2, the two-particle on-site interacting qubit network is reduced into a direct sum of N single-particle models with CPB.

III. SPECTRUM-SYMMETRY MATCHING FOR NONVANISHING ON-SITE INTERACTION

To see whether the engineered Hubbard model can serve as a quantum data bus to transfer quantum information coherently, we now study the influences of the on-site interaction on the eigenstate structure. With a vanishing on-site interaction, the matrix representation of the Hamiltonian (1) in the single-particle subspace is equivalent to a high spin with angular momentum processing in transverse magnetic field. Accordingly, in the subspace of two electron with opposite spins, the Hamiltonian of $U=0$ can also be reduced into N single-particle engineered models on l-site chain $(l=1,3,\ldots,2N-1)$ without CPB. Obviously, all the eigenstates in this case meet the SSMC that guarantees a perfect QST.

It is crucial for our analysis that, when the on-site interaction U vanishes, the Hamiltonian (20) becomes $H_0^{(L)}=-2J_x$ and satisfies $[H^{(L)},R]=0$ and thus the eigenstates could be classified into two sets with different parities, where the parity operator R is a reflection operation in one dimension

$$R|J,-M\rangle = |J,M\rangle. \tag{22}$$

When U is switched on, the shifts of energy levels arising from U should determine the fidelity of QST via such system. Here, a set of the energy levels remains unchanged while other set of levels deviated by nonzero values

$$\Delta_M = |E_M(U) - E_M(0)|. \tag{23}$$

This observation can be proved exactly by a straightforward calculation for the action of $W^{[L]}$ on the eigenvectors

$$\left|L,M\left(\frac{\pi}{2}\right)\right\rangle = \exp\left(i\pi\frac{J_y}{2}\right)|L,M\rangle = \sum_{M'} d^L_{M'M}\left(\frac{\pi}{2}\right)|L,M'\rangle. \tag{24}$$

From the identity about the d function $d^L_{M'M}(\theta)$

$$d^L_{M'M}\left(\frac{\pi}{2}\right) = (-1)^{L-M}d^L_{-M'M}\left(\frac{\pi}{2}\right), \tag{25}$$

one has $d^L_{0M}(\pi/2)=0$ for odd $L-M$, and then

$$W^{[L]}\left|L,M\left(\frac{\pi}{2}\right)\right\rangle = Ud^L_{0M}\left(\frac{\pi}{2}\right)|L,0\rangle = 0. \tag{26}$$

This means that states $|L,M(\pi/2)\rangle$ (L-M is odd) are also the eigenstates of $H^{(L)}$, i.e., the corresponding levels are free of the on-site interaction.

However, another set of eigenvalues should be shifted by the on-site interaction. Imagine that if the energy deviation Δ_M is not so sensitive to M, the shifts of the levels are same approximately. There may exist an appropriate U to ensure that the final levels satisfy the SSMC with another greatest common divisor. It will result in the perfect QST in the invariant subspace

$$V^{[L]}:\{|L,M\rangle, M=-L,-L+1,\ldots,L\}. \tag{27}$$

In order to verify our conjecture, the numerical simulation is employed for small size systems. Exact diagonalization results for the energy levels as functions of U for $L=5$ system, plotted in Fig. 3(a), shows that the spectrum of $H^{(L)}$ consists of two sets of energy levels, one is independent of U, while the other is shifted by the repulsion. Furthermore,

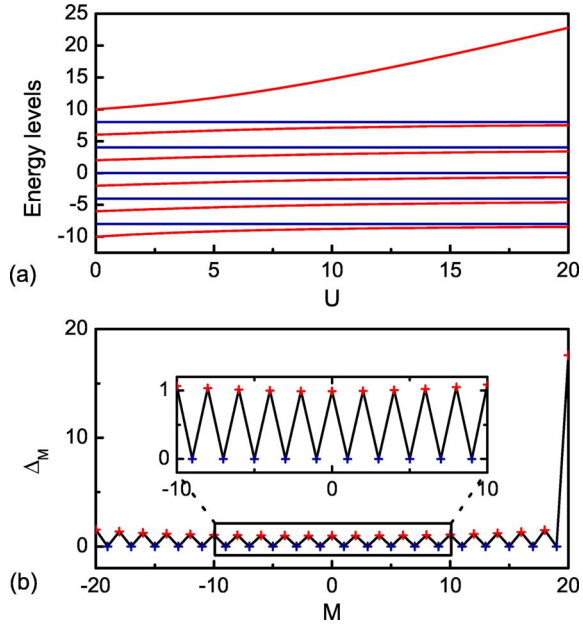

FIG. 3. (Color online) (a) Numerical simulation of the energy levels affected by U for $H^{(L)}$ with $L=5$, $U=0-20$. (b) The level shift Δ_M of $H^{(L)}$ for $L=20$ system. It shows that for small M, Δ_M are approximately uniform. Notice that for the optimal $U=40.5$, the level shifts for small M are approximately equal to the half of the level difference with $U=0$.

numerical calculation for Δ_M of $H^{(L)}$ with $U=40.5$ on 21-site lattice plotted in Fig. 3(b) also shows that, for small M, Δ_M is approximately uniform, which is partially in agreement with our conjecture. On the other hand, numerical calculations also indicate that in the invariant subspace $V^{[L]}$, the components of the state $|L,-L\rangle$ on the basis $V^{[L]}$ for small M are dominant. Based on these two aspects, we have the conclusion that the effective levels of such state should be shifted uniformly. In other words, for such kind of initial state

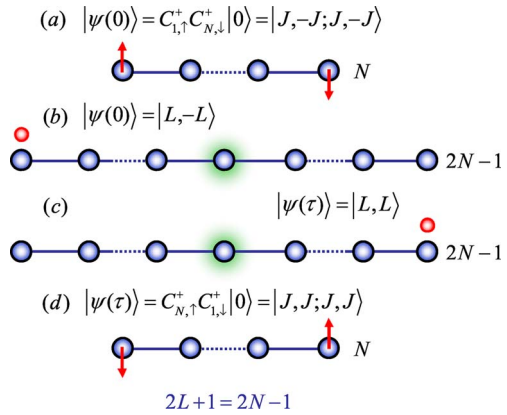

FIG. 4. (Color online) Schematic illustration for the process of swap: (a) The initial state: two electrons with opposite spins located on the two ends of the Hubbard chain with N site. (b) The mapping of the initial state (a): a single spinless particle located on one end of the $(2N-1)$-site model with central potential barrier U. (c) For optimal U, the initial state (b) can evolve to the final state into $|L,L\rangle$ near perfectly at a certain instant τ. (d) The mapping of the final state (c) to the swapped state in the Hubbard chain.

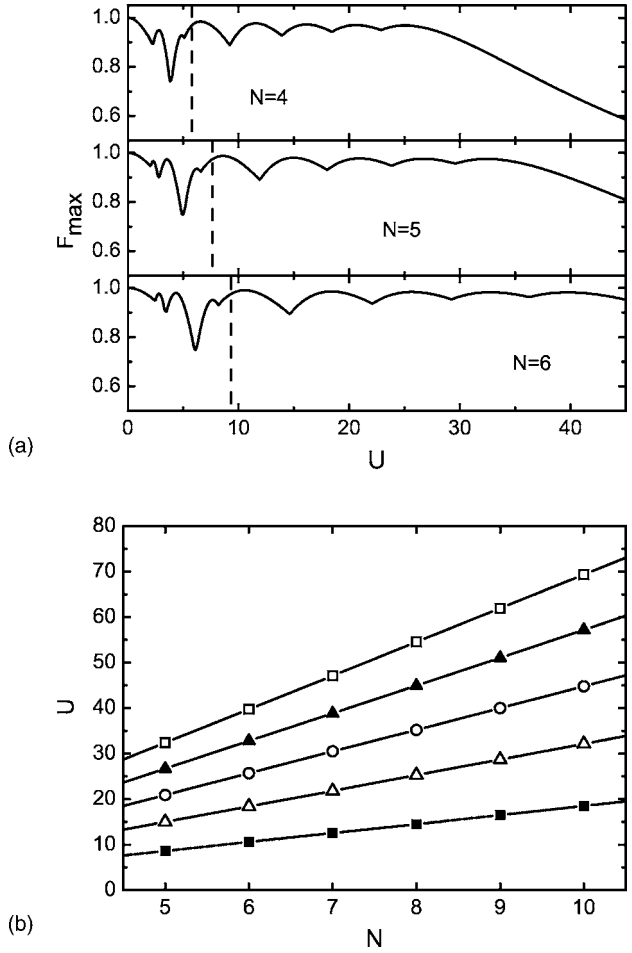

FIG. 5. (a) Numerical simulation of the maximal $F_{\max}$ as the functions of the interaction strength U for $N=4,5,6$ sites system. (b) The optimal U, first (solid square), second (triangle), third (circle), fourth (solid triangle), and fifth (square), as the functions of the sizes N obtained by the numerical simulations. It indicates that the optimal interaction strengths U are directly proportional to N approximately.

$|L,-L\rangle$, when U takes an appropriate value, the effective levels can satisfy the SSMC approximately, then it can be transferred into $|L,L\rangle$ near perfectly.

IV. NEAR-PERFECT SWAP OF TWO ELECTRONS

Now we consider the spin state swapping of two electrons located on the two ends of the Hubbard chain. According to the above analysis, for the initial state

$$|\psi(0)\rangle = c^\dagger_{1,\uparrow}c^\dagger_{N,\downarrow}|0\rangle = |J,-J;J,-J\rangle, \tag{28}$$

the quantum state swapping can be performed by the time evolution under the optimal U, i.e., at certain instance τ

$$|\psi(\tau)\rangle = c^\dagger_{N,\uparrow}c^\dagger_{1,\downarrow}|0\rangle = -|J,J;J,J\rangle. \tag{29}$$

Note that states $c^\dagger_{1,\uparrow}c^\dagger_{N,\downarrow}|0\rangle$ and $c^\dagger_{N,\uparrow}c^\dagger_{1,\downarrow}|0\rangle$ belong to a same invariant subspace $V^{[L]}$, and then the process of swapping is equivalent to the propagation of a single particle over two ends of the $(2N-1)$-site model (20). As discussed above, it

TABLE I. The maxima of fidelity $F_{\max}$, the corresponding optimal U and revival time T_r obtained by numerical simulations for the engineered Hubbard model on four-site system.

Peaks	U	$F_{\max}$	T_r
1	6.6	0.9847	$3.14 \simeq \pi$
2	11.6	0.9768	$4.71 \simeq 3\pi/2$
3	16.2	0.9724	$6.28 \simeq 2\pi$
4	20.6	0.9698	$7.85 \simeq 5\pi/3$
5	25.0	0.9683	$9.42 \simeq 3\pi$

can be approximately realized in a dynamic process since the SSMC can be approximately satisfied for the appropriate U.

The whole process involving the mapping of dynamic evolution from the initial to final states in two pictures is illustrated schematically in Fig. 4. Here, Figs. 4(a) and 4(d) represent the initial state $c^{\dagger}_{1,\uparrow}c^{\dagger}_{N,\downarrow}|0\rangle$ and the final state $c^{\dagger}_{N,\uparrow}c^{\dagger}_{1,\downarrow}|0\rangle$, respectively, in the practical picture: two electrons with opposite spins located on the two ends of the N-site Hubbard chain. Figures 4(b) and 4(c) represent the initial state $a^{\dagger}_{-L}|0\rangle$ and the final state $a^{\dagger}_{L}|0\rangle$, respectively, in the equivalent picture with TBM for the virtual spinless Bloch electron: a single spinless Bloch electron located on one end of the $(2N-1)$-site model with central potential barrier U. The perfect swapping process from (a) to (d) is equivalent to the time evolution from (b) to (c).

In order to confirm the above prediction, a numerical simulation is performed for the swapping fidelity

$$F(U,t) = |\langle J,J;J,J|e^{-iHt}|J,-J;J,-J\rangle|^2. \tag{30}$$

In the time range $t \in [0,10]$, the maxima of the fidelity

$$F_{\max}(U) = \max\{F(U,t), t \leqslant 10\} \tag{31}$$

are plotted in Fig. 5(a) as the functions of U for $N=4$, 5, and 6. One can see for the same N that $F_{\max}$ have several regular peaks on the right side of the dashed lines corresponding to different optimized U. It shows that there indeed exist some optimized U to get very high $F_{\max}(U)$, which seem to be scattering free for the propagation of electrons. For a four-site engineered Hubbard model, we list the optimized U and the corresponding revival time T_r for each regular peak in Table I. In addition, for different peaks the optimized U as functions of the sizes N are plotted in Fig. 5(b). Interestingly, they are linear functions in the concerned range.

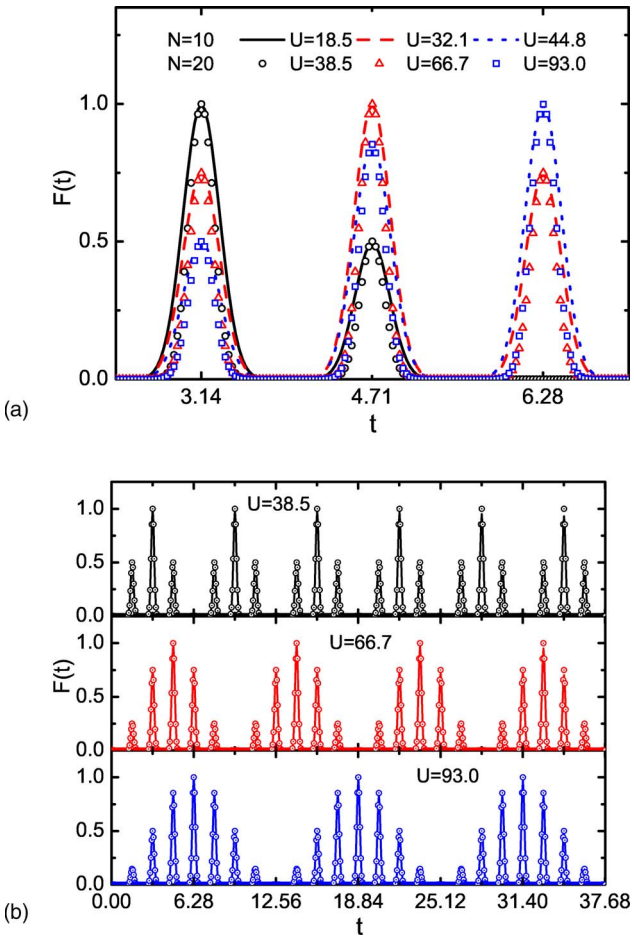

FIG. 6. (Color online) (a) Numerical simulation of the fidelities as a function of time for $N=10$ (line, dashed line, dot line) and $N=20$ (circle, triangle, square) engineered Hubbard models with the first three appropriate U, respectively. It shows that the revival times obey the experience formula $T_{rn}=0.5\pi(n+1)$ approximately in spite of the size of the system. (b) The comparison between the numerical results (line) and the experiment formula $F_n(t)$ (circle) for the swapping fidelity in the 20-site Hubbard chains with optimal U as $n=1,2,3$ in (a).

TABLE II. The maximal fidelities and the corresponding revival time T_r of the first two peaks obtained by numerical simulations for the systems with N and U.

	1st Peaks			2nd Peaks		
N	U	$F_{\max}$	T_r	U	$F_{\max}$	T_r
2	2.3	0.9999	2.72	3.6	0.9999	3.50
3	4.9	0.9926	3.18	8.0	0.9929	4.74
4	6.6	0.9847	3.14	11.6	0.9768	4.71
5	8.6	0.9873	3.14	15.0	0.9802	4.71
6	10.6	0.9906	3.14	18.4	0.9856	4.71
7	12.6	0.9931	3.14	21.8	0.9894	4.71
8	14.5	0.9948	3.14	25.2	0.9920	4.71
9	16.5	0.9960	3.14	28.7	0.9938	4.71
10	18.5	0.9968	3.14	32.1	0.9950	4.71

On the other hand, for different $N=2-10$, more detailed data for the first and second peaks are also obtained numerically to reveal the hidden relationships between them. In Table II, these numerical results are listed with the following prominent characters for the cases of $N=4-10$. First, the optimized U is linearly proportional to N with slopes approximately 2.0 and 3.4 for the first and second peak, respectively. Second, for a given optimized U, the larger N is, the higher the maximum of the fidelity $F_{\max}$ becomes. Third, the revival times are π and $3\pi/2$ for the first and second peak, approximately. Further numerical results shows the revival

times for the nth peak of $F_{\max}$ obey the experience formula

$$T_{rn} = \frac{\pi}{2}(n+1) \tag{32}$$

approximately. It implies that the possible greatest common divisors are $2/(n+1)$, which is determined by the shifts of the effective levels as U increases. In addition, it is also interesting to see how much does the maximal fidelities depend on the exact timing. In Fig. 6(a) we plot the fidelity as a function of time for engineered Hubbard models of $N=10$ and $N=20$ with the first three optimal U respectively. It shows that the functions $F(t)$ for this two systems are universal approximately, which has the features as follows: (i) $F(t)$ get the maxima around $\pi m/2$ for integer m; (ii) for nth optimal U, $F(t)$ approach to 1 around $T_{rn}=\pi(n+1)/2$ for $n=1\sim3$; (iii) the peaks of $F(t)$ are not so sharp, which provide a wild time interval of high-fidelity measurement. Furthermore, based on the numerical results, it is also found that the fidelity obeys the experience formula approximately

$$F_n(t) = e^{-4\pi^2\xi(t)^2}\sin^2\left(\frac{\pi t}{2T_{rn}}\right), \tag{33}$$

where

$$\xi(t) = \frac{2t}{\pi} - nint\left(\frac{2t}{\pi}\right), \tag{34}$$

with $nint(x)$ being the nearest integer function of x. In Fig. 6(b), the numerical results and the experience formula (33) of the peaks $n=1,2,3$ for a $N=20$ system are plotted. It shows that they are well in agreement with each other, which implies there should be a theory to explain the nearly perfect swapping process.

V. SUMMARY

In summary, we have investigated the QST in the engineered Hubbard model with on-site interaction analytically and numerically. It is proved that the engineered Hubbard model for two electrons with opposite spins can be reduced rigorously to the direct sum of N irreducible sub-Hamiltonians with various lengths, which depict a single fermion model with a CPB. When $U=0$, the perfect swapping of two electrons can be obtained due to the zero CPB. Numerical simulations show that one can also perform perfect quantum swapping approximately between two electrons with opposite spins at the ends of the engineered Hubbard chain with nonzero on-site interactions. The angular momentum reduction method could be expected to work for the QST in an engineered quantum models with more electrons.

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