Quantum switch for single-photon transport in a coupled superconducting transmission-line-resonator array

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We propose and study an approach to realize quantum switch for single-photon transport in a coupled superconducting transmission-line-resonator (TLR) array with one controllable hopping interaction. We find that the single photon with arbitrary wave vector can transport in a controllable way in this system. We also study how to realize controllable hopping interaction between two TLRs via a Cooper-pair box (CPB). When the frequency of the CPB is largely detuned from those of the two TLRs, the variables of the CPB can be adiabatically eliminated and thus a controllable interaction between two TLRs can be obtained.

Coupled cavity arrays (CCAs) [1] have recently attracted considerable attentions of both theorists and experimentalists. The CCAs have been proposed to implement quantum simulators for many-body physics, such as discovering new matter phases of photons [2–4] and providing a new platform to study spin systems [5,6]. The CCAs are also suggested to manipulate photons for optical quantum information processing [7–9]. Moreover, photon transport in the CCAs has been investigated [10–14]. There are several possible ways to construct the CCAs, for example: (i) coupled defect cavities in photonic crystals [15]; (ii) coupled toroidal microresonators [16]; and (iii) coupled superconducting transmission-line resonators (TLRs) [11,12].

In CCAs, there have been many proposals to realize quantum switch [17,18], which is used to control single-photon transport [11,19–21]. For example, the reflection and transmission of photons in a coupled resonator waveguide can be controlled by a tunable two-level quantum system [11,18], acting as a controller.

Here, we study another approach to control the single-photon transport in a CCA, which consists of a chain of TLRs [22,23]. In our proposal, the controllable transport is realized by a tunable coupling. As we know, how to control coupling between two solid devices is a major challenge in scalable quantum computing circuits [24–30]. To obtain a tunable coupling, we propose that a Cooper-pair box (CPB) acts as a coupler. When the frequency of the coupler is largely detuned from those of the two resonators, the variables of the coupler can be adiabatically eliminated and thus a controllable interaction can be induced. Compared with previous approach [11], this approach has following advantage: dynamical variables of the coupler are adiabatically eliminated, therefore the coupler is a passive controlling element, which makes robust to prevent from the environment of the coupler.

As shown in Fig. 1, one-dimensional CCA is a chain of N cavities, each is only coupled to its nearest-neighbor ones, Figs. 1(a) and 1(b) are the site lattice model and the schematic of coupled TLR array, respectively. The TLRs are as-

sumed to have the same frequency. We also assume that the coupling strength between two nearest-neighbor TLRs is the same, except one between the lth and $(l+1)$th TLRs. The Hamiltonian of the system is

$$H = \omega \sum_n a_n^\dagger a_n - t \sum_n (a_n^\dagger a_{n+1} + a_{n+1}^\dagger a_n) - \lambda t(a_l^\dagger a_{l+1} + a_{l+1}^\dagger a_l),$$

$$(1)$$

hereafter we take $\hbar = 1$. Here, we assume that all TLRs have the same frequency ω. $a_n^\dagger$ and a_n are the creation and annihilation operators of the nth TLR; t is the coupling strength between the nth ($n \neq l$) and $(n+1)$th TLRs; $\lambda = (t'-t)/t$ is introduced to denote the relation between t and t', where t' is the coupling strength between the lth and $(l+1)$th TLRs. Obviously, $-1 < \lambda < 0$ corresponds to $0 < t' < t$, while $\lambda \geq 0$ implies $t' \geq t$. Below we will first study how to control the single-photon transport by changing coupling strength t', and then answer question how to realize controllable coupling t'.

In the case of $t' = t$, the Hamiltonian in Eq. (1) is reduced to the usual bosonic tight-binding model $H_{\text{btb}} = \omega \sum_n a_n^\dagger a_n - t \sum_n (a_n^\dagger a_{n+1} + a_{n+1}^\dagger a_n)$ as shown in Ref. [31], which describes an N-site lattice model with nearest-neighbor coupling. It is
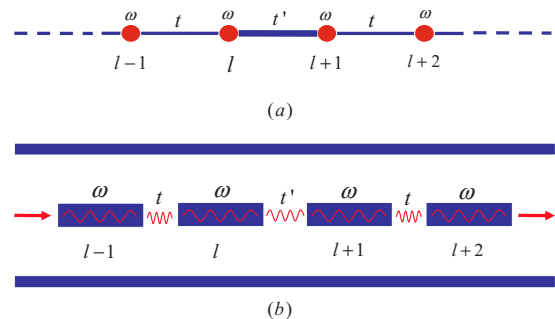

FIG. 1. (Color online) Schematic configuration for controllable transport of single photon: (a) one-dimensional site lattice model for the coupled cavity array; (b) schematic of coupled superconducting transmission-line-resonator array.

well known that, under the periodic boundary condition, the bosonic tight-binding Hamiltonian can be diagonalized as $H_{\mathrm{btb}} = \Sigma_k \Omega_k a_k^\dagger a_k$ by using the Fourier transformation $a_k = \Sigma_n \exp(iknd_0)a_n/\sqrt{N}$, where d_0 is the site distance. Below, d_0 is taken as unit. We choose the wave vectors $k = 2\pi m/N$ within the first Brillouin zone, i.e., $-N/2 < m \le N/2$. The corresponding dispersion relation is $\Omega_k = \omega - 2t \cos k$, which is an energy-band structure. For $t > 0$, the wave vectors $k = \pm \pi/2$ correspond to the energy-band center, while the wave vectors $k=0$ and $k = \pm \pi$ correspond to the bottom and top of the energy band, respectively.

Let us now define a total excitation number operator $\hat{N} = \Sigma_n a_n^\dagger a_n$. It is straightforward to show that $\hat{N}$ commutes with the model Hamiltonian (1), i.e., $[\hat{N}, H] = 0$, which implies that the total excitation number $\hat{N}$ is a conserved observable. We now restrict our discussion to the single excitation subspace since we only consider the single-photon transport. In this case, a general state can be written as, $|\Omega\rangle = \Sigma_n A_n |1_n\rangle$, where we have introduced the basis state $|1_n\rangle = |0\rangle \otimes \cdots \otimes |1\rangle_n \otimes \cdots \otimes |0\rangle$, which represents the state that the nth TLR has one photon while other TLRs have no photon. A_n is the probability amplitude of the state $|1_n\rangle$. Using the discrete scattering method proposed in Ref. [11] and according to the eigenequation $H|\Omega\rangle = \Omega|\Omega\rangle$, we have

$$-t(A_{n+1} + A_{n-1}) = (\Omega - \omega)A_n, \quad n \ne \{l, l+1\}, \quad (2a)$$

$$-t'A_{l+1} - tA_{l-1} = (\Omega - \omega)A_l, \quad (2b)$$

$$-A_{l+2} - t'A_l = (\Omega - \omega)A_{l+1}. \quad (2c)$$

For the coherent transport of a single-photon with the energy $\Omega = \omega - 2t \cos k$, we can assume the following forms for the probability amplitudes:

$$A_n = e^{ikn} + re^{-ikn} \quad (n \le l), \quad (3a)$$

$$A_n = se^{ikn} \quad (n \ge l+1). \quad (3b)$$

Here r and s are the reflection and transmission amplitudes, respectively. Obviously, Eqs. (3a) and (3b) are the solutions of Eq. (2a). Substituting Eqs. (3a) and (3b) into Eqs. (2b) and (2c), we can obtain the transmission coefficient

$$T(\lambda, k) = \frac{4(\lambda+1)^2 \sin^2 k}{\lambda^2(\lambda+2)^2 + 4(\lambda+1)^2 \sin^2 k}, \quad (4)$$

and the reflection coefficient $R(\lambda, k) = |s|^2 = 1 - T(\lambda, k)$. Equation (4) shows that the reflection and transmission coefficients $R(\lambda, k)$ and $T(\lambda, k)$ are function of the parameter λ and the wave vector k of the incident photon, and they are independent of other variables, e.g., the site position parameter l, the cavity frequency ω, and the coupling constant t.

Equation (4) shows two symmetry relations $T(\lambda, k) = T(\lambda, -k)$ and $T(\lambda, \pi/2 - k) = T(\lambda, \pi/2 + k)$. Therefore we need only to analyze the transmission coefficient within the region $0 \le k \le \pi/2$. In this region, there are four special cases: (1) $T(\lambda \ne 0, 0) = 0$, when the wave vector $k=0$, for $\lambda \ne 0$, the input single photon is reflected completely; (2) $T(\lambda = -1, k) = 0$, when $\lambda = -1$, the coupling between the lth
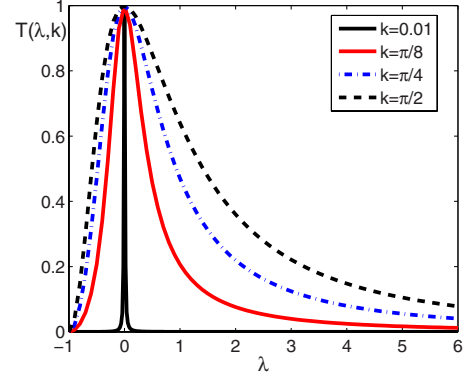

FIG. 2. (Color online) The transmission coefficient T versus the parameter λ for different wave vectors $k = 0.01$, $\pi/8$, $\pi/4$, and $\pi/2$ is plotted.

and $(l+1)$th cavities is switched off, so for any value of the wave vector k, the transmission coefficient is zero; (3) $T(\lambda \to \infty, k) = 0$, when $\lambda \to \infty$, namely, $t' \gg t$, the transmission coefficient is zero for any k. Physically, when $t' \gg t$, the Hamiltonian (1) is approximated to $H(t' \gg t) \approx -t'(a_l^\dagger a_{l+1} + a_{l+1}^\dagger a_l)$. The input photon will stay in the lth and $(l+1)$th cavities once it arrives the lth cavity; (4) $T(\lambda = 0, k) = 1$, $\lambda = 0$ implies $t' = t$, the present model reduces to the usual bosonic tight-binding model, so the photon with any wave vector can be perfectly transported.

To observe the effect on the transmission coefficient T for general wave vector k and parameter λ, in Fig. 2, the transmission coefficient T is plotted as a function of the parameter λ for wave vectors $k = 0.01$, $\pi/8$, $\pi/4$, and $\pi/2$. Figure 2 indicates that there are two regions, $-1 \le \lambda \le 0$ and $0 \le \lambda$, in which controllable transport of single photon can be achieved. The transmission coefficient T can be tuned from 0 to 1 by changing the coupling strength t', namely, λ. When $t' = 0$, the transmission coefficient $T = 0$. With the increase of the coupling strength $t' \to t$, the transmission coefficient T gradually approaches to 1. For $t' \ge t$, the transmission coefficient T approaches to 0 with the increase of the coupling $t' \to \infty$. In this region, the larger wave vector k corresponds to the larger parameter range of λ. In both regions, the controllable transport of single-photon with arbitrary wave vector k can be realized. Therefore, our approach for single-photon transport can cover complete bandwidth.

Let us now focus the problem on how to realize controllable coupling between two TLRs [18,27]. The system we considered is shown in Fig. 3. Two TLRs are coupled to a CPB through capacitors C_l and C_r, respectively. We assume that the two TLRs are identical, that is, they have the same length d and capacitance C_0 (inductance L_0) per unit length. We consider only single-modes of the two TLRs in near resonant with the CPB. The free Hamiltonian of the two TLRs is

$$H_{\mathrm{TLR}} = \omega a_l^\dagger a_l + \omega a_r^\dagger a_r, \quad (5)$$

where $a_l^\dagger$ $(a_r^\dagger)$ and a_l (a_r) are the creation and annihilation operators of the resonant modes with frequency ω for the left (right) TLR, respectively.
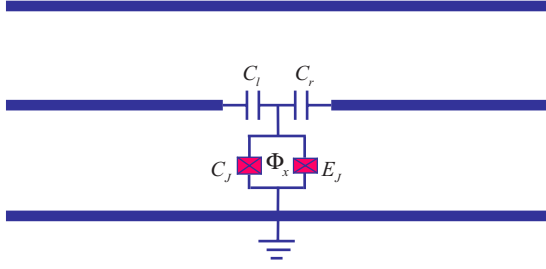
FIG. 3. (Color online) Schematic for two TLRs (the left and the right ones), which are coupled to a CPB through two capacitors C_l and C_r, respectively. The CPB is biased by a magnetic flux Φ_x

The CPB is a superconducting loop interrupted by two identical Josephson junctions with the capacitance C_J and the Josephson energy $E_J^{(0)}$. To obtain a tunable Josephson coupling energy, an external magnetic flux Φ_x is applied through the superconducting loop. The Hamiltonian of the CPB is

$$H'_{\mathrm{CPB}} = E_C n^2 - E_J(\Phi_x)\cos\varphi, \qquad (6)$$

where n is the number operator of Cooper-pair charges on the island connected to the CPB, and φ is the superconducting phase difference across the Josephson junction. The charging energy E_C and effective Josephson energy $E_J(\Phi_x)$ of the CPB are $E_C = 2e^2/(C_l + C_r + 2C_J)$ and $E_J(\Phi_x) = 2E_J^{(0)}\cos(\pi\Phi_x/\Phi_0)$, respectively. Here, we assume that the charging energy and the effective Josephson energy satisfy the condition $E_J(\Phi_x) \gg E_C$. Under this condition, the spectrum of the lowest energy levels of the CPB can be described approximately by a harmonic oscillator [29]. That is, we expand $E_J(\Phi_x)\cos\varphi$ around $\varphi = 0$ up to $O(\varphi^2)$, and then Eq. (6) becomes

$$H_{\mathrm{CPB}} = \omega_b b^\dagger b, \quad \omega_b = \sqrt{2E_C E_J(\Phi_x)}. \qquad (7)$$

The annihilation and creation operators b and $b^\dagger$ in Eq. (7) are defined in terms of $\varphi = \sqrt[4]{E_C/(2E_J(\Phi_x))}(b + b^\dagger)$ and $n = -i\sqrt[4]{E_J(\Phi_x)/(8E_C)}(b - b^\dagger)$.

We assume that the linear dimension of the CPB is much smaller than wavelengths of the TLRs, and choose the position of the CPB at the origin of the axis. Then the quantized voltages at the left and right TLRs are

$$V_j(0) = -i\sqrt{\frac{\omega}{dC_0}}(a_j - a_j^\dagger), \quad j = l, r. \qquad (8)$$

According to circuit theory, we know that the voltage at the island is $\Phi_0\dot\varphi/(2\pi)$. Therefore, the Coulomb interaction induced by the two capacitors C_l and C_r is

$$H_I = \sum_{j=l,r} \frac{C_j}{2}\left(V_j(0) - \frac{\Phi_0}{2\pi}\dot\varphi\right)^2. \qquad (9)$$

In fact, capacitors C_l and C_r induce a direct Coulomb interaction between the two TLRs with the strength $\propto C_l C_r$. However, this direct interaction is much smaller than the interaction between the two TLRs and the CPB given by Eq. (9) with strengths $\propto C_{\Sigma l} C_r$ and $\propto C_{\Sigma r} C_l$ under the condition $\{C_{\Sigma l}, C_{\Sigma r}\} \gg \{C_l, C_r\}$, where $C_{\Sigma l} = C_0 d/2 + C_l$ and $C_{\Sigma r} = C_0 d/2 + C_r$ are the sum capacitors connected to the left and

right TLRs, respectively [32]. For instance, using current experimental parameters [33] $C_0 d/2 \sim 1.6$ pF and $C_l = C_r = 6$ fF, we find that the interaction between the TLRs and the CPB is larger than the direct interaction between two TLRs by three orders of magnitude.

Using Eqs. (5)–(9), the total Hamiltonian of the system described in Fig. 3 is

$$H = \omega_l a_l^\dagger a_l + \omega_r a_r^\dagger a_r + \omega_b' b^\dagger b + g_l(a_l b^\dagger + b a_l^\dagger) + g_r(a_r b^\dagger + b a_r^\dagger), \qquad (10)$$

where we have introduced the renormalized frequencies

$$\omega_j = \omega\left(1 + \frac{C_j}{dC_0}\right), \quad j = l, r, \qquad (11a)$$

$$\omega_b' = \omega_b + (C_l + C_r)\omega_b^2\left(\frac{\Phi_0}{2\pi}\right)^2\left(\frac{E_C}{2E_J(\Phi_x)}\right)^{1/2}, \qquad (11b)$$

and the coupling strengths

$$g_j = -C_j\omega_b\frac{\Phi_0}{2\pi}\sqrt{\frac{\omega}{dC_0}}\left(\frac{E_C}{2E_J(\Phi_x)}\right)^{1/4}, \quad j = l, r. \qquad (12)$$

It should be noted that we have made the rotation wave approximation when Eq. (10) is derived.

Equation (10) describes that two TLRs are coupled to the CPB, which serves as a coupler. To obtain controllable coupling between the two TLRs, we restrict the system in the large detuning regime, where the frequency differences between the two TLRs and the CPB are much larger than their coupling constants, i.e., $\Delta_l \gg g_l$ and $\Delta_r \gg g_r$. Here, $\Delta_j = \omega_b' - \omega_j$ for $j = l, r$ are the detuning between the frequencies of the TLRs and that of the CPB. By adiabatically eliminating the degree of freedom of the CPB, we obtain an effective interaction between the two TLRs. That is, we perform a unitary transform $U = \exp[g_l(a_l b^\dagger - b a_l^\dagger)/\Delta_l + g_r(a_r b^\dagger - b a_r^\dagger)/\Delta_r]$ for the Hamiltonian in Eq. (10) and use the Hausdorff expansion up to the first order in the small parameter g_j/Δ_j with $j = l, r$, then we obtain an effective Hamiltonian

$$H_{\mathrm{eff}} = \omega_l' a_l^\dagger a_l + \omega_r' a_r^\dagger a_r + g(a_r a_l^\dagger + a_l a_r^\dagger), \qquad (13)$$

where we have defined the Stark-shifted frequencies $\omega_j' = \omega_j + g_j^2/\Delta_j$ for $j = l, r$, and the effective coupling strength

$$g = \frac{g_l g_r(\Delta_l + \Delta_r)}{2\Delta_l\Delta_r}. \qquad (14)$$

Note that the effective Hamiltonian of the CPB $H_{\mathrm{CPB}} = \omega_b'' b^\dagger b$ with $\omega_b'' = \omega_b' - g_l^2/\Delta_l - g_r^2/\Delta_r$ has been neglected in Eq. (13). It is obvious that the Hamiltonian (13) describes an effective interaction between the two TLRs. According to Eqs. (7) and (11b), the frequency of the CPB can be tuned by the external magnetic flux Φ_x. Correspondingly, the detunings Δ_l and Δ_r between the TLRs and the CPB can be tuned, thus the coupling constant g can be tuned. When the detunings are very larger than the coupling constants between the TLRs and the CPB, the effective coupling constant g between the two TLRs is negligibly small, and then the interaction between the two TLRs is switched off. For example, if we assume that the two transmission-line resonators are iden-

tical, i.e., $\Delta_l = \Delta_r = \Delta$ and $g_l = g_r = g'$, and we take the parameters [33]: $\omega = 2\pi \times 3$ GHz, $C_l = C_r = 6$ fF, $C_0 d = 1.6$ pF, $E_C = 2\pi \times 0.35$ GHz, $E_J^{(0)} \sim 10^3 E_C$, then we calculate $g \approx 1.1 \sim 23$ MHz corresponding to $\cos(\pi \Phi_x / \Phi_0) \approx 0.02 \sim 1$. In this region, the conditions $E_J(\Phi_x) \gg E_C$ and $\Delta \gg g'$ are satisfied.

In conclusion, we have studied a quantum switch for single-photon transport in a coupled TLR array with one controllable hopping interaction. We have found that the controllable single-photon transport, for an arbitrary wave vector of photons, in the coupled TLR array can be realized by tuning one of the coupling constants. How to realize the controllable coupling between two TLRs is also studied. We have proposed that a CPB serves as a coupler to connect the two TLRs. In the regime of $E_J(\Phi_x) \gg E_C$, the CPB is approximately described as a harmonic oscillator. Under the large detuning condition, we have obtained an effective interaction between the TLRs by adiabatically eliminating the variables of the CPB. This induced effective coupling can be controlled by the external magnetic flux Φ_x through the CPB.

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