Fast entanglement of two charge-phase qubits through nonadiabatic coupling to a large Josephson junction

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We propose a theoretical protocol for quantum logic gates between two Josephson junction charge-phase qubits through the control of their coupling to a large junction whose Josephson coupling energy is much larger than its Coulomb charge energy. In the low excitation limit of the large junction, it behaves effectively as a quantum data-bus mode of a harmonic oscillator. Our protocol can be fast since it does not require the data-bus to stay adiabatically in its ground state, as such it can be implemented over a wide parameter regime independent of the data-bus quantum state.

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I. INTRODUCTION

Significant progress has been made in improving the quantum coherence of Josephson junction (JJ) based qubits[1,2] since the first experiment breakthrough in 1999.[3] The coherent interaction of two JJ qubits has been implemented and the initial indications of their entanglement have been detected.[4,5] Even the demonstration of 2-bit conditional gate of charge qubits was reported most recently.[6] These developments have paved the way towards the realization of the two important elements of universal quantum computation; the ability to implement arbitrary single-bit rotations and the controlled logic gates between two qubits. Various theoretical schemes have been proposed for quantum logic gate operations of JJ qubits. The important measure of their success depends on the proposed mechanism for a controlled coupling between the JJ qubits. Most protocols involve the coupling of each JJ qubit to an auxiliary data-bus (such as variable transformer) and affect an effective coupling between two qubits through the elimination of dynamic variables of the data-bus.[7–13]

It is well known that the dissipation induced decoherence is fatal to the effecieny of quantum computing. More gate operations should be performed before decoherence happens. However, to obtain the effective interaction of the qubits, many existing schemes of the 2-bit gate[7–13] require the data-bus to be adiabatic, i.e., to stay in its ground state or a different pure state during gate operation, thus apply to just weak coupling parameters. This leads to slow gates with long operation times. In this article, we suggest a protocol capable of efficient and fast gate operations between two charge-phase qubits.[2] As to be shown in detail below, our protocol is insensitive to the state of the data-bus, thus is not restricted to the weak coupling limit as required by the adiabatic condition. In fact, the resulting strong effective coupling leads to short gate operation time. Therefore more gate operation can be performed before the dissipation induced decoherence happens. Furthermore, all control parameters in our protocol can be modulated within current experiments.

II. THE CHARGE QUBIT-LARGE JUNCTION SYSTEM

Our main idea is to construct a system of two JJ qubits linked by a quantum data-bus of a harmonic oscillator with the creation (annihilation) operator $\hat{a}(\hat{a}^{\dagger})$. Each of the two qubits is linearly coupled to this data-bus mode and described by the following form of the Hamiltonian $H(t) = \hat{a}\hat{F}(t) + \text{h.c.}$ with a general force $\hat{F}(t) = = \beta_1(t)\hat{F}_1 + \beta_2(t)\hat{F}_2$. $\hat{F}_j(j=1,2)$ is a dynamic variable (not necessarily an observable) of the jth qubit. It is known that when $\hat{F}_j$ commutes with its own conjugate $\hat{F}_j^{\dagger}$, the time evolution of $H(t)$ is given by

$$U(t) = \exp\left[i\sum_{k,j} \mu_{k,j}(t)\hat{F}_k^{\dagger}\hat{F}_j\right] e^{\alpha(t)a} e^{\alpha^*(t)a^{\dagger}}, \tag{1}$$

where the expressions for $\mu_{kj}(t)$ and $\alpha(t)$ can be explicitly obtained using the Wei-Norman algebra.[14,15] The crucial observation is, under certain conditions, the qubit-data bus interaction part of the above evolution operator is cancelled and thus the time evolution can be simply described by

$$U(T) = \exp\left[i\sum_{k,j} \mu_{kj}(T)\hat{F}_k^{\dagger}\hat{F}_j\right]. \tag{2}$$

Formally, the quadratic terms of F_j correspond to the effective nonlinear interactions between the two qubits. Clearly our strategy does not require the state of the data-bus to be adiabatic. Similar approaches have been adopted before in protocols of quantum computing with thermal ions“[16] and by Wang *et al.*[17]

We consider a system of coupled charge-phase qubits as illustrated by the electronic circuit shown in Fig. 1. Similar systems have been recently discussed by You *et al.*[12] Our idea works in the limit when the large junction (of capacitance C) stays at a low excitation or even a thermal state. The same model with only one qubit was used to demonstrate the progressive decoherence by us.[18] Each of the Cooper pair box is split into two small junctions of capacitance C_k' and C_k'' $(k=1,2)$ that form a superconducting loop. C_{gk} is the capacitance of the gate, and E_{Jk} is the Josephson coupling energy. For simplicity we set $C_k' = C_k'' \equiv C_k$ and $E_{Jk}' = E_{Jk}'' \equiv E_{Jk}$. If C is much larger than all the other capacitances in the circuit, then the Coulomb energy can be approximated by a nonentangled form that has no Coulomb interaction between the two qubits and the large junction. The total Hamiltonian con-

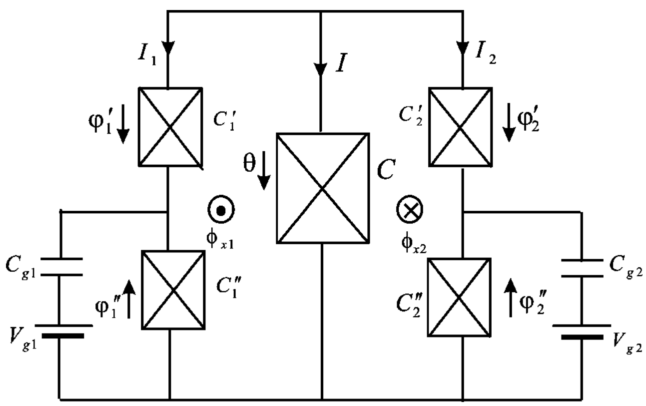

FIG. 1. Schematic diagram of two charge-phase qubits are coupled through a large junction.

taining the reduced Coulomb energy and the three Josephson coupling energies, then reads

$$H = \sum_{k=1,2} [E_{ck}(n_k - n_{gk})^2 - E_{Jk}(\cos \varphi_k' + \cos \varphi_k'')] + E_c N^2 - E_J \cos \theta, \tag{3}$$

where $E_{ck}=2e^2/C_{\Sigma k}$, $n_{gk}=C_{gk}V_{gk}/2e$, $E_c=2e^2/C$, and $C_{\Sigma k} = C_{gk}+C_k'+C_k''$. Here n_k is the number of Cooper pairs on the kth island, while N is the number of Cooper pairs on the Coulomb island connected with the large junction. φ_k', φ_k'', and θ superconduction phase differences across the relevant junctions. They are related through the fluxoid quantization condition around the loop $\theta+\varphi_k''-\varphi_k'=2\Theta_k$ and $\Theta_k = \pi\Phi_{xk}/\phi_0$. Introducing $\varphi_k=(\varphi_k'+\varphi_k'')/2$, we rewrite Eq. (3) to

$$H = \sum_{k=1,2} E_{ck}(n_k - n_{gk})^2 + E_c N^2 - E_J \cos \theta - 2\sum_{k=1,2} E_{Jk} \cos\left(\frac{\theta}{2} - \Theta_k\right)\cos \varphi_k, \tag{4}$$

now with the quantization condition $[\varphi_k, n_l]=i\delta_{lk}$. As is well known for such Josephson junction circuit, when $n_{gk}=0.5$ and when E_{jk} is not much larger than E_{ck}, the linear combinations of the two lowest charge eigenstates $\{|0\rangle_i, |1\rangle_i\}$ for each of the split Cooper pair box consist a good representation of a qubit. The two-level approximation has been verified in experiment.[2]

We now consider the "coherent" regime when $E_J \gg E_c$. As was found before in the study of quantum phase transitions[20] and also some other references such as Ref. 12, the spectrum of the low energy part of the large junction can be described approximately by a harmonic oscillator. Within this approximation, we expand the above Hamiltonian around $\theta=0$ up to $O(\theta^2)$ and obtain an effective spin-boson Hamiltonian

$$H = H(g_k, f_k) \equiv \sum_{k=1,2} [g_k(a^\dagger + a)\sigma_{xk} + f_k\sigma_{xk}] + \Omega a^\dagger a, \tag{5}$$

where

$$\Omega = \sqrt{2E_cE_J},$$

$$g_k = -\left(\frac{E_c}{32E_J}\right)^{1/4} E_{Jk} \sin \Theta_k,$$

$$f_k = -E_{Jk} \cos \Theta_k, \tag{6}$$

and the quasi-spin and boson operators are defined as

$$\sigma_{xk} = |1\rangle_{kk}\langle 0| + |0\rangle_{kk}\langle 1|,$$

$$a = \left(\frac{E_J}{8E_c}\right)^{1/4}\theta + i\left(\frac{E_c}{2E_J}\right)^{1/4} N. \tag{7}$$

The validity of this approximation can be verified through a straightforward numerical calculation. In fact, we first calculate the evolution wave function $|\psi_1\rangle$, which is governed by the approximate Hamiltonian (5). Then we compare it with the exact wave function $|\psi_2\rangle$ governed by the exact Hamiltonian (4) to a sufficiently high precision in expansion of the cosine function of θ [e.g., up to $O(\theta^6)$]. In this way we showed that, during one gate operation time ($\sim 10^{-10}$ s as calculated in this paper later), the deviation of $|\psi_1\rangle$ from $|\psi_2\rangle$ is 10^{-4} at most.

III. TWO WAYS TO CREATE THE ENTANGLEMENT OF QUBITS

This equivalent spin-boson system can be solved exactly since the interaction term $g_k(a^\dagger + a)\sigma_{xk}$ commutes with the free spin-part. For each spin eigenstate, the σ_{xk} acts as a linear external force on the boson part. With the Wei-Norman algebraic method,[14] we find in the interaction picture

$$U_I(t) \equiv U_I[t; g_k] = \exp[-iC(t) - iA(t)\sigma_{x1}\sigma_{x2}] \times \prod_{k=1,2} \exp[-iB_k(t)a\sigma_{xk}]\exp[-iB_k^*(t)a^\dagger\sigma_{xk}], \tag{8}$$

where the time-dependent parameters are given by

$$B_k(t) = \frac{g_k}{-i\Omega}(e^{-i\Omega t} - 1),$$

$$A(t) = \frac{2g_1g_2}{\Omega}\left(\frac{1}{i\Omega}(e^{i\Omega t} - 1) - t\right),$$

$$C(t) = \frac{g_1^2 + g_2^2}{\Omega}\left(\frac{1}{i\Omega}(e^{i\Omega t} - 1) - t\right). \tag{9}$$

The first way to eliminate the interaction between the qubit and the data-bus comes from the fact that when $B_k(t)=0$, the evolution operator $U_I(t) \equiv \exp[-iC(t) - iA(t)\sigma_{x1}\sigma_{x2}]$ becomes independent of variables of the large junction. It takes the canonical form capable of two-qubit gate operations. $B_i(t)$ is a periodic function of time and it vanishes at $t_n = 2n\pi/\Omega$ for integer $n=0, \pm 1, \pm 2, \ldots$. At these instants of time, the time evolution operator becomes explicitly as

$$U_I(t_n) = \exp\left[-i\frac{4n\pi g_1 g_2}{\Omega^2}\sigma_{x1}\sigma_{x2}\right],$$

up to a phase factor $\exp[-i2n\pi(g_1^2+g_2^2)/\Omega^2]$ in the interaction picture. This is equivalent to a system of two coupled qubits with an interaction of the form $\propto\sigma_{x1}\sigma_{x2}$.

For a two-qubit system governed by a Hamiltonian of the form $g\sigma_{x1}\sigma_{x2}$, one usually adjusts the evolution time to realize an arbitrary two-bit controlled rotation $U_{xx}(\xi) \equiv \exp[-i\xi\sigma_{x1}\sigma_{x2}]$ at $t=\xi/g$. In our system, the evolution time is fixed at t_n by the requirement of $B_i(t_n)=0$. This is in fact not a problem as experimentally one can vary g_1 and g_2 to affect the desired rotation $U_{xx}(\xi)$ at t_n. We note that g_i depends on the external flux Φ_{xk} of the loop $\Theta_k=\pi\Phi_{xk}/\phi_0$, so it is easy to adjust to the maximum of $g_{k\ \max}=g_k(\Theta_k=(\pi/2))=-(E_c/32E_J)^{1/4}E_{Jk}$. The minimal time for one operation $t_{n\ \min}=2n_{\min}\pi/\Omega$ is given by

$$n_{\min} = \left[\frac{\Omega^2}{2g_1\left(\frac{\pi}{2}\right)g_2\left(\frac{\pi}{2}\right)}\right] + 1, \tag{10}$$

where $[x]$ denotes the integer part of x.

At the same time, we note that the free Hamiltonian part $H_0=\Sigma f_k\sigma_{xk}$ does not vanish during the above discussed two-qubit operation. However, it commutes with the interaction term $\propto\sigma_{x1}\sigma_{x2}$ and simply leads the evolution operator being

$$U_{xx}(\xi) = [e^{-i\xi\sigma_{x1}\sigma_{x2}}e^{-i\Sigma f_k(\xi)\sigma_{xk}}]e^{i\Sigma f_k(\xi)\sigma_{xk}}, \tag{11}$$

i.e., augmented by two single-bit operations with $f_k(\xi)=-E_{Jk}\cos\Theta_k(\xi)$. $\Theta_k(\xi)$ satisfies the equation

$$\frac{n\pi}{4\Omega^2}\left(\frac{8E_c}{E_J}\right)^{1/2}E_{J1}E_{J2}\sin\Theta_1\sin\Theta_2 = \xi. \tag{12}$$

Alternatively the influence of the large junction on the two-qubit logic operation can be removed by using two operations in succession, a concept as presented recently for a two-qubit gate on trapped ions.[19] The two steps are described as follows.

First, we evolve the system with the Hamiltonian H for a duration $\tau/2$. The evolution operator (in the interaction picture) is

$$U_I\left(\frac{\tau}{2}\right) = U_I\left[\frac{\tau}{2};g_k\right]. \tag{13}$$

Second, at time $t=\tau/2$, we instantly reverse the direction of the magnetic field such that a sudden change of flux from Φ_{xk} to $-\Phi_{xk}$ occurs that leads to $g_k'=-g_k$ and $f_k'=f_k$. The system is then driven by a new Hamiltonian $H'=H(-g_k,f_k)$, with which we evolve for another $\tau/2$. Since $H_0=H_0'$ and $H_I=-H_I'$, the evolution operator (in the interaction picture) becomes

$$U_I'\left(\frac{\tau}{2}\right) = U_I\left[\frac{\tau}{2};-g_k\right] = \exp[-iC'(t)-iA'(t)\sigma_{x1}\sigma_{x2}] \times \prod_{k=1,2}\exp[-iB_k'(t)a\sigma_{xk}]\exp[-iB_k'^*(t)a^\dagger\sigma_{xk}]. \tag{14}$$

The combined dynamics from the above two steps is now described by the following evolution operator:

$$\begin{aligned}\widetilde{U_I}(\tau) &= U_I'\left(\frac{\tau}{2}\right)U_I\left(\frac{\tau}{2}\right)\\ &= U_I\left(\frac{\tau}{2};-g_k\right)U_I\left(\frac{\tau}{2};g_k\right)\\ &= \exp[-iM(\tau)\sigma_{x1}\sigma_{x2}]\exp[-iN(\tau)],\end{aligned} \tag{15}$$

where

$$M(\tau) = A'\left(\frac{\tau}{2}\right) + A\left(\frac{\tau}{2}\right) = \frac{2g_1g_2}{\Omega}\left(\frac{2}{\Omega}\sin\frac{\Omega\tau}{2} - \tau\right), \tag{16}$$

and

$$N(\tau) = C'\left(\frac{\tau}{2}\right) + C\left(\frac{\tau}{2}\right) = \frac{g_1^2+g_2^2}{\Omega}\left(\frac{2}{\Omega}\sin\frac{\Omega\tau}{2} - \tau\right). \tag{17}$$

Again we see that after the above two successive operations, we realize an effective controlled rotation of the two qubits, i.e., $U_{xx}(\xi)\equiv\exp[-i\xi\sigma_{x1}\sigma_{x2}]$ can be realized by fixing $\Theta_k=\pi/2$ or $\Phi_{xk}=\phi_0/2$ and for a time t satisfying $M(\tau)=\xi$. As a bonus we find that $f_k=0$ in this case, i.e. the single-bit rotation vanishes automatically during the two-bit rotation. With both of the above approaches, single bit operations can be cleanly implemented by setting $\Theta_k=\pi$ or $g_k=0$.

IV. DISCUSSION AND CONCLUSION

We now consider the implementation of our protocol in realistic experiments. We note the typical time for gate operation is

$$\tau_o = \frac{2\pi\Omega}{g^2} = \rho\frac{2\pi}{g}, \tag{18}$$

with g to be roughly understood as the order of magnitude of g_k and $2\pi/g$ approximately the single qubit operation time. One can increase the ratio $\rho=g/\Omega$ to shorten these operation times similar to other JJ coupling schemes.[7–12] It is important to emphasize, however, most of these other schemes are based on an adiabatic evolution of the data-bus, thus are limited to a weak coupling, or, a small ρ. In contrast, our protocol is not confined to the adiabatic dynamics, thus can operate with a larger ρ for a faster gate. In fact, even when the capacitance of the large junction is much larger than the other capacitances, the ratio g/Ω can still be made large with realistic experimental parameters. For example, if we take $E_J=800$ (μeV), $E_c=10$ (μeV), $E_{ci}=200$ (μeV), $E_{Jk}=200$ (μeV), $\Phi_{xk}=\phi_0/2$,[2,21] we find $g/\Omega\simeq0.23$, a limit may be prohibited for other schemes yet works well for our protocol.

A key requirement for our protocol is the precise control of the operation time, especially in the second approach which needs a sudden switch of the magnetic field during each two-qubit operation. However, the second scheme need not to change the magnitude of the magnetic field but just to switch the direction of it. Practically the instantaneous switch never happens since each manipulation costs time. This will bring certain errors to the desired operation.

Before concluding, we wish to remark that, although our circuit resembles that of You *et al.*,[12] our operating scheme and the underlying physics is different. The essential difference is that our gate does not require an adiabatic operation, thus is faster.

In conclusion, we have presented an efficient protocol for implementing controlled interactions between two charge-phase qubits. We have discussed two alternative approaches to realize our protocol. Our scheme seems to work over a wide parameter range and is faster than most existing protocols. From the experimental point of view, our protocol seems advantageous as it only needs the control of one system parameter, that is the external applied magnetic field. We have adopted a setup involving charge-phase qubit, thus our protocol is less sensitive to charge fluctuations along with phase fluctuation.[2] In addition, the Cooper pair box of each qubit could be replaced with dcSQUID and the coupling with the large junction can be turned on and off for further convenience. The circuit arrangement then can be scaled up to larger number of JJ qubits with similar controls.

Since the spin-boson structure is quite common for many kinds of Josephson junction entanglement problems, the underlying mathematics used here may be generalized to some other entanglement schemes, for example, with the nanomechanical resonator as a data bus to entangle qubits.

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