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Controllable single-photon frequency converter via a one-dimensional waveguide

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We propose a single-photon frequency converter via a one-dimensional waveguide coupled to a three-level V-type atom. An on-demand classical driving field is used to couple to the atom, allowing it to absorb a photon with a given frequency and then emit a photon with a different carried frequency. We study such a single-photon frequency conversion mechanism in two kinds of realistic physical systems: the system of coupled-resonator waveguide with cosine dispersion relation and the one of waveguide with linear dispersion relation. To demonstrate the single-photon transfer efficiency, we introduce the concept of scattering flows via the calculation of group velocities and find that the driving field prefers to be weak in the coupled-resonator waveguide but arbitrarily strong in the linear waveguide to achieve an optimal transfer efficiency. Furthermore, we demonstrate that our theoretical model is experimentally feasible with currently available technologies.

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I. INTRODUCTION

Photon frequency conversion [1–3] refers to transducing the input photons with a given frequency into the output photons with a different frequency while preserving the nonclassical properties. Experimentally, the photon frequency conversion has been achieved in the nonlinear medium by frequency mixing technologies [4–8]. In particular, the single-photon frequency conversion [9] has many applications in quantum communication and quantum information processing, and the effective single-photon frequency conversion scheme [10] has been proposed in a waveguide channel with the assistance of the Sagnac interferometer, which couples to a multilevel emitter [11–13].

Recently, the coherent control of single photons has been studied in the one-dimensional (1D) waveguide(s) with linear [14] or nonlinear [15–17] dispersion relation, where an additional two-level or three-level system acts as a quantum switch. In particular, when a three-level system is applied to couple to two 1D waveguides simultaneously [18,19], the single photons can be transferred from one waveguide to the other, and the carried frequency can be the same or different, depending on whether or not the atom experiences the internal state transition in the scattering process. In the present study, we aim to investigate a single-photon frequency conversion mechanism within only a single 1D waveguide.

To this end, we theoretically propose a single-photon transmission network composed of a waveguide and an embedded three-level atom (or artificial atom) with V-type configuration. One of the two atomic transitions is coupled to the electromagnetic field in the waveguide and the other to the on-demand classical driving field, which is applied to allow the atom to absorb a photon with a given frequency and reemit it with the same or a different frequency. In this sense, the atom provides two scattering channels for the incident photon and functions as a single-photon frequency converter. In our scheme, the transfer efficiency and the frequency difference between the two channels can be controlled by readily adjusting the frequency and strength of the driving field, instead of the atomic energy level configuration, which is usually difficult to tune in realistic physical systems.

In this paper, first we give a general description for the single-photon transmission and frequency conversion mechanism in a 1D waveguide based on the Lippman-Schwinger equation [20]. Then we consider two explicit models: the coupled-resonator waveguide (CRW) [15–19] and linear waveguide [21–23]. For the case of CRW, the single-excitation spectrum of each of the two channels has the structure of one energy band and two discrete bound states, with one of them being above the energy band and the other below it. For the case of a linear waveguide, the related spectrum structures of the channels have only the lower energy bounds with the absence of the upper ones and the bound states. In both cases, the frequency converter only works when the energy of the incident state is within the overlap regime of the spectra of the two channels. To demonstrate the efficiency for the single photon transferring from one channel to the other, we introduce the concept of the scattering flows via calculation of the group velocities in different channels. We find that the classical driving field prefers to be weak in the

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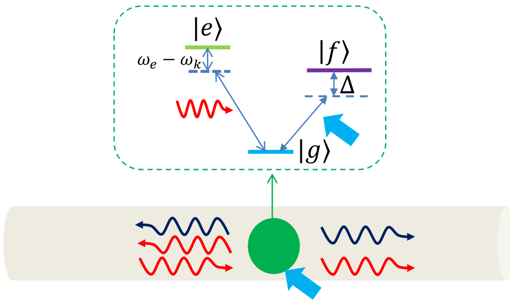

FIG. 1. (Color online) The schematic diagram of the model. A V-type atom is located in a 1D waveguide, serving as a frequency converter for the single photon injected from the left side of the waveguide.

case of CRW, but arbitrarily strong in the case of a linear waveguide to achieve an optimal transfer efficiency.

The rest of the paper is organized as follows. In Sec. II, we present the Hamiltonian in a general waveguide-atom coupled system and give the scattering amplitudes based on the Lippman-Schwinger equation. With these results, we discuss the single-photon scattering and frequency conversion in the 1D CRW with cosine dispersion relation and linear waveguide in Secs. III and IV, respectively. Subsequently, the experimental feasibility of our theoretical model is discussed in Sec. V. In Sec. VI, we draw the conclusions.

II. FORMALISM OF THE FREQUENCY CONVERTER

A. The model and Hamiltonian

As shown in Fig. 1, the system we consider contains a 1D waveguide and a three-level V-type atom. The V-type atom is characterized by a ground state $|g\rangle$, an intermediate state $|f\rangle$, and an excited state $|e\rangle$, whose energies are denoted as ω_g, ω_f, and ω_e, respectively. The ground-state energy ω_g is set to zero as reference. The kth electromagnetic mode in the waveguide couples to the transition $|g\rangle \leftrightarrow |e\rangle$ with the strength J_k, while the classical driving field with frequency ν drives the transition $|g\rangle \leftrightarrow |f\rangle$ with the Rabi frequency η. In the rotating frame with respect to $H_0' = \nu|f\rangle\langle f|$, the Hamiltonian of the system is written as ($\hbar = 1$)

$$H = \Delta|f\rangle\langle f| + \omega_e|e\rangle\langle e| + \eta(|g\rangle\langle f| + |f\rangle\langle g|) + \sum_k \omega_k a_k^\dagger a_k + \sum_k (J_k a_k|e\rangle\langle g| + J_k^* a_k^\dagger|g\rangle\langle e|), \quad (1)$$

where a_k is the annihilation operators for the kth-mode electromagnetic field with frequency ω_k in the waveguide. The dispersion relation between the frequency ω_k and the wave vector k depends on the realistic physical system. $\Delta \equiv \omega_f - \nu$ is the detuning between the atomic transition $|g\rangle \leftrightarrow |f\rangle$ and the classical field.

The first line of the Hamiltonian (1), which describes the coupling between the classical field and the transition $|g\rangle \leftrightarrow |f\rangle$, leads to two dressed states,

$$|\phi_+\rangle = \sin\frac{\theta}{2}|g\rangle + \cos\frac{\theta}{2}|f\rangle, \quad (2a)$$

$$|\phi_-\rangle = -\cos\frac{\theta}{2}|g\rangle + \sin\frac{\theta}{2}|f\rangle, \quad (2b)$$

with the corresponding eigenenergies

$$\nu_\pm = \frac{\Delta \pm \sqrt{\Delta^2 + 4\eta^2}}{2}, \quad (3)$$

where $\tan\theta = 2\eta/\Delta$. In the dressed-state representation, the Hamiltonian can be separated as $H = H_0 + V$, where

$$H_0 = \sum_k \omega_k a_k^\dagger a_k + \omega_e|e\rangle\langle e| + \sum_{n=\pm} \nu_n|\phi_n\rangle\langle\phi_n| \quad (4)$$

is the free Hamiltonian for the waveguide and the atom, and

$$V = \sum_k \left[J_k a_k \left(\sin\frac{\theta}{2}|e\rangle\langle\phi_+| - \cos\frac{\theta}{2}|e\rangle\langle\phi_-| \right) + \text{H.c.} \right] \quad (5)$$

represents their interaction.

B. The Lippman-Schwinger equation and scattering amplitudes

We assume the single photon is initially input from the left end of the 1D waveguide and take the input state as $|k,\phi_n\rangle \equiv a_k^\dagger|0\rangle \otimes |\phi_n\rangle$ ($|0\rangle$ is the photonic vacuum state in the waveguide and $n = \pm$), which represents the atom in the internal state $|\phi_n\rangle$ and the photon in the kth mode. Then the scattering state $|\psi_{kn}^{(+)}\rangle$ is given by the Lippman-Schwinger equation [20,24,25] as

$$|\psi_{kn}^{(+)}\rangle = |k,\phi_n\rangle + \frac{1}{\omega_{kn} - H_0 + i0^+} V|\psi_{kn}^{(+)}\rangle, \quad (6)$$

where the energy of the incident state is $\omega_{kn} = \omega_k + \nu_n$.

Since the excitation number $N = \sum_k a_k^\dagger a_k + |e\rangle\langle e|$ is conserved in this system, the eigenstate with one excitation can be written as

$$|\psi_{kn}^{(+)}\rangle = \sum_p (\alpha_{pn}|p,\phi_-\rangle + \beta_{pn}|p,\phi_+\rangle) + u_{kn}|0,e\rangle. \quad (7)$$

Here, u_{kn} is the probability amplitude of the atom in the excited state, α_{pn} (β_{pn}) is the amplitude for finding one output photon with wave vector p in the waveguide and the atom in the state $|\phi_-\rangle(|\phi_+\rangle)$.

By combining Eqs. (6) and (7), one can get the amplitude for the atom in the excited state,

$$u_{kn} = \frac{\left(\sin\frac{\theta}{2}\delta_{n,+} - \cos\frac{\theta}{2}\delta_{n,-}\right)J_k}{\omega_{kn} - \omega_e - \sin^2\frac{\theta}{2}A_+ - \cos^2\frac{\theta}{2}A_- + i0^+}, \quad (8)$$

with

$$A_\pm = \sum_p \frac{|J_p|^2}{\omega_{kn} - \omega_{p\pm} + i0^+}. \quad (9)$$

The single-photon scattering process can be formulated as follows. The atom initially prepared in one of the dressed states $|\phi_n\rangle$ ($n = \pm$) absorbs the incident photon with frequency ω_k, and emits an outgoing photon with frequency $\omega_{k'}$, with itself passing into either of the dressed states $|\phi_l\rangle$ ($l = \pm$). The energy conservation of the waveguide-atom coupled system implies that $\omega_{k'} = \omega_k$ if $|\phi_l\rangle = |\phi_n\rangle$, and $\omega_{k'} \neq \omega_k$ if $|\phi_l\rangle \neq |\phi_n\rangle$. The evidence on the conservation of energy in the scattering process can be found in the element of the S matrix,

$$S_{k',l\leftarrow k,n} = \delta_{l,n}\delta_{k,k'} - 2\pi i\delta(\omega_{k',l} - \omega_{k,n})T_{k',l\leftarrow k,n}, \quad (10)$$

where the elements of the on-shell T matrix are obtained as [20]

$$T_{k',l\leftarrow k,n} \equiv \langle k',\phi_l|V|\psi_{kn}^{(+)}\rangle = u_{kn}J_{k'}^{*}\left(\sin\frac{\theta}{2}\delta_{l,+} - \cos\frac{\theta}{2}\delta_{l,-}\right). \quad (11)$$

It can be found from Eq. (10) that when the photonic flow is confined to the incident channel, the frequency of the emitted photon is equal to the absorbed one. However, when the incident photon is transferred to another channel, the frequency of the emitted photon will be lowered or raised by the amount $|\nu_+ - \nu_-|$. Consequently, the three-level atom acts as a frequency converter for single photons propagating in the 1D waveguide.

According to the above discussions, the absorption and emission of the single photon by the atom can be formalized as a two-channel scattering process due to the existence of the two available atomic dressed states. Hereafter, we will call them "negative channel" and "positive channel" according to the related atomic states $|\phi_-\rangle$ and $|\phi_+\rangle$, respectively. In the following discussions, we restrict our consideration to the case in which the single photon is incident from the negative channel with wave vector k (>0); then the element of the S matrix in the negative channel is

$$S_{k',-\leftarrow k,-} = r_-\delta_{k',-k} + t_-\delta_{k',k}, \quad (12)$$

where r_- (t_-) is the reflection (transmission) amplitude. The element of the S matrix in the positive channel is

$$S_{k',+\leftarrow k,-} = t_+[\delta_{k',q(k)} + \delta_{k',-q(k)}], \quad (13)$$

where the forward and backward transfer amplitudes are equal and denoted by t_+. For the sake of simplicity, in what follows we will write the wave vector $q(k)$ (>0) as q, whose dependence on the input wave vector k is given by the implicit relation

$$\omega_q = \omega_k - \nu_+ + \nu_-. \quad (14)$$

It shows from Eqs. (12) and (13) that the wave vector of the scattering photon k' satisfies $k' = \pm k$ when the incident photon is still confined in the negative channel, and $k' = \pm q$ when it is transferred to the positive channel.

III. SINGLE-PHOTON SCATTERING IN 1D CRW

A 1D CRW is typically made of single-mode resonators. We assume that all of the resonators have the same frequency ω and that the hopping energies ξ between any two nearest-neighbor resonators, which are determined by the inter-resonator coupling, are the same. Then the 1D CRW is characterized by the dispersion relation [15]

$$\omega_k = \omega - 2\xi\cos kl, \quad (15)$$

where l is the distance between arbitrary two-nearest-neighbor resonators. In the rest of this section, the wave vector k is dimensionless by setting $l = 1$.

For a three-level V-type atom located in the ath resonator, the atomic $|e\rangle \leftrightarrow |g\rangle$ transition couples with the kth field mode in the CRW with the coupling strength $J_k = Je^{ika}$ (J is assumed real). As we have mentioned above, the atomic $|f\rangle \leftrightarrow |g\rangle$ transition couples with the driving field, which

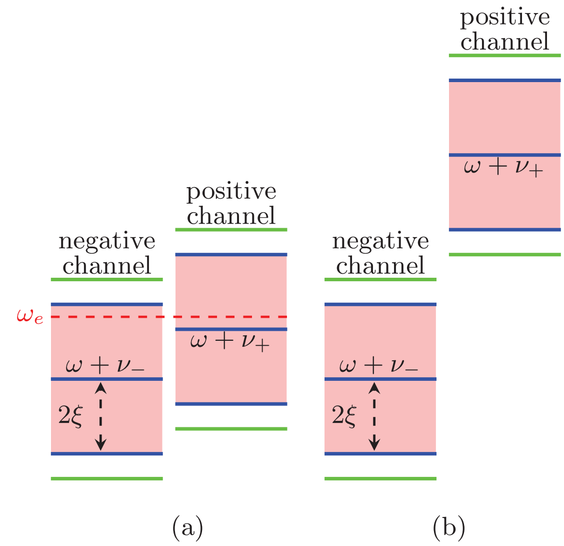

FIG. 2. (Color online) The energy-band configurations for the two channels in the CRW. (a) The energy bands are partially overlapped. (b) The energy bands are completely separated. In this case, the dispersion relation of the incident photon is $\omega_k = \omega - 2\xi\cos k$. The green thick lines represent the bound states in the corresponding channels and the red dashed line represents the energy of the atomic excited state $|e\rangle$.

forms two dressed states with energies $\nu_\mp$ (with respect to the rotating frame), corresponding to the negative and positive channels in the scattering process, respectively. The energies for the free particle states of the atom-CRW system (governed by H_0) in the negative (positive) channel form an energy band with the bandwidth 4ξ, which is centralized at $\omega + \nu_-$ ($\omega + \nu_+$). The broken translation symmetry of the CRW due to the existence of the atom supports a pair of bound states below and above the energy band, respectively, for each channel [18]. There are two following band configurations, as shown in Fig. 2: (a) partial overlap between the two bands, and (b) no overlap between them.

For a state of incident photon confined in the negative channel, the reflection and transfer amplitudes are obtained from Eqs. (12) and (13) utilizing the residue theorem as

$$r_-^c = \frac{e^{-2ika}\cos^2\frac{\theta}{2}}{2i\xi\left(\frac{\omega_k-\omega_e}{J^2} + \frac{i\sin^2\frac{\theta}{2}}{2\xi\sin q}\right)\sin k - \cos^2\frac{\theta}{2}}, \quad (16a)$$

$$t_+^c = \frac{e^{i(q-k)a}\cos\frac{\theta}{2}\sin\frac{\theta}{2}}{2i\xi\left(\frac{\omega_k-\omega_e}{J^2} + \frac{i\cos^2\frac{\theta}{2}}{2\xi\sin k}\right)\sin q - \sin^2\frac{\theta}{2}}, \quad (16b)$$

where the superscript c refers to CRW and the subscript $-$ ($+$) refers to the negative (positive) channel.

The cosine-type dispersion in Eq. (15) characterizes the nonlinear relation between the frequency and the wave vector of the traveling photon in the CRW. For the incident photon with a fixed wave vector k, the group velocity is $v_g = 2\xi\sin k$. Meanwhile, the scattered photon in different channels will have different group velocities, that is, $v_g = 2\xi\sin k$ in the negative channel and $v_g = 2\xi\sin q$ in the positive channel, where the wave vector q is defined in Eq. (14). Note that the group velocities have the same units as the frequency.

In this sense, we define the scattering flows of the single photons as the square modulus of the scattering amplitudes multiplying the group velocities in the corresponding channels. For the sake of simplicity, we set the incident flow as unit; then the reflection and transmission flows in the negative channel are calculated as $J_c^r = |r_-^c|^2$ and $J_c^t = |t_-^c|^2$, respectively. In addition, the transfer flow to the positive channel is obtained as $J_c^{tr} = 2|t_+^c|^2 \sin q / \sin k$ or $J_c^{tr} = 0$, depending on whether the energy of the incident photon is inside or outside the energy band of the positive channel. It then follows from Eqs. (16) that the scattering flows are independent of the atom position.

Now, we consider the case where the two energy bands partially overlap, as shown in Fig. 2(a). Here, the incident frequency may be either inside or outside the continuous regime of the positive channel. In Fig. 3(a), we plot the reflection flow J_c^r, the transmission flow J_c^t, the summation $J_c^r + J_c^t$ in the negative channel, and the total flow $J_c^r + J_c^t + J_c^{tr}$ as functions of the frequency of the incident photon. It can be observed that the single photon is confined in the negative channel when the energy of the incident state is out of the overlap region of the two continuum bands. Consequently, the flow's conservation is described by $J_c^r + J_c^t = 1$. When the incident energy is within the overlap region of the two continuum bands, the flow in the negative channel satisfies $J_c^r + J_c^t < 1$, which means the incident photons can be transferred to the positive channel, and the flow conservation is changed as $J_c^r + J_c^t + J_c^{tr} = 1$.

In Fig. 3(a), we can also observe that the photon incident from the negative channel is completely reflected when the incident frequency is $\omega_k = 0.988$ (in units of ω). The complete reflection arises from the Feshbach resonance mechanism which is predicted by Feshbach [26]: when the energy of the particle incident from the open channel is fine tuned to match that of the bound state in the closed channel, it will be completely reflected. In our system, the open channel is provided by the negative channel and the closed channel is provided by the positive channel. The bound state in the closed (positive) channel arises from the existence of the atom, which breaks the translation symmetry of the CRW. The energy of this bound state can be obtained by the transcendental equation

$$\omega_{k-} - \omega_e = \frac{J^2 \sin^2(\theta/2)}{2i\xi \sin q}. \tag{17}$$

Actually, the bound-state energy of the positive channel corresponds to the poles of the scattering matrix of the system studied in Ref. [15], i.e., a two-level system embedded in the positive channel.

In Fig. 3(b), we plot the transfer flow J_c^{tr} as a function of the frequency of the incident photon in the negative channel when the driving field is resonant with the atomic transition $|g\rangle \leftrightarrow |f\rangle$. It can be observed that when the frequency of the incident photon is resonant with the transition frequency between the atomic excited state $|e\rangle$ and the dressed state $|\phi_-\rangle$ ($\omega_k = \omega_e - \nu_-$), the transfer flow reaches its maximum. As the Rabi frequency η increases, the maximum of the transfer flow decreases monotonically. When the driving field is strong enough such that the two energy bands are completely separated [as shown in Fig. 2(b)], the transfer flow vanishes, i.e., the incident single photon remains in the initial negative channel and cannot travel in the positive channel. In Fig. 3(c), we have plotted the transfer flow versus the detuning Δ and the incident frequency ω_k by fixing the atomic $|g\rangle \leftrightarrow |f\rangle$

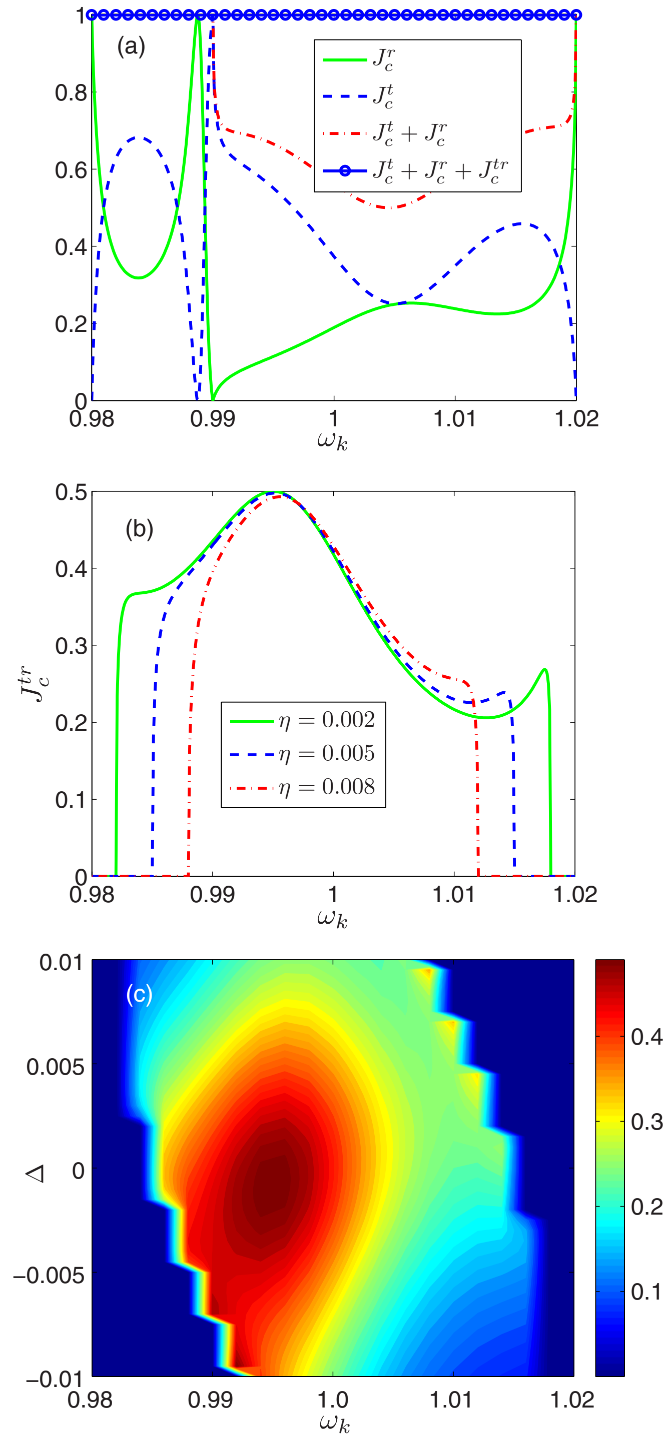

FIG. 3. (Color online) The scattering flows (in units of the incident flow) as functions of the frequency of incident photon in the CRW when the two energy bands are partially overlapped. (a) The reflection and transmission flows when $\Delta = 0$ and $\eta = 0.005$. (b) The transfer flows for different η under the resonance situation $\Delta = 0$. (c) The contour of the transfer flow vs Δ and ω_k with $\omega_f = 0.95$ and $\eta = 0.005$. The other parameters are set as $\xi = 0.01, \omega_e = 0.9995$ and $J = 0.015$. All the frequencies and the energies are in units of $\omega = 1$.

transition frequency $\omega_f = 0.95$ and the Rabi frequency $\eta = 0.005$ in units of $\omega = 1$. It can be found that the transfer flow reaches its maximum only when the classical field is resonant with the atomic transition $|g\rangle \leftrightarrow |f\rangle$, that is, $\Delta = 0$.

The above discussions show that the overlap of the two bands is necessary for the atom to fulfill the function of a photonic frequency converter. Here, the driving field prefers to be weak such that the frequency difference in the two bands cannot be too large. However, the preference for weak driving field is not necessary when the CRW is replaced by a 1D waveguide with a linear dispersion relation, which will be discussed in the next section.

IV. SINGLE-PHOTON SCATTERING IN 1D LINEAR WAVEGUIDE

In this section, we focus on the single-photon scattering in a 1D waveguide with the linear dispersion relation $\omega_k = v_g|k|$, where k is the wave vector and v_g is the group velocity of the light in the waveguide. We consider that the V-type atom is located at the position $x = a$, and the atomic $|e\rangle \leftrightarrow |g\rangle$ transition couples to the kth field mode in the waveguide with the coupling strength $J_k = Je^{ika}$, where J is assumed to be real. As before, a classical field is applied to couple to the atomic $|g\rangle \leftrightarrow |f\rangle$ transition, and the induced dressed states support two channels for the photons in the waveguide. As shown in Fig. 4, the energy bands of the negative and positive channels have the lower energy bounds at ν_- and ν_+ (in the rotating frame), respectively, but without the upper bounds.

For single photons with the wave vector k incident from the negative channel, the reflection and transfer amplitudes in the linear waveguide are calculated from Eqs. (12) and (13) utilizing the residue theorem as

$$r_-^l = \frac{e^{-2ika}J^2\cos^2\frac{\theta}{2}}{iv_g(\omega_{k-}-\omega_e)/L - J^2}, \tag{18a}$$

$$t_+^l = \frac{J^2 e^{i(|q|-k)a}\cos\frac{\theta}{2}\sin\frac{\theta}{2}}{iv_g(\omega_{k-}-\omega_e)/L - J^2}. \tag{18b}$$

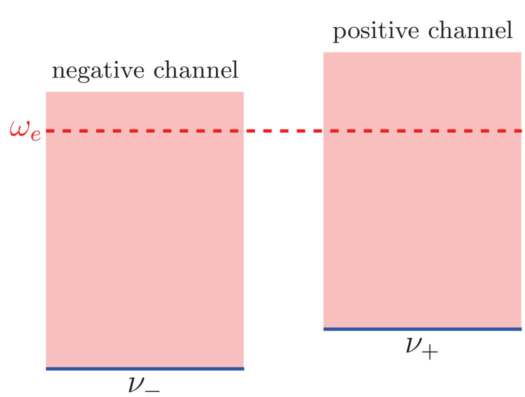

FIG. 4. (Color online) The energy-level configurations for the two channels in the linear waveguide with the dispersion relation $\omega_k = v_g|k|$. The red dashed line represents the frequency of the atomic excited state.

Here, the superscript l implies the "linear" waveguide and L is the length of the waveguide. The relationship between the wave vector q in the positive channel and k in the negative channel is obtained from Eq. (14) as

$$|q| = |k| - \frac{\nu_+ - \nu_-}{v_g}. \tag{19}$$

Similar to Sec. III, we can also define the scattering flows. Since the group velocities in both of the channels are the same, the reflection and transmission flows in the negative channel are $J_l^r = |r_-^l|^2$ and $J_l^t = |t_-^l|^2$, and the transfer flow to the positive channel is $J_l^{tr} = 2|t_+^l|^2$ as if the energy of the incident photon locates inside the overlap regime of the energy bands for the two channels. It is the same as the case of the CRW in which the scattering flows are independent of the atomic position.

In Fig. 5, we plot the scattering flows as functions of the frequency of the incident photon with the assumption that the classical field resonantly drives the atomic $|g\rangle \leftrightarrow |f\rangle$ transition ($\Delta = 0$). The reflection flow, the transmission flow, the flow in the negative channel, and the total flow in the system are displayed in Fig. 5(a) and the transfer flow is depicted in Fig. 5(b). It can be observed from Fig. 5(a)

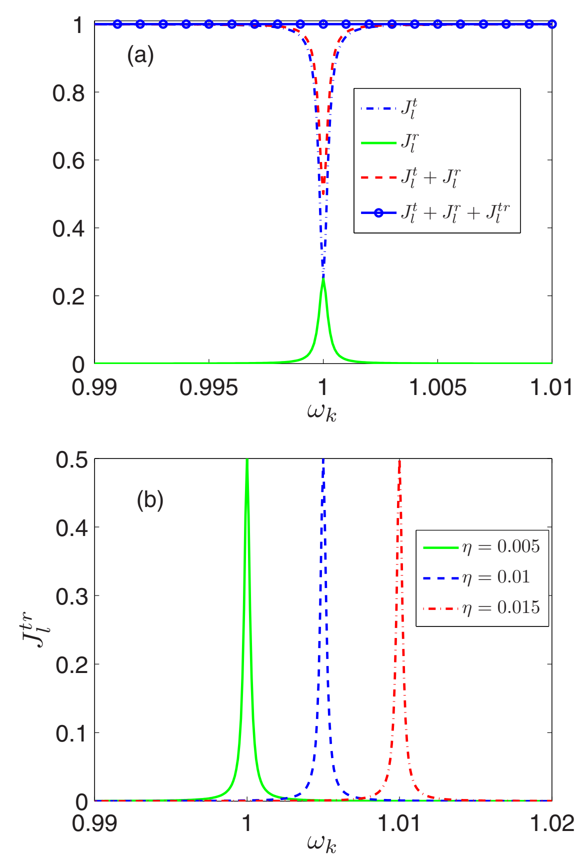

FIG. 5. (Color online) The scattering flows (in units of the incident flow) as functions of the frequency of the incident photon in the linear waveguide. The parameters are set as $\omega_e = 0.9995$, $J = 0.015$, and $\Delta = 0$. Under these parameters, the energy for the atomic dressed ground states are $\nu_\pm = \pm\eta$. (a) The reflection and transmission flows when $\eta = 0.005$. (b) The transfer flows for different η. All the frequencies and the energies are in units of $v_g/L = 1$.

[also in Fig. 5(b)] that when the incident frequency is below the difference between the lower bounds of the positive and negative channels ($\omega_k < \nu_+ - \nu_- = 2\eta$), the photon is confined in the negative channel. Consequently, $J_l^{tr} = 0$ and the flow conservation equation is $J_l^r + J_l^t = 1$. However, the single photon is transferred to the positive channel once the incident frequency surpasses the energy difference of the lower bounds of the two channels and the flow conservation equation becomes $J_l^r + J_l^t + J_l^{tr} = 1$. Seen from Fig. 5(b), when the incident photon is resonant with the $|e\rangle \leftrightarrow |\phi_-\rangle$ transition, i.e., $\omega_k = \omega_e - \nu_-$, the transfer flow J_l^{tr} achieves its maximum with the value 0.5, which is independent of the strength of the driving field. However, when the frequency of the incident photon is far off resonance from the atomic $|e\rangle \leftrightarrow |\phi_-\rangle$ transition, the transfer flow is close to zero, and the photon is nearly perfectly transmitted in the initial channel [as shown in Fig. 5(a)]. When the classical field is off resonant with the atomic $|g\rangle \leftrightarrow |f\rangle$ transition, the maximum of the transfer flow is lowered, i.e., always smaller than 0.5. These can be understood from Eq. (18b): For a resonant classical field, $\Delta = 0$ and $\theta = \pi/2$, the value of the numerator in Eq. (18b) achieves its maximum. For an off-resonant classical field ($\Delta \neq 0$ and $\theta < \pi/2$), the value of the numerator in Eq. (18b) is lowered.

The above discussions show that the V-type atom functions as a photonic frequency converter in a 1D linear waveguide as long as the energy of the incident state lies inside the overlap region in the energy spectrum of the two channels. Due to the disappearance of the upper energy bounds in the linear waveguide, the driving field can be arbitrarily strong in principle. This is evidently different from the case of CRW, where the driving field is expected to be weak.

V. EXPERIMENTAL FEASIBILITY

To demonstrate our theoretical results about the single-photon frequency conversion mechanism, we now propose an experimentally accessible quantum device which is composed of the superconducting transmission line resonator(s) and the spin of nitrogen-vacancy (NV) center in the diamond. The superconducting transmission line resonator supports the single-mode electromagnetic field with the resonant frequency $\omega/2\pi \approx 3$ GHz [27], and serves as a linear waveguide or a CRW by being cut into N equal segments. For the CRW situation, the coupling between the neighbor segments can be realized via the tunable capacitances and the coupling strength can be achieved, $\xi/2\pi = 5$–100 MHz [27,28]. The NV center, which acts as the V-type atom, has the $S = 1$ triplet ground states (as shown in Fig. 6). The Hamiltonian for the single NV center is written as [29]

$$H_{NV} = DS_z^2 + E\left(S_x^2 - S_y^2\right) + g_e\mu_B\vec{B}\cdot\vec{S}, \qquad (20)$$

where $D/2\pi = 2.88$ GHz is the ground-state zero-field splitting between the $m_s = 0$ (denoted by $|0\rangle$) and $m_s = \pm 1$ (denoted by $|1\pm\rangle$) states, $\vec{S}$ are the conventional Pauli spin-1 operators, and E is the ground-state strain-induced splitting coefficient. $g_e = 2$ is the ground-state Lande factor and $\mu_B/2\pi = 14$ MHz/mT is the Bohr magneton. In our consideration, the strain-induced splitting is much smaller than the Zeeman splitting induced by the external magnetic field, and the second term in the Hamiltonian H_{NV} can be neglected.

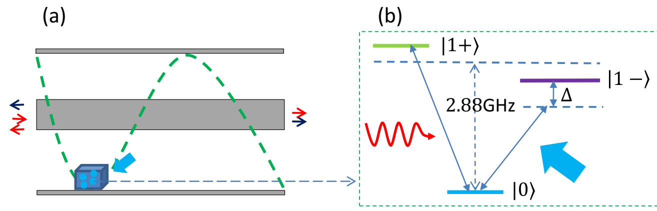

FIG. 6. (Color online) (a) The possible realistic system to implement one-photon frequency conversion: the superconducting transmission line coupled with NV or ensemble of NVs. (b) Schematics of the $S = 1$ ground states of the NV center applied as the single-photon scatterer.

The superconducting transmission line resonator magnetically couples to the NV's transition between the states $|0\rangle$ and $|1+\rangle$ and the coupling strength for a single NV center is $J_{\text{single}}/2\pi \approx 20$ Hz [30], which can be enhanced by $\sqrt{N}$ times via introducing an ensemble of N ($\approx 10^{12}$) such NV centers and the collective coupling strength achieves $J/2\pi \approx 20$ MHz. An on-demand classical microwave field drives the transition between the states $|0\rangle$ and $|1-\rangle$ with the Rabi frequency $\eta/2\pi \approx 5$ MHz [31]. These parameters are suitable for our consideration in the theoretical discussions in Secs. III and IV.

In addition to the NV center(s) mentioned above, the superconducting Josephson junction can also be used to couple with the transmission line resonator(s) [32] and function as a single-photon frequency converter. In addition, the system of high-Q photonic crystal resonators coupled with defect [33] or NV centers [34–36] is a candidate to demonstrate the single-photon frequency conversion mechanism we propose.

VI. DISCUSSIONS AND CONCLUSIONS

In this paper, we discussed the single-photon frequency conversion mechanism in a 1D waveguide coupled to a three-level V-type atom. The on-demand classical driving field introduces a pair of dressed states, and supplies the negative and positive channels for the incident single photons. In our studies, we have shown that the single-photon incident from the negative channel can be transferred to the positive channel with the carried frequency being red-shifted. Also, the single-photon incident from the positive channel can be transferred to the negative channel. However, the carried frequency will be blue-shifted instead of red-shifted due to the energy conservation of the atom-waveguide coupled system.

In conclusion, we have studied the single-photon scattering in the system composed of 1D waveguide and an embedded three-level V-type atom. In our system, the absorption and reemission processes of single photons is formulated by a two-channel scattering process. To demonstrate the transfer efficiency from one channel to the other with a different carried frequency, we analytically investigate the scattering flows in a 1D CRW with cosine dispersion and in a 1D waveguide with linear dispersion. It is found that converting the frequency of single photons in a 1D waveguide requires that (1) there is overlap between the continuums of the two channels, and (2) the energy of the atomic excited state is in the continuums of the two channels. As long as these requirements are satisfied, the strength of the classical field can be arbitrarily large for the waveguide with linear dispersion relation to achieve the maximal probability of the outgoing photon with different frequency; however, a weak classical field is preferred in the 1D CRW.

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