

# Wave-packet transmission of Bloch electrons manipulated by magnetic field

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We study the phenomenon of wave-packet revivals of Bloch electrons and explore how to control them by a magnetic field for quantum-information transfer. It is shown that the single electron system can be modulated into a linear dispersion regime by the “quantized” flux and then an electronic wave packet with the components localized in this regime can be transferred without spreading. This feature can be utilized to perform the high-fidelity transfer of quantum information encoded in the polarization of the spin. Beyond the linear approximation, the relocalization and self-interference occur as the interesting phenomena of quantum coherence.

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## I. INTRODUCTION

Most recently, many theoretical investigations about quantum-information transfer (QIT) based on quantum spin systems are carried out in order to implement scalable quantum computation [1–13]. Here, the quantum spin system usually behaves as a quantum data bus to integrate many qubits. These investigations mainly aim at transferring the quantum state through a solid-state data bus with minimal spatial and dynamical control over the on-chip interactions between qubits. In this paper we will pay attention to a fundamental aspect of QIT and generally study the wave-packet spreading and revival of Bloch electrons in one-dimensional lattice systems.

For the problems of wave-packet evolution, we can cast back for much earlier investigations by Schrödinger and others about the quantum-mechanical descriptions of localization of macroscopic objects [14]. They demonstrated that a class of wave packets (now we call them coherent and squeezed states) of the harmonic oscillator can keep their shapes during propagation and their centers of mass (CM) follow a classical trajectory. As a semiclassical solution of the Schrödinger equation, a superposition of the much higher excitation states with an almost-homogeneous spectrum form a coherent-state-type wave packet in the Coulomb potential, which can show the phenomena of nonspreading evolution and self-interference on classical orbits [15]. This prediction has been demonstrated in the experiment involving the laser-induced excitation of atomic Rydberg wave packets [16,17].

We can refer such nonspreading wave-packet evolution with a complete autocorrelation [14] as a perfect QIT if we could encode the quantum information in the spin polarization of an electron. The investigation in this paper is motivated by our recent explorations about the QIT based on the quantum system possessing a commensurate structure of energy spectrum matched with a symmetry (SMS), which en-

sure a perfect QIT both in one- and higher-dimensional cases [3,18]. Actually the almost-homogeneous spectrum for the coherent-state-type wave packet just satisfies the condition of SMS. In particular the nonspreading transfer of a zero-momentum wave packet is attractive for the task of quantum information transmission since a static superposition can behave as a quantum storage. This is very similar to the scheme of the quantum storage of a photon based on an atomic ensemble where two stored photonic wave packets localized in the same position with different polarizations can function to decode the information of a qubit [19,20].

This paper will focus on a realistic, but simplest, Bloch electron system (see Fig. 1) in a magnetic field where the on-site Coulomb interactions are ignored. In this sense the spin polarization is always conserved during the time evolution of an arbitrary state and then quantum information encoded in the spin polarization of an electron can be well protected. Thus the locality of electron wave packet becomes a crucial element to maximize the fidelity of QIT. We show that, by the “quantized” flux threading the ring lattice, the effective dispersion relation of a Bloch electron can be modulated into a linear dispersion regime that possesses SMS structure, and then an electronic wave packet with the components localized in this linear regime can be transferred without changes of its shape. This feature can be utilized to perform the high-fidelity QIT encoded in the polarization of the spin. The phenomena of wave packet revivals and self-interference can also be demonstrated for the cases beyond

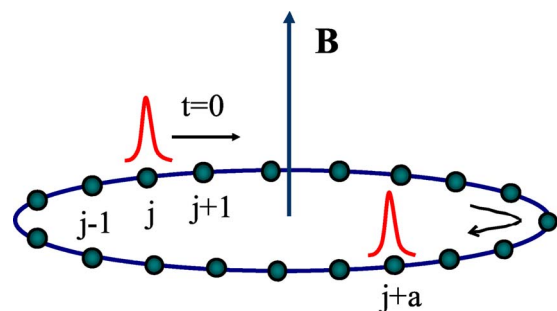


FIG. 1. (Color online) The schematic illustration for the time evolution of a wave packet in a ring threaded by a magnetic field.

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the linear dispersion regime. These interesting quantum coherence effects may suggest a feasible protocol to implement the perfect QIT of Bloch electrons manipulated by the external magnetic field.

## II. MODEL OF FLUX-CONTROLLED BLOCH ELECTRON IN A RING AND ITS LINEARIZATION

In this section, we present the Bloch electron model under consideration, a simple tight-binding model in an external magnetic field. Here, the Coulomb interaction is ignored for simplicity. We restrict our attention to the influence of the applied field on the propagation of the Bloch electrons.

Consider a ring lattice with  $N$  sites threaded by a magnetic field illustrated schematically in Fig. 1. The Hamiltonian of the corresponding tight-binding model,

$$H[\phi] = -J \sum_{j,\sigma} (e^{i2\pi\phi/N} a_{j,\sigma}^\dagger a_{j+1,\sigma} + \text{H.c.}), \quad (1)$$

depends on the magnetic flux  $\phi$  through the ring in the unit of flux quantum  $\Phi_0 = h/e$ . Here,  $a_{j,\sigma}^\dagger$  is the creation operator of Bloch electron at the  $j$ th site with spin  $\sigma = \uparrow, \downarrow$ . The flux  $\phi$  does not exert force on the Bloch electrons, but can change the local phase of its wave function due to the Aharonov-Bohm (AB) effect. Note that the interaction between the field and electrons is independent of the intrinsic degree of freedom spin. This will be crucial to employ such kind of setup to transfer quantum information encoded in the polarization of the spin. Because of the AB effect, the role of the magnetic flux cannot be removed trivially.

Now we consider the evolution of the GWP,

$$|\psi_\sigma(k_0, N_A)\rangle = \frac{1}{\sqrt{\Omega_1}} \sum_j e^{(\alpha^2/2)(j-N_A)^2} e^{ik_0j} |j\rangle_\sigma \quad (2)$$

with the momentum  $k_0$ , where  $|j\rangle = a_{j,\sigma}^\dagger |0\rangle$ , the half width of the wave packet,

$$2\sqrt{\ln 2/\alpha} \ll N, \quad (3)$$

and the normalization factor  $\Omega_1 = \sum_j \exp[-\alpha^2(j-N_A)^2]$ . The limitation for the width of the GWP ensures the locality of the state and avoids the overlap between the head and the tail of the wave packet. We will see that as time evolves, the head and tail do meet in certain situations and the interference phenomenon occurs.

In the following, we will show that the appropriate magnetic flux can ensure the transfer of the wave packet without spreading. The well-known Bloch dispersion relation

$$\varepsilon_k = -2J \cos\left(k + \frac{2\pi\phi}{N}\right), \quad (4)$$

where

$$k = \frac{2\pi l}{N} \quad (l = 1, \dots, N), \quad (5)$$

can be obtained through the Fourier transformation

$$a_{k,\sigma}^\dagger = \frac{1}{\sqrt{N}} \sum_j e^{ikj} a_{j,\sigma}^\dagger, \quad (6)$$

which can be employed to diagonalize  $H[\phi]$  as

$$H[\phi] = \sum_{k,\sigma} \varepsilon_k n_{k,\sigma}, \quad (7)$$

with the number operator  $n_{k,\sigma} = a_{k,\sigma}^\dagger a_{k,\sigma}$ .

It is observed that, when we tune the flux  $\phi$  into each discrete value

$$\phi = \phi_n \equiv \left(\frac{1}{2}n + \frac{1}{4}\right)N, \quad (8)$$

for  $n=0, 1, 2, \dots$ , there is a linear dispersion regime with momenta  $k$  around zero, i.e.,  $\varepsilon_k \sim k$ . For the wave packets as a superposition of those eigenstates with momenta just in this region, the effective Hamiltonian becomes

$$H_{eff} = vp, \quad (9)$$

which is of the ‘‘ultrarelativistic’’ type with the effective ‘‘light velocity’’  $v = (-1)^n 2J$  and

$$p = \sum_{k,\sigma} ka_{k,\sigma}^\dagger a_{k,\sigma} \quad (10)$$

is the Bloch momentum operator. Obviously, this can directly result in a nonspreading wave-packet transmission in the linear dispersion regime. Figure 3 illustrates how the ‘‘quantized’’ magnetic flux  $\phi = \phi_n$  where  $n=0, 1, 2, \dots$ , can speed up the zero-momentum Gaussian wave packet (GWP)  $|\psi(0, N_A)\rangle$  centered at site  $N_A$  with the width  $1/\alpha$ . The details of it will be given in the next section.

Actually, with the linearized Hamiltonian  $H_{eff}$ , the time evolutions of some states can be described as a spatial translation by the evolution operator

$$U(t) = \exp(-ipvt) \equiv T(vt), \quad (11)$$

with a displacement  $x=vt$ . Here the translational operator is defined by

$$T(x_0)|j\rangle_\sigma = |j+x_0\rangle_\sigma \quad (12)$$

for arbitrary  $|j\rangle_\sigma$ .

For small  $\alpha$ ,  $|\psi(0, N_A)\rangle_\sigma$  is a GWP of width  $\alpha$  around  $k=0$  in  $k$  space, and then the wave function at instance  $t$  is a translational GWP,

$$|\Phi(t)\rangle_\sigma = T(vt)|\psi(0, N_A)\rangle_\sigma = e^{i\varphi} |\psi(0, N_c(t))\rangle_\sigma, \quad (13)$$

which is centered at  $N_c(t) = N_A + vt$ . The overall phase factor  $e^{i\varphi}$  has no effect on the final result. It is seen that the wave packet moves with velocity  $v$ . Since all the wave functions satisfy the periodic boundary condition

$$|\psi(0, N+j)\rangle_\sigma = |\psi(0, j)\rangle_\sigma, \quad (14)$$

we have

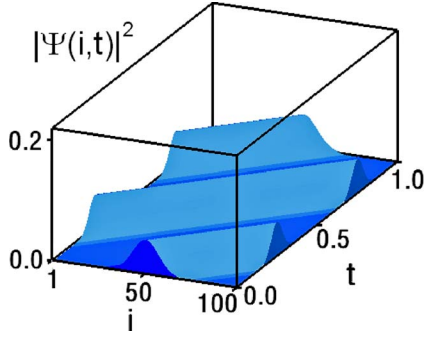


FIG. 2. (Color online) Numerical simulation of the time evolution of a zero-momentum Gaussian wave packet (GWP) with  $\alpha = 0.1$  in position-space  $i$  of the 100-site ring with  $\phi = N/4$ . The time  $t$  is in the unit of  $100/J$ .

$$\begin{cases} \langle \sigma | j | \Phi(t) \rangle_{\sigma} = \langle \sigma | j - N | \Phi(t) \rangle_{\sigma} & \text{for } j > N, \\ \langle \sigma | j | \Phi(t) \rangle_{\sigma} = \langle \sigma | j + N | \Phi(t) \rangle_{\sigma} & \text{for } j < 1. \end{cases} \quad (15)$$

Obviously, the wave packet moves along the ring keeping the initial shape without any spreading as illustrated in Fig. 2 schematically.

### III. NONSPREADING WAVE-PACKET EVOLUTION AND SOLID-STATE FLYING QUBIT

To analyze the cyclic motion of Bloch electrons, the numerical simulation is performed for a zero-momentum GWP with  $\alpha = 0.1$  in the system of  $N = 100$  and  $\phi = N/4 = 25$ . The simulated time evolution of the wave packet is plotted in Fig. 3. For the cases with different values of  $\phi$  and  $\alpha$ , the autocorrelation functions

$$|A(t)| = \sum_{\sigma} |\langle \sigma | \Phi(t) \psi(0, N_A) \rangle_{\sigma}|, \quad (16)$$

which can be used to describe the properties of the electron propagation, are investigated numerically. The results for  $\phi = 20, 25, 33$  and  $\alpha = 0.1, 0.3$  are plotted in Figs. 3(a) and 3(b), which show that a zero-momentum GWP with small  $\alpha$  can be transferred without spreading when  $\phi$  are around each  $\phi_n$ .

Meanwhile, the flux threading the ring can control the shape and destination of the final wave packet. It is observed from the above analysis that the flux plays an important role for manipulating the nonspreading wave packet. Actually, such a phenomenon can also be understood by the following transformation of the basis vectors of Hilbert space. The existence of the flux is equivalent to adding a speed to boost the zero-momentum wave packet since the magnetic flux provides an extra phase to the basis in the position space, i.e.,

$$|j\rangle_{\sigma} \rightarrow |\psi(\phi)\rangle_{\sigma} = e^{2\pi i \phi j / N} |j\rangle_{\sigma}. \quad (17)$$

In other words, a GWP with small  $\alpha$  and momentum  $k_0 = \pi(2n+1)/2$  can be transferred along the ring without spreading approximately. We will also demonstrate this in the last section about open chain systems.

In the above studies, the spin state of the Bloch electron is a conserved quantity that cannot be influenced during the propagation no matter how the spatial shape of the wave

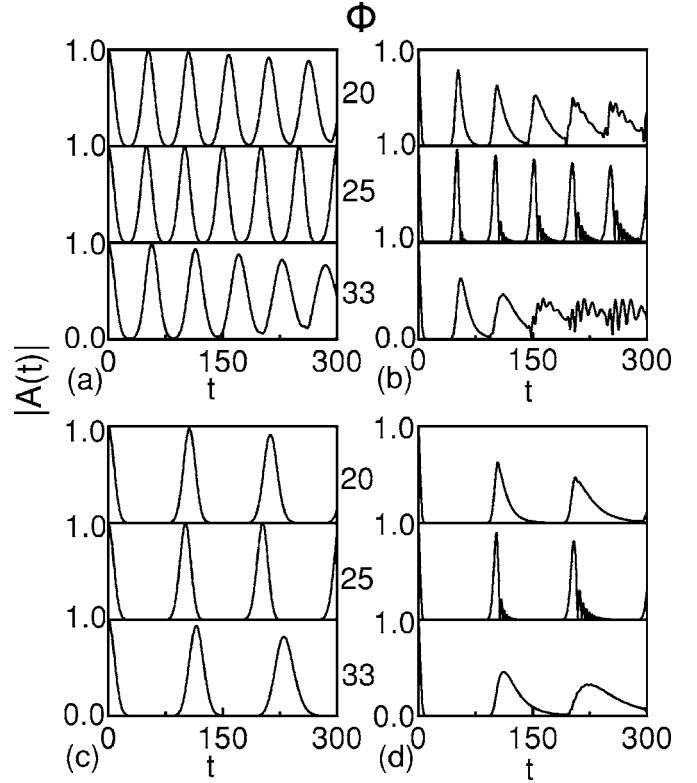


FIG. 3. Numerical simulation of the autocorrelation functions  $|A(t)|$  of the zero-momentum GWPs (or  $k_0 = 2\pi\phi/N$  for the system without external field) with  $\alpha = 0.1$  (a, c) and  $\alpha = 0.3$  (b, d) in the 100-site ring (a, b) and chain (c, d) with  $\phi = 20, 25$ , and  $33$  (or  $\phi = 0$  but the GWP with corresponding speed). It shows that for small  $\alpha$  and  $2\pi\phi/N = \pi/2$  the GWP can be transferred without spreading. The unit of time  $t$  is  $1/J$ .

function changes. From an abstract point of view, the spatial properties of the carrying particle, i.e., the Bloch electron, seems to be irrelevant since only amplitudes and relative phases are used to encode quantum information. However, when the propagation of the Bloch electron can be exploited to transfer the information of qubit, the nonspreading propagation of the carrier is very crucial for the expected high-fidelity of quantum state transfer from one location to another.

With the above consideration we can imagine the electronic wave packets with spin polarization as an analog of photon “flying qubit,” the type-II (polarized) photon qubit where the quantum information was encoded in its two polarization states. We define the solid-state flying qubit, at a single location  $A$  in a quantum wire, as the two Bloch electronic wave packets  $|1\rangle_A = |1\rangle_{N_A}$  and  $|0\rangle_A = |0\rangle_{N_A}$  be encoded as

$$\begin{aligned} |1\rangle_{N_A} &= \frac{1}{\sqrt{\Omega_1}} \sum_j e^{-(\alpha^2/2)(j-N_A)^2} e^{i(\pi/2)j} a_{j,1}^{\dagger} |0\rangle, \\ |0\rangle_{N_A} &= \frac{1}{\sqrt{\Omega_1}} \sum_j e^{-(\alpha^2/2)(j-N_A)^2} e^{i(\pi/2)j} a_{j,1}^{\dagger} |0\rangle. \end{aligned} \quad (18)$$

Because of the intrinsic linearity of the Schrödinger equation, it is self-consistent to encode an arbitrary state

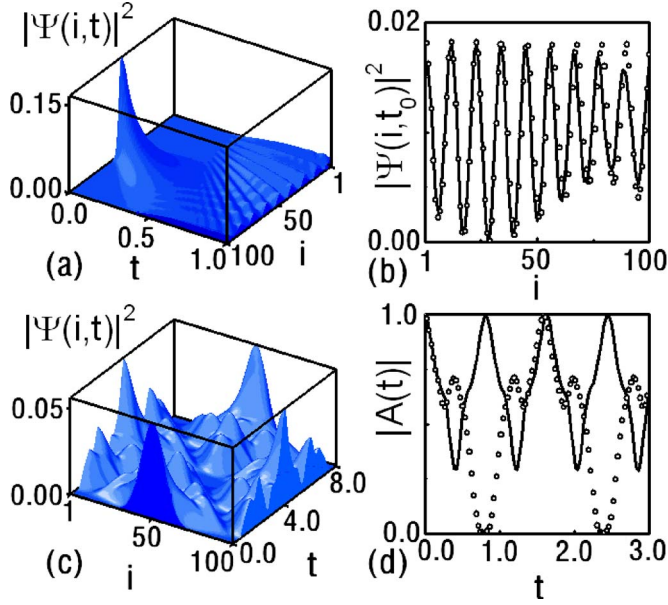


FIG. 4. (Color online) The self-interference (a) and quantum revival (c) phenomena of the GWPs obtained by numerical simulation. (b) Plots of the self-interference fringe for 100-site ring obtained by numerical simulation (solid line) and theoretical analysis (circle) at  $t_0 = \delta\tau = 90/J$ . (d) Plots of the autocorrelation functions,  $|A(t)|$ , for the zero-momentum GWPs with  $\alpha = 0.1$  in 100-site ring (circle) and chain (solid line) systems. The unit of time  $t$  is  $100/J$  in (a), (c), and  $1000/J$  in (d).

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|1\rangle_A + \sin\left(\frac{\theta}{2}\right)e^{i\varphi}|0\rangle_A \quad (19)$$

as

$$|\psi(\theta, \varphi)\rangle_A = \frac{1}{\sqrt{\Omega_1}} \sum_j e^{-(\alpha^2/2)(j-N_A)^2} e^{i(\pi/2)j} a_{j,\sigma}^\dagger |0\rangle \quad (20)$$

and then it is transferred to another place  $B$  with a very high fidelity due to the feature of nonspreading propagation of Bloch electron.

#### IV. SELF-INTERFERENCE AND REVIVAL OF SPREADING WAVE PACKET

We now turn our attention to the problem of nonspreading propagation of Bloch electronic wave packets beyond the linear dispersion regime. We consider a zero-momentum GWP in an external field with  $\phi$  far from  $\phi_n$  (or a GWP with small momentum  $k_0$  but  $\phi=0$ ). Because of the nonlinear dispersion relation, such kind of wave packet spreads while its center is moving. It is clear that, when the head of the wave packet catches up with its tail, quantum interference phenomena set in.

In order to demonstrate this phenomenon, numerical simulation is performed for a GWP with  $\alpha=0.3$  and  $k_0 = 0.05\pi$  (or a zero-momentum GWP with the external magnetic flux  $\phi = k_0 N / 2\pi$ ) in the 100-site ring system. The time evolution of the GWP obtained by numerical simulation is plotted in Fig. 4(a). The interference fringe appears when the

GWP spreads, which demonstrates the self-interference phenomenon. The profile of the fringe can be estimated analytically as follows.

Consider a GWP  $|\psi(k_0, N_A)\rangle$  at  $t=0$  in the coordinate space. When  $\phi=0$ , the Hamiltonian  $H[0]$  can be approximately written as

$$H_{eff} = -J \sum_{k,\sigma} \varepsilon_k a_{k,\sigma}^\dagger a_{k,\sigma} \quad (21)$$

for  $\varepsilon_k \sim (2-k^2)$ . On the other hand,  $|\psi(k_0, N_A)\rangle_\sigma$  is also a GWP around  $k_0$  in the  $k$  space. Then for GWP with small  $k_0$  it will evolve into a GWP,

$$|\Phi(t)\rangle_\sigma = A_2 \sum_j e^{i\varphi(j,t)} e^{-(\alpha'^2/2)(j-N_c)^2} |j\rangle_\sigma \quad (22)$$

with the spreading width

$$\alpha' = \alpha / \sqrt{1 + 4\alpha^4 J^2 t^2}, \quad (23)$$

centered at  $N_c = N_A + 2Jk_0 t$ , where  $A_2$  is the normalization factor, and

$$\varphi(j,t) = k_0 j + 2Jt - Jk_0^2 t + Jt(j - N_c)^2 \alpha^2 \alpha'^2 \quad (24)$$

is the time-dependent phase, i.e., the momentum of the moving GWP. Such kind of wave packet spreads while its center is still moving. Since all the wave functions satisfy the periodic boundary condition  $|N+j\rangle = |j\rangle$ , at a certain instant  $t$ , there is an overlap between the head and tail parts of the wave packet and then the quantum interference phenomenon occurs. In other words, when the GWP spreads over the circumference of the ring, we need to consider the virtual superposition of the ‘‘head’’ and ‘‘tail.’’

Since the widely spreading GWP can wind the ring many times, the virtual superposition can be considered as the renormalized wave function

$${}_\sigma \langle j | \Phi_{vs}(t) \rangle_\sigma = \frac{1}{\sqrt{\Omega_{vs}}} {}_\sigma \langle j \pm lN | \Phi(t) \rangle_\sigma, \quad (25)$$

where

$$\Omega_{vs} = \sum_j |{}_\sigma \langle j | \Phi_{vs}(t) \rangle_\sigma|^2$$

is a normalization factor. For small  $t = \delta\tau$ , one can only take the summation over  $l=0,1$  as an approximation, which results in spatial interference fringe

$$|{}_\sigma \langle j | \Phi_{vs}(\delta\tau) \rangle_\sigma|^2 = \frac{1}{\sqrt{\Omega_{vs}}} |{}_\sigma \langle j | \Phi(\delta\tau) \rangle_\sigma|^2 \times [1 + c^2 + 2c \cos(Kj + \varphi_0)] \quad (26)$$

with the effective wave vector

$$K = 2NJ\alpha^2 \alpha'^2 \delta\tau \quad (27)$$

and initial phase

$$\varphi_0 = K(N/2 - N_c) + k_0 N. \quad (28)$$

Here,  $|{}_\sigma \langle j | \Phi(\delta\tau) \rangle_\sigma|^2$  and

$$c = \exp[-\alpha'^2 N(j - N_c + N/2)] \quad (29)$$

only provide the modulation to the fringe. The spatial period

$$\Delta = \left| \frac{2\pi}{K} \right| = \left| \frac{\pi}{NJ\alpha^2\alpha'^2\delta\tau} \right| \quad (30)$$

characterizes the interference fringe.

In Fig. 4(b), the interference fringe at  $\delta\tau=90/J$  obtained by numerical simulation and the analytical approximate result are plotted. It shows that the theoretical analysis is in agreement with the result of numerical simulation.

Now we consider the special case of zero-momentum GWP moving along a lattice without magnetic field. In this case, although the dispersion relation for such kind of GWP is nonlinear, the quantum revival is still possible since the  $k^2$  dispersion also meets the condition of SMS [3,18]. To demonstrate this numerical simulation for the time evolution of a GWP with  $\alpha=0.1$ , a ring of 100 sites is performed and the density probability of the GWP as the function of the position  $i$  and time  $t$  (in the unit of  $100/J$ ) are plotted in Fig. 4(c). It shows that revival occurs after the GWP spreads. According to the theory of SMS, the revival time is  $\tau=2\pi/\Delta E$  in the general cases, where  $\Delta E$  is the greatest common divisor of energy-level spacing between any two eigenstates. Then in the general case, we have

$$\tau = \frac{2}{\pi J}(N+1)^2 \quad (31)$$

for the chain while  $\tau=N^2/(2\pi J)$  for the ring. Now we consider a special case, in which the initial zero-momentum GWP is centered at the middle of the chain. Obviously, the parity of the GWP with respect to the reflection symmetry is even. Thus expansion coefficients of the GWP for all the eigenstates with odd parity are all zero, which means that the effective levels driving the GWP are only the half set. Thus this fact results in a particular revival time

$$\tau = \frac{1}{4\pi J}(N+1)^2. \quad (32)$$

To verify the above analysis, the autocorrelation functions are also calculated for the initial zero-momentum GWP with  $\alpha=0.1$  and  $N_A=(N+1)/2$  in the ring of 100 sites and chain. The results are plotted in Fig. 4(d), which shows that the revival time is well in agreement with the analytical estimation.

## V. WAVE PACKET DYNAMICS IN THE OPEN CHAIN

In this section, we consider the dynamics of a GWP in an open chain in the absence of external field. The single-particle spectrum is

$$\varepsilon_k = -v \cos k \quad (33)$$

where  $v=2J$ , and the corresponding eigenvectors are

$$|\psi_{k,\sigma}\rangle = a_{k,\sigma}^\dagger |0\rangle = \sum_{j=1}^N \sqrt{\frac{2}{N+1}} \sin(kj) |j\rangle_\sigma, \quad (34)$$

where

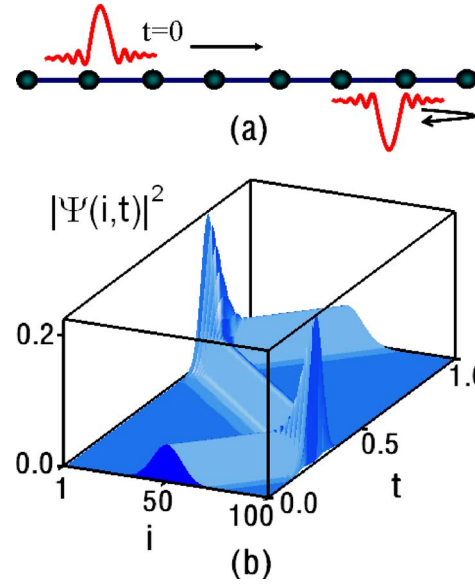


FIG. 5. (Color online) (a) The schematic illustration for the time evolution of a GWP in a chain. The  $\pi$  shift occurs at the boundary. (b) Numerical simulation of the time evolution of a moving GWP with  $\alpha=0.1$  and  $k_0=2\pi\phi/N=\pi/2$  in the 100-site chain. The unit of time  $t$  is  $100/J$ .

$$k = \frac{\pi l}{N+1} \quad (l=1, \dots, N) \quad (35)$$

can be regarded as a pseudomomentum. Nevertheless, The GWP  $|\psi_\sigma(k_0, N_A)\rangle$  located at  $N_A$  with momentum  $k_0 \sim \pi/2$  at  $t=0$  will also evolve into

$$|\Phi_{k,\sigma}(t)\rangle = |\psi_{k,\sigma}(k_0, N_A + vt)\rangle. \quad (36)$$

Since all the eigenvectors satisfy the open boundary condition, we have

$$|\psi_{k,\sigma}(k_0, N_A + vt)\rangle = -|\psi_{k,\sigma}(k_0, 2N+2 - N_A - vt)\rangle \quad (37)$$

for  $N_A + vt > N$ . It indicates that the wave packet reflects at the boundaries with “ $\pi$ -phase shift.” Then the wave packet bounds back and forth along the chain as illustrated in Fig. 5(a) schematically.

Similarly, under the transformation

$$e^{ik_0 j} |j\rangle_\sigma \rightarrow |j\rangle_\sigma, \quad (38)$$

the propagation of a moving GWP with  $k_0=2\pi\phi/N$  is equivalent to that of a zero-momentum GWP in the system with extra phase  $\exp(i2\pi\phi/N)$  on the hopping term. Numerical simulation for the time evolution of a GWP with  $\alpha=0.1$  and  $k_0=\pi/2$  in a chain of  $N=100$  is plotted in Fig. 5(b). The autocorrelation functions  $|A(t)|$  are also calculated for  $\alpha=0.1, 0.3$  and  $k_0=2\pi\phi/N$ ,  $\phi=20, 25, 33$ , which are plotted in Figs. 3(c) and 3(d). It shows that a GWP with small  $\alpha$  and momentum  $k_0=\pi(2n+1)/2$  can be transferred along the chain without spreading approximately. From the autocorrelation functions for rings and chains, it is easy to find that the period of the revivals of GWP in a ring is approximately the half of that in a chain. This is in agreement with the analytical results that the period is  $\tau=(N+1)/J$  for the chain and

$\tau=N/(2J)$  for the ring. Comparing the revival times for GWP with linear and nonlinear dispersion relation, we have the conclusion that the former is suitable for implementing the fast QIT in the solid.

## VI. SUMMARY

In summary, the quantum transmission of a Bloch electron in the one-dimensional lattice is studied by theoretical analysis and numerical simulation. It is found that a zero-momentum GWP can be transferred without spreading approximately if an optimal magnetic flux is applied. This feature can be employed to perform the high-fidelity QIT encoded in the polarization of the Bloch electrons. Mean-

while, beyond such optimal range of the field, the time evolution of the GWP is also investigated in the nonlinear dispersion regime. The interesting quantum coherence effects found in this paper, such as the wave packet revivals and self-interference, can motivate a feasible protocol based on the practical systems to implement the perfect QIT of Bloch electrons controlled by the external magnetic field.

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