Effective boson-spin model for nuclei-ensemble-based universal quantum memory

Zhi Song,[1,*] Peng Zhang,[2] Tao Shi,[1] and Chang-Pu Sun[1,2,†,‡]
[1]*Department of Physics, Nankai University, Tianjin 300071, China*
[2]*Institute of Theoretical Physics, The Chinese Academy of Science, Beijing 100080, China*
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We study the collective excitation of a macroscopic ensemble of polarized nuclei fixed in a quantum dot. Under the approximately homogeneous condition that we explicitly present in this paper, this many-particle system behaves as a single-mode boson interacting with the spin of a single conduction-band electron confined in this quantum dot. Within this effective spin-boson system, the quantum information carried by the electronic spin can be coherently transferred into the collective bosonic mode of excitation in the ensemble of nuclei. In this sense, the collective bosonic excitation can serve as a stable quantum memory to store the quantum spin information of the electron.

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I. INTRODUCTION

In the current development of quantum information science and technologies, people have devoted much effort searching for the optimal system serving as a long-lived quantum memory to store the quantum information carried by a quantum system with short decoherence time.[1] A universal quantum information storage can be understood as a physical process to encode the states of each qubit (rather than the general quantum state) into the states of the quantum memory with much longer decoherence time than the lifetime of qubit,[2] or transform the quantum information carried by a quantum system (such as photons), which is difficult to manipulate, to an easily controllable system (such as the localized atomic ensemble).[3] Such quantum information storage is absolutely necessary in both measurement-based quantum computation schemes[4,5] and two-qubit gate-based computation schemes.[6]

In the past years the collective excitation of the ensemble of atoms have been proposed to serve as quantum memory for photon information.[7] Several experiments[8–10] have already demonstrated the central principle of this scheme. These schemes work to record the Fock states of photon or their coherent superpositions. In this paper we will pay attention to the universal quantum storage (called qubit storage) that stores the basic two-level state, the state of qubit rather than a general quantum state.[2] The universality of the qubit storage lies on the fact that a general quantum state can be encoded as the state of multiqubits, and the corresponding quantum logic operations can be decomposed into the "quantum networks," which are the product of the fundamental operations defined with respect to the qubits.[6]

As usual, the foundation of a universal scheme of quantum information storage depends on whether one can discover a quantum system with very long decoherence time as the universal quantum memory. Recently a protocol for universal quantum information storage was presented based on the nanomechanical resonator interacting with charge qubits. As the universal quantum memory, the nanomechanical resonator behaves as a single-mode harmonic oscillator and its coupling to charge qubit is just described by the Jaynes-Cummings (JC) model.[11] Such spin-boson interaction forms the basis for ion-trap-based computation schemes as well.[12] These idealized schemes motivate us to seek another more practical protocol based on collective bosonic excitation in various physical systems. We note that a mesoscopic system that consists of finite nuclear spins attached in a quantum dot has been proposed to realize a long-lived quantum memory in this universal way.[13–15] The present paper will start from this basic idea and then work on the macroscopic limit that the number of polarized nuclei is very large so as to be treated approximately as infinite.

We will show that, under two independent sufficiently approximately homogeneous conditions, the collective excitation of a macroscopic ensemble of polarized nuclei fixed in a quantum dot can behave as a single bosonic mode. In this sense, confined in this quantum dot, the spin of a single conduction-band electron interacts with this collective excitation and then forms an effective spin-boson system. It demonstrates a dynamic process to coherently store the quantum information carried by the electronic spin in the collective bosonic mode of the nuclei ensemble. Then the collective excitation of the nuclei ensemble can serve as a universal quantum memory to store the quantum information of spin state of electrons.

II. BOSON REALIZATION OF COLLECTIVE EXCITATION IN THE ENSEMBLE OF POLARIZED NUCLEI

We can consider the ensembles of $N \sim 10^{3-5}$ polarized nuclei with spin I_0, which are fixed in a charged quantum dot and interact with a single conduction-band electron confined in this dot (Fig. 1). There exists a hyperfine contact interaction between the s-state conduction electron and the fixed nuclei. When a static magnetic field is applied to the dot, the effective Hamiltonian for the total system reads

$$H = \Omega_z \sigma_z + \omega_z \sum_{j=1}^{N} I_z^{(j)} + \sigma_z \sum_{j=1}^{N} g_j I_z^{(j)} + \sigma_+ \sum_{j=1}^{N} \frac{g_j}{2} I_-^{(j)} + h.c, \tag{1}$$

where the operators

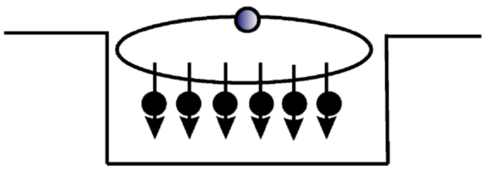

FIG. 1. Polarized nuclei interacting with an electron in a quantum dot. Because of the overlap between the electronic wave function and the nuclear wave functions, an effective spin-spin coupling between the electron and the nuclei is induced.

$$\mathbf{I}^{(i)} = (I_x^{(i)}, I_y^{(i)}, I_z^{(i)}) \tag{2}$$

and

$$\sigma = (\sigma_x, \sigma_y, \sigma_z) \tag{3}$$

describe the spins of the nucleus at ith site and the conduction electron, respectively. The coefficients Ω_z and ω_z are the Larmor precession frequencies of nucleus and electron, which are linearly determined by the applied external magnetic field. The strength g_i of the hyperfine interaction depends on the local value of the norm $|\psi(x_i)|^2$ at the position x_i of ith nucleus, whereas $\psi(x_i)$ is the wave packet of a single electron inside the dot. In this paper, we take the average of couplings as $\overline{g}=\Sigma_i g_i/N=A/N$. Here, we have also used the spin-flip operators

$$\sigma_\pm = (\sigma_x \pm i\sigma_y)/2 \tag{4}$$

and

$$I_\pm^{(j)} = (I_x^{(j)} \pm iI_y^{(j)})/2. \tag{5}$$

In the following discussion, the eigenstates of σ_z and $I_z^{(j)}$ are denoted as $|\uparrow\rangle_e$, $|\downarrow\rangle_e$, and $|m\rangle_j$, which satisfy

$$\sigma_z|\uparrow\rangle_e = |\uparrow\rangle_e, \quad \sigma_z|\downarrow\rangle_e = -|\downarrow\rangle_e \tag{6}$$

and

$$I_z^{(j)}|m\rangle_j = m|m\rangle_j, \quad m = I_0, I_0 - 1, \ldots, -I_0. \tag{7}$$

An obvious observation seen from the above expression of the Hamiltonian (1) is that all nuclei wholly couple to a single electron spin. Then we can introduce a pair of collective operators

$$B = \frac{\sum_{i=1}^N g_i I_-^{(i)}}{\sqrt{2I_0 \sum_j g_j^2}} \tag{8}$$

and its conjugate B^+ to depict the collective excitations in the ensemble of nuclei with spin I_0 from its polarized initial state

$$|G\rangle = |M = -NI_0\rangle = \prod_{i=1}^N |-I_0\rangle_i, \tag{9}$$

where M is the eigenvalue of the z component of total nuclei spin $I_z=\Sigma_{i=1}^N I_z^{(j)}$, which denotes the saturated ferromagnetic state of nuclei ensemble.

Now we can show that the collective excitations depicted by B and B^+ can behave as bosons under the "quasihomogeneity" conditions in the low-excitation limit (we will explicitly present this as follows). In fact, in the previous investigations,[3,16,17] we have proved that, if the coupling is homogeneous or in a periodic way, the collective operators B and $B^\dagger$ can indeed be considered as boson operators in the low-excitation and macroscopic limit $n/N \to 0$, where n is the number of excitation from ground state $|G\rangle$. The number n characterizes the number of collective excitations of the nuclei ensemble, which is defined through the eigenvalue $m_n=-NI_0+1/2+n$ of the z component of total spin

$$J_z = \sigma_z + I_z, \tag{10}$$

where $n=0,1,2,\ldots,2NI_0-1$. It is obvious that J_z is a "good quantum number" for the Hamiltonian (1) since $[J_z,H]=0$. Here, we do not include the saturated ferromagnetic states with $J_z=\pm\left(NI_0+\frac{1}{2}\right)$. It is noted that n is actually the number of excitations in the system, regardless of mode B or others. There is an intuitive argument that if the g_is have different values, but the distribution is "quasihomogeneous," B and $B^\dagger$ can also be considered as boson operators satisfying

$$[B,B^+] \to 1, \tag{11}$$

approximately. In the following discussion, we will provide two descriptions for the quasihomogeneity condition, under which Eq. (11) holds in the limit $n/N \to 0$.

To this end we rewrite the commutator of B and $B^\dagger$ as

$$[B,B^+] = -\frac{\sum_j g_j^2 I_z^{(j)}}{I_0 \sum_j g_j^2} \equiv 1 - F(N,n), \tag{12}$$

where

$$F(N,n) = \frac{\sum_j g_j^2 [I_z^{(j)} + I_0]}{I_0 \sum_j g_j^2}. \tag{13}$$

Since $g_j^2 \leqslant g_{\max}^2$, $F=F(N,n)$ can be estimated as

$$F \leqslant \frac{g_{\max}^2 (I_z + NI_0)}{\overline{g^2} I_0 N} \leqslant \frac{g_{\max}^2 n}{\overline{g^2} I_0 N}, \tag{14}$$

where $\overline{g^2}=\Sigma_j g_j^2/N$ is the average over set $\{g_j^2\}$. Here we have used the definition of n: $I_z+I_0N=n$ or $n+1$ with respect to the electronic spin-up or spin-down and the condition $NI_0 \gg 1$. Therefore, it is easy to see that when $g_{\max}^2/\overline{g^2} \sim 1$ in the limit $n/N \to 0$, we have $F(N,n) \to 0$ and then $[B,B^+] \simeq 1$. Based on the above argument, the first quasihomogeneity condition can be obtained as

$$\frac{g_{\max}^2}{\overline{g^2}} \sim 1. \tag{15}$$

Note that the above condition corresponds to the physically relevant case of a quantum dot since g_i is proportional to the norm $|\psi(x_i)|^2$ at the position of nuclei for the wave packet $\psi(x_i)$ of a quasifree electron moving inside the dot.

However, the above quasihomogeneity condition is not necessary and we can find another independent one as follows. By a straightforward calculation we can also reexpress $F(N,n)$ as

$$F = 1 - \frac{(I_z + I_0 N)}{I_0 N} - \frac{\sum_j I_z^{(j)}[g_j^2 - \overline{g^2}]}{I_0 N \overline{g^2}}. \tag{16}$$

Since the term $(I_z+I_0N)/(I_0N)\sim n/(I_0N)$ for $N\gg 1$ and $|\langle I_z^{(j)}\rangle|\leqslant I_0$, the upper limit of the second term on the right-hand side of Eq. (16) can be estimated as

$$\left|\frac{\sum_j I_z^{(j)}(g_j^2 - \overline{g^2})}{I_0 N \overline{g^2}}\right| \leqslant (\overline{\delta g^2})/\overline{g^2}, \tag{17}$$

in terms of the absolute value deviation

$$\overline{\delta g^2} = \frac{1}{N}\sum_j |g_j^2 - \overline{g^2}|, \tag{18}$$

of g_j^2. Therefore it is obvious that $[B,B^+]\to 1$ in the low-excitation limit $n/N\to 0$ when another quasihomogeneity condition

$$\frac{\overline{\delta g^2}}{\overline{g^2}} \to 0 \tag{19}$$

holds.

It is pointed out that, both of the two quasihomogeneity conditions (15) and (19) are sufficient conditions for the boson commutation relation $[B,B^+]=1$, but they are independent from each other, and we can obtain neither of them from the other. There are some cases in which one of the two conditions is satisfied, but another is violated. For instance, in the case with $N\sim 10^4$, if $g_1=g_2=\cdots=g_{N-1}=g$ and $g_N=10g$ then we have $\overline{g^2}=1.01g^2$, $g_{\max}^2=100g^2$, and $\overline{\delta g^2}\approx 0.02g^2$. It is apparent that the condition (15) is violated, but the condition (19) is satisfied since $g_{\max}^2/\overline{g^2}\approx 100$ and $\overline{\delta g^2}/\overline{g^2}\approx 0.02$. In another example, we take $g_1=g_2=\ldots=g_{N/2}=g$ and $g_{N/2+1}=g_{N/2+2}=\ldots=g_N=3g$, then we have $\overline{g^2}=5g^2$, $g_{\max}^2=9g^2$, and $\overline{\delta g^2}\approx 4g^2$. This is a physically relevant case when the size of the electron wave function is fixed and the nuclear spin density is increased. It indicates that the condition (19) is violated, but the condition (15) is satisfied in this case.

III. EFFECTIVE HAMILTONIAN DECORATED BY EXTERNAL MAGNETIC FIELD

As mentioned above the z component of total spin $J_z=\sigma_z+I_z$ is conserved and thus we can classify the total Hilbert space for the ensemble of polarized nuclei according to the excitation number n. In the following we denote the eigenspace of J_z with eigenvalue m_n by V_n. Then V_n can be decomposed into a direct sum of two eigenspaces

$$V_{n+} = \mathrm{Span}\{|g_k^{(n)}\rangle | k = 1,2,\ldots,\}, \tag{20}$$

where the basis vectors

$$|g_k^{(n)}\rangle \in \left\{|\uparrow\rangle \otimes |l_1 l_2,\ldots,l_N\rangle \Big| \sum_j l_j = -NI_0 + n\right\} \tag{21}$$

and

$$V_{n-} = \mathrm{Span}\{|f_k^{(n)}\rangle | k = 1,2,\ldots,\}, \tag{22}$$

where the basis vectors

$$|f_k^{(n)}\rangle \in \left\{|\downarrow\rangle \otimes |l_1 l_2 \cdots l_N\rangle \Big| \sum_j l_j = -NI_0 + n + 1\right\} \tag{23}$$

of I_z, i.e., $V_n=V_{n+}\oplus V_{n-}$. Then the Hamiltonian (1) can be decomposed into three parts in the invariant subspace V_n, namely,

$$H = H_R + H_S + H_p. \tag{24}$$

Each part H_R, H_S, and H_p can be described as follows. The first part

$$H_R = \Omega(\sigma_+ B + \sigma_- B^+) \tag{25}$$

is a resonate JC Hamiltonian with the collective Rabi frequency

$$\Omega = \sqrt{I_0 \sum_j \frac{g_j^2}{2}} \tag{26}$$

coupling the electron spin to the collective excitation. Associated with the noncollective excited states $|g_k^{(n)}\rangle$, $|f_k^{(n)}\rangle$ and the corresponding composite energies

$$\mu_g(n) = \frac{\Omega_z}{2} + \omega_z\left(m_n - \frac{1}{2}\right) - \frac{N\overline{g}I_0}{2},$$

$$\mu_f(n) = -\frac{\Omega_z}{2} + \omega_z\left(m_n + \frac{1}{2}\right) + \frac{N\overline{g}I_0}{2}. \tag{27}$$

The second part

$$\begin{aligned} H_S &= \Omega_z\sigma_z + \omega_z\sum_{j=1}^N I_z^{(j)} - \sigma_z\sum_{j=1}^N g_j I_0 \\ &= \mu_g(n)\sum_k |g_k^{(n)}\rangle\langle g_k^{(n)}| + \mu_f(n)\sum_k |f_k^{(n)}\rangle\langle f_k^{(n)}| \end{aligned} \tag{28}$$

is derived from the first and second terms of the original Hamiltonian and also operates within the subspace V_n In the third part

$$\begin{aligned} H_p &= \sigma_z\sum_{j=1}^N g_j(I_z^{(j)} + I_0) \\ &= \sum_k \sum_{j=1}^N \frac{g_j}{2}(M_{kn}^{(j)} + I_0)|g_k^{(n)}\rangle\langle g_k^{(n)}| \\ &\quad - \sum_k \sum_{j=1}^N \frac{g_j}{2}(M_{kn}'^{(j)} + I_0)|f_k^{(n)}\rangle\langle f_k^{(n)}|, \end{aligned} \tag{29}$$

$M_{kn}^{(j)}$ ($M_{kn}'^{(j)}$) is the c number, which describes the z component of the jth nuclear spin in the state $|g_k^{(n)}\rangle$ ($|f_k^{(n)}\rangle$).

We observe that the interaction part $H_{JC}=H_R+H_S$ is very similar to the JC Hamiltonian in cavity QED describing the interaction between the two-level atom and single-mode electromagnetic field in the rotating-wave approximation. In order to create the entanglement between electron spin and collective bosons, one can adjust the external field B_0 so that $\mu_f(n)=\mu_g(n)$; that is,

$$\Omega_z = \omega_z + N\overline{g}\overline{I}_0. \tag{30}$$

In this case $H_{JC}=\oplus_n H_{JC}^{[n]}$ on the whole space $V=\oplus_n V_n$ can be reduced to the irreducible parts

$$H_{JC}^{[n]} \sim H_R + n\omega_z, \tag{31}$$

for $\omega_z=\Omega_z-N\overline{g}\overline{I}_0$. Correspondingly, the dynamics of the total system can also be constrained within the invariant subspace V_n and then the last term $n\omega_z$ independent of σ_+ and B can be ignored since it can not contribute to the dynamics of the system significantly.

To consider the effectiveness of the above qubit storage protocol, we need to analyze the role of other collective modes orthogonal to the basic collective model defined by B and B^+. These auxiliary collective modes complement the B mode to generate a complete Hilbert space of the nuclei ensemble. In fact, we can generally construct the complete set of creation and annihilation operators C_k^+ and C_k $(k=1,2,\ldots,N)$ including $C_0=B$ and all auxiliary modes as

$$C_k = \frac{\sum_{i=1}^N h_i^{[k]} I_+^{(i)}}{\sqrt{2I_0\sum_j h_j^2}}, \tag{32}$$

where

$$\mathbf{h}^{[k]} = (h_1^{[k]}, h_2^{[k]}, \ldots, h_N^{[k]}) \tag{33}$$

(for $k=1,2\ldots,N$) are N orthogonal vectors in the N-dimension space $\mathbf{R}^N$, which can be systematically constructed by making use of the Gramm-Schmidt orthogonalization method starting from

$$\mathbf{h}^{[1]} = (g_1, g_2, \ldots, g_N) \in \mathbf{R}^N. \tag{34}$$

Since $\mathbf{h}^{[k]}\cdot\mathbf{h}^{[j]}=\delta_{kj}$ and the Gramm-Schmidt orthogonalization can also result in the quasihomogeneity conditions

$$\frac{\overline{\delta h^{[k]2}}}{\overline{h^{[k]2}}} \sim 0, \quad or \quad \frac{h_{\max}^{[k]2}}{\overline{h^{[k]2}}} \sim 1, \tag{35}$$

we have

$$[C_k, C_j^+] \to \delta_{kj} \tag{36}$$

in the large N limit.

For example, one can construct a boson mode with respect to the existing mode by the collective excitation B by choosing a distribution of coupling constants

$$\left\{ h_i \middle| h_i = g_{N-i}, h_{N-i} = -g_i, \forall\, i < \frac{N}{2} \right\} \tag{37}$$

as a permutation of $\{g_i\}$ and then define a independent boson mode by the collective operator C as Eq. (32). We can check that both the orthogonal relation $\overline{\Sigma_{i=1}^N g_i h_i = 0}$ and the quasihomogeneity conditions $\overline{\delta h^2}/\overline{h^2}\sim 0$ or $h_{\max}^2/\overline{h^2}\sim 1$ can be obviously satisfied. Then one can prove that in each invariant subspace V_n with $n\ll N$, there are the typical boson commutation relations

$$[C, C^+] = 1, \quad [C, B^+] = 0. \tag{38}$$

Apparently, from the above generalized the Gramm-Schmidt orthogonalization, the auxiliary boson operators can be expressed as the linear combination of the spin operators through a matrix transformation $\mathbf{C}=U\mathbf{I}$ for

$$\mathbf{C} = (B, C_2, \ldots, C_N)^T, \tag{39}$$

$$\mathbf{I} = (I_-^{(1)}, I_-^{(2)}, \ldots, I_-^{(N)})^T,$$

where U is a unitary (or orthogonal) matrix. Since $\mathbf{C}^+\mathbf{C} = \mathbf{I}^+\mathbf{I} = \Sigma_{j=1}^N I_+^{(j)} I_-^{(j)}$ under such transformation one can prove that there exists a constraint

$$B^+B + \sum_k C_k^+ C_k \approx I_z + I_0 N, \tag{40}$$

when they work on the subspace V_n with $n\ll N$

$$\Omega_z = \omega_z + N\overline{g}\overline{I}_0. \tag{41}$$

Formally, the Hamiltonian in Eq. (31) can be rewritten in the whole space as

$$H_{JC} = H_R + \omega_z I_z + \Omega_z\sigma_z - \sigma_z \sum_{j=1}^N g_j I_0 \sim H_R + \omega_z \sum_k C_k^+ C_k + \omega_z(B^+B + \sigma_z) \tag{42}$$

according to the above constraint. The above argument implies no coupling between the basic mode B and the auxiliary modes C_k $(k=1,2,\ldots,N-1)$, and thus the dependence on C_k is trivial in the above equation. However, all mode-coupling terms between the auxiliary modes C_k and the electron-spin qubit occur only in H_p, which can cause decoherence of the qubit.[18] In Sec. VI we will explore this decoherence mechanism in detail.

IV. VALIDITY OF THE SINGLE-MODE APPROXIMATION

First, let us assume that B mode is independent of those auxiliary collective modes C_k. To formally diagonalize H_{JC} by straightforward calculations, one can obtain the eigenvalues of H_{JC}

$$E_\pm(m_i, m) = \left(m + \sum_i m_i\right)\omega_z \pm \sqrt{I_0(m+1)\sum_j \frac{g_j^2}{2}} \tag{43}$$

and the corresponding eigenstates

$$|\psi^{(\pm)}(\{m_i\}, m)\rangle = \frac{1}{\sqrt{2m!m_i!}}\left(|\uparrow\rangle_e \pm |\downarrow\rangle_e \frac{B^+}{\sqrt{m+1}}\right) \otimes (B^+)^m (C_i^+)^{m_i}|G\rangle, \tag{44}$$

where we have $\Sigma_i m_i + m = n$.

It is also pointed out that, only the "excited" nuclear spins whose z component values are not $-I_0$ have contributions to the summation in the definition of H_p (29). Therefore, because of the low-exciton condition, there is an intuitive argument to show that H_p is only a perturbation term. In the following, this guess can be proved explicitly. Under the quasihomogeneity conditions (15) and (19), one has

$$H_p \sim \frac{\overline{g}}{2}(I_z + NI_0)\sigma_z, \tag{45}$$

where $\overline{g}=A/N$ is the average coupling strength between the electron and nuclei. Then the first-order energy correction for H_p can be estimated with perturbation theory

$$\delta E(n) \sim \langle \psi_n^{(\pm)}|H_p|\psi_n^{(\pm)}\rangle \sim \frac{n}{4}\overline{g}. \tag{46}$$

On the other hand, the energy gap between $E_\pm(n)$ is

$$\Delta E(\{m_i\},m) = \sqrt{I_0(m+1)\sum_j \frac{g_j^2}{2}} \sim \frac{\overline{g}}{\sqrt{2}}\sqrt{NI_0(m+1)}, \tag{47}$$

where we have used the relation $\overline{g^2}\sim\overline{g}^2$, which meets the conditions (15) and (19). Therefore, the magnitude of the contribution of H_p can be described by the ratio

$$\left|\frac{\delta E(n)}{\Delta E(\{m_i\},m)}\right| \sim \sqrt{\frac{n}{N}}. \tag{48}$$

The above estimation implies that H_p can indeed be regarded as perturbation to H_{JC} in the low-excitation case and macroscopic limit $n \ll N$. Therefore, we can take H_{JC} as the effective Hamiltonian of the total system. It is noted that, there is no coupling between mode B and the auxiliary modes C_k. Hence we can write the effective Hamiltonian of the electron spin and the B mode as

$$H_c = \Omega(\sigma_+ B + \sigma_- B^+) + \omega_z\left[B^+B + \sigma_z - \tfrac{1}{2}\right]. \tag{49}$$

In order to quantitatively evaluate the extent of approximation of the single-mode boson approach and obtain the effective Hamiltonian, the numerical method is employed to verify our approximate analytical result. We compute the eigenstates of the original spin-exchange Hamiltonian

$$H_s = \frac{1}{2}\sum_j g_j(\sigma_+ I_-^{(j)} + \sigma_- I_+^{(j)}) \tag{50}$$

and

$$H_c - n\omega_z = \Omega(\sigma_+ B + \sigma_- B^+) \tag{51}$$

for finite N system. Without loss of generality we take a Gaussian-type distribution, which satisfies the conditions (15) and (19). In Fig. 2, the spectrums of H_s and $H_c - n\omega_z$ for $N=80$ and $I_0=1/2$ system in the subspace $J_z=-N/2+1$ are plotted in Figs. 2(a) and 2(b). It shows that the spectrums are in agreement with each other. By comparing the numerically exact results with the analytically approximate ones for the effective spin-boson system, the numerical result shows that in the low-excitation and macroscopic limit with the quasihomogeneity condition, the single mode boson effective Hamiltonian (49) can work well in describing the collective excitation of the nuclei ensemble stimulated by the conduction band electron. We can also understand the difference of the spectrum structures in Fig. 2 in terms of the concept of "hardcore boson."[19] We imagine a model Hamiltonian H_b $=H_s(I_-^{(j)}\to b^{(j)})$, which is obtained by replacing $I_-^{(j)}$ ($I_+^{(j)}$) in Eq. (50) with a set operators $b^{(j)}(b^{(j)+})$. If they satisfy the usual commutation relation of bosons that the operators $b^{(j)+}(b^{(j)})$ and $b^{(k)}(b^{(k)+})$ on different sites commute with each other, then B and B^+ automatically satisfy the boson commutation relation and then result in the regular spectrum same as to that illustrated in Fig. 2(b). However, if the bosons are of hardcore, i.e., they are repulsive with each other at a same site, one can describe them with vanishing anticommutators $\{b^{(j)+}, b^{(j)+}\}=0$ for the same site and the vanishing commutators $[b^{(j)\pm}, b^{(k)\pm}]=0$ for different sites. In this case the repulsive interaction with hardcore feature will widen the original spectral lines to form the similar band structure in the energy spectrum.

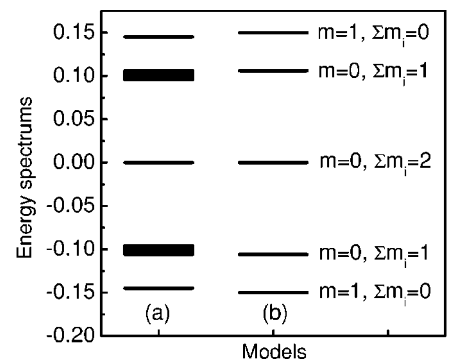

FIG. 2. Spectrums of H_s (a) and $H_c-\omega_z$ (b) for $N=80$ and $I_0=1/2$ system in the subspace $J_z=-N/2+1$. The slight deviation between the two spectrums is because of the hardcore effect of two bosons and should vanish in macroscopic limit.

V. QUANTUM INFORMATION STORAGE AS A DYNAMICAL PROCESS

We note that the above effective Hamiltonian is just of the JC type on the resonance and then it can be used to produce the entanglement between the qubit state of the electron and the bosonic mode of the collective excitation of the nuclei ensemble. Thus this entanglement induces a writing process of qubit information into the collective excitation.

We assume that the initial state of the total system

$$|\psi(0)\rangle = |d(0)\rangle_e \otimes |G\rangle \tag{52}$$

contains the arbitrary superposition

$$|d(0)\rangle_e = \alpha|\uparrow\rangle_e + \beta|\downarrow\rangle_e \tag{53}$$

of electronic states and define the Fock state

$$|m\rangle_b = \frac{1}{\sqrt{m!}}(B^+)^m|G\rangle \tag{54}$$

for the collective excitation. We now consider the long time evolution by projecting the wave function onto each invariant subspace spanned by the states $\{|\uparrow\rangle_e|m\rangle_b, |\downarrow\rangle_e|m+1\rangle_b\}$ (we denote this subspace by Z_m). Then this evolution of mode B and the electron spin can be explicitly characterized by reduced evolution matrices

$$U_m(t) = e^{-i\omega_z mt}\begin{bmatrix} \cos \Omega_m t & -i \sin \Omega_m t \\ -i \sin \Omega_m t & \cos \Omega_m t \end{bmatrix} \quad (55)$$

for $m=0,1,2,\ldots$. Here the dressed Rabit frequency

$$\Omega_m = \sqrt{(m+1)}\Omega \quad (56)$$

depends on the number of collective excitation. A storage process of qubit information is expressed by the factorization map

$$|d(0)\rangle_e \otimes |G\rangle \to |\downarrow\rangle_e(\beta|G_1\rangle - i\alpha e^{-i\omega_z \pi/2\Omega}|G_2\rangle) = |\downarrow\rangle_e W(\alpha|G_1\rangle + \beta|G_2\rangle) \quad (57)$$

at $t=\pi/2\Omega$. Here $|G_1\rangle=|G\rangle$, $|G_2\rangle=(B^+)|G\rangle$ are two collective excitation states that are orthogonal to each other, and

$$W = \begin{bmatrix} 0 & 1 \\ -ie^{-i\omega_z \pi/2\Omega} & 0 \end{bmatrix} \quad (58)$$

is a unitary transformation.

The same case was also encountered in Ref. 10 (see the difference between the initial and final states in Eq. (2) of Ref. 10). In quantum information theory this can be easily implemented by a local unitary transformation independent of the initial state $|d(0)\rangle_e$ (or the coefficients α and β). So the decoding process can easily map back from the final state of the quantum memory

$$|F\rangle = \beta|G_1\rangle - i\alpha e^{-i\omega_z \pi/2\Omega}|G_2\rangle \quad (59)$$

by its inverse transformation W^{-1}. In this sense, we say that the above map implements the quantum information storage. We note that this is very similar to the case in quantum teleportation, in which the initial state-independent transformation can easily be implemented by Bob for the teleported state once the two-bit classical information is told by Alice.

However, if one prepares the quantum memory not in its perfectly polarized state $|G\rangle$ (e.g., $|m\rangle_b$ $(m\neq 0)$), the general initial state

$$|\psi(0)\rangle = (\alpha|\uparrow\rangle_e + \beta|\downarrow\rangle_e) \otimes |m\rangle_b \quad (60)$$

will evolve into

$$\begin{aligned}|\psi(t)\rangle &= \alpha U_m(t)|\uparrow\rangle_e|m\rangle_b + \beta U_{m-1}(t)|\downarrow\rangle_e|m\rangle_b \\ &= \alpha e^{-im\omega_z t}\cos(\Omega_m t)|\uparrow\rangle_e|m\rangle_b + \beta e^{-i(m-1)\omega_z t}\cos(\Omega_{m-1}t) \\ &\quad \times|\downarrow\rangle_e|m\rangle_b - i\beta e^{-i(m-1)\omega_z t}\sin(\Omega_{m-1}t)|\uparrow\rangle_e|m-1\rangle_b \\ &\quad - i\alpha e^{-im\omega_z t}\sin(\Omega_m t)|\downarrow\rangle_e|m+1\rangle_b. \end{aligned} \quad (61)$$

It is noted that in order to obtain the above result, we have considered $|\uparrow\rangle_e\otimes|m\rangle_b$ and $|\downarrow\rangle_e\otimes|m\rangle_b$, which belong to different subspaces Z_m and Z_{m-1}. Hence they are driven by two different blocks $U_m(t)$ and $U_{m-1}(t)$ of the block-diagonal evolution matrix $U=\mathrm{diag}[U_m(t)]$, respectively. The above result from a straightforward calculation shows that only the ensemble of nuclei, which is prepared in the collective ground state, the polarized ensemble, can serve as a quantum memory. Otherwise there must exist the systematic error for quantum information processing.

VI. DECOHERENCE DUE TO THE COUPLINGS WITH AUXILIARY MODES

Finally, we need to revisit the quantum decoherence in the process of quantum information storage based on the collective excitation of the polarized nuclei ensemble. The main source of decoherence is due to the existence of the single particle motion described by the perturbation Hamiltonian H_p. The similar situation was ever considered for the collective excitation in the ensemble of free atoms by one of the present authors (CPS) and his collaborators.[21] The condition under which we can ignore the perturbation result from noncollective excitations is just of preserving quantum coherence. By making use of the boson modes C_k, we can expect that the part H_p contains the coupling between the spin qubit and $N-1$ auxiliary C_k modes. This will realize a typical quantum decoherence model for a two-level system coupled to a bath of many harmonic oscillators.

In order to analyze this problem more quantitatively, we describe the single-particle motion from the perturbation Hamiltonian H_p in terms of the excitation of auxiliary modes. We consider a quasihomogeneous case $g_j\approx g$, in which

$$H_p \approx \sum_k \left(\omega_z + \tfrac{1}{2}g\sigma_z C_k^+ C_k\right), \quad (62)$$

where we ignore the coupling term $gB^+B\sigma_z/2$ since it can only lead a phase shift in the spin qubit. We consider the nuclear ensemble prepared in a thermal equilibrium state

$$\rho_R = \frac{1}{Z}\prod_k \sum_{n_k} \exp\left(-\frac{\omega_z n_k}{k_B T}\right)|n_k\rangle\langle n_k|,$$

where

$$Z = \prod_k \sum_{n_k} \exp\left(-\frac{\omega_z n_k}{k_B T}\right) \quad (63)$$

the partition function at the temperature T where k_B is the Boltzmann constant. Let the spin qubit be initially in a pure state $|\phi\rangle=\alpha|0\rangle+\beta|1\rangle$. After a straightforward calculation we obtain the density matrix at time t

$$\rho(t) = U(t)(|\phi\rangle\langle\phi| \otimes \rho_R)U^{-1}(t) \quad (64)$$

and the corresponding reduced-density matrix $\rho_S(t) = Tr_B\rho(t)$ by tracing over the auxiliary modes $\{C_k\}$. The off-diagonal elements of $\rho_S(t)$ can be given explicitly as

$$\rho_S^\star(t)_{10} = \rho_S(t)_{01} = \frac{\alpha^\star\beta(e^{(\omega_z/k_BT)}-1)^{N-1}e^{i(N-1)\theta}}{\sqrt{(e^{(2\omega_z/k_BT)} - 2e^{(\omega_z/k_BT)}\cos(gt)+1)^{N-1}}}, \quad (65)$$

where

$$\theta = \arctan\frac{\sin(gt)}{\exp(\omega_z/k_BT) - \cos(gt)}. \quad (66)$$

In the zero-temperature limit or $T\to 0$, there is no decoherence since the off-diagonal elements

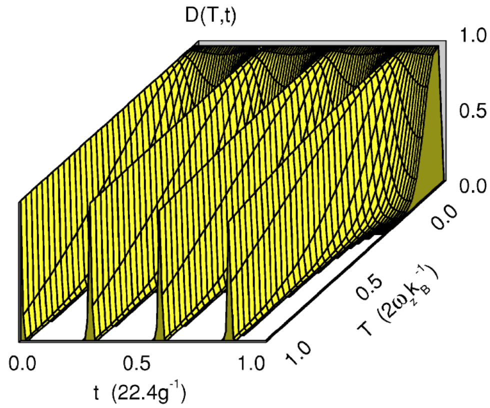

FIG. 3. The decoherence factor $D(T,t)$ for a small-size system as the function of time and temperature.

$$\rho_S^\star(t)_{10} = \rho_S(t)_{01} = \alpha^\star \beta \tag{67}$$

do not change. But in a finite temperature, the norm of off-diagonal element is proportional to the so-called decoherence factor

$$D(T,t) = d(T,t)^{N-1} \equiv \frac{(e^{(\omega_z/k_BT)} - 1)^{N-1}}{\sqrt{(e^{(2\omega_z/k_BT)} - 2e^{(\omega_z/k_BT)}\cos(gt) + 1)^{N-1}}}. \tag{68}$$

This result illustrates that thermal excitation of the auxiliary modes will block the implementation of the macroscopic nuclear ensemble-based quantum memory since in the large N limit $D(T,t) \to 0$ except for the special instances at

$$t = \frac{2k\pi}{g}, \quad k = 0,1,2,\ldots \tag{69}$$

In these instances, $D(T,t)=1$ and there is no decoherence at all. Besides, since in these instances $\rho_S^\star(t)_{10}=\alpha^\star\beta$ is just the initial values and then we implement a ideal quantum information storage to recover the stored state. To further consider the temperature dependecne of the auxiliary mode induced decoherence, we plot a 3D graphic of $D(T,t)$ for a small size $N=20$ system. Figure 3 shows that the case $D(T,t)=1$ appears periodically as time t, and $D(T,t)\to 1$ all the time when $T\to 0$. According to the experimental data,[14] the period is roughly estimated as $2\pi/g \sim 10^{-7}s$.

VII. SUMMARY WITH REMARKS

In summary we have studied the possibility of quantum memory by using collective excitation of ensemble of polarized nuclei surrounding a single electronic spin in a quantum dot. We explicitly present the quasihomogeneous independent conditions, under which the many-particle system, a macroscopic ensemble of polarized nuclei, can be treated as a single-mode bosonic system. Thus the interaction is of the similar form of Jaynes-Cummings model. Based on this fact, the collective excitation can serve as a quantum memory to store the spin state of a conduction electron.

We also pointed out that the physical system for quantum information storage is the same as that in Ref. 10, which first showed that electronic-spin coherence can be reversibly mapped onto the collective state of the surrounding nuclei. But our studies emphasize that the collective excitation-based quantum memory can be understood in terms of the spin-boson model with essential simplicity in physics. Especially the valid conditions are discovered in present paper. That is, the collective operators are explicitly invoked to depict the bosonic collective excitations and then we can present an effective boson-spin model, which reveals physical mechanism with collective quantum coherence behind the original conceptual protocol for the long-lived quantum memory.

There are two sources of quantum decoherence in such quantum information processing, one is due to noncollective mode and the other is due to the nuclear spin diffusion or coupling with environment. The latter is dominate and has been well considered in Ref. 20, but the former can still play a role in certain cases. So we stress the former in this paper since the same situation was even considered for the collective excitation in the ensemble of free atoms by us.[21] In principle, the latter can also be treated in our spin-boson model with similar approach by adding diffusion terms. We also noted that the systematic errors in transferring quantum information can occur because of the appearance of higher excitation by illustrating that only the ensemble of nuclei prepared in the collective ground state rather than the excited ones can serve as a quantum memory. How to avoid the higher excitation of the collective boson mode and how to correct the error due to the appearance of higher excitation are open questions that need further investigations.

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*Electronic address: songtc@nankai.edu.cn
†Electronic address: suncp@itp.ac.cn
‡Internet www site: http//www.itp.ac.cn/suncp

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