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Supersymmetric response of a Bose-Fermi mixture to photoassociation

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We study supersymmetric (SUSY) responses to a photoassociation (PA) process in a mixture of Bose molecules b and Fermi atoms f which turn to mutual superpartners for a set of proper parameters. We consider the molecule b to be a bound state of the atom f and another Fermi atom F with different species. The b-f mixture and a free F atom gas are loaded in an optical lattice. The SUSY nature of the mixture can be signaled in the response to a photon-induced atom-molecule transition: While two new types of fermionic excitations, an individual b particle–f hole pair continuum and the Nambu-Goldstone-fermion-like (or "goldstino-like") collective mode, are concomitant for a generic b-f mixture, the former is completely suppressed in the SUSY b-f mixture and the zero-momentum mode of the latter approaches an exact eigenstate. This SUSY response can be detected by means of the spectroscopy method, for example, the PA spectrum which displays the molecular formation rate of $Ff \to b$.

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Introduction. Recently, studies in the supersymmetry (SUSY) for a mixture of cold Bose and Fermi atoms have made spectacular progress [1–3]. In such a cold atomic system, however, a Bose atom never transits to a Fermi atom, its superpartner, or vice verse. In addition to the nonrelativity, this is another essential difference of this low-energy SUSY from the SUSY in high-energy physics. For the latter, such SUSY decay processes are always anticipated, for example, a quark (lepton) may emit or absorb a gaugino and decays to a squark (slepton), the superpartner of the quark (lepton) [4].

To expose the interesting SUSY nature of the mixture, the effective "decay" process must be introduced. For a cold atomic SUSY mixture with Bose-Einstein condensation, there is an effective decay of SUSY generators since they behave as the fermion annihilation and creation operators [3]. Therefore, the SUSY excitations can be simulated by a boson-enhanced fermionic excitation. As a result, a Nambu-Goldstone-fermion-like (or "goldstino-like") collective mode in the condensation phase of bosons could be observed by means of the single-particle spectroscopy [5,6].

To achieve an exact SUSY mixture, the system parameters must be fine-tuned, which requires elaborate experimental setups and then loses the generality. In this article, we explore how to observe the SUSY response by means of a spectroscopy measurement, even if the mixture deviates slightly from the SUSY and the bosons do not condense to form a whole ordered phase. This can resolve the fine-tuning restraints in measuring the SUSY response. On the other hand, the explicit breaking of the SUSY may create new excitations, the bosonic particle–fermionic hole individual continuous excitations, other than the collective goldstino-like mode. Although our theory is nonrelativistic, the creation of these new excitations due to SUSY explicit breaking should be quite general. This may be a helpful point in the study of SUSY in relativistic theory.

We consider a mixture of Bose molecules b and Fermi atoms f with on-site interaction in a d-dimensional optical lattice ($d = 2, 3$) [see Fig.1(a)]. With properly tuned interactions and hopping amplitudes, this b-f mixture may become SUSY [3]. We are interested in a special kind of molecule b, a bound state of f, and another species of Fermi atom F with binding energy E_b, and we restrict our analysis to the normal phase of the b-f mixture [7]. To probe the SUSY behaviors, we load a free Fermi atom F gas, which does not interact with both b and f directly. In a photoassociation (PA) process [8], the transitions between two atoms and one molecule, that is, $Ff \leftrightarrow b$, are induced by two laser beams with frequencies ω_1 and ω_2. For the SUSY b-f mixture, this resembles a high-energy physics process: a quark or a lepton (f) absorbs a fermionic gaugino ("absorbs" an F and emits a photon) and decays to a squark or a slepton (b) or vice verse. (One can also consider f to be a Fermi molecule formed by the bound state of a Bose atom b and a Fermi atom F, i.e., processes $Fb \leftrightarrow f$. We study these processes separately.)

For a negative detuning, $\delta_0 = \omega_2 - \omega_1 - E_b$, we show that the molecule dissociation process $b \to Ff$ is forbidden. In the formation process $Ff \to b$, two types of new fermionic excitations, an individual (bosonic) particle–(fermionic) hole pair continuum and a collective mode, emerge when the SUSY in the b-f mixture is slightly broken. For a SUSY b-f mixture, the former is completely suppressed while the latter in zero-momentum becomes an exact eigenstate, the goldstino-like mode [3]. In this sense, we regard these excitations as *the SUSY responses*. The PA spectrum is directly related to the the molecular formation rate varying as the detuning and faithfully describes these two types of excitations. The position of peak in the PA spectrum determines the frequency of the collective zero-momentum mode. This molecular formation rate is measured by the number variation of the F atoms in time. Experimentally, the number counting of atoms is much simpler than detecting the single atom spectrum.

Model setup. The system illustrated in Fig. 1(a) is described by a Hamiltonian $H = H_0 + H_{\rm ex}$, where $H_0 = H_{bf} + H_F$ with $H_{bf} = H_b + H_f + V$. By means of the Feshbach resonance [9], the scattering lengths between F and the b-f mixture can be adjusted to negligibly small. In the tight-binding approximation, one has

$$\begin{aligned} H_\alpha &= -\sum_{\langle ij\rangle} t_\alpha a_i^{\alpha\dagger} a_j^\alpha - \mu_\alpha \sum_i a_i^{\alpha\dagger} a_i^\alpha, \\ V &= \frac{U_{bb}}{2}\sum_i n_i^b(n_i^b - 1) + U_{bf}\sum_i n_i^b n_i^f, \end{aligned} \tag{1}$$

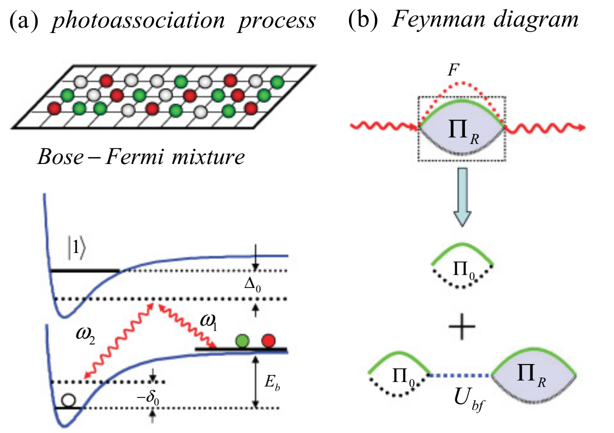

FIG. 1. (Color online) (a) Upper: The optical lattice with cold particles. The gray (green), black (red), and white dots denote Fermi atoms f, F, and the molecule b, respectively. Lower: The PA processes of two atoms to one molecule with the binding energy E_b. (b) The Feynman diagram for the linear response theory. The Green function of Q is calculated by random-phase approximation. Upper: The wavy (red) lines and dotted (red) line denote the free Green functions of photon and F, respectively. Lower: The solid (green) curves and dotted (black) curves denote the free Green functions of f and b, respectively.

where $a_i^\alpha = b_i$, f_i, and F_i ($\alpha = b$, f, and F) are the annihilation operators of b, f, and F at site i; and μ_α and $n_i^\alpha = a_i^{\alpha\dagger} a_i^\alpha$ are chemical potentials and the number operators at site i. The definitions of the hopping amplitudes t_α and the interaction strengths $U_{\alpha\beta}$ by the Wannier function $w_\alpha(\mathbf{r})$ can be found in the literature [10]. The spatial inhomogeneity of optical lattices trapping the atoms and molecules has been omitted. For the subsystem b-f mixture, the Hamiltonian H_{bf} is SUSY invariant if $t_b = t_f$, $U_{bb} = U_{bf}$, and $\mu_b = \mu_f$ [1,3]. In order to prevent the phase separation, the parameters obey, e.g., $4\pi t_f \rho_f U_{bb} > U_{bf}^2$ in two dimensions, where ρ_f is the density of f atoms [11]. We choose the parameters of the system obeying this condition.

The PA processes are realized by simultaneously shining two laser beams with frequencies ω_1 and ω_2 into the lattice [shown in Fig. 1(a)]. The ω_1 beam may turn two free atoms f and F into a higher energy bound state $|1\rangle$ which then may transit to the molecule b by emitting a photon with frequency ω_2. Meanwhile, the molecule b may also be excited to $|1\rangle$ by the ω_2 beam and then is unbound with some probability by emitting a photon with frequency ω_1. For large detuning Δ_0, the state $|1\rangle$ can be eliminated adiabatically, so that the PA is modeled by the tight-binding Hamiltonian

$$H_{\rm ex} = \sum_i (g_i b_i^\dagger f_i F_i e^{i\delta_0 t} + \text{H.c.}), \tag{2}$$

where the detuning $\delta_0 = \omega_c - E_b$, with the effective driven frequency $\omega_c = \omega_2 - \omega_1$, and $g_j = g_0 \exp(-i\mathbf{k}_0 \cdot \mathbf{r}_j)$, with $g_0 \propto \int d^d\mathbf{r} \exp(-i\mathbf{k}_0 \cdot \mathbf{r}) w_b^*(\mathbf{r}) w_f(\mathbf{r}) w_F(\mathbf{r})$ being the coupling intensity independent of the site.

In the $\mathbf{k}$ space, the Hamiltonian $H_{\rm ex}$ is rewritten as

$$H_{\rm ex} = g_0\sqrt{\rho}\left(\sum_{\mathbf{k}} Q_{\mathbf{k}-\mathbf{k}_0}^\dagger F_{\mathbf{k}} e^{i\delta_0 t} + \text{H.c.}\right), \tag{3}$$

where $Q_{\mathbf{k}}^\dagger = \sum_{\mathbf{p}} b_{\mathbf{p}+\mathbf{k}}^\dagger f_{\mathbf{p}}/\sqrt{N}$ and $a_{\mathbf{k}}^\alpha = \sum_j a_j^\alpha \exp(-i\mathbf{k} \cdot \mathbf{r}_j)/\sqrt{\mathcal{V}}$ (where $\mathcal{V}$ stands for the volume and $a_{\mathbf{k}}^\alpha$ stand for $b_{\mathbf{k}}$, $f_{\mathbf{k}}$, and $F_{\mathbf{k}}$). $\rho = N/\mathcal{V}$ is the total density of b and f with the particle number $N = \sum_i (n_i^b + n_i^f)$.

Molecular formation rate. The formation rate of the molecules b can be counted by the PA variation of F-fermion number $R = \partial_t \langle \psi(t)|N_F|\psi(t)\rangle$ for $|\psi(t)\rangle$ being the time evolution from the ground state $|G\rangle = |g\rangle|F\rangle$ of H_0. It follows from the linear response theory that

$$R = 2g_0^2 \rho \sum_{\mathbf{k}} \text{Im} D_R(\mathbf{k}, -\delta_0), \tag{4}$$

where the retarded Green function is given by

$$D_R(\mathbf{k}, \omega) = \int_{-\infty}^{\infty} dx A(\mathbf{k} - \mathbf{k}_0, x) \frac{n_f(x) - n_f(\varepsilon_{\mathbf{k}}^F)}{x - \varepsilon_{\mathbf{k}}^F - \omega - i0^+}, \tag{5}$$

in terms of one loop calculations [Fig. 1(b)]. The single-particle dispersions are $\varepsilon_{\mathbf{k}}^\alpha = -2t_\alpha \sum_{s=1}^d \cos k_s - \mu_\alpha$, where the lattice spacings are set to be the unit. $n_f(x)$ is the Fermi distribution at temperature T and the spectral function $A(\mathbf{k}, \omega) = -\text{Im}\Pi_R(\mathbf{k}, \omega)/\pi$ is defined by the retarded Green function $\Pi_R(\mathbf{k}, \omega) = -i\int_0^\infty dt \langle g|\{Q_{\mathbf{k}}(t), Q_{\mathbf{k}}^\dagger(0)\}|g\rangle e^{i\omega t}$.

At sufficiently low temperature, the pole and branch cut in Eq. (5) are not qualitatively affected by T and nor is the molecular formation rate. For simplicity, we take a zero-temperature approximation in our calculation. It follows from Eq. (5) that the rate $R = R_{b\to Ff} - R_{Ff\to b}$ contains two parts:

$$\begin{aligned} R_{b\to Ff} &= \sum_{\mathbf{k}} 2\pi g_0^2 \rho A(\mathbf{k} - \mathbf{k}_0, \varepsilon_{\mathbf{k}}^F - \delta_0)\theta(\delta_0 - \varepsilon_{\mathbf{k}}^F), \\ R_{Ff\to b} &= \sum_{\mathbf{k}} 2\pi g_0^2 \rho A(\mathbf{k} - \mathbf{k}_0, \varepsilon_{\mathbf{k}}^F - \delta_0)\theta(-\varepsilon_{\mathbf{k}}^F), \end{aligned} \tag{6}$$

which, respectively, are the dissociation rate for $b \to Ff$ and the formation rate for $Ff \to b$.

Collective and individual fermionic modes. In order to obtain R for the weak interactions, we perturbatively calculate $\Pi_R(\mathbf{k}, \omega) = \rho^{-1}[\Pi_0^{-1}(\mathbf{k}, \omega) + U_{bf}]^{-1}$, which formally results from the equation of motion of $Q_{\mathbf{k}}$. It then follows from the random phase approximation (RPA) illustrated by the "bubble" in Fig. 1(b) that

$$\Pi_0(\mathbf{k}, \omega) = \int \frac{d^d\mathbf{p}}{(2\pi)^d} \frac{n_f(\varepsilon_{\mathbf{p}}^f) + n_b(\varepsilon_{\mathbf{k}+\mathbf{p}}^b)}{\omega - E_{\mathbf{kp}} + i0^+}, \tag{7}$$

where $E_{\mathbf{kp}} = \varepsilon_{\mathbf{k}+\mathbf{p}}^b - \varepsilon_{\mathbf{p}}^f + 2\rho_b \delta U + U_{bf}\rho$, with $\delta U = U_{bb} - U_{bf}$ and $\rho_b = N_b/\mathcal{V}$; $n_b(x)$ is the Bose distribution. The isolated pole and branch cut of $\Pi_R(\mathbf{k}, \omega)$ describe the collective and individual SUSY excitations of $Q_{\mathbf{k}}^\dagger|g\rangle$.

Next we consider the elementary excitations in the two-dimensional lattice with f atoms at half filling, that is, $\rho_f = 0.5$. For the SUSY b-f mixture, that is, $\delta U = 0$ and $\delta t = t_b - t_f = 0$, the dispersion of the collective modes, $E_c(\mathbf{k}) \simeq \Delta\mu - \alpha|\mathbf{k}|^2$ for the small $|\mathbf{k}|$ [3], is read out from the poles of the retarded Green function $\Pi_R(\mathbf{k}, \omega)$ [see Fig. 2(a)], where $\Delta\mu = \mu_f - \mu_b$. For large $|\mathbf{k}|$, the energy $E_c(\mathbf{k}) = E_c(|\mathbf{k}|, \theta)$ depends not only on $|\mathbf{k}|$ but also on the angle $\theta = \arctan(k_y/k_x)$. The energy $E_c(|\mathbf{k}|, \theta)$ of $Q_{\mathbf{k}}^\dagger|g\rangle$ decreases as $|\mathbf{k}|$ increases for a fixed θ. The retarded Green's function

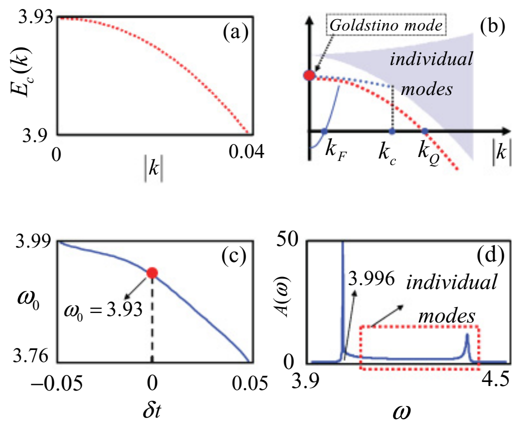

FIG. 2. (Color online) The dispersion, spectrum, and spectral function of the excitations for $\Delta\mu = 3.93$ and $U_{bb} = U_{bf} = 0.1$; t_b is taken as the unit. (a) The dispersion of collective mode for the SUSY mixture. (b) The spectrum for the SUSY system: the dashed (red) and dotted (blue) curves denote the dispersions of collective modes for $\theta = 0$ and $\pi/5$, respectively. The solid (blue) curve denotes the dispersion of atoms F for a small Fermi momentum k_F. (c) The frequencies of $Q_0^\dagger|g\rangle$ for different δt. (d) The spectral function of the zero momentum for $\delta t = -0.1$: The sharp peak is the shifted goldstino-like mode.

$\Pi_R(0,\omega) = (\omega - \Delta\mu + i0^+)^{-1}$ possesses a pole $\omega = \Delta\mu$, which corresponds to the goldstino-like excitation $Q_0^\dagger|g\rangle$. This recovers the result in Ref. [3]. The excitation spectrum is schematically shown in Fig. 2(b).

For the b-f mixture deviating slightly from SUSY, the retarded Green function $\Pi_R(0,\omega)$ has an isolated pole and a branch cut, which correspond to *a collective fermionic mode and individual (bosonic) particle–(fermionic) hole pair continuum modes*, respectively. The pole in $\omega_0 < E_0 - 4\delta t$ for $\delta t > 0$ (or $\omega_0 < E_0$ for $\delta t < 0$) describes the shifted goldstino-like mode. The frequencies ω_0 of the collective zero-momentum mode for different δt are shown in Fig. 2(c). For $\mathbf{k} \neq 0$, the pole of $\Pi_R(\mathbf{k},\omega)$ has the form $E_c'(\mathbf{k}) \simeq \omega_0 - \alpha'|\mathbf{k}|^2$ for small $|\mathbf{k}|$. Remarkably, the branch cut l_0 of $\Pi_R(0,\omega)$ emerges, which describes individual zero-momentum modes. Here, $l_0 = [E_0 - 4\delta t, E_0]$ for $\delta t > 0$ (or $[E_0, E_0 - 4\delta t]$ for $\delta t < 0$), and $E_0 = \Delta\mu + 2\rho_b\delta U + U_{bf}\rho$. Notice that for the weak interactions U_{bb} and U_{bf} the SUSY breaking from δU does not develop a branch cut but only shifts the positions of the pole and the branch cut. The pole and the branch cut can be seen in the spectral function $A(\mathbf{k},\omega)$, that is, the peak and the hump in Fig. 2(d) for $\mathbf{k} = \mathbf{0}$. Note that for the SUSY b-f, the branch cut length l_0 of $\Pi_R(0,\omega)$ shrinks to zero so that the individual continuum modes of zero momentum are completely suppressed. On the other hand, as the b-f mixture deviates from the SUSY, the goldstino-like mode is gradually suppressed. We examine the dependence of the spectral function on the interacting strength and find that the hump height may be depressed as the interaction becomes stronger; for example, the height is lower than 0.5 for $U_{bf} = U_{bb} = 0.5$ compared with ∼10 in Fig. 2(d) for $U_{bf} = U_{bb} = 0.1$.

In order to study the PA spectrum of the molecular formation rate, we discuss the excitation spectrum shown in Fig. 2(b). For some momenta $\mathbf{k}$, there is a collective mode (dashed red curve) below the individual continuum. For other momenta $\mathbf{k}_c$, the dispersion of the collective mode merges into the continuum. However, for small momentum $\mathbf{k}$, there always exists a collective mode below the individual continuum. For convenience, we define a critical momentum $k_Q(\theta)$, so that for a fixed θ, when $|\mathbf{k}| > k_Q(\theta)$, the negative frequencies of the mode $Q_\mathbf{k}$ emerge; that is, $A(\mathbf{k}, \omega < 0) \neq 0$ when $|\mathbf{k}| > k_Q(\theta)$, and $A(\mathbf{k}, \omega < 0) = 0$ when $|\mathbf{k}| < k_Q(\theta)$.

PA spectrum. The rate R varies as detuning δ_0 or the light frequency ω_c. Measurement of the b boson formation rate varying as δ_0 is called the PA spectrum $S(\delta_0)$. For a long wave photon, the coupling g_j varies slowly in space and the rates in Eq. (6) are approximately independent of $\mathbf{k}_0$.

According to Eq. (6), the dissociation rate $R_{b\to Ff}$ does not vanish only if $\delta_0 > \varepsilon_\mathbf{k}^F$ and $A(\mathbf{k}, \varepsilon_\mathbf{k}^F - \delta_0) \neq 0$. Because the spectral function $A(\mathbf{k}, x < 0) \neq 0$ is defined by the retarded Green function, it does not vanish only when the energies for collective modes or individual modes of $Q_\mathbf{k}$ are negative for the large $|\mathbf{k}| > k_Q(\theta)$. We consider a dilute Fermi gas F with the chemical potential $\mu_F \sim -4t_F$; the dispersion relation turns out $\varepsilon_\mathbf{k}^F = t_F|\mathbf{k}|^2 - \mu_{\rm eff}$, where $\mu_{\rm eff} = \mu_F + 4t_F$. In this case, the fermion F possesses a small Fermi momentum $k_F = \sqrt{\mu_{\rm eff}/t_F}$ which is much smaller than $k_Q(\theta)$ for small deviating δt. Therefore, ε_k^F is always positive when $|\mathbf{k}| > k_Q$ [see Fig. 2(b)]. For the negative detuning δ_0, the condition $\delta_0 > \varepsilon_k^F$ is not satisfied in the regime $|\mathbf{k}| > k_Q(\theta)$. That is, $A(\mathbf{k}, \varepsilon_\mathbf{k}^F - \delta_0)$ and $\theta(\delta_0 - \varepsilon_k^F)$ can not be nonzero simultaneously for the negative detuning and small Fermi momentum k_F. This finishes our proof of $R_{b\to Ff} = 0$.

The vanishing of $R_{b\to Ff}$ for the negative δ_0 and small Fermi momentum k_F can be understood in a more straightforward way. The transition $b \to Ff$ is described by the Hermite conjugate term (H.c.) in the Hamiltonian $H_{\rm ex}$, which is a high-frequency oscillation term when $\delta_0 < 0$. Hence, the Fermi golden rule results in $R_{b\to Ff}$ vanishing under the first-order perturbation (linear response).

For the negative δ_0 and small Fermi momentum k_F ($\mu_{\rm eff} \ll t_F$), the molecule formation rate now is reduced to $R = -R_{Ff\to b}$ and

$$R_{Ff\to b} \simeq \begin{cases} Z_0 g_0^2 N/[2(t_F + \alpha')] & \text{for} \quad \delta_0 = -\omega_0, \\ 2\pi g_0^2 \rho N_F A(0, -\delta_0) & \text{for} \quad |\delta_0| \in l_0, \\ 0, & \quad\text{otherwise}, \end{cases}$$

which leads to our main result: *The PA spectrum* $S(\delta_0) = -R_{Ff\to b}$ [see Fig. 3(a)] *displays the spectral function of excitations* $Q_0^\dagger|g\rangle$.

For the SUSY b-f mixture, the length of branch cut $l_\mathbf{0}$ tends to zero and the individual modes are suppressed. Meanwhile, the residue $Z_0 = 1$ and the formation rate $R_{Ff\to b} = g_0^2 N/[2(t_F + \alpha)] \equiv R_0 \propto N$ at $\delta_0 = -\Delta\mu$ and vanishes for the other detunings. There is a sharp peak at $\delta_0 = -\Delta\mu$ in the PA spectrum. For a generic b-f mixture deviating from SUSY, the residue $Z_0 < 1$ decreases as $|\delta t|$ increases. As a result, the peak height is lowered while its position is shifted to $\delta_0 = -\omega_0$. The ratio $R_{Ff\to b}(\omega_0)/R_0$ is shown in Fig. 3(b) for different δt and a small $\mu_{\rm eff}$, where $R_{Ff\to b}(\omega_0)$ is the value of $R_{Ff\to b}$ at $\delta_0 = -\omega_0$. Remarkably, a minor hump develops in the region $\delta_0 \in l_0$ due to the emergence of individual modes

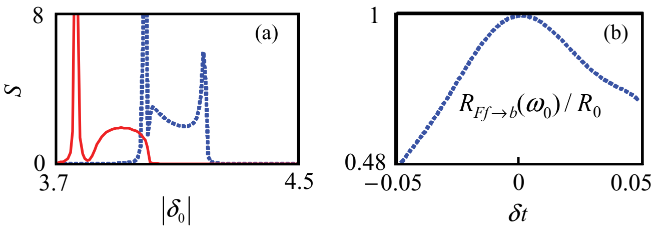

FIG. 3. (Color online) The PA spectra with the same parameters as those in Fig. 2. (a) The detuning dependence for $\delta t = 0.05$ and -0.05, the solid (red) and dashed (blue) lines, respectively. The unit of S is $2\pi g_0^2 \rho N_F$ and N_F/N is taken to be 0.01. (b) The major peak values for different δt. This shows that the peak value is suppressed when the system deviates from SUSY.

[see Fig. 3(a)]. As the system deviates further from SUSY, the individual modes are enhanced due to the sum rules $\int d\omega A(\mathbf{0}, \omega) = 1$. The temperature may suppress and broaden the peak and the hump. These characters of the PA spectrum in the b-f mixture are experimentally measurable SUSY responses to the light field.

Conclusions. We studied how to observe the SUSY nature of the b-f mixture in optical lattices through PA spectra. For the Bose molecules formed with two species of Fermi atoms, we showed that the photon-induced atom-molecule transition displays the signal of SUSY. As the response to the PA processes, a fermionic individual continuum and the goldstino-like mode were found. The PA spectrum can explicitly witness the molecular formation rate of $Ff \to b$. Because the goldstino-like mode in zero momentum turns to be the exact eigenstate for the SUSY mixture, the major peak in the PA spectrum reflects the SUSY response to the light field, even if the mixture is not fine-tuned to a SUSY one.

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