

Indirect driving of a cavity-QED system and its induced nonlinearityYusuf Turek,¹ L. P. Yang,^{1,2} W. Maimaiti,¹ Yong Li,^{2,3,*} and C. P. Sun^{2,3,†}¹*State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, and University of the Chinese Academy of Sciences, Beijing 100190, China*²*Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, Anhui 230026, China*³*Beijing Computational Science Research Center, Beijing 100084, China*

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The linear driving for a single-mode optical field in a cavity can result from the external driving of a classical field even when the coupling between the classical field and the cavity is weak. We revisit this well-known effect with a microscopic model where a classical field is applied to a wall of the cavity to excite the atoms in the wall, and recombination of the low excitations of the wall mediates a linear driving for the single-mode field inside the cavity. With such modeling about the indirect driving through the quantum excitations of the wall, we theoretically predict several nonlinear optical effects for the strong-coupling cases, such as photon antibunching and photon squeezing. We propose a greatly simplified nonlinear quantum photonics model.

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I. INTRODUCTION

Photons are a prime candidate for quantum information processing such as quantum computing and long-distance quantum communication, as they can be easily generated and can travel long distances with high coherence. Due to the ability of obtaining photon-photon interactions, the nonlinear optical process has great advantages in quantum information processing and quantum computation compared with linear optics methods, and possesses great potential for a variety of emerging technologies [1]. However, there is no direct interaction between single photons in physics according to the quantum electrodynamics (QED). Hence, it is of great importance to achieve the interaction between single photons. The most popular method to generate the strong nonlinear effects between photons is spontaneous parametric down conversion, which is used especially as a source of entangled photon pairs [2,3]. Examples of such quantum optical phenomena have been investigated experimentally including generation of quadrature squeezing states and two-photon entanglement states in various degrees of freedom [4–12].

Generally, the typical nonlinear optical phenomenon occurs only at very high optical intensities and the degree of nonlinearity between single photons is very low [13–15]. However, producing high-degree nonlinearity at very low mean-photon level is desirable in many quantum information processing applications [16,17], such as the photon blockade effect, which plays an important role as an effective single-photon source in quantum information processing. Recently, some similar nonlinear effects such as cross phase modulation [18,19] and spontaneous down conversion [20,21] have been observed with a single-photon level pump. These nonlinearities at single-photon level are also obtained through an optical cavity in which photon-photon interaction is relatively strong [4]. Recently, Gupta *et al.* [22] experimentally investigated the Kerr nonlinearity and dispersive optical bistability of a Fabry-Perot

(FP) optical cavity arising from the long-lived coherent motion of ultracold atoms trapped within. They reported that the strong nonlinearity would be observed at low average intracavity photon number level $\bar{n} = 0.05$, and even at as low as $\bar{n} = 10^{-4}$. The photon blockade effect was also found in optomechanical systems [23], where the Kerr interaction between photons is induced by the strong optomechanical coupling.

It is known that the linear coupling between an external classical field and a single-mode cavity can result in a linear driving to create a coherent state of the cavity field. If we assume that the single-mode cavity field is driven by an external driving field with frequency ω_f , then the Hamiltonian can be written as

$$V_f = \omega a^\dagger a + f_0 a^\dagger e^{-i\omega_f t} + \text{H.c.},$$

where a^\dagger (a) is the creation (annihilation) operator of the single-mode radiation field and f_0 is the related driving strength. The corresponding energy spectrum of the output field of the cavity is of the Lorentz form. However, the underlying mechanism to explain this simple phenomenon is not clear until now. In this paper, we revisit this well-known effect by giving a microscopic explanation of physical mechanism for such linear driving. Here, a classical field is applied to one wall of a FP cavity, which is modeled as a two-level atomic ensemble where the two levels can be imagined as excited and nonexcited states of local excitons. When the decay rate of the atomic ensemble is much larger than the decay of the cavity, the recombination of the low excitations of the ensemble in the wall will mediate a linear driving for the single-mode field inside the cavity. Furthermore, if the higher-order excitation of the atomic ensemble is taken into account, the single-mode cavity field exhibits some interesting nonlinear photonic phenomena.

When the external driving field is weak, but the coupling between atomic ensemble (the cavity wall) and the cavity mode is strong, the Kerr nonlinear effect is dominant in the effective Hamiltonian of photons, which produces the photon blockade phenomena. In this case, we found that the strong nonlinearity

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as well as photon blockade of our system would occur at a low intracavity photon number, even though as low as $\bar{n} \simeq 10^{-4}$.

On the contrary, for the case of strong driving and weak coupling (between the atomic ensemble and the cavity), the light-squeezing nonlinear effect is dominant. For weak coupling between the atomic ensemble and single-mode cavity field we found that in our system the optical bistability, even multistability phenomena would appear with increasing the driving strength. From the output intensity spectrum of single-mode cavity field, we found that the squeezed effect of the output field occurs when the driving strength increases. We also found that the maximum squeezing takes place at the vicinity of the resonance point.

This paper is organized as follows: In Sec. II, we describe our model with an effective Hamiltonian in terms of collective low excitation operators of atomic ensemble (cavity wall), and present the clear microscopic explanation to indirect quantum driving. In Sec. III, we consider the effects of higher-order excitation of atomic ensemble (cavity wall) and obtain the effective Hamiltonian of the single-mode cavity field, which describes the very interesting nonlinear photonic phenomena by controlling some corresponding parameters of the system. In Sec. IV, we study the two extreme cases separately, and calculate the second-order correlation function and output spectrum. Finally, we conclude and give some remarks to our work in Sec. V.

II. SIMPLIFIED MODEL FOR NONLINEAR PHOTONICS AND ITS LINEAR LIMIT

In this section, we build a microscopic model to explain the quantum driving. Here, we assume that a wall (left wall) of the cavity consists of a vast number of two-level systems (TLSs), which can be viewed as an atomic ensemble. The two levels can be imagined as the excited and nonexcited states of the local exciton. As shown in Fig. 1, the left wall of the cavity is driven by a classical external field with frequency ω_f . The model Hamiltonian reads as (hereafter we take $\hbar = 1$)

$$H = \omega_c c^\dagger c + \sum_{i=1}^N \left\{ \frac{\omega_a}{2} \sigma_z^{(i)} + [(g c + \Omega e^{-i\omega_f t}) \sigma_+^{(i)} + \text{H.c.}] \right\}, \quad (1)$$

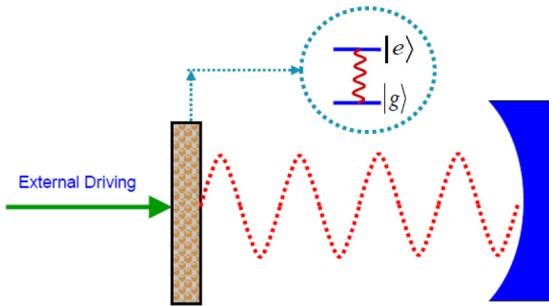


FIG. 1. (Color online) Schematic of indirect quantum driving model. The wall of the cavity consists of N two-level atoms with the same energy difference ω_a . A classical field with frequency ω_f is applied to excite these atoms to generate a linear driving for the single-mode cavity field.

where, c (c^\dagger) is the annihilation (creation) operator of the single-mode cavity field with frequency ω_c , the Pauli matrices $\sigma_z^{(i)} = |e_i\rangle\langle e_i| - |g_i\rangle\langle g_i|$, $\sigma_+^{(i)} = |e_i\rangle\langle g_i|$, and $\sigma_-^{(i)} = |g_i\rangle\langle e_i|$ describe the i th atom with the ground (excited) states $|g_i\rangle$ ($|e_i\rangle$) and energy level spacing ω_a ; N is the number of the two-level atoms, and ω_f is the frequency of the classical driving field. For simplicity, we take the uniform driving strength $\Omega_i = \Omega$ and cavity-atom coupling constant $g_i = g$.

To explore the effects and phenomena resulting from the above, we take the Holstein-Primakoff (H-P) transformation [24] for the collective atomic operators

$$\sum_{i=1}^N \sigma_+^{(i)} = B^\dagger \sqrt{N - B^\dagger B}, \quad (2)$$

$$\sum_{i=1}^N \sigma_-^{(i)} = \sqrt{N - B^\dagger B} B, \quad (3)$$

and

$$\sum_{i=1}^N \sigma_z^{(i)} \equiv 2B^\dagger B - N. \quad (4)$$

Here, the B and B^\dagger represent the atomic collective excitation operators. In the low excitation limit $\langle B^\dagger B \rangle / N \ll 1$, we have [25–27]

$$B^\dagger \approx \frac{1}{\sqrt{N}} \sum_{i=1}^N \sigma_+^{(i)}, \text{ and } B \approx \frac{1}{\sqrt{N}} \sum_{i=1}^N \sigma_-^{(i)}, \quad (5)$$

where the operator B satisfies the standard bosonic commutation relation $[B, B^\dagger] \approx 1$. Using these relations (4) and (5), in the interaction picture with respect to $H_0 = \omega_f (c^\dagger c + B^\dagger B)$ we can rewrite our model Hamiltonian (1) in terms of the atomic collective operators B and B^\dagger as

$$H^{(0)} = \Delta_c c^\dagger c + \Delta_b B^\dagger B + (G c B^\dagger + \chi B + \text{H.c.}), \quad (6)$$

where $\Delta_c = \omega_c - \omega_f$ is the detuning between the single-mode cavity and external driving field and $\Delta_b = \omega_a - \omega_f$ the detuning between the two-level atom and external field, $G = g\sqrt{N}$ and $\chi = \Omega\sqrt{N}$. For simplicity here, we assumed all these coupling strengths are real. We note that during the derivation of Eq. (6) we have neglected a constant term $N\omega_b/2$ since it has no effect on our results in the context.

The quantum Langevin equations of variables of our system are obtained from Eq. (6) as

$$\dot{c}(t) = -i\Delta_c c(t) - iGB(t) - \frac{\kappa}{2}c(t) + \sqrt{\kappa}c_{\text{in}}(t), \quad (7)$$

$$\dot{B}(t) = -i\Delta_b B(t) - iGc(t) - i\chi - \frac{\gamma}{2}B(t) + \sqrt{\gamma}B_{\text{in}}(t), \quad (8)$$

where κ is the decay rate of the cavity, γ is the decay rate of collective mode B , and $c_{\text{in}}(t)$ and $B_{\text{in}}(t)$ are zero-mean noise operators (i.e., $\langle c_{\text{in}} \rangle = \langle B_{\text{in}} \rangle = 0$) satisfying the fluctuation relations

$$\langle c_{\text{in}}(t)c_{\text{in}}^\dagger(t') \rangle = [n(\omega_c) + 1]\delta(t - t'), \quad (9a)$$

$$\langle B_{\text{in}}(t)B_{\text{in}}^\dagger(t') \rangle = [n(\omega_b) + 1]\delta(t - t'), \quad (9b)$$

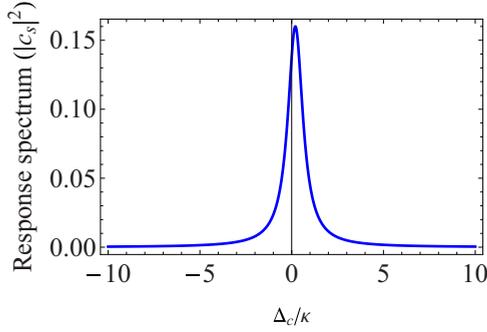


FIG. 2. (Color online) Cavity response vs detuning. Here, $\omega_a - \omega_c = 5 \times 10^4$, $N = 1 \times 10^4$, $\Omega = 1$, $g = 1$. All parameters are in the units of cavity decay rate, κ . In this figure the shift amount is $\delta = 0.2$.

where

$$n(\omega_r) = \frac{1}{\exp\left(\frac{\omega_r}{k_B T}\right) - 1}, \quad (r = b, c) \quad (10)$$

are the average thermal excitation numbers of the cavity mode and atomic collective modes at temperature T , respectively.

In our system, the natural lifetime of the excited atom is much smaller than the lifetime of a photon in the cavity, $\gamma^{-1} \ll \kappa^{-1}$. Thus we can eliminate adiabatically the degrees of freedom of the atomic ensemble by substituting the steady-state solution of Eq. (8) into Eq. (7) and obtain

$$\dot{c}(t) = -i\Delta_{\text{eff}}^{(0)}(t) - \frac{1}{2}\kappa c(t) - if + \sqrt{\kappa}c_{\text{in}}(t). \quad (11)$$

Here, $\Delta_{\text{eff}}^{(0)} = \omega_{\text{eff}}^{(0)} - \omega_f$ is the detuning between the driving field and the effective frequency of the single-mode cavity $\omega_{\text{eff}}^{(0)} = \omega_c - \delta$ with the atomic-ensemble-induced shift $\delta = Ng^2/\omega_{ba}$ and $\omega_{ac} = \omega_a - \omega_c$, and $f = -G\chi/\Delta_b$ is the induced driving amplitude of the single-mode cavity. Here, we assume that the laser is detuned sufficiently far from resonance that $|\Delta_b| \gg \gamma, \Omega$. Under this condition, the driven atomic ensemble only modifies the resonance frequency of the cavity, and the correction of the cavity decay rate has been neglected.

From Eq. (11), we obtain an effective Hamiltonian of the single-mode cavity field as

$$H_{\text{eff}} = \omega_{\text{eff}}^{(0)} c^\dagger c + (f c^\dagger e^{-i\omega_f t} + \text{H.c.}), \quad (12)$$

which describes a typical model of a quantum harmonic oscillator driven by a classical field with strength f and frequency ω_f .

As shown in Fig. 2, the driving on the wall of the cavity induces an effective driving for the cavity mode and the spectrum of the output field is of the Lorentz form centered at the effective frequency $\omega_{\text{eff}}^{(0)}$. This is the so-called indirect driving.

III. NONLINEAR PHOTONIC EFFECT DUE TO LARGER EXCITATION

To investigate the effect of high-order expansion terms of H-P transformation for collective atomic operators to our system we take the first-order expansion term into account,

and obtain

$$\sum_{i=1}^N \sigma_+^{(i)} \approx \sqrt{N} B^\dagger \left(1 - \frac{B^\dagger B}{2N}\right), \quad (13)$$

$$\sum_{i=1}^N \sigma_-^{(i)} \approx \sqrt{N} \left(1 - \frac{B^\dagger B}{2N}\right) B, \quad (14)$$

and

$$S_z = \sum_{i=1}^N \sigma_z^{(i)} = B^\dagger B - \frac{N}{2}. \quad (15)$$

For this case, the Hamiltonian (6) is rewritten as

$$H' = H^{(0)} + H^{(1)}, \quad (16)$$

where the first part is the zeroth-order form as given in Eq. (6) and the second part is

$$H^{(1)} = -\frac{1}{2N} (GcB^{\dagger 2}B + \chi B^{\dagger 2}B + \text{H.c.}), \quad (17)$$

resulting from the first-order expansion of collective excitation operator. We can see that, for the low excitation case $\langle B^\dagger B \rangle/N \ll 1$, the effect of $H^{(1)}$ can be neglected, and the Hamiltonian (16) reduces to the zeroth-order Hamiltonian (6).

We note that the adiabatic elimination does not depend on the number of the atoms in the ensemble [28]. Thus here, we can still use the adiabatic elimination method to get the effective Hamiltonian of the single-mode cavity field. By taking the same procedure as in Sec. II, we obtain the effective Hamiltonian of the single-mode cavity field as

$$\tilde{H}_{\text{eff}} = \Delta_{\text{eff}}^{(1)} c^\dagger c + \chi_{\text{kerr}} c^{\dagger 2} c^2 + [\mu c^2 + \zeta c^\dagger c^2 + Fc + \text{H.c.}]. \quad (18)$$

Here,

$$\Delta_{\text{eff}}^{(1)} = \Delta_c - \frac{Ng^2}{\Delta_b} + \chi_{\text{kerr}} + 4\mu, \quad (19a)$$

$$\chi_{\text{kerr}} = \frac{Ng^4}{\Delta_b^3}, \quad (19b)$$

$$\mu = \frac{Ng^2\Omega^2}{\Delta_b^3}, \quad (19c)$$

$$\zeta = \frac{2Ng^3\Omega}{\Delta_b^3}, \quad (19d)$$

and

$$F = -\frac{Ng\Omega}{\Delta_b} \left[1 - \frac{2\Omega^2 + g^2}{\Delta_b^2}\right]. \quad (20)$$

From this result we can see that if we take the the first-order expansion of the collective atomic operators, the effects of photonic nonlinearity appear. Here, the term $c^{\dagger 2}c^2$ characterizes the Kerr effect with the strength χ_{kerr} , c^2 characterizes the squeezing effect with the strength μ , and $c^\dagger c^2$ denotes the two photon phase-space filling effect with the strength ζ . We note that if $|\Delta_b| \gg \sqrt{Ng}, \Omega$ the effects of these nonlinear terms are negligible. We also note that the strengths of these terms

can be controlled and enhanced separately by mediating the corresponding parameters.

If the atom-cavity coupling strength is much larger than that of the external driving field, i.e., $g \gg \Omega$, the above effective Hamiltonian (18) of the single-mode cavity field reduces to

$$H_1 = \Delta_{\text{eff},1} c^\dagger c + \chi_{\text{kerr}} c^{\dagger 2} c^2 + (F' c + \zeta c^\dagger c^2 + \text{H.c.}), \quad (21)$$

where we have neglected the squeezing term, and

$$\Delta_{\text{eff},1} = \Delta_c - \frac{Ng^2}{\Delta_b} + \chi_{\text{kerr}}, \quad (22)$$

and

$$F' = -\frac{Ng\Omega}{\Delta_b} + \frac{1}{2}\zeta, \quad (23)$$

are the corrected detuning and driving strength, respectively. The strength of the Kerr term χ_{kerr} is related to the number of the atoms N , the atomic detuning Δ_b , and the atom-cavity coupling strength g , but independent of Ω . Thus, we can enhance the Kerr term effect by mediating g , Δ_b with fixed number of the atoms. In this case, we can investigate the photon statistical properties of the single-mode cavity field.

If the strength of external driving field is much larger than the atom-cavity coupling strength, i.e., $\Omega \gg g$, the total effective Hamiltonian (18) reduces to

$$H_2 = \Delta_{\text{eff},2} c^\dagger c + (F'' c + \mu c^2 + \zeta c^\dagger c^2 + \text{H.c.}). \quad (24)$$

Here,

$$\Delta_{\text{eff},2} = \Delta_c - \frac{Ng^2}{\Delta_b} + 4\mu, \quad (25)$$

and

$$F'' = -\frac{Ng\Omega}{\Delta_b} + \frac{2Ng\Omega^3}{\Delta_b^3}. \quad (26)$$

We can see that in this particular case the dominant squeezing effect of light and other correlated photonic nonlinear effects can be directly controlled by the external driving strength. In this case, we can calculate the output squeezing spectrum of the cavity field to investigate the efficiency of our scheme to generate the squeezed photonic state.

Note that in both the cases discussed above we have to consider the two-photon phase-space filling effect term, whose strength is characterized by ζ . Since its strength is related to g and Ω , it will directly effect the investigated both nonlinear phenomena. In the next section, we will investigate the above two particular cases separately.

IV. SECOND-ORDER CORRELATION: PHOTON ANTIBUNCHING

In this section, we study the first case $g \gg \Omega$, where the Kerr effect is dominant. To investigate the photon statistics of the single-mode cavity radiation field, we will calculate the second-order correlation function at zero time delay, $g^{(2)}(0) = \langle a^\dagger a^\dagger a a \rangle / \langle a^\dagger a \rangle^2$. We will begin our calculation by writing the master equation of our system with Hamiltonian (21)

$$\begin{aligned} \dot{\rho} = & -i[H_1, \rho] + \kappa(n_{th} + 1)(2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c) \\ & + \kappa n_{th}(2c^\dagger \rho c - cc^\dagger \rho - \rho cc^\dagger), \end{aligned} \quad (27)$$

where, $n_{th} = n(\omega_c)$ is the thermal occupation number of the single-mode cavity field as defined in (10). As we see, in our system the operator equation is nonlinear, in this case it is useful to use the c -number Fock-Planck equation.

The density matrix of the cavity mode in the generalized P representation function [4] reads

$$\rho = \int \Lambda(\alpha) P(\alpha, \beta) d\mu(\alpha, \beta), \quad (28)$$

where $(\alpha) = (\alpha, \beta) \equiv (\alpha, \alpha^\dagger)$, and in the generalized P representation α and α^\dagger are independent variables. The nondiagonal coherent state projection operator is defined as

$$\Lambda(\alpha) = \frac{|\alpha\rangle\langle\beta^*|}{\langle\beta^*|\alpha\rangle}. \quad (29)$$

The corresponding Fock-Planck equation of ρ in the P representation is written as

$$\begin{aligned} \frac{\partial P(\alpha)}{\partial t} = & \frac{\partial}{\partial \alpha} [\kappa' \alpha + \zeta'(\alpha^2 + 2\alpha^* \alpha) + 2\chi'' \alpha^* \alpha^2 - E] P \\ & + \frac{\partial}{\partial \alpha^*} [\kappa'^* \alpha^* + \zeta'^*(\alpha^{*2} + 2\alpha \alpha^*) + 2\chi''^* \alpha \alpha^{*2} - E^*] \\ & \times P - \frac{\partial^2}{\partial \alpha^2} [\chi'' \alpha^2 + \zeta'' \alpha] P - \frac{\partial^2}{\partial \alpha^{*2}} \\ & \times [\chi''^* \alpha^{*2} + \zeta''^* \alpha^*] P + 2\kappa n_{th} \frac{\partial^2}{\partial \alpha \partial \alpha^*} P. \end{aligned} \quad (30)$$

Here, $\kappa' = \kappa + i\Delta_{\text{eff},1}$, $\chi'' = i\chi_{\text{kerr}}$, $\zeta' = i\zeta$, and $E = -iF'$. In the P representation, α and α^* are independent complex variables and the Fokker-Planck equation has a positive semidefinite diffusion matrix in four-dimensional space. This allows us to define the equivalent stochastic differential equations using the Ito rules [4]

$$\begin{aligned} \frac{\partial}{\partial t} \begin{pmatrix} \alpha \\ \alpha^* \end{pmatrix} = & \begin{pmatrix} E - \kappa' \alpha - 2\chi'' \alpha^\dagger \alpha^2 - \zeta'(\alpha^2 + 2\alpha^\dagger \alpha) \\ E^* - \kappa'^* \alpha^\dagger - 2\chi''^* \alpha^\dagger \alpha^2 - \zeta'^*(\alpha^\dagger \alpha^2 + 2\alpha^\dagger \alpha) \end{pmatrix} \\ & + \begin{pmatrix} -2\chi'' \alpha^2 - 2\zeta' \alpha & 2\kappa n_{th} \\ 2\kappa n_{th} & -2\chi''^* \alpha^\dagger \alpha^2 - 2\zeta'^* \alpha^\dagger \end{pmatrix}^{\frac{1}{2}} \\ & \times \begin{pmatrix} \eta_1(t) \\ \eta_1^\dagger(t) \end{pmatrix}, \end{aligned} \quad (31)$$

where $\eta_1(t)$ and $\eta_1^\dagger(t)$ are the δ correlated stochastic forces with zero mean, namely

$$\langle \eta_1(t) \rangle = \langle \eta_1^\dagger(t) \rangle = 0. \quad (32a)$$

$$\langle \eta_1(t) \eta_1^\dagger(t') \rangle = \delta(t - t'), \quad (32b)$$

$$\langle \eta_1(t) \eta_1(t') \rangle = 0, \quad (32c)$$

The semiclassical or mean value of the above equations can be obtained by replacing α^\dagger by the steady-state value α_0^* determined by

$$E - \kappa' \alpha - 2\chi'' \alpha^\dagger \alpha^2 - \zeta'(\alpha^2 + 2\alpha^\dagger \alpha) = 0, \quad (33)$$

where $|\alpha_0|^2 = n_0$ represents the mean intracavity photon number in the steady state.

To investigate the effect of quantum fluctuation to steady state, we consider the very small fluctuation around the steady state by taking $\alpha = \alpha_0 + \alpha_1$. Linearizing Eq. (31), we obtain

the stochastic differential equation for the fluctuation variable $\alpha_1(\alpha_1^\dagger)$ as [4]

$$\frac{\partial}{\partial t} \boldsymbol{\alpha}_1(t) = -\mathbf{A} \cdot \boldsymbol{\alpha}_1(t) + \mathbf{D}^{\frac{1}{2}} \cdot \boldsymbol{\xi}(t). \quad (34)$$

Here, $\boldsymbol{\alpha}_1 = (\alpha_1, \alpha_1^\dagger)^T$, and

$$\mathbf{A} = \begin{pmatrix} \kappa' + 4\chi''n_0 + 4\zeta'\text{Re}(\alpha_0), & 2\chi''\alpha_0^2 + 2\zeta'\alpha_0 \\ 2\chi''^*\alpha_0^{*2} + 2\zeta'^*\alpha_0^*, & \kappa'^* + 4\chi''^*n_0 + 4\zeta'^*\text{Re}(\alpha_0) \end{pmatrix}, \quad (35)$$

represents the drift matrix,

$$\mathbf{D} = \begin{pmatrix} -2\chi''\alpha_0^2 - 2\zeta'\alpha_0 & 2\kappa n_{th} \\ 2\kappa n_{th} & -2\chi''^*\alpha_0^{*2} - 2\zeta'^*\alpha_0^* \end{pmatrix} \quad (36)$$

is the diffusion matrix, and $\boldsymbol{\xi}(t) = [\eta_1(t), \eta_2(t)]^T$.

According to Ref. [29], the correlation matrices

$$\mathbf{C}_{ss} \equiv \begin{pmatrix} \langle a^2 \rangle - \langle a \rangle^2, & \langle a^\dagger a \rangle - |\langle a \rangle|^2 \\ \langle a^\dagger a \rangle - |\langle a \rangle|^2, & \langle a^{\dagger 2} \rangle - \langle a^\dagger \rangle^2 \end{pmatrix} \approx \begin{pmatrix} \langle \alpha_1^2 \rangle, & \langle \alpha_1^\dagger \alpha_1 \rangle \\ \langle \alpha_1^\dagger \alpha_1 \rangle, & \langle \alpha_1^{\dagger 2} \rangle \end{pmatrix} \quad (37)$$

in the steady state can be evaluated by

$$\mathbf{C}_{ss} = \begin{pmatrix} C_{11} & C_{12} \\ C_{12} & C_{11}^* \end{pmatrix} \quad (38)$$

$$= \frac{\mathbf{D} \text{Det}(\mathbf{A}) + [\mathbf{A} - \text{Tr}(\mathbf{A})\mathbf{I}]\mathbf{D}[\mathbf{A} - \text{Tr}(\mathbf{A})\mathbf{I}]^T}{2\text{Tr}(\mathbf{A})\text{Det}(\mathbf{A})}. \quad (39)$$

Here, $\text{Tr}(\mathbf{A})$ and $\text{Det}(\mathbf{A})$ are the trace and the determinant of matrix \mathbf{A} , respectively. We calculate the correlation matrices elements,

$$C_{11} = \frac{-2\kappa[\kappa' + 4\chi''n_0 + 4\zeta'\text{Re}(\alpha_0)]^*(\chi''\alpha_0^2 + \zeta'\alpha_0)(1 + 2n_{th})}{\text{Tr}(\mathbf{A})\text{Det}(\mathbf{A})}, \quad (40)$$

and

$$C_{12} = \frac{2\kappa n_{th}|\kappa' + 4\chi''n_0 + 4\zeta'\text{Re}(\alpha_0)|^2 + 4\kappa|\chi''\alpha_0^2 + \zeta'\alpha_0|^2}{\text{Tr}(\mathbf{A})\text{Det}(\mathbf{A})}, \quad (41)$$

respectively. Then, we obtain the total photon number inside the cavity including the quantum fluctuation effect as

$$\bar{n} = n_0 + C_{12}. \quad (42)$$

For the zero-temperature case $n_{th} = 0$, the above equation changes into

$$\bar{n} = n_0 + \frac{2|\chi_{\text{kerr}}\alpha_0^2 + \zeta\alpha_0|^2}{\text{Det}(\mathbf{A})}. \quad (43)$$

It follows from Eq. (43) that if $g = 0$ the above intercavity photon number is zero since all the parameters in our system are proportional to the coupling coefficient g .

The second-order correlation function can also be calculated easily from the above correlation matrices elements as

$$\begin{aligned} g^{(2)}(0) &\approx 1 + 2\frac{\langle \alpha_1^\dagger \alpha_1 \rangle}{n_0} + 2\text{Re}\left(\frac{\langle \alpha_1^2 \rangle}{\alpha_0^2}\right) \\ &= 1 + \frac{2C_{12}}{n_0} + 2\text{Re}\left(\frac{C_{11}}{\alpha_0^2}\right). \end{aligned} \quad (44)$$

Generally, the thermal fluctuation would increase the second-order correlation function at zero time delay $g^{(2)}(0)$

to above unity. To optimize $g^{(2)}(0)$ to investigate the photon antibunching effect of the system, we only consider the zero-temperature case, $n_{th} = 0$. Thus the above second-order correlation Eq. (44) is only related to the quantum fluctuation effect. The intracavity photon number and second-order correlation function at zero time delay vs the coupling strength g is shown in Fig. 3. We just consider the case where the driving field is resonant with the cavity field $\Delta_c = 0$, but largely detuning from the atoms. As shown in Fig. 3(a), the intracavity photon number will be much lower than one as increasing the coupling strength g , and the corresponding $g^{(2)}(0)$ displays the typical antibunching behavior as shown in Fig. 3(b). The strong nonlinear effects appear at very low photon number, i.e. as low as $\bar{n} \simeq 10^{-4}$.

V. OUTPUT INTENSITY AND SQUEEZING SPECTRA

In this section, we investigate the second extreme case $\Omega \gg g$, where the squeezing effect of the single-mode cavity field is dominant. To calculate the output fluctuation spectrum, we write down the quantum Langevin equation of the cavity mode

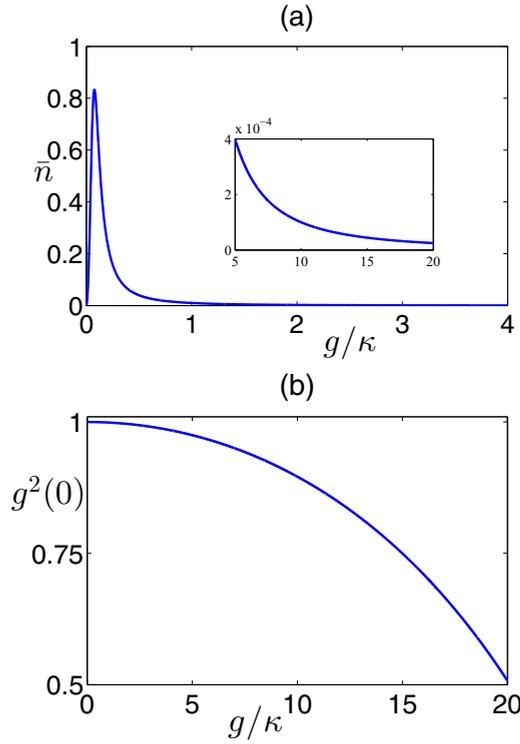


FIG. 3. (Color online) The second-order correlation function $g^{(2)}(0)$, in Eq. (44) and total intracavity photon number in Eq. (42) vs coupling strength g . Here we take the parameters as : $\Delta_c = 0$, $\Delta_b = 45$, $N = 1 \times 10^4$, $\Omega = 0.1$. All the parameters are in the units of κ .

according to the Hamiltonian (24) as

$$\dot{c} = -\left(\frac{\kappa}{2} + i\Delta_{\text{eff},2}\right)c - 2i\mu c^\dagger - i\zeta(2c^\dagger c + c^2) - iF'' + \sqrt{\kappa}c_{\text{in}}. \quad (45)$$

Here, $c_{\text{in}}(t)$ is the noise operator and satisfies the fluctuation relations as listed in Eq. (9a). The steady state value of c is determined by

$$F'' - i\left(\frac{\kappa}{2} + i\Delta_{\text{eff},2}\right)c_s + 2\mu c_s^* + \zeta(2|c_s|^2 + c_s^2) = 0. \quad (46)$$

To study the influence of the quantum fluctuation, we split the operator c into two parts $c = c_s + \delta c$. Here, δc represents the fluctuation operator, which has a vanishing mean value, i.e., $\langle \delta c \rangle = 0$. Thus, after the linearization, the Langevin equation (45) is rewritten as

$$\delta \dot{c} = -\left[\frac{\kappa}{2} + i\Delta_{\text{eff},2} + 4i\zeta \text{Re}(c_s)\right]\delta c - 2i(\mu + \zeta c_s)\delta c^\dagger + \sqrt{\kappa}c_{\text{in}}. \quad (47)$$

By taking the Fourier transformation, we have

$$\delta c(\omega) = \frac{\sqrt{\kappa}}{D(\omega)}[-iBc_{\text{in}}^\dagger(\omega) + A^*(-\omega)c_{\text{in}}(\omega)]. \quad (48)$$

where

$$A(\omega) = -i\omega + \left[\frac{\kappa}{2} + i\Delta_{\text{eff}}^{(2)} + 4i\zeta \text{Re}(c_s)\right], \quad (49a)$$

$$B = 2(\mu + \zeta c_s), \quad (49b)$$

$$D(\omega) = A(\omega)A^*(-\omega) - |B|^2. \quad (49c)$$

In the frequency space, the noise operators satisfy the following relations

$$\langle c_{\text{in}}(\omega)c_{\text{in}}^\dagger(\omega') \rangle = [n(\omega_c) + 1]\delta(\omega + \omega'), \quad (50a)$$

$$\langle c_{\text{in}}^\dagger(\omega)c_{\text{in}}(\omega') \rangle = n(\omega_c)\delta(\omega + \omega'), \quad (50b)$$

$$\langle c_{\text{in}}(\omega)c_{\text{in}}(\omega') \rangle = \langle c_{\text{in}}^\dagger(\omega)c_{\text{in}}^\dagger(\omega') \rangle = 0. \quad (50c)$$

The input-output relationship is given by $c_{\text{out}} = \sqrt{\kappa}c - c_{\text{in}}$. After a linearization of the input-output fields around the steady-state value, the corresponding relationship between input and output fluctuation operators in the frequency space reads as

$$\delta c_{\text{out}}(\omega) = \sqrt{\kappa}\delta c(\omega) - c_{\text{in}}(\omega). \quad (51)$$

The output intensity spectrum $S_I(\omega)$ [30] is defined as

$$S_I(\omega) = \frac{1}{|c_{\text{out}}|^2} \int d\omega' \langle \delta I_{\text{out}}(\omega)\delta I_{\text{out}}(\omega') \rangle, \quad (52)$$

where

$$\delta I_{\text{out}}(\omega) = c_{\text{out}}^*\delta c_{\text{out}}(\omega) + c_{\text{out}}\delta c_{\text{out}}^\dagger(\omega). \quad (53)$$

By substituting Eq. (48) and Eq. (51) into Eq. (52) and using the noise fluctuation relations (50a)–(50c), we obtain the explicit expression of output intensity spectrum of single-mode cavity field as

$$S_I(\omega) = \left|1 - \frac{\kappa}{D(\omega)}[C(\omega) + ie^{2i\varphi}B^*]\right|^2. \quad (54)$$

Here, φ is the phase of the output field, and its value is determined by the input-output relationship. We note that in

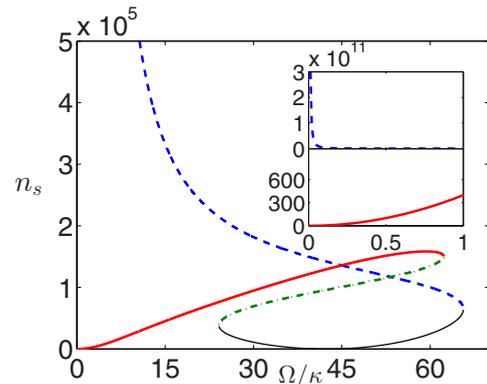


FIG. 4. (Color online) Intracavity photon number vs the driving strength. Here $N = 1 \times 10^4$, $g = 0.1$, $\Delta_c = 1$, $\Delta_b = 60$. All parameters in the unit of the cavity decay rate, κ . The different colors in this figure represents the different steady-state values under same parameters, and the inset in this figure depicts the solid red and dashed blue curves in the interval $\Omega \in (0, 1]$.

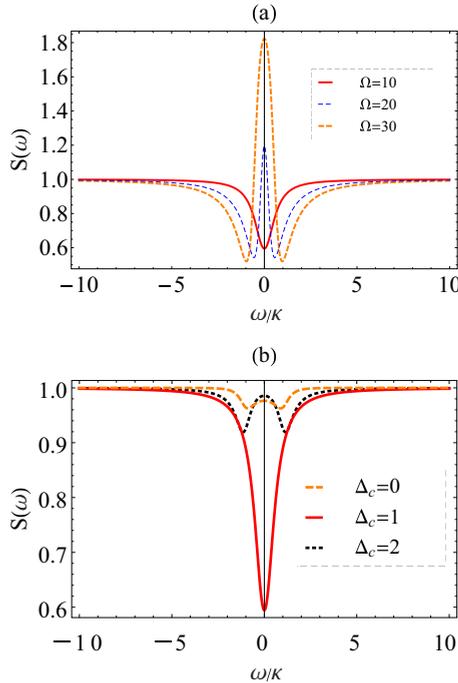


FIG. 5. (Color online) The output intensity spectrum of the single-mode cavity field for (a) different Ω with fixed $\Delta_c = 1$ and (b) different Δ_c with fixed $\Omega = 10$. Here, $N = 1 \times 10^4$, $g = 0.1$, and $\Delta_b = 100$. All the parameters are in the units of the cavity decay rate, κ .

the above calculation the temperature T of the cavity field is assumed to be zero, i.e., $n(\omega_c) = 0$.

The variation of the steady-state field intensity $n_s = |c_s|^2$, which is determined by Eq. (46), as a function of the driving field is given in Fig. 4. It is clear that the bistability and even multistability would occur in our system as increasing the driving field strength. The four lines with different colors represent four different steady-state solutions of Eq. (46). As shown in the subplot of Fig. 4, the solid red line means n_s starts from 0 and increases with the driving strength monotonically, and the dashed blue line corresponds to that n_s starts at infinite

and decreases with the driving strength monotonically. In the following, we choose the value of n_s on the solid red line to do the forward calculations.

To investigate the squeezing effect of the system, we numerically calculate the output intensity spectrum $S(\omega)$. As shown in Fig. 5(a), there would occur the squeezing effect at detuning $\Delta_c = 1$ and the single minimum peak is split into two peaks when the driving strength increases. From Fig. 5(b), we also find that the detuning between the cavity mode and the driving frequency Δ_c also affects the squeezing effect of the output intensity spectrum when the driving strength is fixed.

VI. CONCLUSION AND REMARKS

In this paper we have studied the microscopic mechanism of the external driving for a single-mode cavity field based on an indirect driving model. In this simplified model a wall of the cavity is imagined as an ensemble of local two-level systems. Through this modeling we investigated the nonlinear effects of the single-mode cavity field, which is induced by the recombination of the higher-order excitations of atomic ensemble. By adjusting some parameters there will occur the typical nonlinear phenomena of the single-mode cavity field, such as the photon blockade and squeezing effects.

Our scheme in this paper is closely related to the microscopic description of laser, and can be considered as a simplified nonlinear quantum optical model [31]. Actually, generating entangled photons is very important in quantum information processing and quantum communication. Several schemes to generate entangled photon pairs from three-level systems have been proposed [32–34]. Thus our setup may provide a potential source of entangled photons if we consider the cavity wall consisting of three-level atoms.

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