Quantum storage and information transfer with superconducting qubits

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We theoretically design a rather simple device to realize the general quantum storage based on dc superconducting quantum interference device charge qubits. The distinct advantages of our scheme are analyzed in comparison with existing storage scenarios. More arrestingly, an easily controllable *XY* interaction has been realized in superconducting qubits, which may have more potential applications in addition to those in quantum information processing. The experimental feasibility is also elaborated.

As solid state quantum devices, Josephson junctions and superconducting quantum interference devices (SQUIDs) have manifested arresting and robust macroscopic quantum behaviors. They can be used to develop new quantum bits and logic gates in the context of quantum information science.[1] Since the favorable elements of good coherence, controllability, and scalability are integrated in these superconducting devices, they are very promising for the realization of quantum information processing. Recently, a series of exciting experimental progresses have been made in this field, including high quality single qubits,[2–5] the quantum entanglement between the two qubits,[6,7] and the CNOT gate[8,9] realized in various superconducting devices. In addition, both experimental and theoretical efforts have also been devoted to explore new quantum information processing devices based on the coupling of superconducting qubits with other quantum modes/degrees.[10–13] Nevertheless, most interests have been focused on the design/implementation of single and multiqubit logic gates, while little attention has been paid to quantum storage in superconducting qubits.[14]

As is well known, memory storage is an indispensable part of information processing; its quantum counterpart is even more important because of the fragility of quantum coherence.[15] Roughly speaking, there are two kinds of quantum memory storage: a basic one is to temporarily store the intermediate computational results, just as the role played by the RAM (random access memory) in classical computers; the other is used to store the ultimate results, playing a similar role of the classical hard disks. To fully accomplish quantum information processing, a certain bus is required to transfer the information from these basic temporary memory units to other types of memory units as well as among themselves. Therefore, it is timely and significant to design basic storage units based on superconducting qubits and connect them via an appropriate bus to achieve a workable storage network. In this paper, we design an experimentally feasible basic storage unit based on Josephson charge qubits and propose to couple them with a one-dimensional (1D) transmission line to physically realize a quantum storage network. The distinct advantages of our scheme include (i) the $1/f$ noise caused by background charge fluctuation may be significantly suppressed because the bias voltage for the charge

qubit can be set to degeneracy point in the proposed storage process,[2,16] (ii) it is not necessary to adjust the magnetic flux instantaneously, (iii) in sharp contrast to dynamic quantum storage scenarios, no restriction has to be imposed on the initial state of our temporary memory units, and (iv) the relevant fabrication technique of the designed circuits are currently available. All of these enable our scheme of quantum storage and information transfer to be more promising for the future solid state quantum computing.

A basic storage unit. A basic storage unit is designed to consist of three symmetrical dc superconducting quantum interference devices (dcSQUIDs) as shown in Fig. 1. The original Hamiltonian of the system includes Coulomb energy and Josephson coupling energy, i.e.,

$$H = H_c - \sum_{i=1}^{3} E_{Ji} \cos \pi \frac{\phi_{xi}}{\Phi_0} \cos \theta_i, \qquad (1)$$

where E_{Ji}, ϕ_{xi}, and θ_i are the Josephson coupling energy, the magnetic flux, and the phase difference in the ith SQUID,
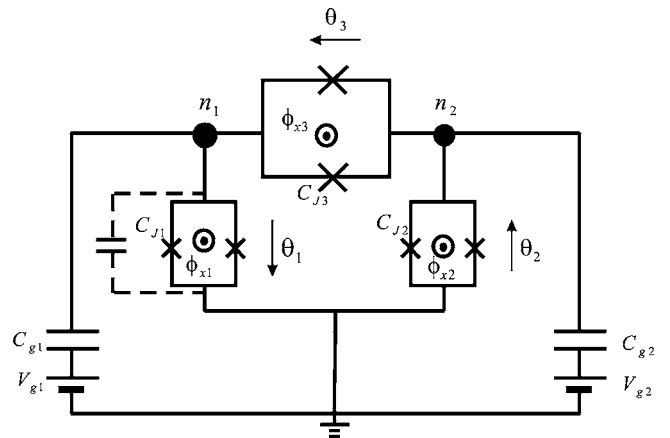

FIG. 1. A schematic circuit of a basic quantum storage unit, where three dcSQUIDs are penetrated by controllable magnetic fluxes, respectively. Each cross denotes a Josephson junction and the black dot with label n_1 (n_2) corresponds the first (second) Cooper pair box.

$\Phi_0 = h/2e$ is the usual superconducting flux quantum. The Coulomb energy part $H_c = E_{c1}(n_1 - n_{g1})^2 + E_{c2}(n_2 - n_{g2})^2 + 4E_3(n_1 - n_{g1})(n_2 - n_{g2})$. Here n_i is the number of the excess Cooper pair in the ith Cooper pair box and $n_{gi} = C_{gi}V_{gi}/2e$ with V_{gi} and C_{gi} as the corresponding gate voltage and capacitance. The coefficients E_α are derived as $E_{c1} = 2e^2 C_{\Sigma2}/(C_{\Sigma1}C_{\Sigma2} - C_{J3}^2)$, $E_{c2} = 2e^2 C_{\Sigma1}/(C_{\Sigma1}C_{\Sigma2} - C_{J3}^2)$, $E_3 = e^2 C_{J3}/2(C_{\Sigma1}C_{\Sigma2} - C_{J3}^2)$ with $C_{\Sigma i} = C_{Ji} + C_{J3} + C_{gi}$ as the summation of all the capacitances connected to the ith Cooper pair box.

When $E_{ci} \gg E_{Ji}$ ($i = 1, 2$), the charging energy dominates the system and the state evolution is approximately confined in the subspace spanned by the two eigenstates $\{|0\rangle_i, |1\rangle_i\}$ of charge number operator. Then the Pauli operators can be introduced to express the dynamic variables, and the reduced Hamiltonian is written as

$$\hat{H} = \sum_{i=1}^{2} \Omega_i \sigma_{zi} + E_3 \sigma_{z1}\sigma_{z2} - \sum_{i=1}^{2} E_{Ji} \cos \pi \frac{\phi_{xi}}{\Phi_0} \sigma_{xi}$$
$$- E_{J3} \cos \pi \frac{\phi_{x3}}{\Phi_0}(\sigma_{x1}\sigma_{x2} - \sigma_{y1}\sigma_{y2}), \quad (2)$$

where $\Omega_i = E_{ci}(n_{gi} - \frac{1}{2}) + 2E_3(n_{gj} - \frac{1}{2})$ ($i \neq j$). In the derivation of Eq. (2), we have used the constraint $\theta_1 + \theta_2 + \theta_3 = 0$. Here, the Pauli matrices are defined as $\sigma_{xi} = |1\rangle_{ii}\langle 0| + |0\rangle_{ii}\langle 1|$, $\sigma_{yi} = -i(|1\rangle_{ii}\langle 0| - |0\rangle_{ii}\langle 1|)$, and $\sigma_{zi} = |0\rangle_{ii}\langle 0| - |1\rangle_{ii}\langle 1|$ in the bases $|1\rangle_i$ and $|0\rangle_i$, which are the eigenstates of the number operator of Cooper pair on the ith box with one and zero Cooper pair.

In this setup, the first SQUID is a computational qubit and the second one is used for storage, while the third one serves as the controllable coupling element between qubits 1 and 2. Prior to the storage process, the two qubits are set to be uncoupled by simply letting $\phi_{x3} = \Phi_0/2$.

We now illustrate that the storage process begins whenever the flux in the third dcSQUID is switched away from $\Phi_0/2$. In fact, the coupling between the two qubits is turned on for $\phi_{x3} \neq \Phi_0/2$. If both of the bias voltages are set to let $n_{g1} = n_{g2} = 1/2$ and the magnetic fluxes ϕ_{xi} threading the first two SQUIDs equal to $\Phi_0/2$, the first and third terms in Eq. (2) vanish. Moreover, if $C_{\Sigma i}/C_{J3}$ ($i = 1$ or 2) is sufficiently large such that $E_3 \ll E_{J3}$, the third term in Eq. (2) is negligibly small (here we shall neglect it first for simplicity and address its influence on the results later). As a result, we have

$$\hat{H} = -E_{J3} \cos \pi \frac{\phi_{x3}}{\Phi_0}(\sigma_{x1}\sigma_{x2} - \sigma_{y1}\sigma_{y2}). \quad (3)$$

Defining the Pauli operators of the second qubit in another representation $\{|\tilde{1}\rangle_2, |\tilde{0}\rangle_2\}$ with $|\tilde{1}\rangle_2 = |0\rangle_2$, $|\tilde{0}\rangle_2 = -|1\rangle_2$, one has $\sigma_{x2} = -\tilde{\sigma}_{x2}$, $\sigma_{y2} = \tilde{\sigma}_{y2}$, $\sigma_{z2} = -\tilde{\sigma}_{z2}$. The corresponding Hamiltonian becomes

$$\hat{H} = E_{J3}(\sigma_{x1}\tilde{\sigma}_{x2} + \sigma_{y1}\tilde{\sigma}_{y2}), \quad (4)$$

where we set $\phi_{x3} = 0$ to maximize the interaction strength between two qubits. This is a central result of the present work. It is notable that this interaction is a typical *XY* coupling of spin-1/2 systems often addressed in many-body spin physics and is of significance in solid state quantum computing.[17] In particular, it is remarkable to achieve such an easily controllable *XY*-interaction in the physical implementation of quantum storage with a rather simple circuit of superconducting qubits. In addition, this controllable coupling may have applications in exploring in-depth spin physics because it can be easily manipulated in the present system.

It is straightforward to find the time evolution operator in the two qubit charge basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ as

$$U(t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \xi(t) & i \sin \xi(t) & 0 \\ 0 & i \sin \xi(t) & \cos \xi(t) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (5)$$

where $\xi(t) = 2E_{J3}t/\hbar$. We can see that at the time $t = \pi\hbar/(4E_{J3})$ the evolution leads to $|00\rangle \rightarrow |00\rangle$, $|01\rangle \rightarrow i|10\rangle$, $|10\rangle \rightarrow i|01\rangle$ and $|11\rangle \rightarrow |11\rangle$. That is to say, the quantum states of the two qubits are swapped (with an unimportant phase shift).[18] For example, if the density matrix of the first qubit is initially $\rho_1(0) = \Sigma_{n,m=0}^{1} c_{mn}|m\rangle_1\langle n|$ while the second qubit is prepared in $|\tilde{0}\rangle_2$, the final state at $t = \pi\hbar/4E_{J3}$ is

$$\rho\left(t = \frac{\pi\hbar}{4E_{J3}}\right) = |0\rangle_1\langle 0| \otimes \sum_{n,m=0}^{1} c_{mn}|\overline{m}\rangle_2\langle\overline{n}|, \quad (6)$$

where $|\overline{m}\rangle_2 = e^{i(\pi/2)m}|\tilde{m}\rangle_2$. Therefore the quantum information carried by the first qubit (the computational one) has been stored in the second one. In the meanwhile the first qubit is set to the ground state to prepare for the next round of computation.

After the state of the computational qubit has been stored in the temporary memory, the flux threading the third SQUID is tuned back to be $\Phi_0/2$ and the two qubits are decoupled. The first qubit can perform new computational task.

It is worth pointing out that the qubit 2 is not necessarily restricted to be in its ground state. Actually our storage protocol works for any state of the second qubit even for the mixed state $\rho_2(0) = \Sigma_{n,m=0}^{1} a_{mn}|\overline{m}\rangle_2\langle\overline{n}|$. This feature is quite different from most existing dynamical storage schemes,[14,19,20] in which a prerequisite is to prepare the storage qubit in the ground state. Also note that although some adiabatic quantum storage schemes[21–23] do not have this restriction they are seriously flawed by the adiabatic condition that demands rather long time to complete the whole storage process.

Another advantage of this protocol is a comparatively loose requirement on the adjustment of the magnetic flux ϕ_{x3} during the storage process. In most quantum computing proposals controlled by the magnetic flux, the instantaneous switch of magnetic flux is normally required. In our protocol, even if ϕ_{x3} is dependent of t, rather than a step function, namely, the Hamiltonian (3) depends on time, since $H(t)$ at different time commute with each other, the time dependence modifies only the definition of $\xi(t)$ in Eq. (5) as
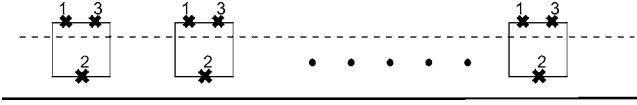
FIG. 2. A schematic diagram of the storage network coupled through 1D transmission line (solid).

$$\bar{\xi}(t) = 2E_{J3}\int_0^t \cos\left(\pi\frac{\phi_{x3}(t')}{\Phi_0}\right)dt'. \tag{7}$$

In this case, one can adjust the storage time τ to satisfy $\bar{\xi}(\tau) = \pi/2$. As for the other external magnetic fluxes ϕ_{x1} and ϕ_{x2}, it is obvious that they do not require the instantaneous manipulation. An additional merit lies in that the bias voltage is set to the degeneracy point during the whole storage process, which strongly suppresses the charge fluctuation induced $1/f$ noise, the most predominant resource of noise in Josephson charge qubits.[16]

All of the above three distinct features make our protocol more arresting and fault tolerant than most existing storage schemes. We also wish to remark that a two-qubit system similar to our setup[6,8] and a three-junction loop circuit[5] have already been fabricated experimentally and illustrated to have good quantum coherence. Therefore the designed architecture of basic storage unit is likely experimentally feasible with current technology and thus is quite promising for near future experimental realization.

Information transfer between the units. Generally speaking, a computational task requires the cooperation of several (or more) qubits. The state of one qubit usually needs to be transferred to another in order to conduct further computations. Also, it is necessary to store the final results to certain physical systems with longer coherence time. Therefore a storage network is indispensable in quantum information processing. One possible scenario to realize such a network is to use a common data bus with controllable coupling to all basic units. Through this data bus, the communication of any two basic units becomes feasible.

Currently, there are some alternative suggestions for possible common data buses including a microcavity, a nanomechanical resonator,[14] and a large junction, etc. Another promising one is the so-called 1D transmission line,[11,12] which has been illustrated to have several practical advantages including strong coupling strength, reproducibility, immunity to $1/f$ noise, and suppressed spontaneous emission.[12]

As an example, here we elaborate the transfer process with the 1D transmission line. Consider an array of identical basic units placed along a 1D transmission line (see Fig. 2). The information stored in the second qubit of any unit can be transferred to another unit via the transmission line. The coupling between the transmission line and the units can be either electrical or magnetic. For concreteness, here we focus only on the magnetic coupling. Different from the 3D microcavity where the magnetic dipole interaction is usually too weak to be considered, the present interaction can be sufficiently strong to accomplish the transfer task by an appropriate design of the circuit.

For an ideal 1D transmission line with the boundary conditions $j(0,t) = j(L,t) = 0$, the quantized magnetic field at $x = nL/2n_0$, where n_0 is the mode resonant with the qubits, n is an arbitrary integer, and L is the length of the line along the x direction, is

$$B_y\left(x = \frac{n}{n_0}\frac{L}{2}\right) = \frac{1}{d}\sqrt{\frac{\hbar l\omega_{n_0}}{L}}(a_{n_0} + a_{n_0}^\dagger), \tag{8}$$

while the electric field is zero at these points. Here $\omega_{n0} = n_0\pi/(L\sqrt{lc})$, d is the distance between the qubit and the transmission line, l (c) the inductance (capacitance) per unit length. The flux induced by the transmission line in a dcSQUID with an enclosed area S reads

$$\Phi_x = \frac{S}{d}\sqrt{\frac{\hbar l\omega_{n_0}}{L}}(a_{n_0} + a_{n_0}^\dagger). \tag{9}$$

It is a reasonable approximation to consider only the effect of the transmission line on the SQUID 2 if the distance between the third (or first) SQUID is significantly longer than d or we simply insert a magnetic shield screen (dotted line in Fig. 2). Under this consideration and the Lamb-Dicke approximation ($g \ll 1$), the Hamiltonian for the qubit 2 in the kth unit with $\phi_{x2} = \Phi_0/2$ becomes

$$H^{(k)} = \Omega_2^{(k)}\sigma_{z2} - gE_{J2}(a + a^\dagger)\sigma_{x2}^{(k)} + \hbar\omega\left(a^\dagger a + \frac{1}{2}\right), \tag{10}$$

where $g = S\sqrt{\hbar l\omega}/(d\Phi_0\sqrt{L})$ (here, for simplicity, we denote a_{n_0} as a and ω_{n0} as ω). During the storage process for the basic units, the second term in the above equation can be neglected because the qubit is largely detuned from the transmission line.

Under the condition $|\Omega_2^{(k)} - \omega|/(\Omega_2^{(k)} + \omega) \ll 1$, the terms oscillating with the frequency $\pm(\Omega_2^{(k)} - \omega)$ are singled out under the rotating-wave approximation, i.e.,

$$H^{(k)} = \Omega_2^{(k)}\sigma_{z2}^{(k)} + \hbar\omega a^\dagger a - (gE_{J2}a\sigma_{+2}^{(k)} + \text{H.c.}). \tag{11}$$

For each qubit, this is a typical Jaynes-Cummings model[24] and there exist many two-dimensional invariant subspaces. Driven by this Hamiltonian, if the qubit 2 of the kth unit is resonant with the cavity by adjusting $n_{g2}^{(k)}$, any state of this qubit can be mapped onto the subspace $\{|0\rangle_{\text{TLR}}, |1\rangle_{\text{TLR}}\}$ of the transmission line resonator.[19] This information can also be retrieved by the qubit 2 of another k'th unit. Consequently, the information carried by the kth unit is transferred to the k'th unit, with the whole process being detailed as below.

Prepare first the transmission line in its ground state $|0\rangle$. Tune $n_{g2}^{(k)}$ to have $\Omega_2^{(k)} = \omega$ for a period $\pi/2gE_{J2}$ such that the state of the kth unit is stored in the transmission line. Then let this qubit be largely detuned with the transmission line resonator while make the frequency of another qubit to satisfy $\Omega_2^{(k')} = \omega$ for another $t = \pi/2gE_{J2}$. This process can be explicitly illustrated as

$$(\alpha|1\rangle_2^{(k)} + \beta|0\rangle_2^{(k)}) \otimes |0\rangle_{\text{TLR}} \otimes |0\rangle_2^{(k')}$$

$$\rightarrow |0\rangle_2^{(k)} \otimes (\beta e^{i\xi}|0\rangle_{\text{TLR}} - i\alpha e^{-i\xi}|1\rangle_{\text{TLR}}) \otimes |0\rangle_2^{(k')}$$

$$\rightarrow |0\rangle_2^{(k)} \otimes |0\rangle_{\text{TLR}} \otimes (\alpha|1\rangle_2^{(k')} + \beta|0\rangle_2^{(k')}).$$

In this way the information is transferred from the kth to the k'th unit.

Discussions and remarks. To see the experimental feasibility, we now examine the used conditions and approximations based on the available/possible experimental parameters. We indeed verified that these conditions and approximations are reasonable and acceptable. For example, if we take $C_{\Sigma2} \sim 500$ aF, $C_{J3} \sim 100$ aF, and $C_{\Sigma1} \sim 1 \times 10^4$ aF, where the large capacitance of $C_{\Sigma1}$ can be achieved by shunting an additional large capacitance (see Fig. 1) and the small Josephson coupling energy of E_{J1} may be realized by using the tunable SQUID (compound Josephson junction) coupling. Then $E_{c1} \sim 32$ μeV, $E_{c2} \sim 640$ μeV, $E_3 \sim 1.6$ μeV, $E_{J2} \sim 100$ μeV, $E_{J3} \sim 100$ μeV, and $g \sim 0.1$.[25] With these parameters, we can see that $E_3 \ll gE_{J2}, E_{J3}$ and the Lamb-Dicke approximation is also justified. In addition, the operation time is estimated to be ~ 30 ps for one basic storage in a unit and ~ 1 ns for one information transfer process, being much shorter than the coherence time for charge qubits at the degeneracy point (~ 800 ns currently). Therefore this process can be completed before the quantum decoherence happens.

Finally, we address the effect of the E_3 neglected earlier. First, it is worthwhile to point out that even if E_3 is not negligible the basic unit part of our protocol still works. This is because an additional term $E_3 \sigma_{z1} \sigma_{z2}$ commutes with Eq. (3), and thus just brings an additional phase to the storage process. Secondly, although this term represents also an unremovable correlation between the two qubits in one unit, fortunately, following the same technique used by the NEC group,[6,8] a single qubit behavior can still be achieved in this system with an appropriate pulse, provided that E_3 is small. This setting makes the two qubits approximately independent. On the other hand, the transfer process may not be implemented successfully if E_3 is not so small. In this case, the first qubit of a unit has to be set in a certain state when the second qubit is transferring information to the transmission line, though this may reduce the efficiency of the transfer process.

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[25] Based on the parameters in Ref. 11, this coupling strength was estimated by setting $L \sim 0.2$ mm, $S \sim 200$ μm^2, $d \sim 1$ μm, and $n_0 = 5$.