Single-photon scattering on a strongly dressed atom

Z. H. Wang,[1] Yong Li,[2] D. L. Zhou,[1] C. P. Sun,[2,3] and Peng Zhang[4,*]

[1]*Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China*
[2]*Beijing Computational Science Research Center, Beijing 100084, China*
[3]*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China*
[4]*Department of Physics, Renmin University of China, Beijing 100872, China*

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We use the generalized rotating-wave approximation approach [Irish, Phys. Rev. Lett. **99**, 173601 (2007)] to study single-photon scattering on a two-level system (TLS) with arbitrarily strong coupling to a local mode in a one-dimensional (1D) coupled-resonator array. We obtain the scattering amplitudes by an analytical method, which works well in a broad parameter region, confirmed by independent numerical results. In particular, when the resonator mode is far off resonance with the TLS, our results appear more reasonable than the ones from the standard adiabatic approximation. The approach is further extended to cases with a 1D resonator array strongly coupled to more than one TLS.

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I. INTRODUCTION

Recently, much attention has been paid to photon transport in a low-dimensional array of coupled resonators [1–25] or a one-dimensional (1D) wave guide [24–49], which plays crucial roles in the realization of all-optical quantum devices. In these systems, two-level or multilevel devices coupled to the resonators or wave guides can be used as quantum switches to control the scattering or transport of the photons.

Up to now, the single-photon scattering amplitudes in 1D resonator arrays or wave guides coupled to a single two-level system (TLS) [1–6,25,28–33] or a single three-level system [15,41,47,48] or multiple quantum devices [8–12,34–38] have been well investigated. The relevant multiphoton scattering amplitudes [21–27,42–46] have also been studied. To our knowledge, all these studies are based on the rotating-wave approximation, which is applicable under the conditions that the frequencies of the two-level or multilevel system are very close to the photon frequencies in the resonators or wave guides, and the coupling strengths between the two-level or multilevel system and the photons are much smaller than their frequencies, namely, in the weak-coupling limit.

Steady progress in related experiments has also been made. D resonator arrays or wave guides can be realized with photonic crystals [50,51], superconducting transmission line resonators [12–14,29,47,52], or other solid devices, while two-level or multilevel systems can be implemented with either natural atoms or solid-state artificial atoms. In hybrid systems of solid-state devices, it has been predicted [53–56] that one can realize ultrastrong TLS-photon coupling with intensities comparable to or even higher than the photon frequencies. Furthermore, the frequency of a solid-state TLS can be controlled easily in a broad region. Therefore it is possible to reach the strong-coupling and far-off-resonance regime in a quantum network based on solid-state devices. In these parameter regions where the conditions of rotating-wave approximation are violated, we need to use effective methods for the scattering between the flying photon and the TLS, and then investigate the possible new effects caused by the strong TLS-photon coupling to the photon transport. Fortunately, for the TLS-coupled single-mode bosonic field, several theoretical methods beyond the rotating-wave approximation [57–73] have been developed. In particular, the generalized rotating-wave approximation (GRWA) [73] developed by Irish is a very effective method with a clear physical picture.

In this paper, we study single-photon scattering on a TLS which strongly couples to the local mode of a 1D single-mode-resonator array. Using the GRWA approach we obtain the single-photon scattering amplitudes under the condition that the photon hopping between different resonators is weak enough. We show that the approach works significantly well in a very broad parameter region, including the region where the rotating-wave approximation is applicable and the one where the TLS-photon coupling is strong while the frequency of the TLS is close to or smaller than the photon frequency.

In particular, in the far-off-resonance region where the TLS frequency is much smaller than that of the flying photon, the standard adiabatic approximation does not work well in the current hybrid system, while the GRWA approach still provides good results. In addition, when the photon-TLS coupling is strong enough, the GRWA approach shows that the single-photon scattering by the TLS becomes equivalent to the transport of a single photon in a 1D resonator array in which the frequency of a certain resonator is shifted. Then our results are further simplified and one can make reasonable qualitative estimations of the characteristics for the photon transport, even without quantitative calculations. We also show that the GRWA approach can be generalized to systems with a 1D single-mode-resonator array coupled with more than one TLS.

This paper is organized as follows. In Sec. II, we apply the GRWA approach to a hybrid system of a 1D resonator array coupled to a single TLS. In Sec. III, we analytically calculate the single-photon scattering amplitudes in such a system with the GRWA approach and compare our results with numerical results. In Sec. IV, we discuss the single-photon scattering problem in the case of strong TLS-photon coupling. In Sec. V, we show the GRWA approach in a system with more than one

*pengzhang@ruc.edu.cn

TLS. We give several discussions and a brief conclusion in Sec. VI.

II. THE GRWA FOR THE TLS-COUPLED 1D RESONATOR ARRAY

In this paper we consider the transmission of a single photon in a 1D single-mode-resonator array coupled to a TLS located in a specific resonator. To obtain reasonable analytical results beyond the rotating-wave approximation, in this section we will generalize the GRWA approach proposed by Irish in Ref. [73] for a single resonator coupled to a TLS to our current hybrid system of a resonator array. We will first make the adiabatic approximation in our system for the cases where the frequency of the TLS is much smaller than the photon frequency, and then introduce the GRWA approach as an improvement of the adiabatic approximation. For the reader's convenience, in Appendix A we review Irish's GRWA approach for a system with a single resonator from the viewpoint of the adiabatic approximation.

A. The system and Hamiltonian

As shown in Fig. 1, we consider a 1D array of an infinite number of identical single-mode resonators with a TLS located inside a certain resonator, which is marked as the zeroth resonator in the array. We further assume that the photons can hop between neighboring resonators. Then the total system is modeled by the Hamiltonian

$$H = H_C + H_A + H_I, \tag{1}$$

where the tight-binding Hamiltonian H_C of the resonator array is

$$H_C = \omega \sum_{j=-\infty}^{+\infty} a_j^\dagger a_j - \xi \sum_{j=-\infty}^{+\infty} (a_j^\dagger a_{j+1} + \text{H.c.}). \tag{2}$$

Here ω is the frequency of the photons in the resonators, ξ is the inter-resonator coupling strength, and a_j and $a_j^\dagger$ are the annihilation and creation operators of the photons in the jth resonator, respectively. Throughout this paper, we set $\hbar = 1$.

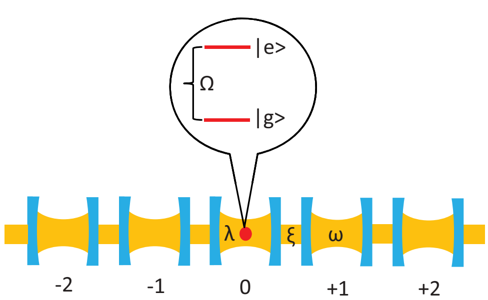

FIG. 1. (Color online) Schematic configuration for the hybrid system of a 1D resonator array interacting with a TLS. The frequency of the photon in each resonator is ω, while the intensity of the photon hopping is ξ. The TLS with frequency Ω is localized at the zeroth resonator and coupled to the photon with coupling strength λ.

In addition, we assume the weak-hopping condition

$$|\xi| \ll \omega \tag{3}$$

is satisfied. Therefore the term $a_j^\dagger a_{j+1}^\dagger + \text{H.c.}$ has been neglected in our consideration. This condition also provides us a small parameter ξ/ω which is very useful in the following calculation.

The Hamiltonian H_A of the TLS and the interaction H_I between the TLS and the photons in the zeroth resonator are

$$H_A = \frac{\Omega}{2}\sigma_z \tag{4}$$

and

$$H_I = \lambda\sigma_x(a_0^\dagger + a_0), \tag{5}$$

respectively. Here Ω is the energy difference between the ground state $|g\rangle$ and the excited state $|e\rangle$ of the TLS, λ is the coupling strength, and the Pauli operators σ_z and σ_x are defined as $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ and $\sigma_x = |e\rangle\langle g| + |g\rangle\langle e|$.

B. The GRWA approach for a TLS-coupled 1D resonator array

Now we apply the GRWA approach to a system of a TLS-coupled 1D resonator array. To this end, we first briefly recall the original GRWA for the single-mode bosonic field coupled to a TLS. As shown in Appendix A, in the GRWA for that system, the Rabi Hamiltonian

$$H_{\text{Rabi}} = \omega a^\dagger a + \frac{\Omega}{2}\sigma_z + \lambda\sigma_x(a^\dagger + a) \tag{6}$$

is approximated as

$$H_{\text{GR}} = U_R \left[\sum_{n=0}^{\infty} \left[\hat{P}_{\text{Rabi}}^{(n)} \left(U_R^{-1} H_{\text{Rabi}} U_R\right) \hat{P}_{\text{Rabi}}^{(n)}\right]\right] U_R^{-1}. \tag{7}$$

Here a and $a^\dagger$ are the annihilation and creation operators of the single-mode bosonic field, respectively. The unitary transformation U_R is defined as

$$U_R = \exp\left[-\frac{\lambda}{\omega}\sigma_x(a^\dagger - a)\right]. \tag{8}$$

It can be used to remove the linear term $\lambda\sigma_x(a^\dagger + a)$ of a and $a^\dagger$ in the Hamiltonian $\omega a^\dagger a + \lambda\sigma_x(a^\dagger + a)$ for the single-mode bosonic field, i.e., we have

$$U_R^{-1}[\omega a^\dagger a + \lambda\sigma_x(a^\dagger + a)]U_R = \omega a^\dagger a. \tag{9}$$

The projection operator $\hat{P}_{\text{Rabi}}^{(n)}$ in Eq. (7) is defined as

$$\hat{P}_{\text{Rabi}}^{(n)} = \begin{cases} |0g\rangle\langle 0g| & \text{if } n = 0, \\ |n,g\rangle\langle n,g| + |n-1,e\rangle\langle n-1,e| & \text{if } n \geqslant 1, \end{cases} \tag{10}$$

where $|n,g(e)\rangle$ is the direct product of the Fock state with n photons and the atomic state $|g(e)\rangle$. Therefore, in Eq. (7) the "counter-rotating-wave" transitions between the states $|n,g\rangle$ and $|m,e\rangle$ with $m \neq n-1$, or the transitions between the eigenstates of the total excitation operator $a^\dagger a + |e\rangle\langle e|$ with different eigenvalues, are removed by the operation $\sum_{n=0}^{\infty}[\hat{P}_{\text{Rabi}}^{(n)} \cdot \hat{P}_{\text{Rabi}}^{(n)}]$ for the rotated Hamiltonian $U_R^{-1} H_{\text{Rabi}} U_R$.

In this sense, the GRWA can be understood as "the rotating-wave approximation for the rotated Hamiltonian." It is pointed out that, as shown in Appendix A, the GRWA converges to the normal rotating-wave approximation under the near-resonance condition $|\omega - \Omega| \ll |\omega + \Omega|$ and the weak-coupling condition $|\lambda| \ll \Omega,\omega$. In addition, under the far-off-resonance condition $\Omega \ll \omega$, the GRWA becomes the adiabatic approximation. Therefore, the GRWA smoothly connects the adiabatic approximation and the rotating-wave approximation, and thus can be used in a broad parameter region.

For our system with a resonator array, we can also develop the GRWA approach with a similar scheme. In the GRWA approach the total Hamiltonian H is approximated as H_G, which is defined as

$$H_G = U H_R^{\mathrm{RWA}} U^{-1}. \tag{11}$$

Here the operator H_R^{RWA} is defined as

$$H_R^{\mathrm{RWA}} = \sum_{n=0}^{\infty} [\hat{P}_n (U^{-1} H U) \hat{P}_n] \tag{12}$$

and the unitary transformation U is given by

$$U = \prod_{j=-\infty}^{+\infty} \exp[\alpha_j \sigma_x (a_j^\dagger - a_j)], \tag{13}$$

where

$$\alpha_j = \frac{\lambda \omega_1}{2\xi^2 - \omega\omega_1} \left(\frac{\xi}{\omega_1}\right)^{|j|} \tag{14}$$

with

$$\omega_1 = (\omega + \sqrt{\omega^2 - 4\xi^2})/2. \tag{15}$$

A straightforward calculation in Appendix B shows that this unitary transformation can remove the linear terms of a_j and $a_j^\dagger$ in the Hamiltonian $H_C + H_I$ of the photons, i.e., we have

$$\begin{aligned} U^{-1}&(H_C + H_I)U \\ &= \omega \sum_{j=-\infty}^{+\infty} a_j^\dagger a_j - \xi \sum_{j=-\infty}^{+\infty} (a_j^\dagger a_{j+1} + \mathrm{H.c.}) - \mathcal{C}, \end{aligned} \tag{16}$$

with $\mathcal{C}$ an unimportant c number. The operator $\hat{P}_n$ is defined as

$$\hat{P}_n = \begin{cases} |0\rangle\langle 0| \otimes |g\rangle\langle g| & \text{if } n = 0, \\ \sum_{\{m_l\}} \prod_l |m_l\rangle_l \langle m_l| \otimes |g\rangle\langle g| \delta_{\sum_l m_l, n} & \\ + \sum_{\{m_l\}} \prod_l |m_l\rangle_l \langle m_l| \otimes |e\rangle\langle e| \delta_{\sum_l m_l, n-1} & \text{if } n \neq 0. \end{cases} \tag{17}$$

Therefore, in Eq. (12) the counter-rotating-wave transitions between the eigenstates of the total excitation operator $\sum_j a_j^\dagger a_j + |e\rangle\langle e|$ with different eigenvalues are removed by the operation $\sum_{n=0}^{\infty} [\hat{P}_n \cdot \hat{P}_n]$ for the rotated Hamiltonian $U^{-1}HU$. In this sense the GRWA approach can also be considered as "the rotating-wave approximation for the rotated Hamiltonian $U^{-1}HU$."

Obviously, the GRWA Hamiltonian H_G in Eq. (11) is a direct generalization of the original GRWA Hamiltonian H_{GR}. Furthermore, a straightforward calculation shows that, the GRWA Hamiltonian H_G also converges to the normal rotating-wave approximation under the near-resonance and weak-coupling conditions. In addition, under the far-off-resonance condition $\Omega \ll \omega$, as shown in Appendix C, the GRWA becomes an improved adiabatic approximation. Therefore, the GRWA approach smoothly connects the adiabatic approximation and the rotating-wave approximation, and thus can be used in a broad parameter region.

III. THE SINGLE-PHOTON SCATTERING AMPLITUDES

In the above section we applied the GRWA to a hybrid system with a 1D resonator array coupled to a single TLS. The single-photon scattering amplitudes in such a system have been calculated analytically under the rotating-wave approximation [2]. In this section we calculate the single-photon scattering amplitudes with the GRWA approach, which is applicable in a broader parameter region.

A. The single-photon scattering amplitudes

The single-photon scattering amplitudes can be extracted from the asymptotic behavior of the eigenstate of the Hamiltonian H, which is approximated as H_G in the GRWA approach. To this end, we need to solve the eigenequation

$$H_G|\Psi(k)\rangle = E(k)|\Psi(k)\rangle \tag{18}$$

with boundary conditions

$$\begin{aligned} |\Psi(k)\rangle = &(e^{ik(-j)} + r_k e^{-ik(-j)})|1\rangle_{-j}|0\rangle_j|\Phi'(k)\rangle \\ &+ t_k e^{ikj}|0\rangle_{-j}|1\rangle_j|\Phi'(k)\rangle + |0\rangle_{-j}|0\rangle_j|\Phi(k)\rangle \end{aligned} \tag{19}$$

in the limit of $j \to +\infty$. Here $|0\rangle_{-j}|0\rangle_j$ is the vacuum state of the resonator modes in the jth and $-j$th resonators, and $|1\rangle_{-j}|0\rangle_j$ and $|0\rangle_{-j}|1\rangle_j$ are defined as $a_{-j}^\dagger|0\rangle_{-j}|0\rangle_j$ and $a_j^\dagger|0\rangle_{-j}|0\rangle_j$, respectively. $|\Phi(k)\rangle$ and $|\Phi'(k)\rangle$ are the quantum states of the TLS and other resonators except the $\pm j$th ones. r_k and t_k are the single-photon reflection and transmission amplitudes, or the single-photon scattering amplitudes.

The physical meaning of the boundary condition (19) can be understood as follows. For the scattering state with respect to a single photon input from the left of the 1D resonator array, there are three possible relevant states for the $-j$th and jth resonators with large $|j|$, i.e., $|0\rangle_{-j}|0\rangle_j$, $|1\rangle_{-j}|0\rangle_j$, and $|0\rangle_{-j}|1\rangle_j$. Furthermore, the probability amplitude with respect to $|1\rangle_{-j}|0\rangle_j$ is $e^{ik(-j)} + r_k e^{-ik(-j)}$, since the photon in the $-j$th resonator can be either the input one or the reflected one. Similarly, the probability amplitude with respect to $|0\rangle_{-j}|1\rangle_j$ is $t_k e^{ikj}$. It is easy to prove that the boundary condition used in the calculation of the single-photon scattering state with the rotating-wave approximation [Eq. (5) of Ref. [2]] can be reexpressed as the one in Eq. (19).

Usually the expression of H_G in Eq. (11) is complicated and it is difficult to solve the eigenequation (18) directly. However, due to Eq. (11), the Hamiltonian H_G is related to H_R^{RWA} through a unitary transformation. Then the eigenequation (18) of H_G is equivalent to that of H_R^{RWA}:

$$H_R^{\mathrm{RWA}}|\Psi_R(k)\rangle = E(k)|\Psi_R(k)\rangle \tag{20}$$

and the eigenstate $|\Psi_R(k)\rangle$ of H_R^{RWA} is given by

$$|\Psi_R(k)\rangle = U^{-1}|\Psi(k)\rangle. \tag{21}$$

More importantly, with the aid of Eq. (13) and the fact that $\lim_{|j|\to\infty}\alpha_j = 0$, the boundary condition (19) for $|\Psi(k)\rangle$ is transformed to the one of $|\Psi_R(k)\rangle$, i.e., in the limit of $j\to\infty$ we have

$$\begin{aligned}|\Psi_R(k)\rangle = {}&(e^{ik(-j)} + r_k e^{-ik(-j)})|1\rangle_{-j}|0\rangle_j|\Phi'_R(k)\rangle\\ &+ t_k e^{ikj}|0\rangle_{-j}|1\rangle_j|\Phi'_R(k)\rangle + |0\rangle_{-j}|0\rangle_j|\Phi_R(k)\rangle,\end{aligned} \tag{22}$$

where $|\Phi_R(k)\rangle$ and $|\Phi'_R(k)\rangle$ are defined as

$$|\Phi_R(k)\rangle = \prod_{i\neq\pm j}\exp[-\alpha_i\sigma_x(a_i^\dagger - a_i)]|\Phi(k)\rangle \tag{23}$$

and

$$|\Phi'_R(k)\rangle = \prod_{i\neq\pm j}\exp[-\alpha_i\sigma_x(a_i^\dagger - a_i)]|\Phi'(k)\rangle, \tag{24}$$

respectively. Therefore, the scattering amplitudes r_k and t_k can be obtained from the solution of the eigenequation (20) of H_R^{RWA} with boundary conditions (22).

B. The perturbative approach for the single-photon scattering amplitudes

In this section we solve the eigenequation (20) of H_R^{RWA} and calculate the single-photon scattering amplitudes. To this end, we first use the explicit result about the unitary operator U shown in Appendix B to calculate the Hamiltonian $U^{-1}HU$. The result is

$$\begin{aligned}U^{-1}HU = {}&\omega\sum_{j=-\infty}^{\infty}a_j^\dagger a_j - \xi\sum_{j=-\infty}^{\infty}(a_j^\dagger a_{j+1} + \mathrm{H.c.})\\ &+\frac{\Omega}{2}\left[\cosh\left(\sum_i 2\nu_i\right)\sigma_z - i\sinh\left(\sum_i 2\nu_i\right)\sigma_y\right]\end{aligned} \tag{25}$$

with $\nu_i = -\alpha_i(a_i^\dagger - a_i)$. In principle, we can derive the explicit expression for H_R^{RWA} with Eqs. (12) and (25). Here, for simplicity, we expand H_R^{RWA} as a power series of the parameter ξ/ω and keep only the low-order terms under the weak-hopping condition $\xi \ll \omega$. Then we can analytically solve Eq. (20) with the approximated H_R^{RWA} and derive the single-photon scattering amplitudes.

Now we calculate the single-photon scattering amplitudes with a first-order approximation where only the zeroth- and first-order terms of ξ/ω are kept in H_R^{RWA}. As shown in the following, in most cases, this approximation is enough to give good results for the scattering amplitudes. A straightforward calculation shows that, up to the first order of ξ/ω, H_R^{RWA} is approximated as

$$\begin{aligned}H_R^{\mathrm{RWA}} \approx H_R^{\mathrm{RWA(1)}} \equiv {}&\omega\sum_j a_j^\dagger a_j - \xi\sum_j(a_{j+1}^\dagger a_j + \mathrm{H.c.})\\ &+\omega_{0g}^{(0)}\Big(|0g\rangle\langle 0g| + \sum_{j\neq 0}|1_jg\rangle\langle 1_jg|\Big) + \omega_{0e}^{(0)}|0e\rangle\langle 0e|\\ &+\omega_{1g}^{(0)}|1_0g\rangle\langle 1_0g| + J^{(0)}(|0e\rangle\langle 1_0g| + \mathrm{H.c.})\\ &+\omega_{1g}^{(1)}(|1_1g\rangle\langle 1_0g| + |1_{-1}g\rangle\langle 1_0g| + \mathrm{H.c.})\\ &+J^{(1)}(|0e\rangle\langle 1_1g| + |0e\rangle\langle 1_{-1}g| + \mathrm{H.c.})\end{aligned} \tag{26}$$

with the parameters

$$\omega_{0g}^{(0)} = -\omega_{0e}^{(0)} = -\frac{\Omega}{2}\exp\left(-2\frac{\lambda^2}{\omega^2}\right), \tag{27a}$$

$$\omega_{1g}^{(0)} = -\frac{\Omega}{2}\exp\left(-2\frac{\lambda^2}{\omega^2}\right)\left(1 - 4\frac{\lambda^2}{\omega^2}\right), \tag{27b}$$

$$J^{(0)} = \frac{\Omega\lambda}{\omega}\exp\left(-2\frac{\lambda^2}{\omega^2}\right), \tag{27c}$$

$$\omega_{1g}^{(1)} = \frac{2\Omega\lambda^2\xi}{\omega^3}\exp\left(-2\frac{\lambda^2}{\omega^2}\right), \tag{27d}$$

$$J^{(1)} = \frac{\Omega\lambda\xi}{\omega^2}\exp\left(-2\frac{\lambda^2}{\omega^2}\right). \tag{27e}$$

In Eq. (26) the states $|0e\rangle$, $|0g\rangle$, $|1_je\rangle$, and $|1_jg\rangle$ are defined as $|0\rangle|e\rangle$, $|0\rangle|g\rangle$, $|1_j\rangle|e\rangle$, and $|1_j\rangle|g\rangle$, respectively, where $|0\rangle$ is the vacuum state of all the resonators.

The physical meaning of Eq. (26) is very clear. In the zeroth-order terms of ξ/ω, or the terms proportional to $\omega_{0g}^{(0)}$, $\omega_{0e}^{(0)}$, $\omega_{1g}^{(0)}$, and $J^{(0)}$, the effective couplings occur between the TLS and the photon in the zeroth resonator in which the TLS is located. Nevertheless, in the first-order terms proportional to $J^{(1)}$, effective couplings appear between the TLS and the modes in the ± 1st resonators. These terms imply that the non-rotating-wave effects from the coupling between the TLS and the zeroth resonator can indirectly influence the behavior of the modes in the ± 1st resonators. Furthermore, the hopping intensities between the zeroth and ± 1st resonators are also tuned by the terms with $\omega_{1g}^{(1)}$.

As shown above, the single-photon scattering amplitudes are approximately derived from the eigenequation

$$H_R^{\mathrm{RWA(1)}}|\Psi_R(k)\rangle = E(k)|\Psi_R(k)\rangle \tag{28}$$

of $H_R^{\mathrm{RWA(1)}}$ with boundary condition (22). It is apparent that the solution $|\Psi_R(k)\rangle$ of Eq. (28) takes the form

$$|\Psi_R(k)\rangle = \sum_{j=-\infty}^{+\infty}u_k^{(1)}(j)|1_jg\rangle + u_e^{(1)}|0e\rangle \tag{29}$$

with the coefficients $u_k^{(1)}(j)$ given by

$$u_k^{(1)}(j) = \begin{cases} e^{ikj} + r_k^{(1)}e^{-ikj} & \text{if} \quad j \leqslant -1,\\ u_k(0) & \text{if} \quad j = 0,\\ t_k^{(1)}e^{ikj} & \text{if} \quad j \geqslant 1.\end{cases} \tag{30}$$

Substituting Eqs. (29) and (30) into Eq. (28), we obtain the linear equations for the reflection amplitude $r_k^{(1)}$ and transmission amplitude $t_k^{(1)}$. These equations can be solved analytically. Then we obtain the scattering amplitudes $r_k^{(1)}$ and

$t_k^{(1)}$ given by the first-order approximation of H_R^{RWA}:

$$r_k^{(1)} = \frac{\begin{array}{c} e^{ik}\lambda^2\Omega\{4\lambda^2\xi\Omega^2\cos(k) + 2e^{4\lambda^2/\omega^2}\omega^3[\omega^2 + 2\xi\omega\cos(k) - 4\xi^2 - 4\xi^2\cos(2k)] \\ + e^{2\lambda^2/\omega^2}\Omega[8\lambda^2\xi^2 + 2\omega^2\xi^2 - \omega^4 - 4\xi\omega(2\lambda^2+\omega^2)\cos(k) + 2\xi^2(4\lambda^2+\omega^2)\cos(2k)]\} \end{array}}{\begin{array}{l} \{-4e^{2ik}\lambda^4\xi\Omega^3 + e^{6\lambda^2/\omega^2}(-1+e^{2ik})\xi\omega^6[\omega - 2\xi\cos(k)] \\ \quad + e^{4\lambda^2/\omega^2}\omega^3\Omega[8e^{3ik}\lambda^2\xi^2 + 2e^{ik}\lambda^2(4\xi^2-\omega^2) + \xi\omega(2\lambda^2+\omega^2) - e^{2ik}\xi\omega(6\lambda^2+\omega^2)] \\ \quad + e^{2ik+\lambda^2/\omega^2}\lambda^2\Omega^2[4\xi\omega(2\lambda^2+\omega^2) + (\omega^4 - 4\xi^2\omega^2 - 16\lambda^2\xi^2)\cos(k) - i\omega^4\sin(k)]\} \end{array}}, \tag{31}$$

$$t_k^{(1)} = r_k^{(1)} + 1. \tag{32}$$

The above procedure can be straightforwardly generalized to cases with high-order approximations of H_R^{RWA}. For instance, in the second-order approximation, H_R^{RWA} is approximated as $H_R^{\mathrm{RWA}(2)}$ which includes the zeroth-, first-, and second-order terms of ξ/ω. It can be found that in $H_R^{\mathrm{RWA}(2)}$ the TLS is effectively coupled to the zeroth, $\pm$1st, and $\pm$2nd resonators. We can also solve the eigenequation of $H_R^{\mathrm{RWA}(2)}$, and obtain the analytical expressions for the relevant scattering amplitudes $r_k^{(2)}$ and $t_k^{(2)}$. In general, for any integer n, the scattering amplitudes $r_k^{(n)}$ and $t_k^{(n)}$ from the nth-order approximation of H_R^{RWA} can be obtained with a similar approach. In the limit of $n \to \infty$, the results $r_k^{(n)}$ and $t_k^{(n)}$ would converge to fixed values r_k and t_k or the precise values of the single-photon scattering amplitudes.

C. Results and discussion

In Figs. 2 and 3, we illustrate the single-photon scattering amplitude r_k given by the GRWA approach in the first- and second-order approximations, i.e., $r_k^{(1)}$ and $r_k^{(2)}$, the one given by the rotating-wave approximation and the result from the numerical diagonalization of the rotated Hamiltonian

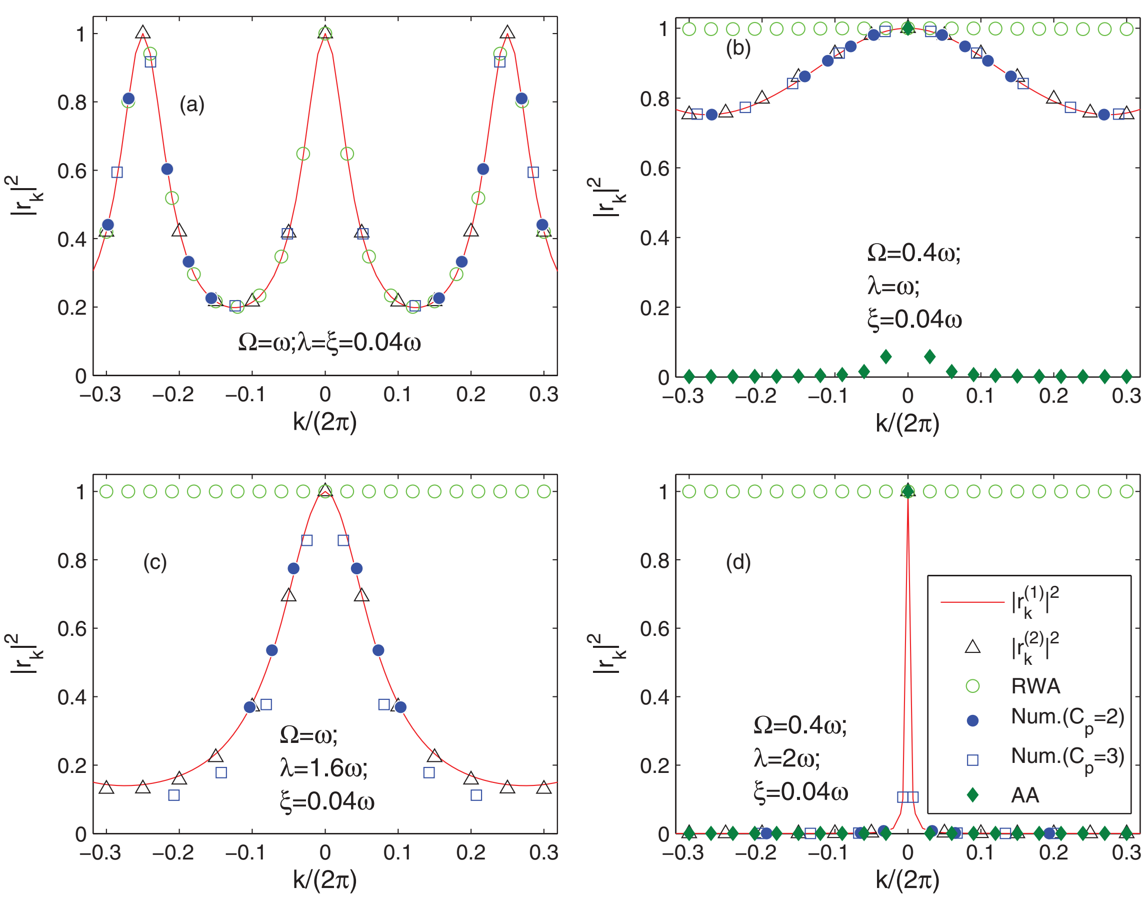

FIG. 2. (Color online) The single-photon reflection rate $|r_k|^2$ given by the GRWA approach with the first- and second-order approximations for (ξ/ω), i.e., $|r_k^{(1)}|^2$ (red solid line) and $|r_k^{(2)}|^2$ (black empty triangles), $|r_k|^2$ from the rotating-wave approximation (RWA) (green empty circles), $|r_k|^2$ from the adiabatic approximation (green filled diamonds), and the numerical calculations with cutoff excitation number $C_p = 2$ (blue filled circles) and $C_p = 3$ (blue empty squares). Here we consider the cases of $\xi = 0.04\omega$ and $\Omega = \omega$, $\lambda = 0.04\omega$ (a), $\Omega = 0.4\omega$, $\lambda = \omega$ (b), $\Omega = \omega$, $\lambda = 1.6\omega$ (c), and $\Omega = 0.4\omega$, $\lambda = 2\omega$ (d).

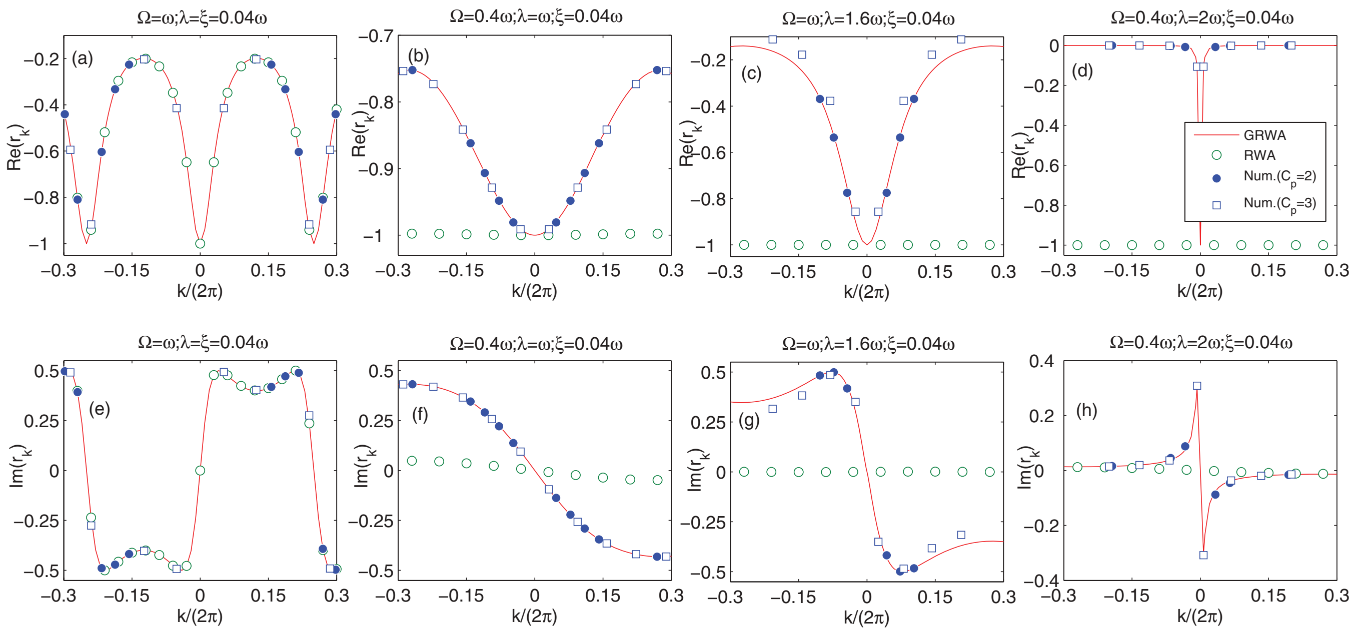

FIG. 3. (Color online) The real (a)–(d) and imaginary (e)–(h) parts of the single-photon scattering amplitude r_k given by the GRWA approach with the first-order approximation for (ξ/ω), i.e., $r_k^{(1)}$ in Eq. (31) (red solid line), the rotating-wave approximation (green empty circles), and the numerical calculations with cutoff excitation number $C_p = 2$ (blue filled circles) and $C_p = 3$ (blue empty squares). Here we consider the cases of $\xi = 0.04\omega$ and $\Omega = \omega$, $\lambda = 0.04\omega$ (a),(e), $\Omega = 0.4\omega$, $\lambda = \omega$ (b),(f), $\Omega = \omega$, $\lambda = 1.6\omega$ (c),(g), and $\Omega = 0.4\omega$, $\lambda = 2\omega$ (d),(h).

$U^{-1}HU$. In our numerical calculations the total excitation $\sum_j a_j^\dagger a_j + |e\rangle\langle e|$ is cut off at a given number C_p for the Hamiltonian $U^{-1}HU$, and the results with $C_p = 2,3$ are shown in our figures.

In Fig. 2 we calculate the reflection rate $|r_k|^2$. It is clearly shown that the results $|r_k^{(1)}|^2$ and $|r_k^{(2)}|^2$ from the first- and second-order approximations for (ξ/ω) are very consistent with each other. Therefore, in most of the cases with $|\xi| \ll \omega$, the first order approximation is good enough for the GRWA approach.

Furthermore, it is shown that in the case of Fig. 2(a) where the weak-coupling and near-resonance conditions are satisfied, both the results from the rotating-wave approximation and the GRWA approach fit well with the numerical calculations. Nevertheless, in the cases of Figs. 2(b)–2(d) where the rotating-wave approximation is not applicable, the results from the GRWA approach are also significantly well consistent with the numerical calculations. This observation is further confirmed by Fig. 3 where the real and imaginary parts of r_k given by different approaches are illustrated.

In Figs. 2(b) and 2(d) with $\Omega = 0.4\omega$, we also compare our results with the ones given by the adiabatic approximation. It is shown that, as we argued in Sec. II, the adiabatic approximation may not be applicable even when $\Omega \ll \omega$, while the GRWA approach can still provide reasonable results.

Therefore, the results in Figs. 2 and 3 show that the GRWA approach with the first-order approximation for ξ/ω, or our results $r_k^{(1)}$ and $t_k^{(1)}$ in Eqs. (31) and (32) can be used as a good analytical approximation for the single-photon scattering amplitudes in the parameter region with $|\xi| \ll \omega$ and $\Omega \lesssim \omega$.

IV. THE SCATTERING AMPLITUDES IN THE STRONG-COUPLING CASE

In the above section we derived the single-photon scattering amplitudes with the GRWA approach. It is pointed out that our results in Eqs. (27a)–(27e) are applicable for arbitrarily large coupling between the TLS and the photon. Now we consider a special case where the TLS is strongly coupled to the photon in the resonator array, so that the condition

$$\frac{\lambda^2}{\omega^2} e^{-2\lambda^2/\omega^2} \ll 1 \tag{33}$$

is satisfied. We further assume that the frequency Ω of the TLS is equal to or smaller than the photon frequency ω, i.e., $\Omega \lesssim \omega$. In this strong-coupling case the expressions in Eqs. (27a)–(27e) can be significantly simplified and then one obtains simple pictures for both the quantitative calculation and the qualitative estimation of the single-photon scattering amplitudes.

Under the condition (33), we keep only the leading term proportional to $(\lambda^2/\omega^2)\exp(-2\lambda^2/\omega^2)$ in $\omega_{0g,e}^{(0)}$, $\omega_{1g}^{(0,1)}$, and $J^{(0,1)}$ defined in Eqs. (27a)–(27e). Then we have

$$\omega_{1g}^{(0)} \approx \frac{2\Omega\lambda^2}{\omega^2} \exp\left(-2\frac{\lambda^2}{\omega^2}\right), \tag{34a}$$

$$\omega_{1g}^{(1)} \approx \frac{\xi}{\omega}\omega_{1g}^{(0)}, \tag{34b}$$

$$\omega_{0g,e}^{(0)}, J^{(0,1)} \approx 0. \tag{34c}$$

Therefore, in Eq. (26) of the Hamiltonian H_R^{RWA}, we need to keep only the first two terms and the terms proportional to $\omega_{1g}^{(0)}$ and $\omega_{1g}^{(1)}$. This simplification implies that, in the strong-coupling regime, our system is equivalent to the simple 1D resonator array with the frequency of the zeroth resonator shifted from ω to $\omega + \omega_{1g}^{(0)}$, while the photon hopping intensity between the zeroth and the ± 1st resonators is shifted from ξ to $\xi + \omega_{1g}^{(1)}$. Since we have also assumed $\Omega \lesssim \omega$, it is apparent that $\omega_{1g}^{(1)} \ll \xi$ in the strong-coupling regime. Then the shift $\omega_{1g}^{(1)}$ of the photon hopping intensity is negligible. We need to consider only the effect given by the frequency shift $\omega_{1g}^{(0)}$ of the photon in the zeroth resonator. Namely, our system is finally equivalent to a 1D resonator array, in which the zeroth resonator has the frequency $\omega + \omega_{1g}^{(0)}$, while all the other resonators have the same frequency ω. In this case the Hamiltonian $H_R^{\mathrm{RWA}(1)}$ is approximated as

$$H_R^{\mathrm{RWA}(1)} \approx \omega \sum_j a_j^\dagger a_j - \xi \sum_j (a_{j+1}^\dagger a_j + a_j^\dagger a_{j+1}) + \omega_{1g}^{(0)} a_0^\dagger a_0, \tag{35}$$

which leads to the single-photon scattering amplitudes

$$r_k \approx -\frac{\omega_{1g}^{(0)}}{\omega_{1g}^{(0)} - 2i\xi \sin(k)}, \tag{36}$$

$$t_k \approx r_k + 1. \tag{37}$$

A straightforward result given by the above expressions for the scattering amplitudes is that, when the effective frequency shift $\omega_{1g}^{(0)}$ of the zeroth resonator is much larger than the bandwidth 4ξ of the free Hamiltonian H_C of the array of resonators with the same frequency ω, the zeroth resonator will be far detuned from a photon with any incident momentum k, and thus every photon will be reflected. Namely, in such a limit we have $r_k \approx 1$, $t_k \approx 0$. Likewise, if the effective frequency shift $\omega_{1g}^{(0)}$ is much smaller than 4ξ, the frequency of the zeroth resonator will be approximately the same as that of the other resonators, and then every photon transmits through the zeroth resonator. In this limit we have $r_k \approx 0$ and $t_k \approx 1$.

In Fig. 4 we plot the photon reflection rate $|r_k|^2$ in the strong-coupling case and compare the results given by Eq. (36) and from numerical diagonalization of the rotated Hamiltonian $U^{-1}HU$ with cutoff excitation number $C_p = 2,3$, respectively. It is clearly shown that our results in Eq. (36) fit well with the numerical results. Furthermore, it is illustrated that in the case of Fig. 4(a), where we have $\omega_{1g}^{(0)}/\xi = 15$, the photon reflection rate $|r_k|^2$ is almost unity for all incident momenta k. In the case of Fig. 4(c) with $\omega_{1g}^{(0)}/\xi = 0.06$, we have $|r_k|^2 \approx 0$ in the region with nonzero momentum k. All these observations are consistent with our above qualitative analysis.

At the end of this section, we remark that, since all the quantities defined in Eqs. (27a)–(27e) exponentially decay to zero with (λ/ω), for any given values of Ω, ω, and ξ, when the TLS-photon coupling intensity λ is large enough, we can always neglect all these parameters and approximate the Hamiltonian $H_R^{\mathrm{RWA}(1)}$ as the free Hamiltonian H_C for an array of identical resonators. Therefore, when the TLS-photon coupling is strong enough, the photon scattering effect becomes negligible and we have $r_k = 0$, $t_k = 1$ for a photon with any incident momentum k.

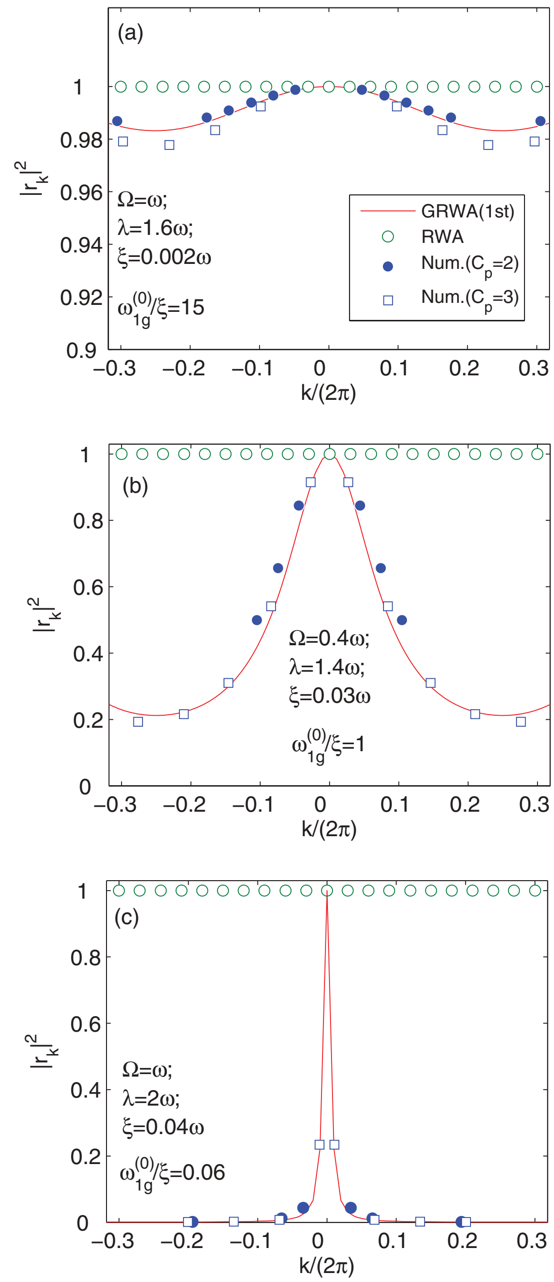

FIG. 4. (Color online) The single-photon reflection rate $|r_k|^2$ in the cases with strong TLS-photon coupling. Here we show the results given by Eq. (36) (red solid line), rotating-wave approximation (green empty circles), and the numerical calculations with cutoff excitation number $C_p = 2$ (blue filled circles) and $C_p = 3$ (blue empty squares) for the cases with $\Omega = \omega$, $\lambda = 1.6\omega$, $\xi = 0.002\omega$ (a), $\Omega = 0.4\omega$, $\lambda = 1.4\omega$, $\xi = 0.03\omega$ (b), and $\Omega = \omega$, $\lambda = 2\omega$, $\xi = 0.04\omega$ (c). In the three cases we have $\omega_{1g}^{(0)}/\xi = 15, 1, 0.06$, respectively.

V. THE GRWA FOR A RESONATOR ARRAY WITH MULTIPLE TLSs

In the above sections, we generalized the GRWA to a system with a 1D resonator array coupled to a single TLS, and calculated the single-photon scattering amplitudes with the GRWA approach. In this section, we extend the GRWA to more general cases with m two-level systems coupled to the resonator array. For simplicity, here we assume that each TLS is individually located in a resonator. Then the Hamiltonian of the total system is written as

$$H_M = H_C + H_{AM} + H_{IM} \tag{38}$$

with the Hamiltonian H_C of the resonator defined in Eq. (2), the Hamiltonian H_{AM} of all the TLSs given by

$$H_{AM} = \frac{\Omega}{2}\sum_{\beta=1}^{m}\sigma_z^{(\beta)}, \tag{39}$$

and the interaction Hamiltonian H_{IM} defined as

$$H_{IM} = \lambda\sum_{\beta=1}^{m}\sigma_x^{(\beta)}(a_{c(\beta)}^{\dagger} + a_{c(\beta)}). \tag{40}$$

Without loss of generality, here we assume that the βth TLS is located in the $c(\beta)$th resonator.

In such a general system, we straightforwardly develop the GRWA approach with the unitary transformation procedure in Secs. II and III. To this end, we first write the Hamiltonian H_M as

$$H_M = H_{M1} + H_{M2}$$

with H_{M1} and H_{M2} defined as

$$H_{M1} = H_C + H_{IM}, \tag{41}$$

$$H_{M2} = H_{AM}. \tag{42}$$

Then we find a unitary operator U_M which can eliminate the linear terms of $(a_j, a_j^{\dagger})$ in H_{M1} and satisfies

$$U_M^{-1}H_{M1}U_M = \omega\sum_{j=-\infty}^{\infty}a_j^{\dagger}a_j - \xi\sum_{j=-\infty}^{\infty}(a_j^{\dagger}a_{j+1} + \text{H.c.}) - \mathcal{C}_M \tag{43}$$

with $\mathcal{C}_M$ a constant c number. The analytical calculation of U_M is obtained in Appendix E.

With the operator U_M, we apply the unitary transformation to the total Hamiltonian H, and make the rotating-wave approximation to the transformed Hamiltonian $U_M^{-1}H_MU_M$. Finally we perform an inverse unitary transformation. Then the GRWA Hamiltonian for the resonator array with multiple TLSs is

$$H_M \approx H_{MG} \equiv U_M\left\{\sum_{n=0}^{\infty}\left[\hat{P}_{Mn}\left(U_M^{-1}H_MU_M\right)\hat{P}_{Mn}\right]\right\}U_M^{-1}, \tag{44}$$

which is a direct generalization of the result in Eq. (11). Here $\hat{P}_{Mn}$ is the projection operator to the eigenspace of the total excitation operator

$$\sum_{\beta=1}^{m}|e\rangle^{(\beta)}\langle e| + \sum_{j=-\infty}^{+\infty}a_j^{\dagger}a_j \tag{45}$$

with respect to the eigenvalue n. With a similar analysis as in Appendix C, one finds that in the case of $\Omega \ll \omega$, such an approach also includes the intraband transitions which are missed in the adiabatic approximation. On the other hand, under the weak-coupling and near-resonance conditions, this approach converges to the rotating-wave approximation.

VI. CONCLUSIONS

In summary, we apply the GRWA to a hybrid system of a 1D single-mode resonator array coupled to a single TLS, and obtain analytical results for the single-photon scattering amplitudes under the conditions $|\xi| \ll \omega$ and $\Omega \lesssim \omega$. It is shown that in comparison with the rotating-wave approximation, the GRWA approach can give better results in a much broader parameter region. In particular, in the far-off-resonance case with $\Omega \ll \omega$, the adiabatic approximation is no longer applicable for our current system, while the GRWA approach still works well. We also discuss how to apply the GRWA approach to a 1D resonator array coupled to multiple TLSs.

In this paper, we assume the resonators in the 1D array are single mode. However, the resonators used in the experiments usually have more than one photon mode. Especially in the cases with strong TLS-photon coupling the multimode effect may be important. Likewise, it may also be necessary to go beyond the two-level approximation and include the higher excited states of the artificial atoms in the strong-coupling cases. These effects will be discussed in a future presentation for the calculation of the photon scattering in a multimode-resonator array or a multimode waveguide beyond the rotating-wave approximation.

ACKNOWLEDGMENTS

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APPENDIX A: GRWA FOR A TLS-COUPLED SINGLE-MODE BOSONIC FIELD

In this appendix, we reformulate the GRWA approach proposed by Irish in Ref. [73] from the viewpoint of the adiabatic approximation. To this end, we begin with the simple Rabi Hamiltonian for the TLS-coupled single-mode bosonic field:

$$H_{\text{Rabi}} = \omega a^{\dagger}a + \frac{\Omega}{2}\sigma_z + \lambda\sigma_x(a^{\dagger} + a). \tag{A1}$$

Here a and $a^{\dagger}$ are the annihilation and creation operators, respectively, of the single-mode bosonic field with frequency ω. Ω is the energy level spacing between the excited state $|e\rangle$ and the ground state $|g\rangle$ of the TLS, and λ is the coupling intensity between the TLS and the bosonic field. The Pauli operators σ_z and σ_x are defined in Sec. II.

Since the Hamiltonian H_{Rabi} does not have simple invariable subspaces, the exact diagonalization of H_{Rabi} is rather complicated [74]. However, Jaynes and Cumming showed that [75], under the near-resonance condition

$$|\omega - \Omega| \ll |\omega + \Omega| \tag{A2}$$

and the weak-coupling condition

$$|\lambda| \ll \omega,\Omega, \tag{A3}$$

the term $|e\rangle\langle g| a^\dagger + \text{H.c}$ can be safely neglected. Then the Hamiltonian H_{Rabi} is approximated as H_{JC}, which is defined as

$$H_{\text{JC}} = \omega a^\dagger a + \lambda(|e\rangle\langle g|a + \text{H.c.}). \tag{A4}$$

That is the so called rotating-wave approximation. After this approximation, the Hamiltonian H_{Rabi} becomes invariant in the two-dimensional subspaces spanned by the states $|g,n\rangle$ and $|e,n-1\rangle$ for $n = 1,2,\ldots,$ as well as in the one-dimensional subspace spanned by $|g,0\rangle$, and thus can be diagonalized easily.

For the convenience of our discussions on the GRWA, here we introduce the projection operators $\hat{P}^{(n)}_{\text{Rabi}}$ defined as

$$\hat{P}^{(n)}_{\text{Rabi}} = \begin{cases} |0g\rangle\langle 0g| & \text{if } n = 0, \\ |n,g\rangle\langle n,g| + |n-1,e\rangle\langle n-1,e| & \text{if } n \geqslant 1. \end{cases} \tag{A5}$$

Then the Jaynes-Cumming Hamiltonian H_{JC} in Eq. (A4) is rewritten as

$$H_{JC} = \sum_{n=0}^{\infty} \hat{P}^{(n)}_{\text{Rabi}} H_{\text{Rabi}} \hat{P}^{(n)}_{\text{Rabi}}. \tag{A6}$$

Now we introduce the GRWA approach, which is developed as an analytical approximate method to diagonalize the Hamiltonian H in Eq. (A1) in a broad parameter region beyond the rotating-wave approximation. The GRWA is closely related to both the rotating-wave approximation and the adiabatic approximation [76–78] for the TLS-coupled single-mode bosonic field [72,73,79] which is used in the far-off-resonance case:

$$\Omega \ll \omega. \tag{A7}$$

Therefore, before introducing the GRWA, we first introduce the adiabatic approximation in the system of a TLS and a single-mode bosonic field [72,73]. In such an approximation, the bosonic field is considered to be the fast-varying part and the TLS is considered as the slowly varying part. Then the Hamiltonian H_{Rabi} is rewritten as

$$H_{\text{Rabi}} = H_{\text{Rabi1}} + H_{\text{Rabi2}}, \tag{A8}$$

where

$$H_{\text{Rabi1}} = \omega a^\dagger a + \lambda\sigma_x(a^\dagger + a) \tag{A9}$$

is the self-Hamiltonian of the fast-varying part together with the interaction between the fast-varying and the slowly varying parts, and

$$H_{\text{Rabi2}} = \frac{\Omega}{2}\sigma_z \tag{A10}$$

is the free Hamiltonian of the slowly varying part.

The Hamiltonian H_{Rabi1} is easily diagonalized with the eigenstates

$$|\pm,n\rangle = |\pm\rangle \otimes |n_\pm\rangle \tag{A11}$$

and the relevant eigenenergies

$$E_{n\pm} = \omega(n - \lambda^2/\omega^2). \tag{A12}$$

Here $|\pm\rangle$ are the eigenstates of σ_x with eigenvalues ± 1 and $|n_\pm\rangle$ are defined as

$$|n_\pm\rangle = \exp[\mp\lambda/\omega(a^\dagger - a)]\,|n\rangle\,. \tag{A13}$$

In the Rabi Hamiltonian the states $|\alpha,n\rangle$ and $|\alpha',n'\rangle$ are coupled by the term H_{Rabi2}.

The spirit of the adiabatic approximation is described as follows [76–78]. Under the far-off-resonance condition $\Omega \ll \omega$, the motion of the fast-varying part or the bosonic field adiabatically follows the slowly varying part or the TLS, and can be frozen in the adiabatic branches with fixed quantum number n, or the two-dimensional subspaces spanned by $|+,n\rangle$ and $|-,n\rangle$ for $n = 1,2,\ldots$. We neglect the H_{Rabi2}-induced transitions between the states $|\alpha,n\rangle$ and $|\alpha',n'\rangle$ with $n \neq n'$. Then the eigenstates and eigenenergies of H are approximated as

$$|\Psi_{\pm,n}\rangle = \frac{1}{\sqrt{2}}(|+,n\rangle \pm |-,n\rangle) \tag{A14}$$

and

$$E_{\pm,n} = \pm\frac{\Omega}{2}\langle n_-|n_+\rangle + \omega(n - \lambda^2/\omega^2), \tag{A15}$$

respectively.

Now we introduce the GRWA. In the "adiabatic basis" $\{|\Psi_{\pm,n}\rangle\}$, the Hamiltonian H is rewritten as

$$H_{\text{Rabi}} = \sum_{n,n'}\sum_{\alpha,\alpha'=\pm} (H_{\text{Rabi}})^{\alpha',n'}_{\alpha,n}\,|\Psi_{\alpha,n}\rangle\langle\Psi_{\alpha',n'}| \tag{A16}$$

with the matrix elements

$$(H_{\text{Rabi}})^{\alpha',n'}_{\alpha,n} = \langle\Psi_{\alpha,n}|H_{\text{Rabi}}|\Psi_{\alpha',n'}\rangle.$$

In the GRWA, the Rabi Hamiltonian H_{Rabi} is approximated as H_{GR} which is defined as

$$H_{\text{GR}} = \sum_{n=0}^{+\infty}\sum_{\alpha=\pm}(H_{\text{Rabi}})^{\alpha,n}_{\alpha,n}\,|\Psi_{\alpha,n}\rangle\langle\Psi_{\alpha,n}| + \sum_{n=1}^{\infty}(H_{\text{Rabi}})^{+,n-1}_{-,n}\,|\Psi_{-,n}\rangle\langle\Psi_{+,n-1}| + \text{H.c.} \tag{A17}$$

Namely, only the matrix elements $(H_{\text{Rabi}})^{\alpha',n'}_{\alpha,n}$ inside each two-dimensional subspace spanned by the states $|\Psi_{-,n}\rangle$ and $|\Psi_{+,n-1}\rangle$ as well as the one in the one-dimensional subspace spanned by the state $|\Psi_{-,0}\rangle$ are kept in the GRWA. In other words, in the GRWA one takes into account only the quantum transitions between the states $|\Psi_{\alpha,n}\rangle$ and $|\Psi_{\beta,n'}\rangle$ with

$$n + N_\alpha = n' + N_\beta, \tag{A18}$$

where $N_\alpha = 1$ for $\alpha = +$ and $N_\alpha = 0$ for $\alpha = -$. Then, as in the rotating-wave approximation, the Hamiltonian is reduced to a 2×2 block-diagonal matrix in the GRWA.

It can be shown that, under the weak-coupling and near-resonance conditions, the GRWA returns to the rotating-wave approximation. In addition, under the far-off-resonance condition, the results given by the GRWA converge to those from the adiabatic approximation. Therefore, the GRWA smoothly connects the ordinary rotating-wave approximation and the adiabatic approximation, and thus works well in a broader parameter regime, especially the region with strong

TLS-photon coupling and far-off-resonant $\Omega \lesssim \omega$ (see, e.g., Ref. [73] and Fig. 8 of Ref. [80]).

It can be proved with a straightforward calculation that the Hamiltonian H_{GR} defined in Eq. (A17) could also be rewritten as [73]

$$H_{\mathrm{GR}} = U_R \left\{ \sum_{n=0}^{\infty} \left[\hat{P}^{(n)}_{\mathrm{Rabi}} \left(U_R^{-1} H_{\mathrm{Rabi}} U_R \right) \hat{P}^{(n)}_{\mathrm{Rabi}} \right] \right\} U_R^{-1} \tag{A19}$$

with the unitary transformation U_R defined as

$$U_R = \exp\left[-\frac{\lambda}{\omega} \sigma_x (a^\dagger - a) \right]. \tag{A20}$$

Comparing Eq. (A19) with Eq. (A6), one can see that the GRWA is nothing but the rotating-wave approximation for the rotated Hamiltonian $U_R^{-1} H_{\mathrm{Rabi}} U_R$.

APPENDIX B: THE UNITARY TRANSFORMATION U

In this Appendix, we calculate the unitary transformation U determined by Eq. (16). It is obvious that U can be expressed as the product of the displacement operators for each resonator mode, i.e., we have

$$U = \prod_{j=-\infty}^{+\infty} \exp[\alpha_j \sigma_x (a_j^\dagger - a_j)]. \tag{B1}$$

Then the Hamiltonian $H_C + H_I$ is transformed into

$$\begin{aligned} &U^{-1}(H_C + H_I)U \\ &\quad = H_C + \sum_{j\neq 0} [\omega\alpha_j - \xi(\alpha_{j+1} + \alpha_{j-1})]\sigma_x (a_j^\dagger + a_j) \\ &\qquad + [\omega\alpha_0 - \xi(\alpha_1 + \alpha_{-1}) + \lambda]\sigma_x (a_0^\dagger + a_0) - \mathcal{C}. \end{aligned} \tag{B2}$$

The exact expression of $\mathcal{C}$ is not required here. Comparing the result Eq. (B2) with Eq. (16), we get the equations for the parameters $\{\alpha_j\}$:

$$\omega\alpha_j - \xi(\alpha_{j+1} + \alpha_{j-1}) = 0, \quad j = \pm 1, \pm 2, \ldots \tag{B3}$$

$$\omega\alpha_0 - \xi(\alpha_1 + \alpha_{-1}) + \lambda = 0, \quad j = 0. \tag{B4}$$

Now we solve Eqs. (B3) and (B4) in two steps. First, we introduce a cutoff for the equations at $j = \pm N$, and solve the equations

$$\omega\alpha_j^{(N)} - \xi\left(\alpha_{j+1}^{(N)} + \alpha_{j-1}^{(N)}\right) = 0, \quad j = \pm 1, \ldots, \pm(N-1), \tag{B5}$$

$$\omega\alpha_0^{(N)} - \xi\left(\alpha_1^{(N)} + \alpha_{-1}^{(N)}\right) + \lambda = 0, \quad j = 0, \tag{B6}$$

$$\alpha_N^{(N)} = \alpha_{-N}^{(N)} = 0. \tag{B7}$$

A straightforward calculation shows that

$$\begin{aligned} \alpha_j^{(N)} = &\left[\left(\frac{\omega}{2\xi} - \frac{\xi}{\omega_1} \right) \alpha_0^{(N)} + \frac{\lambda}{2\xi} \right] \frac{\left(\frac{\xi}{\omega_1}\right)^{|j|-1} - \left(\frac{\omega_1}{\xi}\right)^{|j|+1}}{1 - \left(\frac{\omega_1}{\xi}\right)^2} \\ &+ \left(\frac{\xi}{\omega_1} \right)^{|j|} \alpha_0^{(N)} \end{aligned} \tag{B8}$$

with ω_1 given by

$$\omega_1 = \tfrac{1}{2}\left(\omega + \sqrt{\omega^2 - 4\xi^2}\right). \tag{B9}$$

Therefore, under the weak-hopping assumption $|\xi| \ll \omega$, we have $\xi/\omega_1 < 1$. Substituting Eq. (B7) into Eq. (B8), we obtain

$$\alpha_0^{(N)} = \frac{-\lambda\omega_1\left[\left(\frac{\omega_1}{\xi}\right)^{2|N|} - 1\right]}{-\omega\omega_1 + 2\omega_1^2 - 2\xi^2\left(\frac{\omega_1}{\xi}\right)^{2|N|} + \omega\omega_1\left(\frac{\omega_1}{\xi}\right)^{2|N|}} \tag{B10}$$

and then we obtain all $\alpha_j^{(N)}$ from Eq. (B8).

Second, we consider the solutions of Eqs. (B5)–(B7) in the limit of $N \to \infty$ as a trial solution of Eqs. (B3) and (B4):

$$\alpha_j = \lim_{N\to\infty} \alpha_j^{(N)} = \frac{\lambda\omega_1}{2\xi^2 - \omega\omega_1} \left(\frac{\xi}{\omega_1} \right)^{|j|}. \tag{B11}$$

Substituting Eq. (B11) into Eqs. (B3) and (B4), we find that the latter are exactly satisfied. Therefore, $\{\alpha_j\}$ from Eq. (B11) are realistic solutions of Eqs. (B3) and ((B4). Then the unitary transformation U defined in Eq. (16) takes the form Eq. (13) with α_j given by Eq. (14).

We emphasize that, as shown in Eq. (B1), U is the product of the displacement operators for each resonator; the magnitude of the displacement for the mode in the jth resonator is described by $|\alpha_j|$. Furthermore, since $\xi/\omega_1 < 1$, Eq. (B11) implies that $|\alpha_j|$ exponentially decays with $|j|$. Then we have

$$\lim_{|j|\to\infty} \exp[\alpha_j \sigma_x (a_j^\dagger - a_j)] = 1. \tag{B12}$$

Therefore, for the resonators far away from the TLS, the relevant displacements in U would be negligible, which is consistent with our considerations above.

APPENDIX C: THE RELATIONSHIP BETWEEN THE GRWA APPROACH AND THE IMPROVED ADIABATIC APPROXIMATION

In this Appendix, we show that under the far-off-resonance condition $\Omega \ll \omega$, the GRWA approach developed in Sec. II becomes an improved adiabatic approximation for our TLS–coupled-resonator-array system. To this end, we first develop the improved adiabatic approximation for our system. Under the far-off-resonance condition $\Omega \ll \omega$, the TLS is considered to be the slowly varying part of the system, while the 1D resonator array is the fast-varying part. Then we decompose the total Hamiltonian as $H = H_1 + H_2$, where $H_1 = H_C + H_I$ is the Hamiltonian of the fast-varying part together with the interaction and $H_2 = H_A$ is the Hamiltonian of the slowly varying TLS.

A straightforward calculation shows that H_1 has the eigenstates

$$|\pm,\vec{n}\rangle = U \prod_k \frac{1}{\sqrt{n(k)!}} A(k)^{\dagger n(k)} |0\rangle \otimes |\pm\rangle. \tag{C1}$$

Here the TLS states $|\pm\rangle$ are the eigenstates of the operator σ_x with eigenvalues ± 1, and $|0\rangle$ is the vacuum state of the resonator array. The photon momentum k can take any value in the region $(-\pi,\pi]$, the creation operator $A(k)^\dagger$ for a photon with momentum k is given by

$$A(k)^\dagger = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{+\infty} e^{ikl} a_l^\dagger, \tag{C2}$$

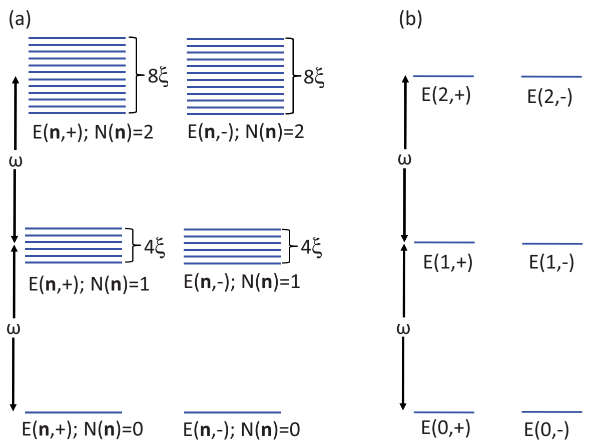

FIG. 5. (Color online) (a) The energy spectrum of the Hamiltonian H_1 of the 1D resonator array and the interaction between the resonator array and the TLS. The low-excitation spectrum has clear band structure. The intraband transitions are missed unreasonably in the adiabatic approximation. (b) The energy spectrum of the Hamiltonian H_1 of a single-mode bosonic field and the interaction between the single-mode bosonic field and a TLS. In such a simple system there is no band structure in the spectrum of H_1, and then the adiabatic approximation is applicable when $\Omega \ll \omega$.

and $\vec{n} = (n(k_1), n(k_2), \ldots)$ is the set of all the numbers $n(k)$. The operator U in Eq. (C1) is defined in Eq. (13).

Obviously, the eigenenergy of H_1 with respect to the eigenstate $|\pm,\vec{n}\rangle$ is

$$E(\pm,\vec{n}) = N(\vec{n})\omega + 2\xi \sum_k n(k)\cos k + \mathcal{C} \tag{C3}$$

with

$$N(\vec{n}) = \sum_k n(k). \tag{C4}$$

As shown in Fig. 5(a), in the weak-tunneling case with $\xi \ll \omega$ the low-excitation spectrum $E(\pm,\vec{n})$ of the eigenenergies of H_1 has a clear band structure. Each energy band includes all the energy levels with the same total photon number $N(\vec{n})$ and different photon-momentum distribution $\vec{n}$. The Nth energy band is centered at $N\omega$ with bandwidth $4N\xi$. Therefore, in the low-excitation cases the interband energy gap has the same order as ω.

The spirit of the adiabatic approximation [76–78] is that, during the quantum evolution, the motion of the slowly varying part of the system follows the motion of the fast-varying part. Then the quantum state of the fast-varying 1D resonator array is frozen in each adiabatic branch with fixed quantum number $\vec{n}$. Mathematically speaking, in the adiabatic approximation for our system, all the H_2-induced quantum transitions between the states $|\pm,\vec{n}\rangle$ and $|\pm,\vec{n}'\rangle$ with $\vec{n} \neq \vec{n}'$ are neglected. This treatment leads to the approximate eigenstates of the Hamiltonian H as

$$|\Psi_{\pm,\vec{n}}\rangle = \frac{1}{\sqrt{2}}(|+,\vec{n}\rangle \pm |-,\vec{n}\rangle). \tag{C5}$$

However, this standard adiabatic approximation is *not* a reasonable approximation for our present system, and should be improved even in the case of $\Omega \ll \omega$. That is because, as shown in Fig. 5(b), the energy spectrum of H_1 has a band structure, and each band includes all the states $|\pm,\vec{n}\rangle$ with the same total photon number $N(\vec{n})$. In addition, in the adiabatic approximation all the H_2-induced transitions between the eigenstates of H_1 with different quantum number $\vec{n}$ are neglected. It means that all the interband and intraband transitions between the states $|+,\vec{n}\rangle$ and $|-,\vec{n}'\rangle$ with $\vec{n} \neq \vec{n}'$ are omitted. In the case of $\Omega \ll \omega$, the omission of interband transitions is reasonable because the energy gap between different bands is of the order of ω, which is much larger than the intensity Ω of H_2. However, the neglect of the intraband transitions is unreasonable because the energy gaps between the levels in the same band can be arbitrarily small.

Furthermore, as proved in Appendix D, we have

$$\langle \Psi_{-,\vec{n}}|H|\Psi_{+,\vec{n}'}\rangle = 0 \quad \text{for } N(\vec{n}) = N(\vec{n}'). \tag{C6}$$

Then the intraband transition occurs only between the states $|\Psi_{\alpha,\vec{n}}\rangle$ and $|\Psi_{\alpha,\vec{n}'}\rangle$ with $N(\vec{n}) = N(\vec{n}')$. After taking into account these intraband transitions, we can improve the adiabatic approximation and approximate the total Hamiltonian H as

$$H_{\mathrm{IA}} = \sum_{\vec{n},\vec{n}'} \sum_{\alpha=+,-} H_{\vec{n}',\alpha}^{\vec{n},\alpha} |\Psi_{\alpha,\vec{n}}\rangle\langle\Psi_{\alpha,\vec{n}}|\delta_{N(\vec{n}),N(\vec{n}')}. \tag{C7}$$

Here the matrix elements $H_{\vec{n}',\beta}^{\vec{n},\alpha}$ are defined as

$$H_{\vec{n}',\beta}^{\vec{n},\alpha} = \langle \Psi_{\alpha,\vec{n}}|H|\Psi_{\beta,\vec{n}'}\rangle, \tag{C8}$$

while the symbol $\delta_{a,b}$ is defined as $\delta_{a,b} = 1$ for $a = b$ and $\delta_{a,b} = 0$ for $a \neq b$.

On the other hand, a straightforward calculation shows that our GRWA Hamiltonian H_G in Eq. (11) can be rewritten as

$$\begin{aligned} H_G = &\sum_{\vec{n},\vec{n}'} \sum_{\alpha=+,-} H_{\vec{n}',\alpha}^{\vec{n},\alpha} |\Psi_{\alpha,\vec{n}}\rangle\langle\Psi_{\alpha,\vec{n}}|\delta_{N(\vec{n}),N(\vec{n}')} \\ &+ \sum_{\vec{n},\vec{n}'} H_{\vec{n}',-}^{\vec{n},+} |\Psi_{+,\vec{n}}\rangle\langle\Psi_{-,\vec{n}'}|\delta_{N(\vec{n}),N(\vec{n}')-1} + \text{H.c.} \end{aligned} \tag{C9}$$

Therefore, it is easy to prove that, under the far-off-resonance condition $\Omega \ll \omega$, we have $H_G \approx H_{\mathrm{IA}}$ and then the GRWA approach converges to the improved adiabatic approximation developed in this Appendix.

APPENDIX D: THE PROOF OF EQ. (C6)

In this Appendix, we prove Eq. (C6) in Appendix C. To this end, we first define the operator $\mathcal{U}_k[\beta_k]$ and the state $|n(k)\rangle$ for the 1D resonator array as a function of the number β_k:

$$\mathcal{U}_k[\beta_k] = \exp\{-2\beta_k[A(k)^\dagger - A(k)]\} \tag{D1}$$

and

$$|n(k)\rangle = \frac{1}{\sqrt{n(k)}} A(k)^{\dagger n(k)}|0\rangle, \tag{D2}$$

respectively. Here $|0\rangle$ is the vacuum state of the resonator array, and $A(k)^\dagger$ is defined in Eq. (C2). We further define the function $f(\vec{\beta})$ as

$$f(\vec{\beta}) = \prod_k \langle n(k)|\mathcal{U}_k[\beta_k]|n'(k)\rangle \tag{D3}$$

with

$$\vec{\beta} = (\beta_{k_1}, \beta_{k_2}, \ldots). \tag{D4}$$

A straightforward calculation shows that

$$f(\vec{\beta}) = \prod_k \langle n(k)|\mathcal{U}_k[\beta_k]|n'(k)\rangle = \prod_k e^{-|2\beta_k|^2/2}\langle n(k)|e^{-2\beta_k A(k)^\dagger} e^{2\beta_k A(k)}|n'(k)\rangle$$
$$= \prod_k \left(e^{-|2\beta_k|^2/2} \sum_{m=\max[0,n'(k)-n(k)]}^{n'(k)} \frac{[2\beta(k)]^m[-2\beta_k]^{n(k)-n'(k)+m}}{m![n(k)-n'(k)+m]!}\langle n(k)|A(k)^{\dagger m}A(k)^m|n'(k)\rangle \right), \tag{D5}$$

which gives

$$f(-\vec{\beta}) = (-1)^{N(\vec{n})-N(\vec{n}')}f(\vec{\beta}) \tag{D6}$$

or

$$f(-\vec{\beta}) = f(\vec{\beta}) \quad \text{for } N(\vec{n}) = N(\vec{n}'). \tag{D7}$$

On the other hand, with the above definitions and straightforward calculations, we have

$$\langle +,\vec{n}|-,\vec{n}'\rangle = f(\vec{\beta}_0), \tag{D8}$$

$$\langle -,\vec{n}|+,\vec{n}'\rangle = f(-\vec{\beta}_0), \tag{D9}$$

where $|\pm,\vec{n}\rangle$ are the eigenstates of the Hamiltonian H_1 defined in Sec. II B and $\vec{\beta}_0 = (\beta_{0k_1},\beta_{0k_2},\ldots)$. Here we have

$$\beta_{0k} = \frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{+\infty}\alpha_l e^{ikl} \tag{D10}$$

with α_l defined in Eq. (14). Then using Eqs. (C5) and (C8), we rewrite the matrix element $H_{\vec{n}',-}^{\vec{n},+}$ as

$$H_{\vec{n}',-}^{\vec{n},+} = \langle\Psi_{+,\vec{n}}|H|\Psi_{+,\vec{n}}\rangle = \langle\Psi_{+,\vec{n}}|H_2|\Psi_{-,\vec{n}'}\rangle$$
$$= \frac{\Omega}{4}(\langle +,\vec{n}|-,\vec{n}'\rangle - \langle -,\vec{n}|+,\vec{n}'\rangle)$$
$$= \frac{\Omega}{4}[f(\vec{\beta}_0) - f(-\vec{\beta}_0)]. \tag{D11}$$

Therefore, our result above in Eq. (D7) directly leads to Eq. (C6).

APPENDIX E: THE UNITARY OPERATOR U_M

In this appendix, we calculate the unitary operator U_M defined in Eq. (43). As in Appendix B, it is easy to prove that U_{TM} is the product of the displacement operators of each resonator mode:

$$U_M = \prod_{j=-\infty}^{+\infty}\exp\left[\alpha_j^M\sigma_x(a_j^\dagger - a_j)\right]. \tag{E1}$$

To derive the expression of α_j^M, we define the column vector

$$\vec{\alpha} = \left(\ldots,\alpha_{-1}^M,\alpha_0^M,\alpha_1^M,\ldots\right)^T. \tag{E2}$$

Then a straightforward calculation shows that the condition (43) is equivalent to the equation

$$\omega\vec{\alpha} - \xi K\vec{\alpha} = \vec{\Lambda}. \tag{E3}$$

Here K is a square matrix with the element K_{ij} in the ith row and jth column given by

$$K_{ij} = \delta_{i,j+1} + \delta_{i,j-1}. \tag{E4}$$

In Eq. (E3), $\vec{\Lambda}$ is a constant column vector with the jth component Λ_j defined as

$$\Lambda_j = \lambda\sum_{\beta=1}^m \delta_{j,c(\beta)}. \tag{E5}$$

Therefore, we formally have the expression for $\vec{\alpha}$:

$$\vec{\alpha} = \frac{1}{\omega - \xi K}\vec{\Lambda}. \tag{E6}$$

Furthermore, we notice that the matrix $\omega - \xi K$ is diagonalized as

$$\omega - \xi K = \int dk(\omega - 2\xi\cos k)\vec{v}(k)\vec{v}(k)^\dagger \tag{E7}$$

with the jth component of the column vector $\vec{v}(k)$ satisfying

$$v_j(k) = \frac{1}{\sqrt{2\pi}}e^{ikj}. \tag{E8}$$

Then we have

$$\frac{1}{\omega - \xi K} = \int dk\frac{\vec{v}(k)\vec{v}(k)^\dagger}{\omega - 2\xi\cos k}. \tag{E9}$$

Substituting (E9) into (E6), we get the expression for α_j^M:

$$\alpha_j^M = \frac{\lambda}{2\pi}\sum_{\beta=1}^m\int dk\frac{\exp\{ik[j-c(\beta)]\}}{\omega - 2\xi\cos k}. \tag{E10}$$

In the case of a single TLS located at the zeroth resonator, we have $m = 1$ and $c(1) = 0$. Then α_j^M is expressed as

$$\alpha_j^M = \frac{\lambda}{2\pi}\int dk\frac{\exp[ikj]}{\omega - 2\xi\cos k}. \tag{E11}$$

On the other hand, in such a single-TLS case, the value of α_j^M is also given by (B11). Therefore, we have

$$\frac{\lambda}{2\pi}\int dk\frac{\exp[ikj]}{\omega - 2\xi\cos k} = \frac{\lambda\omega_1}{2\xi^2 - \omega\omega_1}\left(\frac{\xi}{\omega_1}\right)^{|j|} \tag{E12}$$

with ω_1 defined in Sec. II B. Substituting (E12) into (E10), we finally obtain

$$\alpha_j^M = \sum_{\beta=1}^m\frac{\lambda\omega_1}{2\xi^2 - \omega\omega_1}\left(\frac{\xi}{\omega_1}\right)^{|j-c(\beta)|}. \tag{E13}$$

Therefore, we get the analytical expression of the unitary operator U_M defined in Eqs. (43) and (E1).

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