Master equation and dispersive probing of a non-Markovian process

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For a bosonic (fermionic) open system in a bath with many bosonic (fermionic) modes, we derive the exact non-Markovian master equation in which the memory effect of the bath is reflected in the time-dependent decay rates. In this approach, the reduced density operator is constructed from the formal solution of the corresponding Heisenberg equations. As an application of the exact master equation, we study the active probing of the non-Markovianity of the quantum dissipation of a single bosonic mode of an electromagnetic field in a cavity-QED system. The non-Markovianity of the bath of the cavity is explicitly reflected by the atomic decoherence factor.

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I. INTRODUCTION

The open quantum system approach is of much significance due to its various applications in physics, e.g., quantum information, quantum transport, and quantum chemistry, etc. Since a realistic quantum system is inevitably coupled to many degrees of freedom in its environment, that leads to decoherence of the systems, a general approach to the open quantum system is needed for its dissipative and dephasing processes. The dynamics of an open system is conventionally described by three approaches: effective Hamiltonian [1–5], quantum master equations [6,7], and quantum Langevin equations [8,9]. The last two approaches are both based on modeling with the system plus bath, whereas, the first one is phenomenologically given by a time-dependent or non-Hermitian Hamiltonian, which could lead to the dissipative motion equations.

About 20 years ago, Yu and one (C.P.S.) of the authors revealed an intrinsic relation between the effective Hamiltonian and the quantum Langevin equation, obtained from the Heisenberg equations [3,4]. By discarding the quantum fluctuation for the wide wave packet, they derived the effective Hamiltonian of the system through the formally exact solution for the time-dependent wave function of the total system. However, the resulting effective Hamiltonian ignores the memory effect, which is induced by the backaction of the bath with a time delay. Therefore, if one wanted to recover the non-Markovian phenomenon with a memory effect, the quantum fluctuation of the bath must be taken into account in the above Heisenberg-equation-based approach. To this end, we need to start from the Heisenberg equations of the total system, which can reflect the original role of the bath. In this paper, without any approximation, we derive the exact non-Markovian master equation of the system from the formal solution of the Heisenberg equations. The non-Markovian effect is contained in the time-dependent decay rates in a straightforward way [10].

It is commonly believed that the Markov process happens when the system-bath coupling is weak. However, with the rapid development of experimental technology, the strong-coupling limit can be reached. The theory of open quantum systems in the strong-coupling regime is required for a proper description of the non-Markovian dynamics. Recently, many papers on the exact quantum master equations have been published [10–18]. In particular, one (W.-M.Z.) of the authors and his collaborators derived the exact non-Markovian master equations with a Lindblad form for both Bose [13,14] and Fermi [10,15] systems by a path-integral method in a coherent-state representation. We now revisit these non-Markovian master equations by generalizing our previous approach [5], which was used to derive a partially factorized wave function for open quantum systems. Using the present generalization to derive the reduced density matrix is quite straightforward. Here, we first construct the total density matrix in the coherent-state representation with the help of the formal solution of the Heisenberg equations, instead of using the Feynman-Vernon influence functional, as performed in Refs. [10,14,15]. Then, the reduced density matrix of the system, which covers the detailed information for quantum manipulation, is obtained by tracing over the degrees of freedom of the bath. It reproduces the same reduced density matrix that satisfies a time-local master equation where the non-Markovian memory effect is fully taken into account.

With the help of the exact reduced density matrix, the dynamics of an open quantum system could be well described. Meanwhile, there are several proposals to measure the degree of the non-Markovianity of the open quantum process [19,20]. Very recently, the general non-Markovian dynamics of the environment on its surrounding open quantum system were explored within the exact master equation [21]. The question is how to probe the general non-Markovian dynamics. In this paper, we, thereby, propose a promising approach to probe the time-dependent memory effect of a bath on a damped microcavity by coupling the cavity to a two-level atom dispersively. To probe the non-Markovianity of the dissipation of the single-mode electromagnetic (EM) field in a cavity, we let atoms of large detuning pass through the cavity. We found that the non-Markovianity of the bath is explicitly reflected by the atomic decoherence factor. In the weak-coupling region, the periodically reviving amplitude decreases along with the cavity-bath-coupling strength and decays to zero. On the contrary, in the strong-coupling region, the reviving amplitude increases with the coupling strength and almost does not decay in the ultra-strong-coupling case as a significant non-Markovian effect [21]. This atomic decoherence factor could be detected through the Ramsey interference in experiments.

In the next section, we solve the Heisenberg equations of the unified quantum system plus the bath model (Bose and Fermi) and obtain their formal solutions. In Sec. III, the derivation of the exact master equation of the Bose system is presented. The exact master equation of the Fermi case is addressed in Sec. IV. In Sec. V, we propose probing the non-Markovian dynamics of a damped cavity with largely detuned two-level atoms. Finally, the summary of our main results is given in Sec. VI. Some detailed calculations are displayed in the Appendices.

II. THE UNIFIED QUANTUM BATH MODEL AND THE FORMAL SOLUTION OF THE HEISENBERG EQUATIONS

We consider an open quantum system S, which interacts with another large system B called the bath. The combined system $S+B$ is usually assumed to be closed, thus, is regarded as a universe. The coupling of S to B will lead to the dissipation and dephasing of S. There are various types of baths, but the most commonly employed baths are modeled with noninteracting bosons and fermions. In this paper, we consider the specific cases: A Bose system is surrounded by a Bose bath, or a Fermi system is immersed in a Fermi bath. Here, we solve the Heisenberg equations for both the Bose and the Fermi cases and obtain their formally exact solutions.

The universe Hamiltonian $H = H_s + H_b + H_{\text{int}}$ is decomposed into three parts: The Hamiltonian of the system is taken to be a quadratic form

$$H_s = \left[a_1^\dagger, a_2^\dagger, \ldots, a_{N_s}^\dagger\right] M \left[a_1, a_2, \ldots, a_{N_s}\right]^T, \tag{1}$$

which describes N_s linearly coupled bosons or fermions. $a_i\,(a_i^\dagger)$ is the annihilation (creation) operator of the ith mode of the system satisfying the commutation relation $[a_i, a_{i'}^\dagger]_\mp = \delta_{ii'}$ ($\mp$ corresponds to the boson and fermion, respectively), and M is a positive-definite Hermitian matrix. The Hamiltonian of the Bose or Fermi bath is given by

$$H_b = \sum_{l=1}^{N_b} \omega_l b_l^\dagger b_l, \tag{2}$$

with the number of the uncoupled modes of the bath N_b ($\gg N_s$) and annihilation (creation) operators $b_l\,(b_l^\dagger)$, which satisfy corresponding commutation relations $[b_l, b_{l'}^\dagger]_\mp = \delta_{ll'}$. As proven in Ref. [6], the most usual environment coupled to the open system could be well approximated as a collection of harmonic oscillators with linear quadratic couplings. Here, the interaction Hamiltonian is taken as the form of

$$H_{\text{int}} = \sum_{i=1}^{N_s} \sum_{l=1}^{N_b} \left(\eta_{il} a_i^\dagger b_l + \eta_{il}^* b_l^\dagger a_i\right). \tag{3}$$

In the Heisenberg picture, the dynamics of the system is governed by the Heisenberg equations,

$$\dot{a}_i(t) = -i \sum_j M_{ij} a_j(t) - i \sum_l \eta_{il} b_l(t), \tag{4}$$

$$\dot{b}_l(t) = -i\omega_l b_l(t) - i \sum_i \eta_{li}^* a_i(t). \tag{5}$$

For convenience, we introduce the $(N_s + N_b)$-operator-valued vector,

$$\begin{aligned}\vec{c}(t) &= [\vec{a}, \vec{b}]^T \\ &= \left[a_1(t), a_2(t), \ldots, a_{N_s}(t), b_1(t), b_2(t), \ldots, b_{N_b}(t)\right]^T,\end{aligned}$$

and the $(N_s + N_b) \times (N_s + N_b)$ coefficient matrix,

$$\mathcal{H} = \begin{bmatrix} M & R \\ R^\dagger & E \end{bmatrix}, \tag{6}$$

where

$$R = \begin{bmatrix} \eta_{11} & \eta_{12} & \cdots & \eta_{1N_b} \\ \eta_{21} & \eta_{22} & \cdots & \eta_{2N_b} \\ \vdots & \vdots & \vdots & \vdots \\ \eta_{N_s 1} & \eta_{N_s 2} & \cdots & \eta_{N_s N_b} \end{bmatrix},$$

and

$$E = \text{diag}\left[\omega_1, \omega_2, \ldots, \omega_{N_b}\right].$$

Then, Eqs. (4) and (5) are reexpressed in a compact form

$$\frac{d}{dt}\vec{c}(t) = -i\mathcal{H}\vec{c}(t). \tag{7}$$

It follows from Eq. (6) that $\mathcal{H}$ is a time-independent Hermitian matrix. Consequently, the formal solution of Eq. (7) is given by

$$\vec{c}(t) = \exp[-i\mathcal{H}t]\vec{c}(0) \equiv \mathcal{U}(t)\vec{c}(0),$$

where $\mathcal{U}(t) = \exp[-i\mathcal{H}t]$ is the time-evolution operator. Splitting the matrix $\mathcal{U}(t)$ into four blocks,

$$\mathcal{U}(t) \equiv \begin{bmatrix} [W(t)]_{N_s \times N_s} & [T(t)]_{N_s \times N_b} \\ [P(t)]_{N_b \times N_s} & [Q(t)]_{N_b \times N_b} \end{bmatrix}, \tag{8}$$

we obtain the formal solution of Eq. (7) as

$$\vec{a}(t) = W(t)\vec{a}(0) + T(t)\vec{b}(0), \tag{9}$$

$$\vec{b}(t) = P(t)\vec{a}(0) + Q(t)\vec{b}(0). \tag{10}$$

The dynamics of the total system is governed by these four time-dependent coefficient matrices $W(t)$, $T(t)$, $Q(t)$, and $P(t)$.

Until now, all the results were obtained by formal operations since these coefficient matrices need to be determined by the differential equations. As shown in Appendix B, there are some connections between these coefficient matrices, which play a crucial role in the derivation of the exact master equation. We should also point out that, even though the total Hamiltonian (the system plus the bath) is a quadratic form of annihilation and creation operators, it should not be simply considered as a solvable model since the bath contains a continuous spectrum. Practically, a bilinear system with a continuous number of degrees of freedom is difficult to be solved explicitly.

A. Differential equations of the coefficient matrices

Substituting Eqs. (9) and (10) into Eqs. (4) and (5), we obtain the equations of the coefficient matrices,

$$\dot{W}(t) = -i[MW(t) + RP(t)], \tag{11}$$

$$\dot{T}(t) = -i[MT(t) + RQ(t)], \tag{12}$$

$$\dot{P}(t) = -i[EP(t) + R^\dagger W(t)], \tag{13}$$

$$\dot{Q}(t) = -i[EQ(t) + R^\dagger T(t)], \tag{14}$$

with the initial conditions,

$$W(0) = I, \quad T(0) = \mathbf{0}, \quad P(0) = \mathbf{0}, \quad Q(0) = I. \tag{15}$$

Here, I is the identity matrix, and $\mathbf{0}$ is the null matrix. The differential equations of $P(t)$ and $Q(t)$ are integrated to yield

$$P(t) = -i\int_0^t d\tau\, e^{-iE(t-\tau)} R^\dagger W(\tau) d\tau, \tag{16}$$

$$Q(t) = e^{-iEt}\left[-i\int_0^t d\tau\, e^{iE\tau} R^\dagger T(\tau) + I\right]. \tag{17}$$

Then, we obtain the integrodifferential equations about $W(t)$ and $T(t)$,

$$\dot{W}(t) + iMW(t) + \int_0^t d\tau\, G(t-\tau)W(\tau) = 0, \tag{18}$$

$$\dot{T}(t) + iMT(t) + \int_0^t d\tau G(t-\tau) T(\tau) = -iRe^{-iEt}. \tag{19}$$

Here, the $(N_s \times N_s)$ kernel matrix $G(t) = Re^{-iEt}R^\dagger$ characterizes the non-Markovian memory structure of S. Defining the interaction spectral function,

$$J_{ij}(\omega) = \sum_l \eta_{il}\eta_{lj}^* \delta(\omega - \omega_l),$$

we rewrite the element of the kernel matrix $G(t)$ as

$$G_{ij}(t) = \int d\omega\, J_{ij}(\omega) e^{-i\omega t}.$$

Thus, the matrix $G(t)$ is fully determined by the interaction spectrum.

On the other hand, the coefficient matrices $W(t)$ and $T(t)$ are not independent. By taking the Laplace transform of the integral differential equations (18) and (19), we get

$$W[p] = \mathcal{L}(W) = [p + iM + G(p)]^{-1}, \tag{20}$$

$$T[p] = W[p]\mathcal{L}\left(-iRe^{-iEt}\right), \tag{21}$$

where $\mathcal{L}(\cdots)$ represents the Laplace transform. Consequently, after the inverse Laplace transform, the matrix $T(t)$ is given by

$$T(t) = -i\int_0^t d\tau\, W(t-\tau)\, Re^{-iE\tau}. \tag{22}$$

Thus, the dynamics of S could be completely described by a single coefficient matrix $W(t)$. It is well known that, under the Wigner-Weisskopff approximation, one can obtain the quantum Langevin equations of the operators of S by means of the approximate solution of Eqs. (18) and (19) together with the Heisenberg equations (4) and (5) [9]. In this paper, it will be shown that the exact master equation of the reduced density matrix can also be obtained based on the formal solutions (9) and (10) of the Heisenberg equations. And the Wigner-Weisskopff approximation leads to the quantum Born-Markov master equation.

III. THE BOSON CASE IN THE COHERENT-STATE REPRESENTATION

The formally exact solution based on the Heisenberg equations of motion provides all conceivable information but not for any of the detailed information on the quantum states, which is the central part for quantum-information manipulation. The detailed information on quantum states for an open system is depicted by the reduced density matrix, whose dynamics equation of motion is governed by the master equation. In this section, we derive the exact master equation for N_s-coupled bosons in a Bose bath. In the Schrödinger picture, the total density matrix $\rho(t) = U(t)\rho(0)U^\dagger(t)$ of $S + B$ obeys the Liouville–von Neumann equation $i\hbar\dot{\rho}(t) = [H,\rho(t)]$, where $U(t) = \exp(-iHt)$ is the time-evolution operator of the total system. We assume that the total system is initially in the direct product initial state $\rho(0) = \rho_s(0) \otimes \rho_b(0)$, with density matrices $\rho_s(0)$ and $\rho_b(0)$ of S and B, respectively. Through a lengthy calculation in Appendix C, the reduced density matrix of S is expressed in terms of the coherent state $|\vec{x}\rangle$ of the system,

$$\rho_s(t) = \int d\mu(\vec{\alpha},\vec{\alpha}')d\mu(\vec{\xi},\vec{\xi}')|\vec{\alpha}\rangle\langle\vec{\alpha}'| \times K(\vec{\alpha}^\dagger,\vec{\alpha}',\vec{\xi},\vec{\xi}'^\dagger,t)\langle\vec{\xi}|\rho_s(0)|\vec{\xi}'\rangle, \tag{23}$$

with $\vec{x} = [x_1,x_2,\ldots,x_{N_s}]^T$ $(\vec{x} = \vec{\alpha},\vec{\alpha}',\vec{\xi},\vec{\xi}')$. The propagator, which governs the dynamics of the reduced density matrix, is defined as

$$K(\vec{\alpha}^\dagger,\vec{\alpha}',\vec{\xi},\vec{\xi}'^\dagger;t) = \int d\mu(\vec{z})\langle\vec{\alpha},\vec{z}|U(t)|\vec{\xi}\rangle \times\langle\vec{\xi}'|\rho_b(0)U^\dagger(t)|\vec{\alpha}',\vec{z}\rangle. \tag{24}$$

Here, $|\vec{z}\rangle$ $(\vec{z} = [z_1,z_2,\ldots,z_{N_b}])$ is the coherent state of B. Different from the previous derivation [10,14,15] where the propagating function is obtained using the coherent-state path-integral method and tracing over the environmental degrees of freedom completely through the Feynman-Vernon influence functional, the propagator could also be evaluated in the coherent-state representation by constructing the explicit total wave function [5],

$$U^\dagger(t)|\vec{\alpha}',\vec{z}\rangle = \exp[\vec{a}^\dagger(t)\cdot\vec{\alpha}' + \vec{b}^\dagger(t)\cdot\vec{z}]|0\rangle, \tag{25}$$

as shown in Appendix C. It deserves to be noted that we have used the identities $U^\dagger(t)|0\rangle = |0\rangle$ and $O(t) = U^\dagger(t)OU(t)$.

A. Propagating function

Generally speaking, the bath is initially in its thermal equilibrium state,

$$\rho_b(0) = \left(\prod_l \frac{1}{f_l + 1}\right)\exp[-\beta\vec{b}^\dagger E\vec{b}], \tag{26}$$

where $f_l = 1/[\exp(\beta\omega_l) - 1]$ is the mean-occupation number of the lth bath mode at temperature $T = 1/(k_B\beta)$. In this case, the integral over the bath in the propagator (24) is carried out to give (please refer to Appendix C for the details)

$$K(\vec{\alpha}^\dagger,\vec{\alpha}',\vec{\xi},\vec{\xi}'^\dagger,t) = A(t)\exp[\vec{\alpha}^\dagger J_1(t)\vec{\xi} + \vec{\xi}'^\dagger J_1^\dagger(t)\vec{\alpha}' + \vec{\alpha}^\dagger J_2(t)\vec{\alpha}' + \vec{\xi}'^\dagger J_3\vec{\xi}], \tag{27}$$

where

$$\begin{aligned} A(t) &= \det\{[I + V(t)]^{-1}\},\\ J_1(t) &= [I + V(t)]^{-1}W(t),\\ J_2(t) &= V[I + V(t)]^{-1},\\ J_3(t) &= I - W^\dagger(t)[I + V(t)]^{-1}W(t). \end{aligned}$$

This reproduces the propagating function obtained by the coherent-state path-integral method in the previous papers, e.g., Eq. (31) in Ref. [14]. For convenience, we have introduced a new $N_s \times N_s$ Hermitian matrix $V(t) = T(t)fT^\dagger(t)$. Utilizing the relationship in Eq. (22) between matrices $T(t)$ and $W(t)$, we have

$$V(t) = \int_0^t d\tau_1 \int_0^t d\tau_2 W(\tau_1)\tilde{G}(\tau_2 - \tau_1)W^\dagger(\tau_2), \tag{28}$$

with

$$\tilde{G}(t) = Rfe^{-iEt}R^\dagger. \tag{29}$$

Without any additional hypotheses, the exact propagating function of the reduced density matrix of S is obtained. The dynamics of S is governed by the single coefficient matrix $W(t)$, which is determined by the integral differential equation (18). And the influence of the bath on the dynamics of S is characterized by two memory-kernel matrices $G(t)$ and $\tilde{G}(t)$ [10,14,15].

B. The exact non-Markovian master equation for bosons

In the preceding subsection, we have obtained the exact reduced density matrix of S as in Eq. (23). Now, we construct the master equation through its time derivative,

$$\dot{\rho}_s = \int d\mu(\vec{\alpha},\vec{\alpha}')d\mu(\vec{\xi},\vec{\xi}')|\vec{\alpha}\rangle\langle\vec{\alpha}'| \times \dot{K}(\vec{\alpha}^\dagger,\vec{\alpha}',\vec{\xi},\vec{\xi}'^\dagger;t)\langle\vec{\xi}|\rho_s(0)|\vec{\xi}'\rangle. \tag{30}$$

And it is found that the time differential of the propagating function takes the following form (please refer to Appendix D for the details):

$$\begin{aligned}\dot{K} &= \vec{\alpha}^\dagger\tilde{\Gamma}K\vec{\alpha}' - \mathrm{Tr}[\tilde{\Gamma}]K - \vec{\alpha}^\dagger(\Gamma + i\tilde{\Omega} + \tilde{\Gamma})\vec{\nabla}_{\alpha^*}K\\ &\quad - \big(\vec{\nabla}^T_{\alpha'}K\big)(\Gamma - i\tilde{\Omega} + \tilde{\Gamma})\vec{\alpha}' + \vec{\nabla}^T_{\alpha'}(\tilde{\Gamma} + 2\Gamma)\vec{\nabla}_{\alpha^*}K, \end{aligned}\tag{31}$$

with Hermitian matrices,

$$\tilde{\Gamma}(t) = \dot{V}(t) - \dot{W}(t)W^{-1}(t)V(t) - V[\dot{W}(t)W^{-1}(t)]^\dagger, \tag{32}$$

$$\Gamma(t) = -\tfrac{1}{2}\{\dot{W}(t)W^{-1}(t) + [\dot{W}(t)W^{-1}(t)]^\dagger\}, \tag{33}$$

and

$$\tilde{\Omega}(t) = \frac{i}{2}\{\dot{W}(t)W^{-1}(t) - [\dot{W}(t)W^{-1}(t)]^\dagger\}. \tag{34}$$

For the coherent state defined in Eq. (A1), there exist the following relations [22]:

$$\begin{aligned} \vec{\alpha}t|\vec{\alpha}\rangle &= \vec{a}|\vec{\alpha}\rangle, \quad \vec{\alpha}^\dagger\langle\vec{\alpha}| = \langle\vec{\alpha}|\vec{a}^\dagger,\\ \vec{\nabla}^T_{\alpha'}|\alpha\rangle &= \vec{a}^\dagger|\vec{\alpha}\rangle, \quad \vec{\nabla}_{\alpha^*}\langle\vec{\alpha}| = \langle\vec{\alpha}|\vec{a}. \end{aligned}$$

With these mappings, we can construct the exact master equation of the reduced density matrix of the Bose system S, i.e., Eq. (32) in Ref. [14],

$$\begin{aligned}\dot{\rho}_s(t) &= -i[\tilde{H}_s(t),\rho_s(t)] + \sum_{ij}[\tilde{\Gamma}_{ij}(t) + 2\Gamma_{ij}(t)]\\ &\quad\times\left[a_j\rho_s(t)a_i^\dagger - \frac{1}{2}a_i^\dagger a_j\rho_s(t) - \frac{1}{2}\rho_s a_i^\dagger a_j\right]\\ &\quad+\sum_{ij}\tilde{\Gamma}_{ij}(t)\left[a_i^\dagger\rho_s(t)a_j - \frac{1}{2}a_j a_i^\dagger\rho_s(t) - \frac{1}{2}\rho_s(t)a_j a_i^\dagger\right],\end{aligned}\tag{35}$$

where $\tilde{H}_s = \vec{a}^\dagger\tilde{\Omega}\vec{a}$ is the effective time-dependent Hamiltonian of the system S. The diagonal elements of $\tilde{\Omega}(t)$ are the modified time-dependent frequencies of the different modes of S, and the off-diagonals represent the new interaction strength between the modes of the system. Without Markov approximation, the dissipation of the system and the fluctuation of the bath could not be separated. The original role of the bath is reflected by the time-dependent decay coefficients $\Gamma(t)$ and $\tilde{\Gamma}(t)$ [14].

C. From the Wigner-Weisskopff approximation to the Markov master equation

In this subsection, it will be shown that the Markov master equation can be obtained from the exact master equation by taking the Wiger-Weisskopff approximation [9], instead of performing a direct Markov approximation [16]. Here, the exact master equation is applied to the simplest dissipative system consisting of a single harmonic oscillator with frequency Ω_0 and a Bose environment. In this case, $\tilde{\Omega}$, $\Gamma(t)$, and $\tilde{\Gamma}(t)$ are just time-dependent numbers instead of matrices, which are all determined by $W(t)$ in Eqs. (32)–(34). Under the Wigner-Weisskopff approximation, the solution of Eq. (18) is given by

$$W(t) = \exp[-\Gamma_0 t - i(\Omega_0 + \Delta\omega)t], \tag{36}$$

where

$$\Gamma_0 = \pi J(\Omega_0) \tag{37}$$

is the decay rate of the oscillator induced by the coupling to the vacuum and

$$\Delta\omega = -\mathcal{P}\int\frac{J(\Omega_0)}{\omega - \Omega_0}d\omega \tag{38}$$

is the small frequency shift with the interaction spectrum $J(\omega)$. It is easy to find that, in this case, the parameters of the master equation become time independent,

$$\tilde{\Omega} = \Omega_0 + \Delta\omega, \quad \Gamma = \Gamma_0, \quad \tilde{\Gamma} = 2f(\Omega_0)\gamma_0, \tag{39}$$

where $f(\Omega_0)$ is the mean-occupation number of the oscillator. As we know, Γ characterizes the dissipation of the system, and $\tilde{\Gamma}$ corresponds to the fluctuation in the bath.

Then, the Born-Markov master equation of a damped harmonic resonator is obtained as

$$\dot{\rho}_s(t) = -i[\tilde{\Omega}a^\dagger a,\rho_s(t)] + [1+f(\Omega_0)]\Gamma_0\{2a\rho_s(t)a^\dagger - [a^\dagger a\rho_s(t)+\rho_s(t)a^\dagger a]\} + f(\Omega_0)\Gamma_0\{2a^\dagger\rho_s(t)a - [aa^\dagger\rho_s(t)-\rho_s(t)aa^\dagger]\}. \quad (40)$$

It is known that, for a damped harmonic oscillator, the quantum Langevin equation of the number operator obtained from the Markov approximation is the same as the one from the Wigner-Weisskopff approximation [9]. In this sense, these two approximations are equivalent.

IV. THE FERMI CASE IN THE COHERENT-STATE REPRESENTATION

In the previous section, we obtained the exact master equation of the Bose system. Analogously, in the case of the Fermi system, the reduced density matrix in the fermion coherent-state representation [23,24] reads

$$\rho_s(t) = \int d\mu(\vec{\alpha},\vec{\alpha}')d\mu(\vec{\xi},\vec{\xi}')|\vec{\alpha}\rangle\langle\vec{\alpha}'| \times K(\vec{\alpha}^\dagger,\vec{\alpha}',\vec{\xi},\vec{\xi}'^\dagger,t)\langle\vec{\xi}|\rho_s(0)|\vec{\xi}'\rangle, \quad (41)$$

where the components of vectors $\vec{\alpha}$, $\vec{\alpha}'$, $\vec{\xi}$, and $\vec{\xi}'$ are Grassmann variables and $\rho_s(0)$ is the initial state of S. And the initial state of the bath is still assumed to be the thermal state,

$$\rho_b(0) = \prod_l (1-f_l)\exp[-\beta\vec{b}^\dagger E\vec{b}]. \quad (42)$$

where $f_k = 1/[\exp(\beta\omega_l)+1]$ is the mean-occupation number of the lth Fermi mode with $\beta = 1/(k_BT)$. After tracing over the degrees of freedom of the bath, we find that the propagator is of the same form as the Bose case [10],

$$K = A\,\exp[\vec{\alpha}^\dagger J_1\vec{\xi} + \vec{\xi}'J_1^\dagger\vec{\alpha}' + \vec{\alpha}J_2\vec{\alpha}' + \vec{\xi}'^\dagger J_3\vec{\xi}],$$

but the matrices in K change into

$$\begin{aligned} A(t) &= \det\{[I-V(t)]^{-1}\},\\ J_1(t) &= [I-V(t)]^{-1}W(t),\\ J_2(t) &= [I-V(t)]^{-1} - I,\\ J_3(t) &= W^\dagger(t)[I-V(t)]^{-1}W(t) - I. \end{aligned}$$

After the same procedure as the Bose system, the exact master equation of the Fermi system is obtained as the same one given by Eq. (8) in Ref. [15],

$$\dot{\rho}_s(t) = -i[\tilde{H}_s(t),\rho_s(t)] + \sum_{ij}[2\Gamma_{ij}(t)-\tilde{\Gamma}_{ij}(t)] \times \left[a_j\rho_s(t)a_i^\dagger - \frac{1}{2}a_i^\dagger a_j\rho_s(t) - \frac{1}{2}\rho_s(t)a_i^\dagger a_j\right] + \sum_{ij}\tilde{\Gamma}_{ij}(t)\left[a_i^\dagger\rho_s(t)a_j - \frac{1}{2}a_ja_i^\dagger\rho_s(t) - \frac{1}{2}\rho_s(t)a_ja_i^\dagger\right], \quad (43)$$

where $\tilde{H}_s(t)$, $\Gamma(t)$, and $\tilde{\Gamma}(t)$ are defined in the same way as the bosons' [10,15].

V. PROBING THE NON-MARKOVIANITY OF AN OPEN QUANTUM SYSTEM

In this section, we consider how to probe the non-Markovianity of a quantum dissipation process in a realistic physical system. We understand that such an ideal probing scheme is usually based on the nondemolition measurement [25]. The interaction between the probing apparatus and the system to be detected commutes with the free Hamiltonian of the system, thus, such a measurement does not change the energy of the system. But it will retain the information of the system on the probing apparatus. Such nondemolition interaction can be implemented in the cavity QED as the dispersive interaction between the atom and the cavity [26,27]. On the other hand, it is feasible to prepare and to analyze a two-level Rydberg atom in a state corresponding to an arbitrary point on the Bloch sphere in the quantum optics experiments.

To realize the probing non-Markovianity in the cavity-QED system, we consider an open quantum system: a single-cavity mode coupled to its bath of many bosonic excitation modes resulting from the cavity leakage. Let an atom pass through the cavity, and then, examine the quantum coherence of the atom (see Fig. 1). In this case, the atom could record the intrinsic information of the cavity field to accomplish the probing of the non-Markovianity of the cavity dynamics. This kind of approach was also used to probe the quantum criticality of the many-body system [28] where the sensitive change in the atom decoherence factor, which was characterized by the Loschmidt echo [29], could reflect the quantum criticality of its surrounding environment.

In our case, the frequency of atom ω_a is drastically detuned from the cavity resonance frequency ω_0, i.e., $\Delta = \omega_0 - \omega_a \gg g_{a-f}$, where g_{a-f} is the vacuum Rabi frequency characterizing the atom-cavity coupling. By making use of an adiabatic elimination procedure, we obtain the effective Hamiltonian,

$$H_p = \hbar\omega_0 a^\dagger a + \hbar\omega_a\sigma_z + \hbar\,\delta a^\dagger a\sigma_z \quad (44)$$

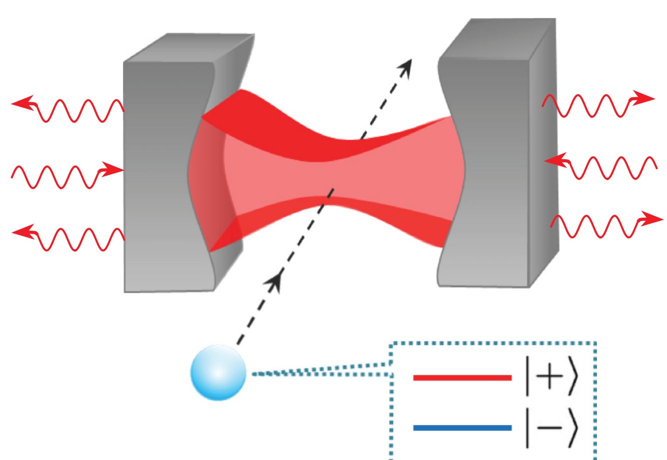

FIG. 1. (Color online) Schematic for probing the non-Markovian dynamics of an open quantum system: a leaking cavity. The two-level atom passing through the cavity is largely detuned from the frequency of the cavity mode to approach the nondemolition measurement.

for our probing scheme from the usual Jaynes-Cummings model [30]. Here, $a(a^\dagger)$ is the annihilation (creation) operator of the cavity, $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ is the Pauli matrix of the atom with the ground (excited) state of atom $|g\rangle(|e\rangle)$, and $\delta = g_{a-f}^2/\Delta$ is the effective dispersive coupling constant [31,32]. Meanwhile, the cavity is coupled to a bosonic bath,

$$H_b + H_{\text{int}} = \sum_l \hbar\omega_l b_l^\dagger b_l + \hbar \sum_l (\eta_l a^\dagger b_l + \text{H.c.}).$$

Here, the atom has enough long coherence time, and we neglect the decay of the atom during the strong probing process.

Before entering the cavity, the atom is initialized in the superposition state $(|e\rangle + |g\rangle)/\sqrt{2}$, and the cavity is initially in the coherent state $|\alpha\rangle$. For simplicity, we assume that the bath is at zero temperature with initial density matrix $\rho_b(0) = |\mathbf{0}\rangle\langle\mathbf{0}|$, where $|\mathbf{0}\rangle$ is the vacuum state of the bath. It is well known that the bath of the cavity decreases the coherence of the atom by disturbing the phase of the cavity field, but it does not change the population of the atom as the result of the dispersive atom-cavity coupling. However, we can detect this decoherence effect by observing the Ramsey interference fringes of the outcoming atom. The exact density matrix of the atom and field is obtained by tracing over the degrees of freedom of the bath,

$$\rho_{a-f} = \text{Tr}_b\{e^{-iHt}[|\psi(0)\rangle\langle\psi(0)|] \otimes \rho_b(0)e^{iHt}\}, \tag{45}$$

where $H = H_p + H_b + H_{\text{int}}$ and $|\psi(0)\rangle = (|e\rangle + |g\rangle) \otimes |\alpha\rangle/2$. In order to describe the decoherence process of the atom, we introduce the decoherence factor [33],

$$D(t) = \tfrac{1}{2}e^{-|\alpha|^2}\text{Tr}_f[\langle g|\rho_{a-f}|e\rangle], \tag{46}$$

where we have added a normalization factor $\exp(-|\alpha|^2)$.

If there were no bath present, the decoherence factor would read

$$D_0(t) = \tfrac{1}{2}\exp[|\alpha|^2(e^{-2i\delta t} - 1)], \tag{47}$$

which is similar to the result in Ref. [32]. Thus, the norm of the decoherence factor will decline to a very small value for $|\alpha|^2 \gg 1$ at the beginning and will revive at $\delta t = n\pi$ ($n = 1,2,3,\ldots$) as depicted by the gray solid lines in Fig. 2. Since the cavity evolves along the two-pronged path in the Hilbert space corresponding to different atomic states, the two paths cross periodically.

When the environment of the cavity is taken into account, we obtain the decoherence factor from Eq. (46),

$$D(t) = \tfrac{1}{2}\exp\{[W_\sigma^*(t)W_{\sigma'}(t) + J_{3,\sigma\sigma'}(t) - 1]|\alpha|^2\}, \tag{48}$$

where W_σ is determined by Eq. (18) with $M = \omega_0 \pm \delta$ ($\pm$ corresponding to $|e\rangle$ and $|g\rangle$ states, respectively), and

$$J_{3,\sigma\sigma'} = \int_0^t d\tau \int_0^t d\tau' W_{\sigma'}^*(\tau)W_\sigma(\tau') \int_0^\infty d\omega\, J(\omega)e^{-i\omega(\tau-\tau')}.$$

Here, we choose the Ohmic spectral density with cutoff frequency Ω_c,

$$J(\omega) = \lambda\omega \exp\left(-\frac{\omega}{\Omega_c}\right),$$

where λ is a dimensionless constant characterizing the cavity-bath-coupling strength.

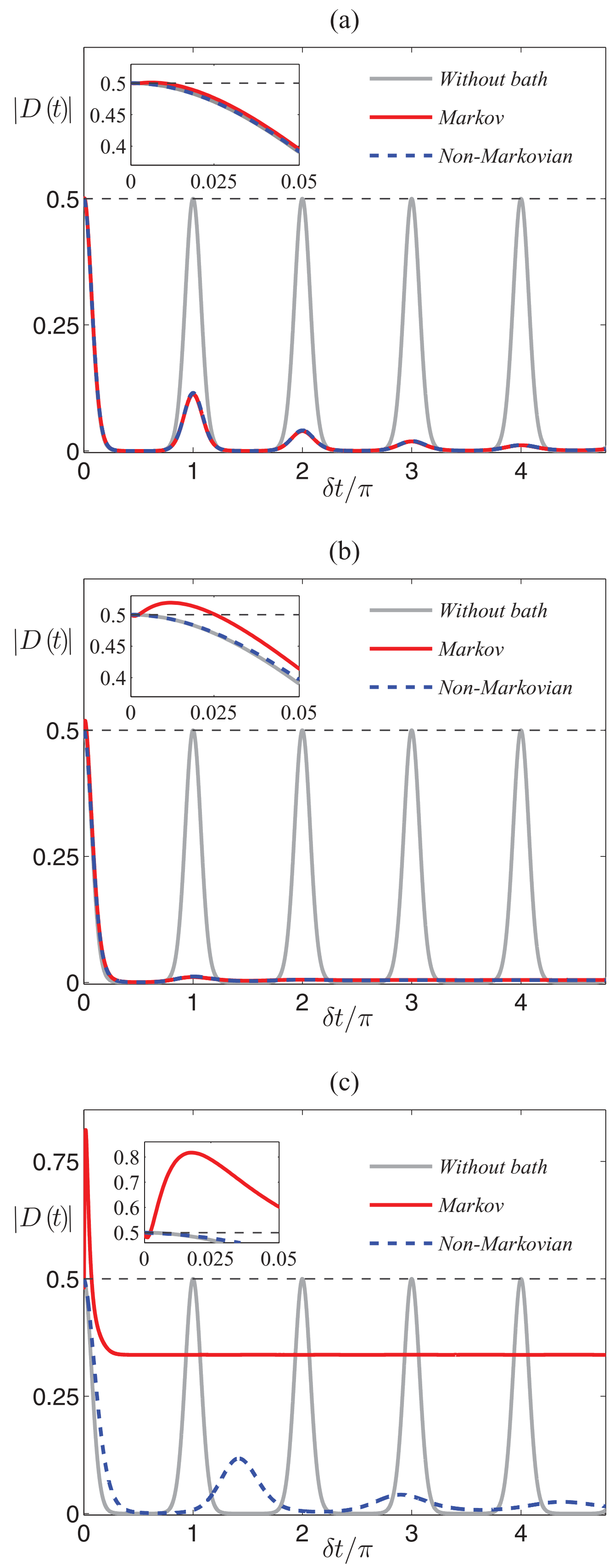

FIG. 2. (Color online) Decoherence factor for different cavity-bath coupling strengths with or without the Markov approximation. (a) $\lambda = 0.002$. (b) $\lambda = 0.01$. (c) $\lambda = 0.1$.

Next, we numerically calculate the norm of the decoherence factor with or without the Markov approximation with parameters: $\omega_0 = 1$, $\delta = 0.1$, $|\alpha|^2 = 5$, and $\Omega_c = 10$. It is found that, when the cavity-bath coupling is small ($\lambda = 0.002$), the decoherence factors with or without the Markov approximation are nearly the same as in Fig. 2(a), but they diverge from each other when the coupling strength becomes large ($\lambda = 0.01$) as in Fig. 2(b). And the Markov approximation loses its validity in the strong-coupling regime ($\lambda = 0.1$). From the insets of Figs. 2(a)–2(c), we find that the Markov approximation also becomes invalid for a short-time dynamics (the norm of the decoherence factor under the Markov approximation exceeds 0.5).

When the cavity-bath coupling is weak, the decoherence factor without the Markov approximation still revives at $\delta t = n\pi$ ($n = 1,2,3,\ldots$), but the recovering amplitude decreases along with the cavity-bath coupling λ and decays to zero [Fig. 3(a)] due to the dephasing of the cavity field induced by the bath. On the contrary, if the cavity-bath coupling becomes strong enough, the reviving magnitude increases with the coupling strength λ [Fig. 3(b)]. Especially, when the coupling strength becomes ultrastrong ($\lambda > 0.1$), the recovering amplitude almost does not decay, just like the fact that the bath does not exist. This is because, when $\lambda > \omega_0/\Omega_c = 0.1$ (for the Ohmic bath), the cavity stays in the system-bath coupling-induced dissipationless localized mode [21]. As a result, the recovering amplitude almost does not decay, but the recovering period is shifted.

Finally, we can utilize the Ramsey interference to detect the decoherence factor. After interacting with the cavity, the atom undergoes an additional resonant microwave $\pi/2$ pulse performing the following transformation:

$$|e\rangle \rightarrow \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle), \quad |g\rangle \rightarrow \frac{1}{\sqrt{2}}(-|e\rangle + |g\rangle).$$

And it is found that (please refer to Appendix E for the detailed calculation),

$$\mathrm{Re}[D(t)] = \tfrac{1}{2}[\Pi_g(t) - \Pi_e(t)], \tag{49}$$

where

$$\Pi_\sigma = e^{-|\alpha|^2}\mathrm{Tr}_f\{\langle\sigma|e^{-i\theta\sigma_y/2}\rho_{a-f}e^{i\theta\sigma_y/2}|\sigma\rangle\} \tag{50}$$

is the population of the atoms in the rotated state $\exp(i\theta\sigma_y/2)|\sigma\rangle$ ($\sigma = g,e$) with rotation angle $\theta = \pi/2$ corresponding to the final $\pi/2$ pulse. Thus, we can measure the real part of the decoherence factor through detecting the population difference of the outcoming atom. As shown in Fig. 4, the real part of the decoherence factor can also reflect the non-Markovianity of the bath.

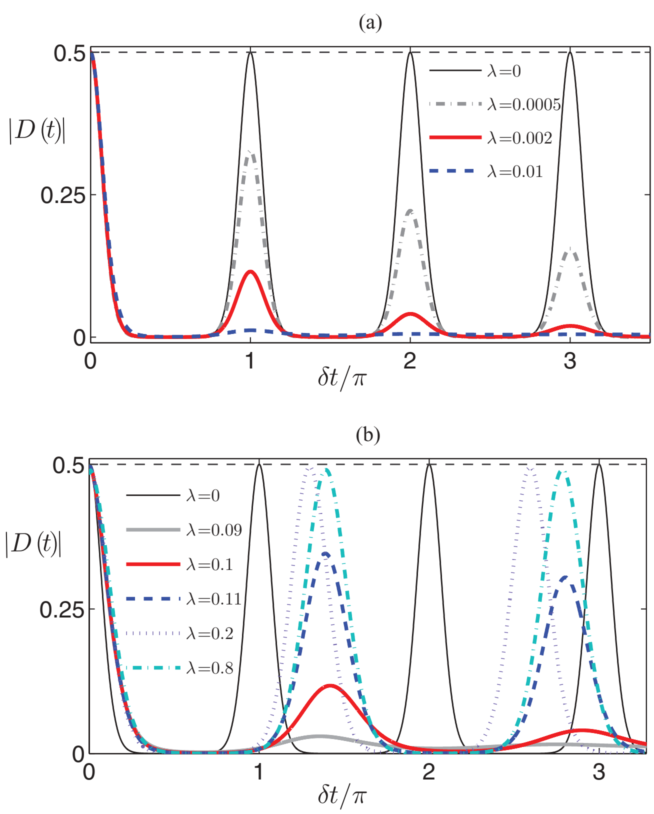

FIG. 3. (Color online) Norm of the decoherence factor without the Markov approximation. (a) If the cavity-bath-coupling strength is weak, the recovering amplitude of the decoherence factor decreases along with λ. (b) When the cavity-bath-coupling strength is large enough, the recovering amplitude of the decoherence factor increases along with λ, but its recovering period is changed by the bath.

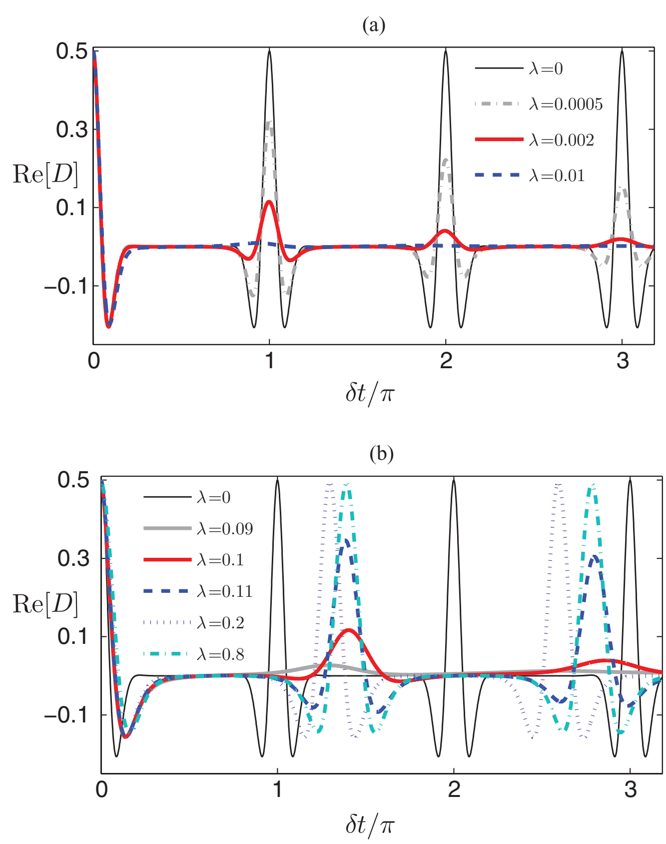

FIG. 4. (Color online) Ramsey interference is used to detect the decoherence factor. (a) The real part of the decoherence factor in the weak-coupling region without the Markov approximation. (b) The real part of the decoherence factor in the strong-coupling region without the Markov approximation.

VI. SUMMARY

By constructing the reduced density matrix from the formal solution of the Heisenberg equations, we revisited the exact non-Markovian master equations for open quantum systems of the Bose or Fermi type. The non-Markovianity can be reflected by the time-dependent decay coefficients, such as $\Gamma(t)$ and $\tilde{\Gamma}(t)$ with historical memory. To probe the non-Markovianity of the dissipation of the single-mode EM field in a cavity, we let large detuning atoms pass through the cavity. It displayed that the non-Markovianity of the bath was explicitly reflected by the atomic decoherence factor. In the weak-coupling regime, the periodically reviving amplitude decreases along with the cavity-bath-coupling strength λ and decays to zero. However, in the strong-coupling regime, the reviving amplitude increases with λ and almost does not decay in the ultra-strong-coupling case. But the recovering period is shifted by the bath. We expect our results to be verified by experiments.

ACKNOWLEDGMENTS

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APPENDIX A: BOSON AND FERMION COHERENT STATES

1. Boson coherent state

For an arbitrary complex number $\alpha = r\,\exp(i\varphi)$, the coherent state of a Bose mode with frequency ω_0 could be defined as

$$|\alpha\rangle \equiv e^{\alpha a^\dagger}|0\rangle = \sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \tag{A1}$$

where $a^\dagger$ is the creation operator of the boson and $|n\rangle$ is the nth Fock state. It is found that the coherent state defined in Eq. (A1) is not normalized, and different coherent states are generally not orthogonal,

$$\langle\alpha|\alpha'\rangle = \langle 0|e^{\alpha^* a}e^{\alpha' a^\dagger}|0\rangle = \exp(\alpha^*\alpha'). \tag{A2}$$

All the coherent states form an overcomplete set,

$$\int d\mu(\alpha)|\alpha\rangle\langle\alpha| = 1, \tag{A3}$$

with the measures,

$$d\mu(\alpha) \equiv e^{-|\alpha|^2}\frac{d^2\alpha}{\pi} = e^{-|\alpha|^2}\frac{r}{\pi}dr\,d\varphi. \tag{A4}$$

And the density matrix of the thermal equilibrium state in this coherent-state representation reads

$$\rho_T = \frac{1}{1+f(\omega_0)}\exp(-\beta\omega_0 a^\dagger a) \tag{A5}$$

$$= \int d\mu(\alpha)\frac{1}{f(\omega_0)}\exp\left[-\frac{|\alpha|^2}{f(\omega_0)}\right]|\alpha\rangle\langle a|, \tag{A6}$$

where $f(\omega_0) = 1/[\exp(\beta\omega_0) - 1]$ is the mean-occupation number with temperature $T = 1/(k_B\beta)$.

2. Fermion coherent state

The fermion coherent state is defined in a similar form as bosons,

$$|\alpha\rangle \equiv e^{-\alpha a^\dagger}|0\rangle. \tag{A7}$$

The only difference lies in the fact that α is a generator of a Grassmann algebra instead of an ordinary complex number and $a^\dagger$ is the creation operator for Fermi particles and they satisfy the anticommutation relations,

$$\{\alpha,\alpha'\} = \{\alpha,a\} = \{\alpha,a^\dagger\} = 0. \tag{A8}$$

The overlap of two fermion coherent states is

$$\langle\alpha|\alpha'\rangle = \exp(\alpha^*\alpha'), \tag{A9}$$

and the completeness relation reads

$$\int d\mu(\alpha)\,|\alpha\rangle\,\langle\alpha| = 1, \tag{A10}$$

with

$$d\mu(\alpha) = d\alpha^* d\alpha\, e^{-\alpha^*\alpha}. \tag{A11}$$

APPENDIX B: CONSTRAINTS OF BLOCKS OF $\mathcal{U}(t)$

Due to the Hermiticity of matrix $\mathcal{H}$, the time-evolution operator $\mathcal{U}(t)$ in Liouville space is a unitary matrix, i.e.,

$$\begin{bmatrix} W(t) & T(t) \\ P(t) & Q(t)\end{bmatrix}\begin{bmatrix} W^\dagger(t) & P^\dagger(t) \\ T^\dagger(t) & Q^\dagger(t)\end{bmatrix} = I, \tag{B1}$$

which leads to

$$WW^\dagger + TT^\dagger = I, \tag{B2}$$

$$PP^\dagger + QQ^\dagger = I, \tag{B3}$$

$$WP^\dagger + TQ^\dagger = \mathbf{0}, \tag{B4}$$

$$PW^\dagger + QT^\dagger = \mathbf{0}. \tag{B5}$$

Except for some special time t, the matrices $W(t)$ and $Q(t)$ are reversible. Then, we have

$$P = -QT^\dagger(W^\dagger)^{-1}, \tag{B6}$$

$$P^\dagger = -W^{-1}TQ^\dagger. \tag{B7}$$

APPENDIX C: CALCULATION OF THE PROPAGATING FUNCTION

The reduced density matrix $\rho_s(t)$ of the system is obtained by tracing over the degrees of freedom of B in $\rho(t)$,

$$\rho_s(t) = \int d\mu(\vec{z})\langle\vec{z}|\rho(t)|\vec{z}\rangle \tag{C1}$$

$$\equiv \int d\mu(\vec{\alpha},\vec{\alpha}')\rho_s(\vec{\alpha},\vec{\alpha}';t)|\vec{\alpha}\rangle\langle\vec{\alpha}'|, \tag{C2}$$

where $|\vec{\alpha}\rangle(|\vec{\alpha}'\rangle)$ and $|\vec{z}\rangle$ are coherent states of S and B, respectively. The element of the reduced density matrix is explicitly given by

$$\rho_s(\vec{\alpha},\vec{\alpha}';t) = \int d\mu(\vec{z})\langle\vec{\alpha},\vec{z}|\rho(t)|\vec{\alpha}',\vec{z}\rangle \tag{C3}$$

$$= \int d\mu(\vec{z})d\mu(\vec{\xi},\vec{z}',\vec{\xi}',\vec{z}'')\langle\vec{\alpha},\vec{z}|U(t)|\vec{\xi},\vec{z}'\rangle$$
$$\times\langle\vec{\xi},\vec{z}'|\rho_s(0)\rho_b(0)|\vec{\xi}',\vec{z}''\rangle\langle\vec{\xi}',\vec{z}''|U^{\dagger}|\vec{\alpha}',\vec{z}\rangle \tag{C4}$$

$$\equiv \int d\mu(\vec{\xi},\vec{\xi}')K(\vec{\alpha}^{\dagger},\vec{\alpha}',\vec{\xi},\vec{\xi}'^{\dagger},t)\langle\vec{\xi}|\rho_s(0)|\vec{\xi}'\rangle, \tag{C5}$$

with

$$K = \int d\mu(\vec{z})\langle\vec{\alpha},\vec{z}|U(t)|\vec{\xi}\rangle\langle\vec{\xi}'|\rho_b(0)U^{\dagger}(t)|\vec{\alpha}',\vec{z}\rangle. \tag{C6}$$

Here, we have used the fact that the initial state of the total system is of the direct product form and the completeness of the coherent states $\{|\vec{z}'\rangle\}$ and $\{|\vec{z}''\rangle\}$ of the bath.

With the help of Eqs. (9), (10), and (24)–(26), the propagator is reexpressed in terms of the coefficient matrices,

$$K(\vec{\alpha}^{\dagger},\vec{\alpha}',\vec{\xi},\vec{\xi}'^{\dagger};t)$$
$$= \int d\mu(\vec{z})\langle W^{\dagger}\vec{\alpha} + P^{\dagger}\vec{z}|\vec{\xi}\rangle\langle\vec{\xi}'|W^{\dagger}\vec{\alpha}' + P^{\dagger}\vec{z}\rangle\left(\prod_l \frac{1}{f_l+1}\right)$$
$$\times\langle T^{\dagger}\vec{\alpha} + !Q^{\dagger}\vec{z}|\exp[-\vec{b}^{\dagger}\beta E\vec{b}]|T^{\dagger}\vec{\alpha}' + Q^{\dagger}\vec{z}\rangle. \tag{C7}$$

Using formulas $\langle\alpha|\exp(\delta b^{\dagger}b)|\alpha'\rangle = \exp[\alpha^{*}\alpha'\exp(\delta)]$ and

$$\int d\mu(\vec{z})e^{\vec{z}^{\dagger}D\vec{z}+\vec{u}^{\dagger}\cdot\vec{z}+\vec{z}^{\dagger}\cdot\vec{v}} = \frac{\exp[\vec{u}^{\dagger}(I-D)^{-1}\vec{v}]}{\det[I-D]} \tag{C8}$$

[for any $N_s\times N_s$ Hermitian matrix, D makes $(I-D)$ positive definite], one goes to

$$K(\vec{\alpha}^{\dagger},\vec{\alpha}',\vec{\xi},\vec{\xi}'^{\dagger},t)$$
$$= A(t)\exp[\vec{\alpha}^{\dagger}J_1(t)\vec{\xi} + \vec{\xi}'^{\dagger}J_1^{\dagger}(t)\vec{\alpha}' + \vec{\alpha}^{\dagger}J_2(t)\vec{\alpha}' + \vec{\xi}'^{\dagger}J_3\vec{\xi}], \tag{C9}$$

where

$$A = \left(\prod_l \frac{1}{f_l+1}\right)\det[I - Qf(I+f)^{-1}Q^{\dagger}]^{-1}, \tag{C10}$$

$$J_1 = W + Tf(I+f)^{-1}Q^{\dagger}[I - Qf(I+f)^{-1}Q^{\dagger}]^{-1}P, \tag{C11}$$

$$J_2 = Tf(I+f)^{-1}T^{\dagger} + Tf(I+f)^{-1}Q^{\dagger}$$
$$\times[I - Qf(I+f)^{-1}Q^{\dagger}]^{-1}Qf(I+f)^{-1}T^{\dagger}, \tag{C12}$$

$$J_3 = P^{\dagger}[I - Qf(I+f)^{-1}Q^{\dagger}]^{-1}P, \tag{C13}$$

and we have introduced a diagonal matrix $f = \mathrm{diag}[f_1,f_2,\ldots,f_{N_b}]$.

Then, we will deal with these four terms one by one. First, we perform some pretreatment to obtain an expanding series. From Eqs. (B3), (B6), and (B7), one finds

$$I - Qf(I+f)^{-1}Q^{\dagger} = Q[T^{\dagger}(WW^{\dagger})^{-1}T + (I+f)^{-1}]Q^{\dagger}. \tag{C14}$$

So that

$$[I - Qf(I+f)^{-1}Q^{\dagger}]^{-1}$$
$$= (Q^{\dagger})^{-1}(I+f)\sum_{n=0}^{\infty}(-1)^n[T^{\dagger}(WW^{\dagger})^{-1}T(I+f)]^nQ^{-1}. \tag{C15}$$

1. $J_1(t)$, $J_2(t)$, and $J_3(t)$

According to Eqs. (B6), (C11), and (C15), $J_1(t)$ is explicitly expanded to

$$J_1 = W - Tf\sum_{n=0}^{\infty}(-1)^n[T^{\dagger}(WW^{\dagger})^{-1}T(I+f)]^nT^{\dagger}(W^{\dagger})^{-1}$$
$$= W - TfT^{\dagger}\sum_{n=0}^{\infty}(-1)^n[(WW^{\dagger})^{-1}T(I+f)T^{\dagger}]^n(W^{\dagger})^{-1}$$
$$= W - V[1 + (WW^{\dagger})^{-1}T(I+f)T^{\dagger}]^{-1}(W^{\dagger})^{-1}$$
$$= W - V[WW^{\dagger} + T(I+f)T^{\dagger}]^{-1}W \tag{C16}$$
$$= (1+V)^{-1}W. \tag{C17}$$

The third step we have introduced is a new $N_s\times N_s$ matrix $V(t) = T(t)fT^{\dagger}(t)$. Similarly, one obtains

$$J_3 = I - W^{\dagger}(1+V)^{-1}W. \tag{C18}$$

The calculation of J_2 is a little more complicated

$$J_2 = T\left\{I + f\sum_{n=0}^{\infty}(-1)^n[T^{\dagger}(WW^{\dagger})^{-1}T(I+f)]^n\right\}$$
$$\times f(I+f)^{-1}T^{\dagger} \tag{C19}$$
$$= V + Tf\left(\sum_{n=1}^{\infty}(-1)^n[T^{\dagger}(WW^{\dagger})^{-1}T(I+f)]^{n-1}\right)$$
$$\times T^{\dagger}(WW^{\dagger})^{-1}TfT^{\dagger} \tag{C20}$$
$$= V + Tf\left(\sum_{n=1}^{\infty}(-1)^n[T^{\dagger}(WW^{\dagger})^{-1}T(I+f)]^{n-1}\right)$$
$$\times T^{\dagger}(WW^{\dagger})^{-1}T(I+f-I)T^{\dagger} \tag{C21}$$
$$= V\left\{I + \sum_{n=1}^{\infty}(-1)^n[(WW^{\dagger})^{-1}T(I+f)T^{\dagger}]^n\right\}$$
$$+V\sum_{n=0}^{\infty}(-1)^n[(WW^{\dagger})^{-1}T(I+f)T^{\dagger}]^n(WW^{\dagger})^{-1}TT^{\dagger} \tag{C22}$$
$$= V[WW^{\dagger} + T(I+f)T^{\dagger}]^{-1}(WW^{\dagger} + TT^{\dagger}) \tag{C23}$$
$$= V(I+V)^{-1}. \tag{C24}$$

2. $A(t)$

Matrix $A(t)$ is determined by the normalization condition,

$$1 = \mathrm{Tr}[\rho_s(t)] \tag{C25}$$
$$= \int d\mu(\vec{\alpha})d\mu(\vec{\xi},\vec{\xi}')K(\vec{\alpha},\vec{\alpha},\vec{\xi},\vec{\xi}',t)\langle\vec{\xi}|\rho_s(0)|\vec{\xi}'\rangle \tag{C26}$$

$$= A\det[I+V]\int d\mu(\vec{\xi},\vec{\xi}')\langle\vec{\xi}|\rho_s(0)|\vec{\xi}'\rangle\exp(\vec{\xi}'^{\dagger}\cdot\vec{\xi}) \tag{C27}$$

$$= A\det[I+V]\int d\mu(\vec{\xi})\langle\vec{\xi}|\rho_s(0)|\vec{\xi}\rangle. \tag{C28}$$

In the second step, we carried out the integral over $\vec{\alpha}$ of Eq. (27) and used the identity,

$$J_3(t)+J_1^{\dagger}(I+V)J_1(t)=I. \tag{C29}$$

And, in the last step, the following formula is used:

$$\int d\mu(\alpha')(\alpha')^n e^{\alpha'^*\alpha}=\alpha^n. \tag{C30}$$

Since the initial density matrix is also normalized, thus,

$$A(t)=\det[(I+V)^{-1}].$$

APPENDIX D: TIME DIFFERENTIAL OF THE PROPAGATING FUNCTION

The time differential of the propagating function is given by

$$\dot{K}=\left[\frac{\dot{A}}{A}+\vec{\alpha}^{\dagger}\dot{J}_1\vec{\xi}+\vec{\xi}'^{\dagger}\dot{J}_1^{\dagger}\vec{\alpha}'+\vec{\alpha}^{\dagger}\dot{J}_2\vec{\alpha}'+\vec{\xi}'^{\dagger}\dot{J}_3\vec{\xi}\right]K. \tag{D1}$$

We define the differential operators,

$$\vec{\nabla}_{\alpha^*}\equiv\left[\frac{\partial}{\partial\alpha_1^*},\frac{\partial}{\partial\alpha_2^*},\ldots,\frac{\partial}{\partial\alpha_{N_s}^*}\right]^T, \tag{D2}$$

and

$$\vec{\nabla}_{\alpha'}^T\equiv\left[\frac{\partial}{\partial\alpha_1},\frac{\partial}{\partial\alpha_2},\ldots,\frac{\partial}{\partial\alpha_{N_s}}\right]. \tag{D3}$$

We are ready to find that

$$\vec{\xi}K=J_1^{-1}(\vec{\nabla}_{\alpha^*}-J_2\vec{\alpha}')K, \tag{D4}$$

$$\vec{\xi}'^{\dagger}K=(\vec{\nabla}_{\alpha'}^T-\vec{\alpha}^{\dagger}J_2)(J_1^{\dagger})^{-1}K, \tag{D5}$$

$$\vec{\xi}'^{\dagger}K\vec{\xi}=(\vec{\nabla}_{\alpha'}^T-\vec{\alpha}^{\dagger}J_2)(J_1^{\dagger})^{-1}J_1^{-1}(\vec{\nabla}_{\alpha^*}-J_2\vec{\alpha}')K. \tag{D6}$$

These relations lead to

$$\begin{aligned}\dot{K}=&\,\vec{\alpha}^{\dagger}\big[\dot{J}_2-\dot{J}_1J_1^{-1}J_2-J_2(J_1^{\dagger})^{-1}\dot{J}_1^{\dagger}+J_2(J_1^{\dagger})^{-1}\dot{J}_3J_1^{-1}J_2\big]K\vec{\alpha}'\\&+\left\{\frac{\dot{A}}{A}-\mathrm{Tr}\big[(J_1^{\dagger})^{-1}\dot{J}_3J_1^{-1}J_2\big]\right\}K\\&+\vec{\alpha}^{\dagger}\big[\dot{J}_1J_1^{-1}-J_2(J_1^{\dagger})^{-1}\dot{J}_3J_1^{-1}\big]\vec{\nabla}_{\alpha^*}K\\&+\big(\vec{\nabla}_{\alpha'}^TK\big)\big[(J_1^{\dagger})^{-1}\dot{J}_1^{\dagger}-(J_1^{\dagger})^{-1}\dot{J}_3J_1^{-1}J_2\big]\vec{\alpha}'\\&+\vec{\nabla}_{\alpha'}^T(J_1^{\dagger})^{-1}\dot{J}_3J_1^{-1}\vec{\nabla}_{\alpha^*}K\end{aligned} \tag{D7}$$

$$\begin{aligned}\equiv&\,\vec{\alpha}^{\dagger}\tilde{\Gamma}K\vec{\alpha}'-\mathrm{Tr}[\tilde{\Gamma}]K-\vec{\alpha}^{\dagger}(\Gamma+i\tilde{\Omega}+\tilde{\Gamma})\vec{\nabla}_{\alpha^*}K\\&-\big(\vec{\nabla}_{\alpha'}^TK\big)(\Gamma-i\tilde{\Omega}+\tilde{\Gamma})\vec{\alpha}'+\vec{\nabla}_{\alpha'}^T(\tilde{\Gamma}+2\Gamma)\vec{\nabla}_{\alpha^*}K,\end{aligned} \tag{D8}$$

with Hermitian matrices,

$$\tilde{\Gamma}=\dot{V}-\dot{W}W^{-1}V-V(\dot{W}W^{-1})^{\dagger}, \tag{D9}$$

$$\Gamma=-\tfrac{1}{2}[\dot{W}W^{-1}+(\dot{W}W^{-1})^{\dagger}], \tag{D10}$$

and

$$\tilde{\Omega}=\frac{i}{2}[\dot{W}W^{-1}-(\dot{W}W^{-1})^{\dagger}]. \tag{D11}$$

In the last step, the following relations have been used:

$$\begin{aligned}\dot{J}_1J_1^{-1}=&\left[\frac{d}{dt}(I+V)^{-1}\right](I+V)\\&+(I+V)^{-1}(\dot{W}W^{-1})(I+V)\end{aligned} \tag{D12}$$

$$\begin{aligned}=&-[(I+V)^{-1}\dot{V}(I+V)^{-1}](I+V)\\&+(I+V)^{-1}(\dot{W}W^{-1})(I+V)\end{aligned} \tag{D13}$$

$$=-(I+V)^{-1}[\dot{V}-(\dot{W}W^{-1})(I+V)], \tag{D14}$$

$$(J_1^{\dagger})^{-1}\dot{J}_3J_1^{-1}=-(I+V)(\dot{W}W^{-1})^{\dagger}-\dot{W}W^{-1}(I+V)+\dot{V}, \tag{D15}$$

and

$$\begin{aligned}&\frac{\dot{A}}{A}-\mathrm{Tr}\big[(J_1^{\dagger})^{-1}\dot{J}_3J_1^{-1}J_2\big]\\&=\frac{d}{dt}\ln A+\mathrm{Tr}[V(\dot{W}W^{-1})^{\dagger}+(\dot{W}W^{-1})V\\&\quad-\dot{V}[I-(I+V)^{-1}]]\end{aligned} \tag{D16}$$

$$=\frac{d}{dt}\mathrm{Tr}[\ln(I+V)^{-1}]+\mathrm{Tr}[-\tilde{\Gamma}+\dot{V}(I+V)^{-1}] \tag{D17}$$

$$=-\mathrm{Tr}[\tilde{\Gamma}]. \tag{D18}$$

APPENDIX E: DECOHERENCE FACTOR

Through the approach in Appendix C, we can obtain the element of the reduced density in Eq. (45),

$$\begin{aligned}\rho_{a-f}(\alpha_f,\sigma;\alpha_f',\sigma')=&\,\tfrac{1}{2}A_{\sigma\sigma'}\exp[\alpha_f^*J_{1,\sigma\sigma'}\alpha+\alpha^*J_{1,\sigma'\sigma}^{\dagger}\alpha_f'\\&+\alpha_f^*J_{2,\sigma\sigma'}\alpha_f'+\alpha^*J_{3,\sigma\sigma'}\alpha],\end{aligned} \tag{E1}$$

where, in the case of the zero-temperature bath,

$$A_{\sigma\sigma'}=1,\quad J_{1,\sigma\sigma'}=W_\sigma(t),\quad J_{2,\sigma\sigma'}=0,\quad J_{3,\sigma\sigma'}=P_{\sigma'}^{\dagger}P_\sigma. \tag{E2}$$

Here, W_σ is determined by Eq. (18) with $M=\omega_0\pm\delta$ ($\pm$ corresponding to $|e\rangle$ and $|g\rangle$ states, respectively), and P_σ is given by Eq. (16). Following from Eq. (46), we find that the population difference of the outcoming atom just gives the decoherence factor,

$$\begin{aligned}\Pi_g(t)-\Pi_e(t)&\\&=e^{-|\alpha|^2}\mathrm{Tr}_f\{\langle g|e^{-i\theta\sigma_y/2}\rho_{a-f}e^{i\theta\sigma_y/2}|g\rangle\\&\quad-\langle e|e^{-i\theta\sigma_y/2}\rho_{a-f}e^{i\theta\sigma_y/2}|e\rangle\}\end{aligned} \tag{E3}$$

$$=D(t), \tag{E4}$$

where $\theta=\pi/2$ and $\sigma_y=i(|g\rangle\langle e|-|e\rangle\langle g|)$. With the help of Eq. (E1), we obtain the decoherence factor as

$$D(t)=\mathrm{Re}(\exp\{[W_\sigma^*(t)W_{\sigma'}(t)+J_{3,\sigma\sigma'}(t)-1]|\alpha|^2\}). \tag{E5}$$

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