Intrinsic cavity QED and emergent quasinormal modes for a single photon

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We propose a special cavity design that is constructed by terminating a one-dimensional waveguide with a perfect mirror at one end and doping a two-level atom at the other. We show that this atom plays the intrinsic role of a semitransparent mirror for single-photon transports such that quasinormal modes emerge spontaneously in the cavity system. This atomic mirror has its reflection coefficient tunable through its level spacing and its coupling to the cavity field, for which the cavity system can be regarded as a two-end resonator with a continuously tunable leakage. The overall investigation predicts the existence of quasibound states in the waveguide continuum. Solid-state implementations based on a dc-superconducting quantum interference device circuit and a defected line resonator embedded in a photonic crystal are illustrated to show the experimental accessibility of the generic model.

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Cavity quantum electrodynamics (QED) [1] essentially reveals the quantum nature of photon confined in an extremely small spatial volume, such that there exists a strong coupling between atom and electromagnetic (EM) field. The atomic spontaneous emission within a microcavity can be enhanced or suppressed. The very property reflects the coherent manipulation of atom-EM field interactions through the boundary condition of cavity. When the cavity boundary is leaky, the set of quasinormal modes (QNMs), which is introduced in the study of the scattering of gravitational waves by a Schwarzschild black hole [2], appears as a discrete spectrum with complex value. Its imaginary part represents the width of the resonant spectrum line [3,4].

In usual experimental setup, the cavity is bounded by two reflective mirrors, which are obviously the external objects independent of the atom inside. In this paper, we consider an alternative cavity setup by localizing an atom at a position inside a one-dimensional (1D) half-resonator waveguide. The cavity is bounded by the termination of the waveguide at one end and the localized atom at the other. Unlike other one- or three-dimensional half-cavity setups [5–8], the characteristic intrinsity of this two-end cavity design arises naturally, in which the atom provides a tunable boundary condition for the cavity EM field, controllable through the parameters of the atom. When the atom resonates with the cavity field, the interference effect between its emission and absorption of photons tunes the boundary to totally reflect the incident EM wave. Hence the atom serves as a perfect mirror and the normal mode emerges. Near resonance, the atom behaves like a semitransparent mirror, resulting in a leaky cavity and the emergence of QNMs. In either case, there exists a strong back action from the emerging cavity field on the atom whose spontaneous emission is thus evidently influenced.

The theoretical considerations above inspire us to design a quantum coherent device, which has the atom responsible for a quantum switch [9] and stores a single photon as the QNMs or the normal modes of the tunable cavity. The recently proposed single-photon transistor [10] was designed upon a similar footing by utilizing the surface-plasmon excitation confined in an infinite waveguide but without any mirror [11].

We further explore two alternative implementations of more laboratory accessibility of the original design, one of which uses a superconductive transmission line resonator and the other is based on a defected line resonator within a photonic crystal. These implementations are useful for showing the detailed phenomena related to the QNM's theory [3,4].

We exhibit our idea about the intrinsic cavity by way of example shown in Fig. 1. The system consists of a 1D semi-infinite waveguide (a half cavity) with a two-level atom localized at a distance a from the termination. The atom has a level spacing Ω between its ground state $|g\rangle$ and its excited state $|e\rangle$, and is coupled to an EM field with strength J. Similar to the case of a finite or two-end cavity in Ref. [12], the model Hamiltonian of our setup reads

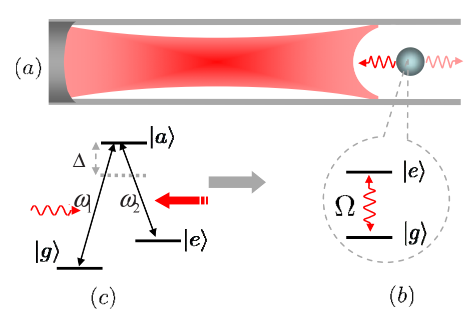

FIG. 1. (Color online) Schematic of the intrinsic cavity. (a) Half-waveguide with a two-level atom as (b) a tunable mirror; to implement the steady two-level atom, we use the stimulated Raman process based on (c) a Λ-type atom behaving as a two-level atom in large detuning to overcome the high-level decay.

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$$H = -iv_g\int_0^\infty dx(\varphi_R^\dagger\partial_x\varphi_R - \varphi_L^\dagger\partial_x\varphi_L) + \Omega|e\rangle\langle e| + J[\varphi_R^\dagger(a) + \varphi_L^\dagger(a)]|g\rangle\langle e| + \text{H.c.}, \quad (1)$$

where $\varphi_R=\varphi_R(x)$ $[\varphi_L=\varphi_L(x)]$ is the bosonic field operator for a right-going (left-going) photon; v_g stands for the group velocity of the photon in the waveguide. We remark here that this Hamiltonian can be obtained from Hamiltonian $H=\Sigma_k\hbar\omega_k a_k^\dagger a_k+\Omega(\sigma_z+1)/2+\Sigma_k J_k(a_k^\dagger+a_k)(\sigma_++\sigma_-)$, by noticing $|k|=k\to -i\partial_x$ for $k>0$ and $|k|=-k\to i\partial_x$ for $k<0$ with assumptions: $\omega_k=v_g|k|$ and $J_k=V$. We define $\varphi_R^\dagger(x)$ as right-going photon for $k>0$ and $\varphi_L^\dagger(x)$ as left-going photon for $k<0$.

The state of the system with either a single photon or an excited atom spans an invariant subspace of H. Thus, corresponding to the eigenvalue $E=v_g k(k>0)$, the stationary eigenstate,

$$|E\rangle = \int_0^\infty dx(u_{k,R}\varphi_R^\dagger + u_{k,L}\varphi_L^\dagger)|0,g\rangle + w_k|0,e\rangle, \quad (2)$$

depicts a single-photon state in the half-cavity, where $|0,g\rangle$ indicates the state without photon while the atom stays on its ground state, and $|0,e\rangle$ indicates the state without photon while the atom is excited. For a photon of momentum k, the probability amplitudes $u_{k,R}=u_{k,R}(x)$, $u_{k,L}=u_{k,L}(x)$, and w_k, corresponding, respectively, to the right-going photon, the left-going photon, and the excited atom, satisfy the following equations:

$$-iv_g\partial_x u_{k,R} + J\delta(x-a)w_k = Eu_{k,R}, \quad (3)$$

$$iv_g\partial_x u_{k,L} + J\delta(x-a)w_k = Eu_{k,L}, \quad (4)$$

$$J[u_{k,R}(a) + u_{k,L}(a)] = (E-\Omega)w_k. \quad (5)$$

We remark here that $u_{k,R}$ and $u_{k,L}$ are not continuous at the position of atom according to Eqs. (3) and (4). However, it will be showed that $u_{k,R}+u_{k,L}$ is actually continuous in the following part. Applying the spatial differentiation to Eqs. (3) and (4), we obtain the second-order differential equation for the photon amplitude on the waveguide as

$$v_g^2\frac{\partial^2}{\partial x^2}u_{k,L}(x) = \left[\frac{J^2E}{E-\Omega}\delta(x-a) - E^2\right]u_{k,L}(x) - iv_g w_k J\frac{\partial}{\partial x}\delta(x-a) + \frac{J^2E}{E-\Omega}u_{k,R}(a)\delta(x-a), \quad (6)$$

$$v_g^2\frac{\partial^2}{\partial x^2}u_{k,R}(x) = \left[\frac{J^2E}{E-\Omega}\delta(x-a) - E^2\right]u_{k,R}(x) + iv_g w_k J\frac{\partial}{\partial x}\delta(x-a) + \frac{J^2E}{E-\Omega}u_{k,L}(a)\delta(x-a). \quad (7)$$

In Eqs. (6) and (7), the amplitudes of right-going and left-going modes are coupled due to the adding atom.

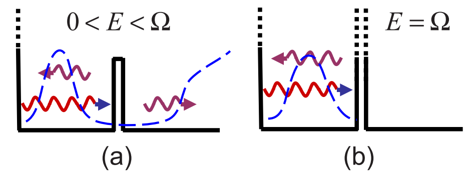

FIG. 2. (Color online) The emergence of the QNMs. The *wavy line* represents the reflective properties of the atom; the *dashed line* shows the typical form of the photon wave function at resonance. (a) When $0<E<\Omega$, the model is equivalent to a leaky cavity. (b) When $E=\Omega$ and $Ea/v_g=n\pi$, the area is well confined and the cavity completely reflects the photon.

To understand the meaning of these amplitudes, we define the sum of the amplitudes $\varphi_k(x)=u_{k,R}(x)+u_{k,L}(x)$ of the EM field and obtain the equivalent Maxwell equation [4] by adding Eqs. (6) and (7) together:

$$\partial_x^2\varphi_k(x) = -\left(\frac{E}{v_g}\right)^2\rho(x)\varphi_k(x), \quad (8)$$

where

$$\rho(x,E) \equiv 1 + \frac{2J^2\delta(x-a)}{E(\Omega-E)} \quad (9)$$

is the singular square of refractive index over the waveguide, which depends on the energy of the system. This energy dependence is illustrated in Fig. 2. Equation (8) can be regarded as a Schrödinger equation where

$$V(x,E) \equiv \rho(x,E) - 1 \quad (10)$$

plays the role of an energy-dependent local potential. The resonance effect occurs at $E=\Omega$ where a δ-type infinite effective potential will emerge. Thus, the atom and the fixed mirror at the termination sandwich a confined 1D space to form a perfect cavity when the cavity photon mode of energy E resonates with the level jumps Ω of the atom. In other words, the resonant atom and cavity gives rise to a spectrum of normal modes. In the case of near resonance, the atom plays the role of a leaky mirror whose rate of leakage is determined by square of refractive index $\rho(x,E)$ which in turn is controlled by the parameters of the atom (such as the level spacing and the coupling constant to the EM field). Actually, the emergence of QNMs in a leaky cavity has been widely investigated for a given refractive index [4] but the alternative we offer here in a 1D continuum bears the spontaneous emergence of QNMs where the effective δ-type potential is given by the localized atom.

Following the routine treatment of QNMs, we assume the field amplitude outside the cavity $(x>a)$ as an outgoing plane wave $\varphi_k(x)=B\exp(ikx)$ and perform an analytical continuation by taking the photonic momentum $k=E/v_g$ as a complex number [4]. The outgoing plane wave here describes the spontaneous radiation of the atom. The corresponding field amplitude $\varphi_k(x)=A\sin(kx)$ inside the emergent cavity $(0<x<a)$ is identical to that of a perfect cavity except that k is complex. Thus, the wave function here reads

$$\varphi_k(x) = \begin{cases} Be^{ikx}, & x > a, \\ A \sin kx, & 0 < x < a. \end{cases} \tag{11}$$

Accordingly, the energy of the QNMs is a complex quantity with its real part being the resonant frequency and its imaginary part the linewidth. These wave functions inside the cavity form a complete set with a definition of general norm [13].

This linewidth determines the leakage rate of the EM field from the cavity [14]. The boundary conditions for the field amplitude at the position of atom $x=a$ implies

$$A \sin\left(\frac{Ea}{v_g}\right) = Be^{iEa/v_g}. \tag{12}$$

Integrating Eq. (8) around $x=a$, the connection boundary condition for the first derivative gives equation

$$\frac{iBE}{v_g}e^{iEa/v_g} - \frac{AE}{v_g}\cos\left(\frac{Ea}{v_g}\right) = \frac{2BJ^2E}{v_g^2(E-\Omega)}e^{iEa/v_g}. \tag{13}$$

By eliminating the coefficients A and B, we obtain the dimensionless equation

$$\tan(\theta) = \left(\frac{\kappa}{W-\theta} + i\right)^{-1}, \tag{14}$$

which determines the complex energy of the QNMs. Here, $\theta = Ea/v_g$, $W = \Omega a/v_g$, and $\kappa = 2J^2 a/v_g^2$ denote a set of dimensionless parameters. To confine the EM field so that an equivalent cavity emerges, the interaction between the EM field and the atom should be sufficiently large, i.e., $1/\kappa \ll 1$. When $\kappa \gg W$, we can expect the solution to around $j\pi (j=0,\pm 1,\pm 2,\ldots)$, which just corresponds to the position of the normal modes of the perfect cavity. Therefore, the solution can be expanded around $j\pi$ with the series of $1/\kappa$ as

$$\theta = j\pi + \alpha_1\kappa^{-1} + \alpha_2\kappa^{-2} + O(\kappa^{-3}). \tag{15}$$

With this expansion, the left-hand side of Eq. (14) is rewritten as

$$\text{l.h.s.} = \alpha_1\kappa^{-1} + \alpha_2\kappa^{-2} + O(\kappa^{-3}), \tag{16}$$

and the right-hand side is rewritten as

$$\text{r.h.s.} = (W - j\pi)\kappa^{-1} - [\alpha_1 + i(W-j\pi)^2]\kappa^{-2} + O(\kappa^{-3}). \tag{17}$$

By substituting the expansion into Eq. (14), we obtain the coefficients α_1 and α_2 as

$$\alpha_1 = W - j\pi, \tag{18}$$

$$\alpha_2 = -(W-j\pi) - i(W-j\pi)^2. \tag{19}$$

Then the QNMs have their energies approximated by the dimensionless complex eigenvalues

$$E_j \simeq \omega_j - i\gamma_j, \tag{20}$$

where for $j=0,\pm 1,\pm 2,\ldots$,

$$\omega_j = \frac{v_g}{a}\left[j\pi + \frac{1}{\kappa^2}(W-j\pi)(\kappa-1)\right] \tag{21}$$

defines the resonant frequencies while

$$\gamma_j = -\frac{v_g}{a\kappa^2}(W-j\pi)^2 \tag{22}$$

are the linewidths of the modes.

The discreteness of the real part ω_j is similar to the case of a perfect cavity with normal modes $\omega_j^0 = jv_g\pi/a$. In both cases, the cavity length a determines the discrete values of the modes. This property results exactly from the so-called shape resonance: when the length a matches the frequency of the EM wave inside the cavity, the reflective wave from the "atom boundary" contains a phase retardation of π relative to the incident wave, destructively interfering the incident wave.

The negativity of the imaginary part γ_j characterizes the decaying of the EM field from inside cavity to the outside. We define the lifetime of the EM field as the inverse $\tau = |\mathrm{Im}\, E_j|^{-1}$, i.e.,

$$\tau = \left(\frac{\kappa}{W-j\pi}\right)^2\frac{a}{v_g}. \tag{23}$$

The lifetime is proportional to J^4 and the inverse square of the detuning, which reaches its maximum at $j=\lfloor W/\pi \rfloor$. This long-living state is selected by the atomic level spacing Ω. The effective refraction index $\rho(x)$ is strongly modified when the system is close to resonance and the confinement of the EM field seems much tighter. In this sense, the near-resonant photonic mode is reflected more fiercely back to the cavity and the leakage is suppressed. Therefore, it is expected that, when the atomic level spacing Ω matches the cavity length a with the value $\Omega = k\pi v_g/a (k\in\mathbb{Z})$, the atom will act like a perfect mirror.

Furthermore, to verify the approximated solution of Eq. (14) above, we solve it numerically and demonstrate the real and the imaginary parts of the solution in the θ-complex plane in Fig. 3(a). The circle points is solved as the cross points of two lines, red dashed line and black dotted line, for the parameters $\kappa=200$ and $W=5$. The analytic solution approximately obtained above is plotted as the dotted dashed blue line, which fits the numerical solution very well in Fig. 3(a). To show this matching effect in details, we plot the curve of the decay rate as a function of the atomic level spacing in Fig. 3(b).

To further describe the QNMs obtained above, we plot the wave function with parameters: $\kappa=200$ and $W=5$ in Fig. 4, which describes the slowest decay process for definite Ω. Here, the resonant frequency $\omega_0 = 3.150\,84 v_g/a$ with the decay rate $\gamma_0 = -8.5\times 10^{-5} v_g/a$. There appears a bound state localized in the intrinsic cavity, however, which diverges exponentially far away outside the cavity. We remark here that the wave function out of the cavity has no meaning. Those characters demonstrate that QNMs are actually resonant states [15]. Due to the long lifetime, it behaves like a bound state even in a large time scales. With these considerations, we conclude that there actually exists a *quasibound state*

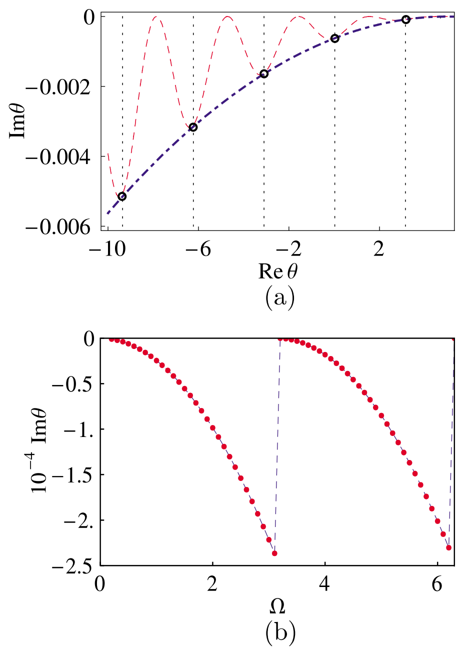

FIG. 3. (Color online) (a) The solution of Eq. (14). The roots of Eq. (14) are marked by the circle. The analytical solution under approximation is shown by a dot-dashed line (blue). The approximation fits numerical point. (b) **Im** θ vs atom frequency. The decay rate reaches its minimum, when $\Omega = n\pi$.

[14,15]. Interestingly, if a single photon is loaded into this cavity, it would take a long time to escape from the cavity. Thus we can propose this artificial fabrication as a storage device for single photon.

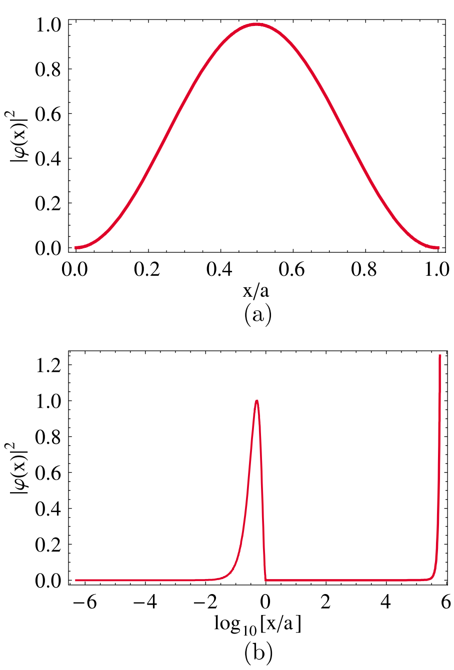

FIG. 4. (Color online) Plot of the wave function of the slowest decay state in real space. The amplitude of the wave function diverges in the infinite far away. The QNM state here is actually a resonant state. (a) Wave function in the intrinsic cavity area. (b) Wave function at the whole area.

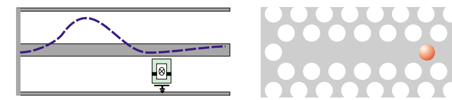

FIG. 5. (Color online) The implementations using, respectively, a superconducting transmission line resonator and a photonic crystal.

To display the quantum nature of the emergent cavity, we study the prominent cavity QED effect—the modification of spontaneous emission rates within the cavity. The radiation of the bounding atom can be controlled by the boundary conditions of the EM field. To this end, we introduce a phenomenological emission rate Γ to the atom, which characterizes the system decay into all other channels of the environment. Then by replacing Ω with $\Omega - i\Gamma$, we formally obtain the decay rate of the atom in the leaky cavity

$$\Gamma_t \simeq \left| \frac{W - j\pi}{\kappa} \right|^2 + \frac{\Gamma}{\kappa} - \frac{\Gamma}{\kappa^2}. \tag{24}$$

In the so-called "strong-coupling regime" where $\kappa \gg 1$, the spontaneous decay is explicitly suppressed. A similar result has been obtained in Ref. [6], when the atom is placed on a node of the EM field.

The emergence of QNMs in the half cavity predicted above can be materialized through some practical laboratory systems. Two such examples, shown in Fig. 5, are the superconducting transmission line resonator with a dc-superconducting quantum interference device (SQUID)-based charge qubit and a photonic crystal with a doped Λ-type three-level atom, in which charge qubit and the three-level atom, respectively, take the roles of the functional mirror.

In the former, the half cavity can be realized through the semi-infinite superconducting transmission line while the two-level system can be realized through the charge qubit with energy eigenstates $|e\rangle$ and $|g\rangle$. The energy-level spacing $\Omega = \sqrt{B_z^2 + B_x^2}$ is defined by the field intensities $B_z = 4E_c(2n_g - 1)$ and $B_x = 2E_J \cos(\pi\Phi_x/\Phi_0)$, where $E_c = e^2/[2(C_g + 2C_J)]$ is the charge energy and $n_g = C_g V_g / 2e$ is the number of charges at the gate. Note that B_z can be controlled by the voltage V_g applied to the gate capacitance C_g whereas B_x can be controlled by the external magnetic flux Φ_x through the SQUID loop. If the effective length of the transmission line resonator is L, the coupling strength between the qubit and the resonator reads $J = e \sin \theta C_g / C_\Sigma \sqrt{\omega/(Lc)}$. Here, c is the capacitance per unit length of the transmission line and ω is the frequency of the quantized EM field. The coupling strength ranges from 5 to 200 MHz and the qubit level spacing is in the range of 5–15 GHz [16]. So long as the coupling between the charge qubit and the line resonator becomes strong, the quasibound state will appear in the transmission line due to the enhancement of reflection by the functional mirror [17]. The coupling strength should reach about several GHz for the observation of the long-life quasibound state.

As for the implementation on a photonic crystal, the doped Λ-type atom in its excited state can decay so fast that we need resort to the stimulated Raman-scattering technology, illustrated in Fig. 1(c). The Hamiltonian here reads

$$H = \omega_1|e\rangle\langle e| + (\omega_1 - \omega_2)|a\rangle\langle a| + \omega b^\dagger b + g(b^\dagger|g\rangle\langle e| + \text{H.c.}) + G(e^{i\omega_c t}|a\rangle\langle e| + \text{H.c.}). \tag{25}$$

With a coupling constant G and a detuning $\Delta = \omega_1 - \omega_c$, a strong drive field is applied to couple the metastable state $|a\rangle$ (with less decay rate) and the excited state $|e\rangle$ with much larger decay rate. The ground state $|a\rangle$ and the excited state $|e\rangle$ are coupled by the EM field $b^\dagger$ inside cavity with coupling constant g and the same detuning $\Delta = \omega_2 - \omega$. In the case with large detuning, an effective tunable coupling $J = -gG/2\Delta$ is induced between EM field, and the ground state $|g\rangle$ and the metastable state $|a\rangle$. It is observed from Eq. (23) that the decay time for single photon in the cavity is proportional to J^4. This high sensitivity to the coupling strength makes it possible to control the leakage by solely adjusting the intensity of the drive field. Experimentally, we study two experimental realizations of the intrinsic cavity based on superconductivity circuit QED system and defected photonic crystal. State (2) is actually the eigenstate of the system. The initial state with a light pulse and a separated ground-state atom is the superposition of these eigenstates. The dynamical evolution of the initial state goes beyond the study in this paper. However, we can expect that the effect of destructive interference can be revealed by detecting the output signals on the right side of the atom. Most recently, an experimental dynamic manipulation of the cavity quality factor was investigated in Ref. [18] based on photonic crystal, where a pump is explored to control the refractive index of waveguide and a probe light is used to check the Q factor of the defected cavity.

In conclusion, we conceive the architecture of a quantum device for coherent manipulation at the single-photon level. An intriguing phenomenon is predicted in which an atom inside a 1D half cavity can act as an end mirror to close the half cavity. This atomic mirror has its reflection coefficient tunable through its level spacing and its coupling to the cavity field, for which the cavity system can be regarded as a two-end resonator with a continuously tunable leakage. As a result, there forms naturally a set of quasinormal modes in the 1D continuum for the single-photon emission process. We also investigate the influence of the quantum dynamics of the atom due to the back action of emerging two-end cavity. Physically, we propose the implementations based on a dc-SQUID circuit and a defected line resonator embedded in a photonic crystal.

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