Nonlinear mechanism of charge-qubit decoherence in a lossy cavity: Quasi-normal-mode approach

Y. B. Gao,[1,2] Y. D. Wang,[2] and C. P. Sun[2,3]
[1]*Department of Applied Physics, Beijing University of Technology, Beijing 100022, China*
[2]*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China*
[3]*Department of Physics, Nankai University, Tianjin 300071, China*
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From the viewpoint of quasinormal modes, we describe a decoherence mechanism of charge qubit of Josephson junctions (JJ) in a lossy microcavity, which can appear in a realistic experiment for quantum computation based on a JJ qubit. We show that nonlinear coupling of a charge qubit to the quantum cavity field can result in additional dissipation of the resonant mode due to the effective interaction between those nonresonant modes and the resonant mode, which is induced by the charge qubit itself. We calculate the characteristic time of the decoherence by making use of the system plus bath method.

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I. INTRODUCTION

The superposition principle is most basically governing the quantum world [1]. It is also the foundation of quantum information technology. The ideal coherent superposition state can only be preserved in the quantum world isolated from external influences. However, the influence of the surrounding environment can never be blocked off completely. Now the coherence is an essential requirement for quantum information and the decoherence will result in errors which reduce the power of quantum computation and quantum communication or even destroys it. The quantum decoherence has become the biggest obstacle to implementation of quantum computation. In practice, we need a qubit with long decoherence time and a longer lifetime medium to implement a quantum logic gate. To overcome quantum decoherence, one should know dynamic details theoretically and experimentally in various physical systems including all kinds of qubits.

Recently in solid state based quantum computation, Josephson junction (JJ) qubits (charge qubit, flux qubit [2] or their hybridizations) have demonstrated large potential as candidates for scalable quantum computation. On the one hand, the Rabi oscillation in a Cooper-pair box (charge qubit) [3], the existence of two-qubit states [4] and the entanglement between a flux qubit and a superconducting quantum interference device (SQUID) [5] have been realized experimentally. Up to now, the decoherence time of a JJ qubit has been the order of 5 μs [6]. On the other hand, to implement quantum computation, one should integrate many qubits to form a quantum coherent network. To this end, a longer lifetime medium is required to transfer quantum information among these qubits in the network. Some investigations have shown that the quantized field in a microwave cavity, whose lifetime is of order 1 ms, might be a good candidate [7]. For this purpose, the integration of JJ qubit and cavity QED has become a focus in exploring the JJ qubit based quantum computing [8–13].

In spite of these exciting advances, the relatively short coherence time is still a problem in implementing the JJ based quantum computing in large scale and the mechanism of decoherence of JJ qubit is not very clear, especially in the presence of external field. The fluctuation of background charge is a well-known source of quantum decoherence [14], but it is not the unique one. For a real dc-SQUID, the fluctuations of gate voltage and magnetic flux produced by the screening current circulating around the dc-SQUID may also cause additional quantum decoherence in the charge-qubit–cavity system. To bring out more clearly the physical mechanism of nonlinear decoherence described in this paper, we can avoid the effect of screening current in the physical case of the dc-SQUID screening parameter $\beta_m = 2LI_c/\Phi_0 \leqslant 2/\pi$ [15]. Here L is the loop inductance of dc-SQUID and I_c the critical current of Josephson junction and Φ_0 flux quanta. To simplify the analysis of our paper and demonstrate more clearly the physics of our nonlinear decoherence mechanism, we do not consider fluctuations of the gate voltage and other sources of decoherence. Most current investigations for integrating and manipulating various kinds of JJ qubits mainly concern the idealized cavity without damping. Naturally one can question what will happen if we place a qubit in a nonideal cavity. That is our direct motivation for this paper. Here we will deal with quantized field in a lossy cavity with the quasi-normal-mode approach [17–20]. In this treatment, cavity modes in a lossy cavity are divided into a single resonant mode and other nonresonant multimodes. Due to the nonlinear coupling of the charge qubit to the cavity field, the effective interaction between those nonresonant modes and the resonant mode causes an additional dissipation of the resonant mode. This is just the mechanism of quantum decoherence for the charge qubits in a lossy cavity.

The paper is organized in the following sections. With the quasi-normal-mode approach, the model in Sec. II describes a charge qubit interacting with a lossy cavity. In Sec. III, we demonstrate how the nonlinear coupling of a charge qubit to cavity field induces the effective interaction between resonant mode and nonresonant ones. In Sec. IV, we find that the nonlinear coupling leads to energy dissipation of resonant mode of cavity field which is linked to quantum decoherence of a charge qubit in a lossy cavity in Sec. V.

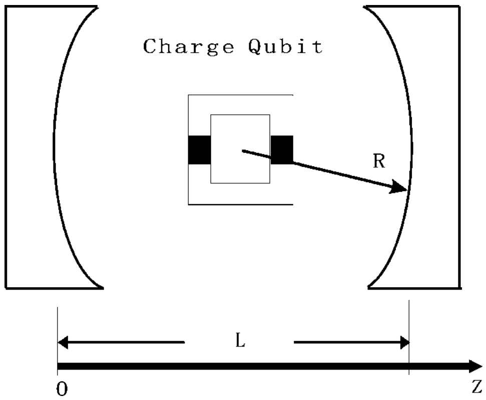

FIG. 1. Schematic of the charge qubit-cavity system. Superconducting microwave cavity with parameters $R=2.55$ mm and $L=0.5$ cm.

II. MODEL: CHARGE QUBIT COUPLED TO THE LOSSY CAVITY

In this paper, we consider a single-mode quantum field in a lossy cavity with frequency $\omega \sim 30$ GHz (typically in the microwave domain) and quality factor $Q \sim 10^6$ coupled to a charge qubit, in which the charging energy $E_C \sim 122$ μeV and the Josephson coupling energy $E_J \sim 34$ μeV [14]. The charge qubit considered in this paper is a dc-SQUID consisting of two identical Josephson junctions enclosed by a superconducting loop. It is located in a lossy cavity which is produced by a semi-transparent mirror. A similar case is discussed for quantum dissipation of semiconductor exciton in a lossy cavity [16]. In this paper, we can describe the magnetic field in a lossy cavity with the quasimode approach [17–20]. And we can divide cavity modes into two parts: a single resonant mode and other nonresonant ones.

In this case, the magnetic flux threading the dc-SQUID is generated by magnetic field $\mathbf{B}=\mathbf{B}_c+\mathbf{B}_q$, which consists of external classical magnetic field $\mathbf{B}_c$ and quantum cavity field $\mathbf{B}_q$ [9]. To demonstrate the physics of our result, we do not consider the effect of the screening current. Similarly the total magnetic flux threading dc-SQUID could be a sum of two parts $\Phi=\Phi_c+\Phi_q$, where $\Phi_c=\int \mathbf{B}_c \cdot d\mathbf{S}$ is the external classical flux threading the dc-SQUID, $\Phi_q=\int \mathbf{B}_q \cdot d\mathbf{S}$ the cavity-induced quantum flux through the dc-SQUID and S the area bounded by the dc-SQUID.

For an ideal cavity, we can describe a cavity field with a set of normal modes with different frequencies. Similarly as in a laser theory [18], we adopt a quasimode approach to describe the quantum cavity field $\mathbf{B}_q$ in terms of a discrete set of quasimodes of the lossy cavity, each of which (resonant mode) has a finite quality factor Q and there exist many modes of the universe (nonresonant modes) corresponding to each quasimode. Then the quantum cavity field $\mathbf{B}_q$ could be composed of resonant mode of cavity field $\mathbf{B}_r$ and nonresonant modes of cavity field $\mathbf{B}_{nr}$, $\mathbf{B}_q=\mathbf{B}_r+\mathbf{B}_{nr}$. In this paper, we assume that the lossy cavity of our interest contains only one quasimode.

Figure 1 shows that the dc-SQUID lies in the x-z plane and the direction of the quantum cavity field is perpendicular to the plane,

$$B_{q,y}(z) = B_r(z) + B_{nr}(z)$$

and

$$B_r(z) = -i\left(\frac{\hbar\omega}{\varepsilon_0 V c^2}\right)^{1/2} \sin\left(\frac{\omega}{c}z\right)(a-a^\dagger),$$

$$B_{nr}(z) = -i\sum_j M_j\left(\frac{\hbar\omega_j}{\varepsilon_0 V_j c^2}\right)^{1/2} \sin\left(\frac{\omega_j}{c}(z-L)\right)(a_j-a_j^\dagger), \tag{1}$$

where $a^\dagger(a)$ and $a_j^\dagger(a_j)$ are the creation (annihilation) operators corresponding to single resonant mode of frequency ω and nonresonant modes of frequencies ω_j, V (V_j) the electromagnetic mode volume corresponding to resonant mode and nonresonant mode, respectively, and L the length of the cavity. The constant M_j in Eq. (1) is proportional to a Lorentzian [19]

$$M_j = \frac{\Lambda\frac{\gamma}{2}}{\sqrt{(\omega_j-\omega)^2+\left(\frac{\gamma}{2}\right)^2}}, \tag{2}$$

where γ is the decay rate of a quasimode of cavity, Λ the bandwidth associated with the cavity wall transparency and ω the central frequency of the resonant mode of cavity. It is obvious that M_j will acquire the maximum value when the frequency of nonresonant mode ω_j is very close to the central frequency of the resonant mode ω. In our investigation, the lossy cavity contained only one quasimode.

The Hamiltonian for a charge qubit (dc-SQUID) can be written as in Ref. [21],

$$H = 4E_C\left(n_g-\frac{1}{2}\right)\sigma_z - E_J\cos\left(\pi\frac{\Phi}{\Phi_0}\right)\sigma_x, \tag{3}$$

where E_C is the charging energy and E_J the Josephson coupling energy, $E_C \gg E_J$ for a charge qubit. Φ is the magnetic flux generated by controlled classical magnetic field and quantum cavity field and $\Phi_0=h/2e$ the flux quanta. As a control parameter, the dimensionless gate charge $n_g=C_gV_g/2e$ is determined by the gate voltage V_g applied on the gate capacitance C_g. Quasispin operators

$$\sigma_z = |0\rangle_q\langle 0|_q - |1\rangle_q\langle 1|_q, \sigma_x = |0\rangle_q\langle 1|_q + |1\rangle_q\langle 0|_q$$

are defined in the charge qubit basis ($|0\rangle_q$ and $|1\rangle_q$).

In Fig. 1, the dc-SQUID is located at the position of the antinode of standing wave field in cavity, i.e., $z=L/2$. Then corresponding to the field decomposition $\mathbf{B}_q=\mathbf{B}_r+\mathbf{B}_{nr}$ the magnetic flux $\Phi_q=\Phi_r+\Phi_{nr}$ enclosed by the dc-SQUID is explicitly given by

$$\Phi_r = -iS\left(\frac{\hbar\omega}{\varepsilon_0 V c^2}\right)^{1/2}(a-a^\dagger),$$

$$\Phi_{nr} = -iS\sum_j M_j\left(\frac{\hbar\omega_j}{\varepsilon_0 V_j c^2}\right)^{1/2}\sin\left(-\frac{\omega_j}{c}\frac{L}{2}\right)(a_j - a_j^\dagger).$$

In a straightforward way, we derive the Hamiltonian of the qubit-cavity system from Eq. (3),

$$H = 4E_C\left(n_g - \frac{1}{2}\right)\sigma_z - E_J\cos(\phi_c + \phi_q)\sigma_x + \hbar\omega a^\dagger a + \sum_j \hbar\omega_j a_j^\dagger a_j, \qquad (4)$$

where ϕ_c and ϕ_q are phase and "phase operator" generated by the flux Φ_c and Φ_q, respectively,

$$\phi_c = \frac{\pi\Phi_c}{\Phi_0},$$

$$\phi_q = -i\phi_0(a - a^\dagger) - i\sum_j \phi_j(a_j - a_j^\dagger)$$

and

$$\phi_0 = \frac{\pi S}{\Phi_0}\left(\frac{\hbar\omega}{\varepsilon_0 V c^2}\right)^{1/2},$$

$$\phi_j = M_j\frac{\pi S}{\Phi_0}\left(\frac{\hbar\omega_j}{\varepsilon_0 V_j c^2}\right)^{1/2}\sin\left(-\frac{\omega_j}{c}\frac{L}{2}\right).$$

In the above discussions to achieve the simplified model we have ignored the effect of the screening current. However, if the inductance of the loop of dc-SQUID is not zero, the screening current will induce the additional decoherence. In presence of the screening current, we cannot neglect the difference between the practical magnetic flux Φ threading the dc-SQUID and the external magnetic flux Φ_x. It can be determined by the following equation [15]

$$\phi = \phi_x + \frac{\pi}{2}\beta_m \sin\phi,$$

where $\beta_m = 2LI_c/\Phi_0$ is usually called screening parameter and $\phi = \pi\Phi/\Phi_0$ and $\phi_x = \pi\Phi_x/\Phi_0$. This equation simply shows that the total flux Φ is the sum of the external flux and the induced flux determined only by Φ itself. If the screening parameter β_m is small enough, we can approximately solve this equation in a single value domain with the technique of perturbation recursion up to the second order

$$\phi = \phi_x + \frac{\pi}{2}\beta_m\sin\phi_x + \left(\frac{\pi}{2}\beta_m\right)^2\sin\phi_x\cos\phi_x. \qquad (5)$$

When $\beta_m\pi/2 \ll 1$, i.e., $\beta_m \ll 2/\pi$, we can ignore the effect of the screening current. But the additional nonlinear terms containing $(\phi_x)^2$ at least should also induce the additional nonlinear interaction between the resonant mode and the nonresonant ones. This is the further result in the decoherence of a charge qubit in the lossy cavity. However, to clearly demonstrate the physics of the central results of our paper, we suppose $\beta_m \ll 2/\pi$ and ignore the effect of the screening current. Therefore we do not give much details for this problem.

III. MODE INTERACTION INDUCED BY NONLINEAR COUPLING TO A CAVITY

As shown in Fig. 1, two spherical mirrors form a microwave cavity [7] containing a single mode standing wave field and an external classical magnetic field is also injected into the cavity. In this paper, the geometry of cavity is described by the parameters: the curvature radius $R = 2.55$ mm, the width between two mirrors $L = 0.5$ cm. By some straightforward calculations, we get that the cavity field $B = (\hbar\omega/\epsilon_0 Vc^2)^{1/2} = 7.52\times10^{-11}$ T and $\phi_0 = \pi\Phi_q/\Phi_0 = 1.14\times10^{-5}$. In a low photon number cavity, we find that $\phi_q \ll \phi_c$, thus there is only a weak polynomial nonlinearity in Eq. (4).

To simplify the Hamiltonian in Eq. (4), we expand $\cos(\phi_c + \phi_q)$ in terms of small quantity ϕ_q up to the second order,

$$\cos(\phi_c + \phi_q) = \left(1 - \frac{1}{2}\phi_q^2\right)\cos\phi_c - \phi_q\sin\phi_c. \qquad (6)$$

Obviously we can know that the second order term $\phi_q^2 = (\phi_r + \phi_{nr})^2$ includes the term $\phi_r\phi_{nr}$ which results in the nonlinear coupling between resonant mode and nonresonant modes of cavity field, on which the results of this paper is based. The first order term ϕ_q is linearly dependent of ϕ_r and ϕ_{nr}, which cannot lead to the coupling between resonant mode and nonresonant ones. Therefore the nonlinear coupling in terms of $\phi_q^2 \sim \phi_r\phi_{nr}$ will induce energy dissipation and quantum decoherence of the charge qubit in a lossy cavity simulataneously.

To clearly demonstrate the effect of quantum decoherence of a charge qubit in a lossy cavity we tune the gate voltage V_g such that $n_g = 1/2$ to eliminate the effect of background charge fluctuation up to the linear order. Then the effective Hamiltonian corresponding to a standard quantum measurement model [22] reads

$$H = H^{(0)}|0\rangle\langle 0| + H^{(1)}|1\rangle\langle 1|, \qquad (7)$$

which is diagonal with respect to eigenstates of quasispin operator σ_x, $|0\rangle = |0\rangle_q + |1\rangle_q$ and $|1\rangle = |0\rangle_q - |1\rangle_q$.

As seen in Eq. (6), the second order term $\phi_q^2 \sim \phi_r\phi_{nr}$ results in the interaction $\sim(a_j - a_j^\dagger)(a - a^\dagger)$ between single resonant mode and other nonresonant ones, while the first order term ϕ_q results in the forced terms $(a - a^\dagger)$ and $(a_j - a_j^\dagger)$ in the above Hamiltonian. With the rotating wave approximation (RWA), we can drop down the terms of a^2 $(a^{\dagger 2})$ and $a_j a(a_j^\dagger a^\dagger)$ in $\cos(\phi_c + \phi_q)$ and get an effective Hamiltonian

$$H^{(k)} = H_s^{(k)} + H_I^{(k)} + H_B^{(k)} + N^{(k)}, \qquad (8)$$

where

$$H_s^{(k)} = \hbar\Omega^{(k)}a^\dagger a - i\xi^{(k)}(a - a^\dagger),$$

$$H_I^{(k)} = \sum_j g_j^{(k)}(a_j a^\dagger + a_j^\dagger a),$$

$$H_B^{(k)} = \sum_j \hbar\omega_j a_j^\dagger a_j - i\sum_j \xi_j^{(k)}(a_j - a_j^\dagger)$$

and the parameters in the above equation can be explicitly expressed as

$$N^{(k)} = \frac{(-1)^k}{\hbar}(\phi_0^2 E_J \cos\phi_c - E_J\cos\phi_c),$$

$$\Omega^{(k)} = \omega + \frac{(-1)^k}{\hbar}\phi_0^2 E_J\cos\phi_c,$$

$$g_j^{(k)} = \frac{(-1)^k}{\hbar}\phi_j\phi_0 E_J\cos\phi_c,$$

$$\xi^{(k)} = \frac{(-1)^k}{\hbar}\phi_0 E_J\sin\phi_c,$$

$$\xi_j^{(k)} = \frac{(-1)^k}{\hbar}\phi_j E_J\sin\phi_c \tag{9}$$

for $k=0,1$. Here $H_s^{(k)}$ describes a system with a forced oscillator of frequency $\Omega^{(k)}$, $H_B^{(k)}$ describes a bath of many forced oscillators of frequency ω_js, and $H_I^{(k)}$ describes the coupling of the resonant mode to nonresonant modes. The coupling constant $g_j^{(k)}$ owns a Lorentz type factor, i.e.,

$$g_j^{(k)} \sim \frac{1}{\sqrt{(\omega_j-\omega)^2 + \left(\frac{\gamma}{2}\right)^2}}.$$

It means that the resonant mode of cavity field dominates the strength of the interaction mostly.

IV. QUANTUM DISSIPATION OF RESONANT MODE OF CAVITY FIELD

In this section, we study quantum dissipation of the resonant mode of cavity field. In each component of the Hamiltonian in Eq. (8), $H^{(k)}$ can result in quantum dissipation of the resonant mode. To solve the dynamic equation governed by the effective Hamiltonian $H^{(k)}$, we rewrite the above Hamiltonian into the new form

$$H^{(k)} = \hbar\Omega^{(k)}b^\dagger b + \sum_j \hbar\omega_j b_j^\dagger b_j + \sum_j g_j^{(k)}(b_j b^\dagger + b_j^\dagger b) + \varphi_k \tag{10}$$

by defining a new set of bosonic operators $b(b^\dagger)$ and $b_j(b_j^\dagger)$, which are the displacements of operators a and a_j,

$$b = a + \lambda, \tag{11}$$

$$b_j = a_j + \lambda_j.$$

Here, φ_k is the constant, λ and λ_j are dependent of the forced terms $(a-a^\dagger)$ and $(a-a^\dagger)$ of the effective Hamiltonian $H^{(k)}$ in Eq. (8).

For any coherent state $|\alpha\rangle_a$ and $|\alpha_j\rangle_{a_j}$ defined with respect to annihilation operators a and a_j, we get coherent state $|\alpha\rangle_b$ and $|\alpha_j\rangle_{b_j}$ defined with respect to annihilation operators b and b_j,

$$|\alpha\rangle_b = e^{-\lambda\alpha^*}|\alpha-\lambda\rangle_a,$$

$$|\alpha_j\rangle_{b_j} = e^{-\lambda_j\alpha_j^*}|\alpha_j-\lambda_j\rangle_{a_j}. \tag{12}$$

Obviously we can see that the effective Hamiltonian $H^{(k)}$ in Eq. (10) describes a typical dissipative system of a single-mode boson soaked in a bath of many bosons (we have studied its wave function structure in details [23]). The well-known solutions of Heisenberg equations for the Hamiltonian $H^{(k)}$ is given in Ref. [23],

$$b^{(k)}(t) = u^{(k)}(t)b(0) + \sum_j v_j^{(k)}(t)b_j(0),$$

$$b_j^{(k)}(t) = e^{-i\omega_j t}b_j(0) + u_j^{(k)}(t)b(0) + \sum_s v_{j,s}^{(k)}(t)b_s(0).$$

Where $b^{(k)}(t)$ and $b_j^{(k)}(t)$ represent the time evolution of operators b and b_j. And we also get the solutions of Heisenberg equations for $a^{(k)}(t)$ and $a_j^{(k)}(t)$ representing the time evolution of operator a and a_j driven by the Hamiltonian $H^{(k)}$ as

$$a^{(k)}(t) = b^{(k)}(t) - \lambda,$$

$$a_j^{(k)}(t) = b_j^{(k)}(t) - \lambda_j, \tag{13}$$

where

$$u^{(k)}(t) = e^{-(\gamma/2)t}e^{-i(\Omega^{(k)}+\Delta\Omega^{(k)})t},$$

$$v_j^{(k)}(t) = -\frac{g_j^{(k)}e^{-i\omega_j t}(1-e^{-i(\Omega^{(k)}+\Delta\Omega^{(k)}-\omega_j)t}e^{-(\gamma/2)t})}{\Omega^{(k)}+\Delta\Omega^{(k)}-\omega_j - i\frac{\gamma}{2}},$$

$$u_j^{(k)}(t) = -\frac{g_j^{(k)}e^{-i\omega_j t}(1-e^{-i(\Omega^{(k)}+\Delta\Omega^{(k)}-\omega_j)t}e^{-(\gamma/2)t})}{\Omega^{(k)}+\Delta\Omega^{(k)}-\omega_j - i\frac{\gamma}{2}},$$

$$v_{j,s}^{(k)}(t) = -\frac{g_j^{(k)}g_s^{(k)}e^{-i\omega_j t}}{\Omega^{(k)}+\Delta\Omega^{(k)}-\omega_s - i\frac{\gamma}{2}} \times\left(\frac{1-e^{-i(\Omega^{(k)}+\Delta\Omega^{(k)}-\omega_j)t}e^{-(\gamma/2)t}}{\Omega^{(k)}+\Delta\Omega^{(k)}-\omega_j - i\frac{\gamma}{2}} + \Lambda\right),$$

$$\Lambda = t, \quad \text{when } s=j,$$

$$\Lambda = \frac{e^{-i(\omega_s-\omega_j)t}-1}{\omega_s-\omega_j}, \quad \text{when } s\neq j, \tag{14}$$

and $\Delta\Omega^{(k)}$ is frequency shift of $\Omega^{(k)}$ corresponding to two different Hamiltonians $H^{(k)}$. In general, $\Delta\Omega^{(k)}$ can be absorbed into $\Omega^{(k)}$, i.e., $\Omega^{(k)}\sim\Omega^{(k)}+\Delta\Omega^{(k)}$. If the forced terms of the Hamiltonian in Eq. (8) are absent, we will get $a^{(k)}(t)=b^{(k)}(t)$ and $a_j^{(k)}(t)=b_j^{(k)}(t)$.

Here we calculate the time evolution of the mean photon number of the resonant mode of cavity field corresponding to two different Hamiltonians $H^{(k)}$. When we assume that the initial state of all modes of cavity field is prepared in Fock state $|n,\{n_j\}\rangle_a = |n\rangle_a \otimes |\{n_j\}\rangle_a$, the mean photon number of resonant mode corresponding to H_k is calculated as

$$n^{(k)}(t) = \langle a^{(k)\dagger}(t) a^{(k)}(t)\rangle = e^{-\gamma t} n + F(\lambda,\{\lambda_j\},\{n_j\},t), \quad (15)$$

where the time dependent constant $F(\lambda,\lambda_j,n_j,t)$ is dependent of λ, λ_j and n_j and the time dependent term $n\exp(-\gamma t)$ characterizing the quantum dissipation induced by nonresonant modes. Thus we can know that the time evolution of the mean number of the resonant mode $n^{(k)}(t)$ is the sum of two parts: (1) quantum dissipation $n\exp(-\gamma t)$ induced by nonresonant modes; (2) the constant $F(\lambda,\{\lambda_j\},\{n_j\},t)$ generated by the first order term ϕ_q and mean number of nonresonant modes n_j.

Through some simple calculations, we find that the constant $F(\lambda,\{\lambda_j\},\{n_j\},t)$ will approach zero when the forced terms vanish and the initial state of the bath (nonresonant modes) in vacuum state $|\{n_j\}\rangle = |\{0_j\}\rangle$. Then we get the same results of Ref. [24] that the time evolution of mean photon number of resonant mode is

$$n^{(k)}(t) = n e^{-\gamma t}. \quad (16)$$

It means that vacuum fluctuation of nonresonant modes leads to quantum dissipation of resonant mode when the forced terms of operators a and a_j are absent. In other words, the nonlinear coupling directly causes quantum dissipation of the resonant mode in a lossy cavity.

In contrast to the model of single boson interacting with a bath of many bosons, the constant $F(\lambda,\lambda_j,n_j,t)$ provides the different effect that the mean number of resonant mode of cavity field does not approach zero when time $t\to\infty$.

V. DECOHERENCE INDUCED BY DISSIPATION OF THE RESONANT MODE

Technically the process of quantum decoherence is described by the time evolution of the reduced density matrix of the coupled qubit-cavity system. To analyze it, we can calculate reduced density matrix for the time evolution of the charge qubit. The pure decoherence process means that the off diagonal elements of the reduced density matrix of the qubit vanish, while the diagonal elements remain unchanged in an ideal case.

Now if the initial state of cavity field is in coherent state, i.e., the resonant mode of cavity field is in a coherent state $|\alpha\rangle$ and other nonresonant modes of cavity field in coherent state $|\{\alpha_j\}\rangle$, the initial state of the total qubit-cavity system can be written as

$$|\Psi(0)\rangle = (C_0|0\rangle + C_1|1\rangle) \otimes |\alpha,\{\alpha_j\}\rangle_a.$$

Then we can easily get the time evolution of the wave function for the qubit-cavity system

$$|\Psi(t)\rangle = U(t)|\Psi(0)\rangle = C_0|0\rangle \otimes |\varphi^{(0)}(t)\rangle + C_1|1\rangle \otimes |\varphi^{(1)}(t)\rangle, \quad (17)$$

where $U^{(k)}(t) = \exp(-iH^{(k)}t)$ are evolution operators for the effective Hamiltonians $H^{(k)}$ and

$$|\varphi^{(k)}(t)\rangle = U^{(k)}(t)|\alpha,\{\alpha_j\}\rangle_a. \quad (18)$$

Then we can obtain time evolution of density matrix for the qubit-cavity system

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)| \quad (19)$$

and calculate the reduced density matrix of the qubit

$$\rho_s(t) = C_0^* C_0|0\rangle\langle 0| + C_1^* C_1|1\rangle\langle 1| + \langle\varphi^{(1)}(t)|\varphi^{(0)}(t)\rangle C_1^* C_0|0\rangle\langle 1| + \text{H.c.} \quad (20)$$

As a measure of the coherence of quantum system [25], the decoherence factor of charge qubit can be calculated as

$$D(t) = |\langle\varphi^{(1)}(t)|\varphi^{(0)}(t)\rangle|. \quad (21)$$

For any coherent state, we have

$$|\alpha\rangle_b = \exp[\alpha b^\dagger(0) - \alpha^* b(0)]|0\rangle_b.$$

Obviously we can see that

$$U^{(k)}(t)|0,\{0_j\}\rangle_b = 0.$$

Formally, we can define

$$O^{(k)}(t) = U^{(k)}(t) O(0) U^{(k)\dagger}(t)$$

for any operator $O^{(k)}$ corresponding to $H^{(k)}$ and get the time dependent equation

$$\frac{dO^{(k)}(t)}{dt} = i[O^{(k)}(t),H_k]. \quad (22)$$

So we have $A^{(k)}(t) = U^{(k)}(t) b(0) U^{(k)\dagger}(t)$ and $A_j^{(k)}(t) = U^{(k)}(t) b_j(0) U^{(k)\dagger}(t)$. By substituting $-i$ into Heisenberg equation with i, we can get the solutions of Eq. (22)

$$A^{(k)}(t) = u^{(k)*}(t) b(0) + \sum_j v_j^{(k)*}(t) b_j(0),$$

$$A_j^{(k)}(t) = e^{i\omega_j t} b_j(0) + u_j^{(k)*}(t) b(0) + \sum_s v_{j,s}^{(k)*}(t) b_s(0).$$

To demonstrate the effect of quantum dissipation of resonant mode induced by nonresonant modes on decoherence of charge qubit, here we do not consider the effect of the forced terms and set $\lambda = 0$ and $\lambda_j = 0$.

In the above section, we have known that the vacuum fluctuation induced by the nonresonant modes can result in quantum dissipation of the resonant mode. So we assume that the initial state of nonresonant modes is in vacuum state $|\{0_j\}\rangle_b = |\{0_j\}\rangle_a$ and initial state of resonant mode coherent state $|\alpha\rangle_b = |\alpha\rangle_a$. And time evolution of decoherence factor of the charge qubit is

$$D(t) = e^{-(|\alpha|^2/2)[|u^{(1)}(t) - u^{(0)}(t)|^2 + \Sigma_j |v_j^{(1)}(t) - v_j^{(0)}(t)|^2]}. \quad (23)$$

With commutation relation $[A^{(k)}(t), A^{(k)\dagger}(t)] = 1$, we have

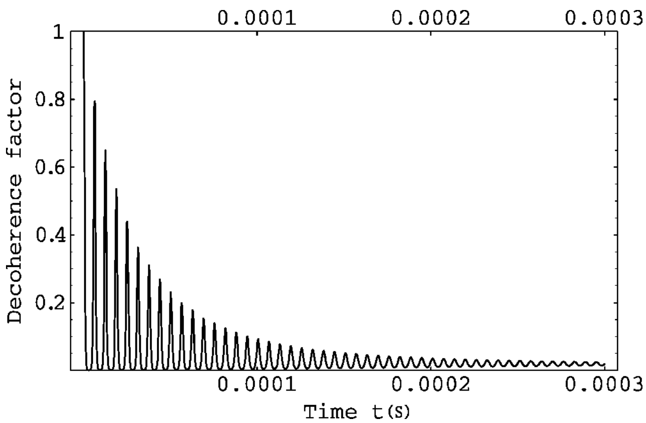

FIG. 2. Decoherence factor $D(t)$ in Eq. (27) as a function of time t with the value of $\alpha=2$.

$$|u^{(k)}(t)|^2+\sum_j |v_j^{(k)}(t)|^2=1. \tag{24}$$

According to Eq. (18), we have

$$|\varphi_k(t)\rangle=|\alpha u^{(k)}(t)\rangle_b\otimes\Pi_j|\alpha v_j^{(k)}(t)\rangle_{b_j}. \tag{25}$$

And the decoherence factor in Eq. (23) becomes

$$D(t)=e^{-|\alpha|^2+(|\alpha|^2/2)[u^{(1)*}(t)u^{(0)}(t)+u^{(1)}(t)u^{(0)*}(t)]}\times e^{(|\alpha|^2/2)\Sigma_j[v_j^{(1)*}(t)v_j^{(0)}(t)+v_j^{(1)}(t)v_j^{(0)*}(t)]}. \tag{26}$$

From the above results in Eq. (9) and Eq. (14), we know that the term $v_j^{(1)*}(t)v_j^{(0)}(t)$ is proportional to $\phi_j^2\phi_0^2\sim\phi_0^4$ which is a 4th order term of nonlinear expansion in Eq. (6). Then we can omit it in the calculation of the decoherence factor and get

$$D(t)=e^{-|\alpha|^2(1-e^{-\gamma t}\cos[(\Omega^{(1)}+\Delta\Omega^{(1)}-\Omega^{(0)}-\Delta\Omega^{(0)})t])}. \tag{27}$$

Figure 2 shows that the decoherence factor $D(t)$ will decrease in the oscillating decay form. In the above equation, the term $\exp(-\gamma t)$ represents quantum dissipation induced by the nonresonant modes. Therefore we obtain the central result of this paper that quantum dissipation of the resonant mode induced by the nonresonant modes directly results in quantum decoherence of the charge qubit in the lossy cavity. At long times, the decoherence factor $D(t)=\exp(-|\alpha|^2)$ is determined by the mean photon number $|\alpha|^2$. At short times, the decoherence factor becomes $D(t)=\exp(-\Gamma t)$. Where the decay rate of decoherence Γ is proportional to the mean photon number of resonant mode $|\alpha|^2$ and decay rate γ of quantum dissipation of resonant mode, $\Gamma=\gamma|\alpha|^2$.

VI. CONCLUSIONS

Before concluding this paper, we would like to note the influence of the fluctuations of gate charge n_g around $1/2$. We notice that the classical fluctuation of gate voltage is not the unique source of decoherence. The most recent investigations have demonstrated that $1/f$ noise is due to the background charge fluctuation, which also plays an important role in the decoherence of a charge qubit [14,26,27]. Maybe there also exists some unknown source of decoherence. For simplicity we deal with the decoherence of a charge qubit by considering that such fluctuations can indeed occur in a real dc-SQUID, but may be ignored so as to bring out more clearly the mechanism. Our investigation only emphasizes the role that nonlinear coupling plays in the decoherence of a charge qubit in some cases.

In this paper, we have discovered the phenomenon of quantum decoherence of a charge qubit in a lossy cavity, where we adopt the quasimode approach to deal with a lossy cavity. We find that the nonlinear coupling between the charge qubit and the cavity field can induce the interaction between resonant mode and nonresonant modes of cavity field. Based on this observation, we achieve a model for this decoherence mechanism that a forced oscillator (resonant mode) interacts with a bath of many forced oscillators (nonresonant modes). The decoherence factor is calculated to demonstrate an oscillating decay of quantum coherence of a charge qubit in the lossy cavity. In addition, we have shown that vacuum fluctuation provided by these nonresonant modes can cause the quantum dissipation of the resonant mode. Consequently, the quantum dissipation of the resonant mode directly results in quantum decoherence of the charge qubit. This analysis describes the source of quantum decoherence for a charge qubit in the lossy cavity.

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