Birefringence lens effects of an atom ensemble enhanced by an electromagnetically induced transparency

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We study the optical control for birefringence of a polarized light by an atomic ensemble with a tripod configuration, which is mediated by the electromagnetically induced transparency with a spatially inhomogeneous laser. The atomic ensemble splits the linearly polarized light ray into two orthogonally polarized components, whose polarizations depend on quantum superposition of the initial states of the atomic ensemble. Accompanied with this splitting, the atomic ensemble behaves as a birefringent lens, which allows one polarized light ray passing through straightly while focuses the other light of vertical polarization with finite aberration of focus.

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I. INTRODUCTION

An atomic ensemble manipulated by electromagnetical field exhibits various quantum coherent properties, such as extremely slow group velocities [1–3], large refractive indexes [4], giant nonlinearities [5–7], laser-induced birefringence [8], and electromagnetically induced transparency (EIT) [9–13]. The phenomena about deflection of light beams in atom vapor have been studied in the earlier work [14–16]. Recently, an enhanced deflection of an unpolarized light beam is observed in an EIT medium with an external gradient field [17,18]. Different from conventional EIT studies, the external fields, which are used to control the light propagation, are transversely inhomogeneous [17–24].

The EIT-induced beam deflection has been explained using quantum approach with dark state polariton possessing an effective magnetic moment [17,23] and using the semiclassical approach [21] with the gradient-index medium caused by the external fields with inhomogeneous profiles. The quantum approach in Ref. [23] exhibits the wave-particle duality of the dark polaritons, where an effective Schrödinger equation is derived to describe the EIT-enhanced spatial motion of the probe field, which is similar to a spinless particle in an inhomogeneous field.

Most recently, we studied an optical analog of the Stern-Gerlach effect for a polarized light ray [24]. It shows that the atom medium is anisotropic to the polarized light when a transversely inhomogeneous field is applied, which makes the linearly polarized probe light split into two parts with opposite circular polarization. However, the optical split in Ref. [24] cannot be exactly referred to as the Stern-Gerlach effect due to the assumption of the incoherent atomic population (i.e., the atoms are initially in a mixed state without the off-diagonal terms). In conventional Stern-Gerlach experiment, atoms are usually prepared in a coherent superposition state. Actually, such field-induced birefringence has been studied extensively [25–28], but most of them refer to the incoherent atomic population.

In this paper, we consider what would happen if atoms are initially in a superposition of submanifold states of the atoms. The atomic coherence offers the possibility to change the polarization of the outgoing wave from linear to any desired polarization state. We show that for atoms with a tripod configuration, our optical control of polarization directly results from the superposition of submanifold states with the intrinsic double-Λ-type EIT structure. Due to the anisotropy of the medium driven by a transversely inhomogeneous field, the EIT-assisted spatial motion of a linearly polarized probe beam exhibits a birefringent phenomenon with the polarization-dependent focusing (defocusing). Namely, a linearly polarized light propagates along a straight line in the medium; the other orthogonally polarized beam converges to different focuses along the z axis. Such phenomenon—light rays parallel to a lens axis fail to converge to the same point—is called aberration. It is more interesting that by changing the frequency of the control light from blue detuning to red detuning, the lenslike object can be adjusted optically from negative (or diverging) to positive (or converging) case. This divergence-to-convergence transition of the lenslike effect is only enhanced in the double-EIT configuration.

The paper is organized as follows. In Sec. II, we describe our scheme and give the interaction Hamiltonian. In Sec. III, we introduce the effective motion equation of the probe light and obtain the exact solution. In Sec. IV, we demonstrate the deflection of the light beam for the two orthogonally polarized components, respectively, and show the birefringence lens effects of the atomic ensemble.

II. SETUP

We consider an ensemble of $2N$ identical and noninteracting atoms, which is confined in a gas cell of length L along z axis. The atoms possess four levels in a tripod configuration as shown in Fig. 1(a). The submanifold of ground states is spanned by two degenerate Zeeman sublevels $|g\rangle$ and $|h\rangle$. Atoms are initially prepared in the superposition

$$|\phi\rangle = \alpha|g\rangle + \beta|h\rangle \tag{1}$$

of the Zeeman sublevels. The linearly polarized probe light is characterized by field operators $\widetilde{E}_1$ and $\widetilde{E}_2$. Due to some se-

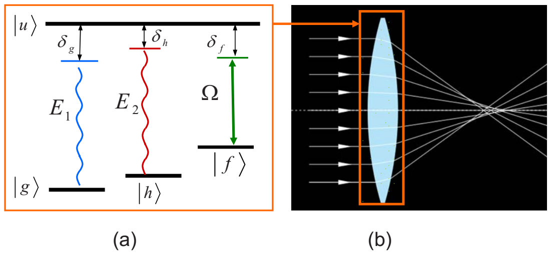

FIG. 1. (Color online) Energy level scheme (a) for the tripod atoms interacting with a coupling field (indicated by Rabi frequency Ω) and a linearly polarized probe field. Such atomic ensemble confined in a gas cell behaves as a converging (or diverging) lens (b).

lection rules, the σ^{+}-(σ^{-}-) component $\tilde{E}_1$ ($\tilde{E}_2$) only couples the Zeeman sublevel $|g\rangle(|h\rangle)$ to excited state $|u\rangle$, while the transition $|f\rangle$-$|u\rangle$ is driven by an intense classical laser field with a Rabi frequency $\Omega=\Omega(x,y)$. The intense laser field has a spatially inhomogeneous profile in the transverse direction. The control beam is detuned from state $|u\rangle$ with detuning δ_f, while the σ^{+}- and σ^{-}-polarized components $\tilde{E}_1$ and $\tilde{E}_2$ have finite detunings δ_g and δ_h, respectively, which can be adjusted by a magnetic field applied along the z direction. Obviously, there exist two Λ configurations consisting of energy levels (g,f,u) and (h,f,u), which are equivalent to the double-EIT structure.

Without loss of generality, we consider the case that both probe and control fields propagate parallel to the z axis. In reality, both the probe and control fields are characterized by wave packets with spatially Gaussian profiles. Therefore, each component of the probe field can be interpreted as a plane wave with a slowly varying operator [29],

$$\tilde{E}_j^{+}(\mathbf{r},t)=\sqrt{\frac{\nu}{2\varepsilon_0 V}}E_j(\mathbf{r},t)e^{i(kz-\nu t)}, \quad (j=1,2). \tag{2}$$

We also introduce the following collective continuous operator $\tilde{\sigma}_{\mu\nu}(\mathbf{r})=\Sigma_{r_j\in N_r}\sigma^{j}_{\mu\nu}/N_r$ for the collective excitations in the atomic medium. It actually describes the average of $\sigma^{j}_{\mu\nu}=|\mu\rangle_j\langle\nu|$ over $N_r[=(2N/V)dr\gg 1]$ atoms in a small but macroscopic volume around position $\mathbf{r}$. The slowly varying operator $\sigma_{\mu\nu}$ for the atomic transition is, respectively, defined as $\tilde{\sigma}_{ug}=\sigma_{ug}\exp(-ikz)$ and $\tilde{\sigma}_{uf}=\sigma_{uf}\exp(-ik_c z)$. Here, k and k_c are the wave numbers to the central frequencies ν and ν_c of the probe and control fields, respectively. For cold atoms, the kinetic energy could be neglected, so the total system can be modeled by the interaction Hamiltonian

$$H_I=\frac{N}{V}\int d^3\mathbf{r}[(\delta_g\sigma_{gg}+\delta_h\sigma_{hh}+\delta_f\sigma_{ff})-(\Omega\sigma_{uf}+gE_1\sigma_{ug} + gE_2\sigma_{uh}+\text{H.c.})]. \tag{3}$$

Due to the symmetry of the states $|g\rangle$ and $|h\rangle$, the transition matrix elements in the above equation are equal

$$g_{ug}=g_{uh}=g=\langle u|d|g\rangle\sqrt{\nu/(2\varepsilon_0 V)} \tag{4}$$

for both circular components, where $\langle u|d|g\rangle$ is the dipole matrix element.

III. EFFECTIVE MOTION EQUATION OF LIGHT

We follow the approach of the effective equation in Ref. [24] to study the atomic response by assuming that the atomic operators are the average over some initial state, e.g., $|\phi\rangle$. To elucidate the induced-lens behavior of the atomic ensemble, we consider the Heisenberg-Langevin equations for the atomic and field operators with the ground-state coherence relaxation rate γ and the decay rate Γ of the excited state $|u\rangle$, which are introduced phenomenologically. Since the intensity of the probe field is much weaker than that of the control field and the number of photons in the signal pulse is much less than the number of atoms in the sample, we treat the atomic equations perturbatively with the perturbative parameters gE_i [24]. For the initially superposition $|\phi\rangle$ of Zeeman sublevels, the averages of atomic operators up to the zeroth order are given by

$$\sigma^{(0)}_{mn}=\text{Tr}(|\phi\rangle\langle\phi|\sigma_{mn}), \tag{5}$$

namely, $\sigma^{(0)}_{gg}=|\alpha|^2$, $\sigma^{(0)}_{hh}=|\beta|^2$, and $\sigma^{(0)}_{gh}=\alpha\beta^*$.

With the above consideration, a straightforward calculation with adiabatic approximation [24] gives the steady-state solution of the atomic linear response

$$\sigma^{(1)}_{gu}=\zeta_g(|\alpha|^2E_1+\alpha\beta^*E_2), \tag{6a}$$

$$\sigma^{(1)}_{hu}=\zeta_h(|\beta|^2E_2+\alpha^*\beta E_1), \tag{6b}$$

where

$$\zeta_s=g(\delta_s-\delta_f)/[2|\Omega(x,y)|^2] \tag{7}$$

for $s=g,h$. The above equations show that when light travels through an atomic ensemble, the atomic response produces collective electric dipole moments. This atomic response also gives a back action on light, thus, leads to the paraxial wave equation

$$\left(i\partial_t+ic\partial_z+\frac{c}{2k}\nabla_T^2\right)\begin{bmatrix}E_1\\E_2\end{bmatrix}=-2g^*N\begin{bmatrix}\sigma^{(1)}_{gu}\\\sigma^{(1)}_{hu}\end{bmatrix}, \tag{8}$$

where $\nabla_T^2=\partial^2/\partial x^2+\partial^2/\partial y^2$, c is the light velocity in vacuum.

Consider the case with degenerate sublevels and without magnetic fields, i.e., $\delta_g=\delta_h$. Equation (8) yields a two-component equation

$$\left(i\partial_t+ic\partial_z+\frac{c}{2k}\nabla_T^2\right)\Phi=V(x,y)\sigma^{(0)}\Phi \tag{9}$$

for the light field envelope $\Phi=(E_1,E_2)^T$, which behaves as a spinor moving in a spin-dependent effective potential

$$V(x,y)\sigma^{(0)}=-\frac{|g|^2N\Delta}{|\Omega(x,y)|^2}\sigma^{(0)}. \tag{10}$$

Here $\Delta=\delta_h-\delta_f$ is the two-photon detuning. This visualized spin represents the polarization state of a probe light. The coupling between atoms and light induces a spin-dependent potential $V(x,y)\sigma^{(0)}$, which obviously affects the propagation of the light with opposite polarized orientation. Consequently, a signal pulse parallel to the control beam may deviate from its original trajectory, when it travels across the medium. We also note that by applying position-dependent fields, an initially isotropic medium becomes anisotropic [24]. However, the magnetic field is necessary for the system to display the circular birefringence. Here, we show that a linear birefringence may also occur.

To describe the propagation of the probe beam clearly, we introduce two polarized components $E_- \equiv \beta E_1 - \alpha E_2$ and $E_+ \equiv \alpha E_1 + \beta E_2$, which are the coherent superpositions of the left- and right-circular polarizations. In the following discussion, we assume that α and β are real. In terms of $E_\pm$, the Schrödinger-like equation (9) becomes

$$\left(i\partial_t + ic\partial_z + \frac{c}{2k}\nabla_T^2\right)E_\pm = \frac{1 \pm 1}{2}V(x,y)E_+. \quad (11)$$

The above equation indicates that the E_- ray propagates along a straight line (since no potential acts on it), which means that the medium is homogenous for E_-. However, the component E_+, which is subject to an effective potential, experiences a declination.

In order to elucidate the spin-dependent lens behavior induced by the coupling laser, we assume that the coupling field has a Gaussian profile $\Omega(r) = \Omega_0 \exp(-x^2/2\sigma^2)$. σ is the width of the driving-field profile. Let the probe beam possess a Gaussian profile

$$E_\pm(0) = \frac{1}{\sqrt{\pi b^2}} e^{-(x-a)^2/2b^2 - (z^2/2b^2)} \quad (12)$$

before it encounters the medium. Here, b is the width of the probe field, and a is the initial location of the wave-packet center of the probe field along the x direction. When b is much smaller than σ, $|\Omega|^{-2}$ is expanded in Taylor series around a, and we retain up to the linear term. Equation (11) reads as

$$i\partial_t E_\pm = -\left(ic\partial_z + \frac{c}{2k}\partial_x^2\right)E_\pm - \frac{1 \pm 1}{2}\zeta(x - a + \eta/\zeta)E_+, \quad (13)$$

where $\zeta = 2a\eta/\sigma^2$ and

$$\eta = \Delta|g|^2 N \exp(a^2/\sigma^2)/\Omega_0^2. \quad (14)$$

Here, we restrict our discussion to the two-dimensional system in a x-z plane. Equation (13) can be exactly solved by the Wei-Norman algebraic method [30]. We give $E_-(x,t) = \Xi(a,t)$ and

$$E_+(x,t) = \Xi\left(a + \frac{t^2\zeta}{2m}, t\right)e^{it\{\eta - [t/3(\zeta^2/2m)] + \zeta(x-a)\}}, \quad (15)$$

where

$$\Xi(a,t) = \frac{\exp\left[-\dfrac{(x-a)^2}{2\left(b^2 + \dfrac{it}{m}\right)} - \dfrac{(z-ct)^2}{2b^2}\right]}{\sqrt{\pi\left(b^2 + \dfrac{it}{m}\right)}}$$

and we have defined the effective mass $m = k/c$.

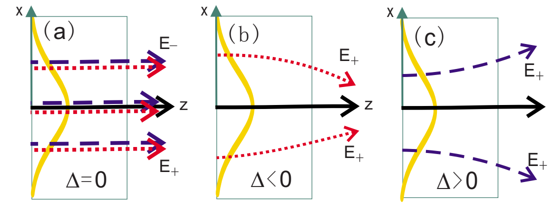

FIG. 2. (Color online) Schematic of a probe ray passing through a EIT medium. The green rectangles represent the atomic medium. The solid yellow lines indicate the profile of the coupling field. The propagation of the E_- component is illustrated by the dashed blue lines in (a). The dotted red lines in (a) indicate the trajectory of the E_+ component when $\Delta=0$ or $a=0$. The medium renders a lenslike effect in (b) and (c). The medium converges the E_+-component beam [the dotted red lines in (b)] when $\Delta<0$ and diverges the E_+-component beam [the dashed blue lines in (c)] when $\Delta>0$.

IV. POLARIZATION-DEPENDENT DEFLECTION AND BIREFRINGENCE LENS EFFECTS

Now we discuss the physics implied in Eq. (15). Equation (15) shows that the profile center $(x,z)=(a,0)$ of the linearly polarized component E_+ at time $t=0$ is shifted to $(x=x_+, z=L)$ at time $t=L/c$, where

$$x_+ = a + \frac{g^2 N a \Delta L^2}{ck\Omega_0^2\sigma^2}\epsilon^{a^2/\sigma^2}, \quad (16)$$

but nothing happens to the E_- component. Therefore, if one tracks the center motion of the probe beam, the trajectory of the linearly polarized E_- component is only a straight line along the z axis. However, its orthogonal component is deflected by the atomic ensemble. E_+ propagates either toward or away from the z axis, which depends on the sign of the two-photon detuning Δ and the incident position a of the probe field.

Figure 2 shows the different cases for the $E_\pm$-ray propagation. Panel (a) indicates the trajectory of the linearly polarized component E_- (dashed blue lines) as well as its orthogonal component E_+ (dotted red lines) under the situation $\Delta=0$ or $a=0$. They all travel across the atomic medium straightly. Panel (b) corresponds to the case that $\Delta<0$, where the medium acts as a lens focusing the E_+ component. Panel (c) describes the case that $\Delta>0$, where the E_+ component experiences a defocusing.

The above analysis implies that the EIT medium behaves as a lens with a varying focus, which can be feasible and optically controlled. Right after light leaves the medium, it obtains a transverse group velocity with magnitude

$$v_x = \frac{2La\Delta|g|^2 N}{k\sigma^2\Omega_0^2}\exp(a^2/\sigma^2). \quad (17)$$

In the case of red detuning $\Delta<0$ [Fig. 3(a)], transverse group velocity v_x is toward the z axis. Therefore, a collimated light ray, starting at points $x=\pm a$ parallel to the lens axis (i.e., z axis), will meet the z axis at

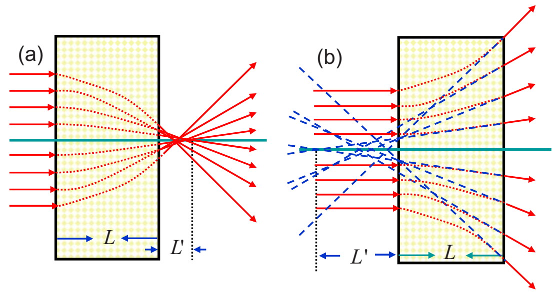

FIG. 3. (Color online) Schematic of an optically controlled lens and its aberration. (a) The EIT medium converges all input parallel rays to different points along the z axis. (b) The EIT medium diverges all input parallel rays, which are brought to different points along the z axis at the same side of the incident rays.

$$z = f_{con}(a) = \frac{L}{2} + F(a), \tag{18}$$

where

$$F(a) = kc\sigma^2\Omega_0^2/(2L|\Delta||g|^2N)\exp(-a^2/\sigma^2). \tag{19}$$

However, as the distance a changes from 0 to h (the height of the cuboid gas cell), this focal point runs from $f_{con}(h)$ to $f_{con}(0)$. This shows a typical aberration with a distortion length $l = F(0) - F(h)$. Obviously, such an EIT-based lens does not form perfect images due to distortions or aberrations introduced by the a-dependent focal points and the width spreading of the wave packet. Therefore, the atomic medium functions as a converging (or positive) lens to some extent. In the blue detuning case [$\Delta > 0$ in Fig. 3(b)], although the transverse velocity has the same magnitude v_x, its direction is opposite to the former case. Here, the light ray will not meet the z axis in the propagating direction, but it virtually crosses the z axis at $z = z_{div} = -[F(a) - L/2]$, which means that the atomic medium acts as a diverging lens. Thus, the atomic ensemble behaves as a negative or diverging lens, i.e., the collimated light rays are diverged when passing through the medium.

Therefore, changing the frequency of the control light from blue detuning to red detuning, we can carry out an optically controlled quantum manipulation based on the EIT for the transition from the diverging lens effect to converging one. The light ray experiences either defocusing or focusing determined by the sign of the two-photon detuning Δ. To realize an optically controlled lens with the transition from divergence to convergence, we would like to consider some experimental data: $\nu = 3 \times 10^{15}$ rad/s, $N/V = 10^{13}$ cm^{-3}, $\Omega_0 = 0.6\Gamma$, $L = 10$ cm, and $\sigma = L/4$. For a ray starting at $a = L/4$, the deflection angle $\alpha \simeq 6.97 \times 10^{-2}$ when two-photon detuning $\Delta = 0.1\Gamma$. This EIT-gas lens seem difficult to put into service currently due to relatively low density of atoms, but the further improvement with higher atomic density, which will induce proportional increase in the defection angle, can lead to the obviously observable effect.

V. CONCLUSION

In this paper, we study the defocusing and focusing effects of probe light by an ensemble of four-level atoms in a tripod configuration driven by a control field with a spatially inhomogeneous profile. We find that the atomic medium serves as a polarization-selective lens for the probe beam. Different from previous proposal [24], the Zeeman splitting of magnetic sublevels does not cause asymmetry atomic susceptibility for left- and right-circular-polarization components of the optical field because no magnetic field is applied in our setup. The present investigation shows that the probe beam splits into two parts, and each polarized component of the outgoing probe field contains the information of the atomic population. Therefore, the preparation of the atomic internal state can also be used to control the deflection, focusing, and defocusing effects. We also point out that there may have a connection to commonly observed nonlinear optical effects, such as cross-phase modulation and self-focusing, as this seems to be a closely related. We will consider the polarization-dependent form of such effects in the future work.

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