Non-Abelian geometric quantum memory with an atomic ensemble

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We study a quantum information storage scheme based on an atomic ensemble with near (also exact) three-photon resonance electromagnetically induced transparency (EIT). Each 4-level-atom is coupled to two classical control fields and a quantum probe field. Quantum information is adiabatically stored in the associated dark polariton manifold. An intrinsic nontrivial topological structure is discovered in our quantum memory implemented through the symmetric collective atomic excitations with a hidden SU(3) dynamical symmetry. By adiabatically changing the Rabi frequencies of two classical control fields, the quantum state can be retrieved up to a non-Abelian holonomy and thus decoded from the final state in a purely geometric way.

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I. INTRODUCTION

Quantum information storage is a physical process to encode the state of a quantum system into the state of another system referred to as a quantum memory [1]. Compared to the original quantum system the quantum memory should possess a large decoherence time for effective storing of quantum information. Moreover, the original state of the quantum system should be retrievable from the encoding quantum memory state. By means of quantum memory one can transport the quantum information from place to place within the decoherence time. Recently, an ensemble of Λ-type atoms has been proposed [2–4] as a candidate for practical quantum memory. The idea is to store and transfer the quantum information contained in photonic states by the collective atomic excitations. This approach is based on the phenomenon of electromagnetically induced transparency (EIT) [5]. Some experiments [6,7] have already demonstrated the central principle of this technique, namely, the reduction of the group velocity of light.

Most recently a system with quasispin wave collective excitations of many Λ-type atoms fixed in "atomic crystal" has been considered as a candidate for a robust quantum memory [8]. A hidden dynamical symmetry of such a system is discovered and it is observed that in certain cases [9] the quantum state can be retrieved up to a non-Abelian Berry phase, i.e., a non-Abelian holonomy [10–15]. This observation extends the concept of quantum information storage. Quantum information storage of photonic states with this topological character can be implemented in an atomic ensemble with off-resonance EIT. In such a case the stored state can be decoded in a purely geometric way. However, this non-Abelian holonomy is in some sense trivial due to the fact that the quantum storage space splits into an orthogonal sum of invariant one dimensional subspaces.

In this work, we shall describe a quantum information storage protocol based on a truly non-Abelian holonomy. To this aim we will consider an ensemble of N 4-level-atoms [13,14], where two meta-stable states are coupled to the excited state by two classical control fields respectively while the ground state is coupled to the excited state by a quantum probe field. In the large N limit with low excitation, a three-excitons system is formed by the symmetric collective excitations from the ground states up to the excited state plus the two virtual excitations from the two metastable states to the excited state. It is easy to prove that these three collective excitations indeed behave as three bosons in the large N limit with low excitation. Intertwining between the excited state and two metastable ones, the collective operators generate an $SU(3)$ algebra. Based on the spectrum generating algebra theory [16] associated with this $SU(3)$, we construct the degenerate eigenstates of the three-mode exciton-photon system. In particular the collective manifold of dark states can be shown to split into dynamically invariant higher-dimensional subspaces. Using these degenerate eigenstates as a quantum memory, quantum information storage of photonic states can be implemented up to a non-Abelian holonomy.

II. THE MODEL

Our system consists of N identical 4-level atoms [13,14], where all the atoms are coupled to two single-mode classical control fields and a quantum probe field as shown in Fig. 1. The atomic levels are labeled as the ground state $|b\rangle$, the excited state $|a\rangle$, and the meta-stable states $|k\rangle$ $(k=1,2)$. The atomic transition $|a\rangle\rightarrow|b\rangle$, with energy level difference $\omega_{ab}=\omega_a-\omega_b$, is coupled to the probe field of frequency ω $(=\omega_{ab}-\Delta_p)$ with the coupling coefficient g; and the atomic transition $|a\rangle\rightarrow|k\rangle$ $(k=1,2)$, with energy level difference ω_{ak}, is driven by the classical control field of frequency ν_k $(=\omega_{ak}-\Delta_k)$ with Rabi-frequency $\Omega_k(t)$.

In the present work we consider the case of δ_k $(=\Delta_k-\Delta_p)$ being very small, that is, those three fields have almost

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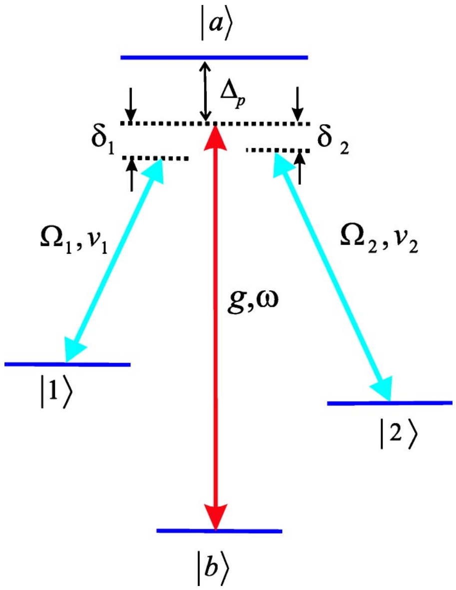

FIG. 1. Four-level atom interacting with a quantum probe field (with coupling constant g, frequency ω, and the detuning Δ_p) and two classic control fields (with frequency ν_k, coupling Rabi frequency Ω_k, and the detuning $\Delta_k=\omega_{ak}-\nu_k$, $k=1,2$). When δ_k $(=\Delta_k-\Delta_p)$ are very tiny, the system satisfies the near 3-photon resonance EIT condition.

the same detuning with respect to the upper level $|a\rangle$. In view of the physical intuition, each metastable state with its relevant control field would constitute a near two-photon resonance EIT if another metastable state and its relevant control field do not exist. With the case of two-photon resonance EIT [17–19] (where the control and probe fields have the same detuning) in mind, we would refer to our case of $\Delta_p \simeq \Delta_k$ as a *near "3-photon resonance" EIT*.

Under the rotating wave approximation the interaction Hamiltonian can be written as (let $\hbar=1$) [8]

$$H_I = \Delta_p S + g\sqrt{N}aA^\dagger + \Omega_1\exp[i\phi_1(t)]T_+^{(1)} + \Omega_2\exp[i\phi_2(t)]T_+^{(2)} + \text{H.c.}, \tag{1}$$

where

$$S=\sum_{j=1}^{N}\sigma_{aa}^{(j)}, \quad A=\frac{1}{\sqrt{N}}\sum_{j=1}^{N}\sigma_{ba}^{(j)},$$

$$T_-^{(k)}=\sum_{j=1}^{N}\sigma_{ka}^{(j)}, T_+^{(k)}=(T_-^{(k)})^\dagger, \quad k=1,2 \tag{2}$$

are symmetrized collective atomic operators. Here $\sigma_{\mu\nu}^{(j)}=|\mu\rangle_{jj}\langle\nu|$ denotes the flip operator of the jth atom from state $|\nu\rangle_j$ to $|\mu\rangle_j$ $(\mu,\nu=a,b,1,2)$; $a^\dagger$ and a the creation and annihilation operators of quantum probe field respectively; and $\phi_k(t)=\delta_k t$. The coupling coefficients g and $\Omega_{1,2}$ are real and assumed to be identical for different atoms in the ensemble. A similar effective Hamiltonian was given in Ref. [8] for the case of an "atomic crystal," in terms of quasi-spin-wave type collective atomic operators and a hidden dynamical symmetry was discovered there. The symmetrized operators (2) are just a special instance of the quasi-spin-wave operators discussed in [8].

Let us first consider a similar dynamical symmetry in the low excitation regime of the atomic ensemble where most of N atoms stay in the ground state $|b\rangle$ and $N\to\infty$. It is obvious that $T_-^{(k)}$ and $T_+^{(k)}$ $(k=1,2)$ generate two mutually commuting $SU(2)$ subalgebras of $SU(3)$ [20]. To form a closed algebra containing $SU(3)$ and $\{A,A^\dagger\}$, we need to introduce two additional collective operators

$$C_k=\frac{1}{\sqrt{N}}\sum_{j=1}^{N}\sigma_{bk}^{(j)}, k=1,2 \tag{3}$$

along with their Hermitian conjugates. These operators have the nonvanishing commutation relations

$$C_k=[A,T_+^{(k)}], \quad [C_k,T_-^{(k)}]=A(k=1,2). \tag{4}$$

As a special case of quasispin wave excitation with zero varying phases, the above three mode symmetrized excitations defined by A and $C_{1,2}$ behave as three independent bosons. Indeed one can check that the operators (3), in the large N limit with low excitation, satisfy the bosonic commutation relations [20]. The commutation relations between the $SU(3)$ algebra and the Heisenberg-Weyl algebra h generated by A, $A^\dagger$, C_k, and $C_k^\dagger$ imply that the dynamical symmetry of evolution governed by H_I can be described by the semidirect product algebra $SU(3)\overline{\otimes}h$.

III. THE DARK STATES

Based on the above hidden dynamical symmetry of the interaction Hamiltonian, we can introduce a dark-state polariton operator

$$D=a\cos\theta - C\sin\theta, \tag{5}$$

where

$$C=C_1\exp[i\phi_1(t)]\cos\kappa + C_2\exp[i\phi_2(t)]\sin\kappa \tag{6}$$

is a coherent mixing of two collective atomic excitations C_1 and C_2, and

$$\kappa=\arctan\frac{\Omega_2}{\Omega_1}, \quad \theta=\arctan\frac{g\sqrt{N}}{\Omega}, \quad \Omega=\sqrt{\Omega_1^2+\Omega_2^2}. \tag{7}$$

In terms of a new operator

$$T_+=T_+^{(1)}\exp[i\phi_1(t)]\cos\kappa + T_+^{(2)}\exp[i\phi_2(t)]\sin\kappa, \tag{8}$$

we can then rewrite the interaction Hamiltonian as

$$H_I=\Delta_p S + g\sqrt{N}aA^\dagger + \Omega T_+ + \text{H.c.} \tag{9}$$

Since $[C,T_-]=A$ and $[A,T_+]=C$, one can readily verify that

$$[D,H_I]=0. \tag{10}$$

To generate the full eigenspace of H_I with zero eigenvalue, i.e., the dark-polariton manifold, we need consider another dark-state polariton operator complementary to D:

$$E=C_2\exp[i\phi_2(t)]\cos\kappa - C_1\exp[i\phi_1(t)]\sin\kappa. \tag{11}$$

It is worthwhile to point out that E satisfies the bosonic commutation relation as well and it is independent of D since

$$[E,E^{\dagger}]=1, \quad [E,D^{\dagger}]=0. \tag{12}$$

Moreover, we have $[E,H_I]=0$ by construction. Our instantaneous quantum storage subspace $\mathcal{V}(t)$ is given by the linear span of the following family of instantaneous dark states, i.e., the eigenstates of $H_I(t)$ with vanishing eigenvalues

$$|D_{m,n}(t)\rangle = \frac{1}{\sqrt{m!\,n!}} D^{\dagger m} E^{\dagger n} |0\rangle, \tag{13}$$

where $|0\rangle = |0\rangle_p \otimes |\mathbf{b}\rangle \equiv |0\rangle_p \otimes |b,b,\ldots,b\rangle$ represents the ground state of the total coupled system with each atom being in the ground state $|b\rangle$ and the quantum probe field being in the vacuum state $|0\rangle_p$. It is easy to prove that any other dark-state polariton operator can be expressed as a linear combination of D and E.

IV. NON-ABELIAN HOLONOMY

Now we study the geometric quantum information storage in the dark-state space $\mathcal{V}(t)$ which is constructed by the above zero-eigenvalue dark states (13).

It is noticed that one can introduce the so-called bright-state polariton operator:

$$B = a \sin\theta + C\cos\theta, \tag{14}$$

which can be used to generate eigenstates involving the excited state $|a\rangle$. Of course, the states obtained by applying B are not absolutely dark since the excited state can spontaneously decay. However, as shown in Ref. [8], under the adiabatic manipulations, these states will not get coupled to the above constructed dark states (13). The adiabatic condition is here given by [21,22]

$$\frac{g\sqrt{N} x_k}{(\sqrt{g^2 N + \Omega^2})^3} \ll 1, \quad x_k = |\Omega_k|, \Omega\,\delta_k \tag{15}$$

for $k=1,2$. So the dark-state space $\mathcal{V}(t)$ can be considered as a reliable storage one.

Let us consider a state vector

$$|\Phi(t)\rangle = \sum_{m,n} c_{mn}(t) |D_{m,n}(t)\rangle \tag{16}$$

belonging to $\mathcal{V}(t)$. A straightforward calculation gives the matrix equation [21,22] for the coefficients $c_{mn}(t)$:

$$\partial_t \mathbf{C}(t) = \mathbf{K}(t)\mathbf{C}(t), \tag{17}$$

where the vector $\mathbf{C}(t)$ of coefficients and the connection matrix $\mathbf{K}(t)$ are respectively defined by

$$\mathbf{C}(t) = [c_{00}(t), c_{01}(t), \ldots; c_{10}(t), c_{11}(t), \ldots]^T \tag{18}$$

and

$$\mathbf{K}(t)_{m,m',n,n'} = -\langle D_{m',n'}(t)|\partial_t D_{m,n}(t)\rangle \tag{19}$$

$(m,m',n,n'=0,1,2,\ldots)$. The quantum storage space $\mathcal{V}(t)$ is, in the considered limit, an infinite dimensional one. Thus in general it is difficult to write down the relevant connection matrix $\mathbf{K}(t)$ explicitly. On the other hand, the adiabatic quantum evolution in $\mathcal{V}(t)$ can be reduced, i.e., this space splits into dynamically invariant finite-dimensional sectors. Let us explain this point now.

We first observe that the following dynamical commutation relations hold:

$$f_{DD}(t) := [D, \partial_t D^{\dagger}] = -i \sin^2\theta(\delta_1 \cos^2\kappa + \delta_2 \sin^2\kappa),$$

$$f_{ED}(t) := [E, \partial_t D^{\dagger}] = -\dot{\kappa}\sin\theta + i(\delta_2 - \delta_1)\sin\theta\cos\kappa\sin\kappa,$$

$$f_{DE}(t) := [D, \partial_t E^{\dagger}] = [-f_{ED}(t)]^*,$$

$$f_{EE}(t) := [E, \partial_t E^{\dagger}] = -i(\delta_1 \sin^2\kappa + \delta_2 \cos^2\kappa). \tag{20}$$

Using these relations for $l' \geqslant m \geqslant 0$ and $l \geqslant n \geqslant 0$, we obtain

$$\begin{aligned}\langle D_{l'-m,m}(t)|\partial_t D_{l-n,n}(t)\rangle &= \delta_{l',l}\delta_{m,n}[(l-m)f_{DD}(t) + m f_{EE}(t)] \\ &+ \delta_{l',l}\delta_{m,n-1}\sqrt{(m+1)(l-m)} f_{DE}(t) \\ &+ \delta_{l',l}\delta_{m,n+1}\sqrt{m(l-m+1)} f_{ED}(t).\end{aligned} \tag{21}$$

Now it is clear from $\delta_{l',l}$ in this expression that the total space $\mathcal{V}(t)$ can be decomposed into a direct sum of subspaces:

$$\mathcal{V}(t) = \oplus_{l=0}^{\infty} \mathcal{V}_l(t), \tag{22}$$

where

$$\mathcal{V}_l(t) = \mathrm{span}\{|D_{l-m,m}(t)\rangle | m = 0,1,\ldots,l \tag{23}$$

has dimension $(l+1)$. Notice that each $\mathcal{V}_l(t)$ is an invariant subspace under the adiabatic manipulation, i.e., if the initial state is given as

$$|\Phi_l(0)\rangle \in \mathcal{V}_l(0), \tag{24}$$

then at time t the state will be

$$|\Phi_l(t)\rangle = \sum_m c_m^{(l)}(t) |D_{l-m,m}(t)\rangle \in \mathcal{V}_l(t). \tag{25}$$

The restricted dynamics in $\mathcal{V}_l(t)$ is governed by the reduced dynamic equation

$$\partial_t \mathbf{C}_l(t) = \mathbf{K}_l(t)\mathbf{C}_l(t), \tag{26}$$

where the subcoefficient vector $\mathbf{C}_l(t)$ and the reduced connection matrix $\mathbf{K}_l(t)$ are respectively given by

$$\mathbf{C}_l(t) = [c_0^{(l)}(t), c_1^{(l)}(t), \ldots, c_l^{(l)}(t)]^T, \tag{27}$$

and

$$\mathbf{K}_l(t) = [-\langle D_{l-m,m}(t)|\partial_t D_{l-n,n}(t)\rangle_{m,n=0,1,2,\ldots,l}. \tag{28}$$

The solution

$$\mathbf{C}_l(t) = \mathbf{W}_l(t)\mathbf{C}_l(0) \tag{29}$$

formally determines the non-Abelian holonomy

$$\mathbf{W}_l(t) = \mathbf{T} \exp\Big[\int \mathbf{K}_l(t)dt\Big], \tag{30}$$

where $\mathbf{T}$ is the time-ordering operator. This non-Abelian holonomy is nondiagonal and thus can mix different instantaneous eigenstates $|D_{l-m,m}(t)\rangle (m=0,\ldots,l)$ inducing in this way a truly non-Abelian gauge structure.

V. GEOMETRIC QUANTUM MEMORY BASED ON THE SIMPLIFIED MODEL

In the following discussion, we consider the simplified model related to the above system as shown in Fig. 1: $\delta_{1,2} \equiv 0$. Such a system has only two controllable parameters $\Omega_{1,2}$ and can be readily realized experimentally. Mathematically the subconnection can be simplified as

$$\mathbf{K}_l(t) = [-\langle D_{l-m,m}(t)|\partial_t D_{l-n,n}(t)\rangle]_{m,n=0,1,\ldots,l} \equiv \dot{\kappa} \sin\theta \mathbf{K}_l^{(0)}, \tag{31}$$

where $\mathbf{K}_l^{(0)}$ is a constant matrix whose (m,n) entry is

$$\delta_{m,n-1}\sqrt{(m+1)(l-m)} - \delta_{m,n+1}\sqrt{m(l-m+1)}. \tag{32}$$

In this simplified case the time-ordering becomes irrelevant and the non-Abelian holonomy can be explicitly computed. In fact we have

$$\mathbf{W}_l(t) = \exp[\phi(t)\mathbf{K}_l^{(0)}], \tag{33}$$

where

$$\phi(t) = \int \dot{\kappa} \sin\theta dt. \tag{34}$$

By noticing that $\mathbf{K}_l^{(0)}$ is proportional to $(E^{\dagger}D - \text{H.c.})$ restricted $\mathcal{V}_l(t)$, we find that this non-Abelian holonomy can be rather easily cast in a diagonal form by introducing a new instantaneous basis. Let us introduce the new set of dark-state polariton operators

$$D' = \frac{1}{\sqrt{2}}(iD + E), \quad E' = \frac{1}{\sqrt{2}}(-iD + E), \tag{35}$$

and the associated dark states

$$|D'_{l-n,n}(t)\rangle = \frac{D'^{\dagger l-n}E'^{\dagger n}}{\sqrt{(l-n)!\,n!}}|0\rangle. \tag{36}$$

A straightforward calculation then gives a diagonal connection matrix

$$\mathbf{K}'_l(t) = -i\dot{\kappa}\sin\theta \,\text{diag}\,(l, l-2, \ldots, -l) \tag{37}$$

and the corresponding holonomy

$$\mathbf{W}'_l(t) = \text{diag}\,(e^{-il\phi(t)}, e^{-i(l-2)\phi(t)}, \ldots, e^{il\phi(t)}). \tag{38}$$

Generally in the EIT-based quantum information storage protocol, the Rabi frequencies $\Omega_{1,2}$ of the two classical control fields are initially set to a very large value compared to $g\sqrt{N}$ and then decreased independently and adiabatically (e.g., as shown in Fig. 2). Thus $\theta(t=0)\to 0$ and $D(0)\to a$.

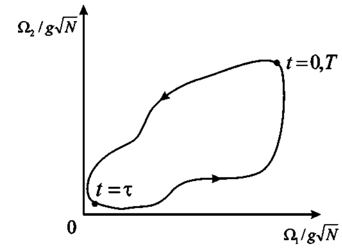

FIG. 2. Cyclic evolution of the parameters $\Omega_{1,2}$. At time $t=\tau$, $\Omega_{1,2} \ll g\sqrt{N}$; at time $t=0$ or $t=T$, $\Omega_{1,2} \gg g\sqrt{N}$.

The initial state $|\Phi(0)\rangle = \Sigma_l\, c_0^{(l)}(0)|l\rangle_p \otimes |\mathbf{b}\rangle$ can be written as

$$|\Phi(0)\rangle \equiv \sum_{l,m} c_m'^{(l)}(0)|D'_{l-m,m}(0)\rangle \tag{39}$$

relative to the new basis $|D'_{l-m,m}(t)\rangle$ with the coefficients

$$c_m'^{(l)}(0) = \frac{(-1)^{l-m}\sqrt{l!}\,c_0^{(l)}(0)}{(i\sqrt{2})^l\sqrt{m!\,(l-m)!}}. \tag{40}$$

Under the adiabatic evolution, the state at time t is given as

$$|\Phi(t)\rangle = \sum_{l,m} c_m'^{(l)}(t)|D'_{l-m,m}(t)\rangle, \tag{41}$$

where

$$c_m'^{(l)}(t) = \exp[-i(l-2m)\phi(t)]c_m'^{(l)}(0). \tag{42}$$

When $\Omega_{1,2}$ become negligible compared to $g\sqrt{N}$ at time τ, $\theta(\tau)\to\pi/2$ and $D(\tau)\to -C_1\cos\kappa - C_2\sin\kappa$. This means that the quantum information, initially encoded in photonic states, has been transferred and written to atomic collective excitations. This accomplishes the quantum information storage protocol.

In order to recover the stored information one needs to drive adiabatically the system parameters $\Omega_{1,2}$ along a cyclic evolution such that at time T the condition $\Omega_{1,2} \gg g\sqrt{N}$ is satisfied in order to guarantee $\theta(T)\to 0$ (see Fig. 2). At the intermediate times $t\in(0,T)$ quantum information is encoded in a combination of photonic and atomic collective excitations. In general, if one wants to recover exactly the initial state after that the adiabatic loop has been completed, she has to perform a unitary transformation to get rid of the effect of the non-Abelian Berry phase factor. In particular, for a cyclic evolution of the parameters $\Omega_{1,2}$ if

$$\phi(T) = \int_0^T \dot{\kappa}\sin\theta dt \equiv \oint \sin\theta d\kappa = 2j\pi \tag{43}$$

(j is an integer), it then follows that $c_m'^{(l)}(T) = c_m'^{(l)}(0)$. In this case the system state at the final time T coincides with the initial state $|\Phi(0)\rangle$.

VI. CONCLUSION

We are now in the position to make a few comments on the relations between the results presented in this paper and the general holonomic approach to quantum information processing [12,14]. In that approach the information is encoded in degenerate eigenstates of a parametric family of Hamiltonians, and in the generic case the universal quantum computation [23] can be achieved by resorting to the non-Abelian holonomies only [12]. By regarding the nontrivial holonomy one gets after an adiabatic loop as a designed quantum state transformation, rather than something one wants to get rid of, it should be then evident that the EIT-based scheme here discussed represents an instance of such general strategy. For example the one exciton space $\mathcal{V}_1$ can encode one *qubit*: $|0\rangle := E'^{\dagger}|0\rangle, |1\rangle := D'^{\dagger}|0\rangle$. In this language the transformation (38) is nothing but a diagonal phaseshift [23]. In order to get the nondiagonal single-qubit operations one would have to relax the condition $\delta_1 = \delta_2 = 0$. Encoding many-qubit states and enacting a controllable geometric coupling between them—as required for realizing the universal computations—along with the robustness of the scheme against the various sources of errors is a more complex problem that calls for further investigations.

In conclusion, we have presented a generalized version of quantum information storage by allowing the quantum state to be retrieved up to an input-independent non-Abelian holonomy. Such a non-Abelian holonomy is independent of both the state to be stored and some dynamic details control of interaction. Thus, to decode the ideal input state, we only need consider the geometry of the parameter space determined by the change of parameters. We also showed the physical process of the geometric quantum storage of photon information with the help of the symmetric collective excitations of the EIT-based 4-level-atom ensemble in the simplified case by adiabatically controlling the classical Rabi frequencies.

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[1] *The Physics of Quantum Information* , edited by D. Bouwmeeste, A. Ekert, and A. Zeilinger (Springer, Berlin, 2000).
[2] M. D. Lukin, S. F. Yelin, and M. Fleischhauer, Phys. Rev. Lett. **84**, 4232 (2000).
[3] M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. **84**, 5094 (2000).
[4] D. F. Phillips, A. Fleischhauer, A. Mair, and R. L. Walsworth, M. D. Lukin, Phys. Rev. Lett. **86**, 783 (2001).
[5] S. E. Harris, Phys. Today **50**, 36 (1997).
[6] C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau, Nature (London) **409**, 490 (2001).
[7] L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, Nature (London) **397**, 594 (1999).
[8] C. P. Sun, Y. Li, and X. F. Liu, Phys. Rev. Lett. **91**, 147903 (2003).
[9] C. P. Sun, P. Zhang, and Y. Li, quant-ph/0311052.
[10] M. V. Berry, Proc. R. Soc. London, Ser. A **392**, 45 (1984); *Geometric Phases in Physics*, edited by A. Shapere and F. Wilczek (World Scientific, Singapore, 1989).
[11] F. Wilczek and A. Zee, Phys. Rev. Lett. **52**, 2111 (1984).
[12] P. Zanardi and M. Rasetti, Phys. Lett. A **264**, 94 (1999);J. Pachos, P. Zanardi, and M. Rasetti, Phys. Rev. A **61**, 010305(R) (2000).
[13] R. G. Unanyan, B. W. Shore, and K. Bergmann, Phys. Rev. A **59**, 2910 (1999).
[14] L. M. Duan, J. I. Cirac, and P. Zoller, Science **292**, 1695 (2001).
[15] J. A. Jones, V. Vedral, A. Ekert, and G. Castagnoli, Nature (London) **403**, 869 (2000).
[16] B. G. Wybourne, *Classical Groups for Physicists* (Wiley, New York, 1974);M. A. Shifman, *Particle Physics and Field Theory* (World Scientific, Singapore, 1999), p. 775.
[17] M. D. Lukin, Rev. Mod. Phys. **75**, 457 (2003).
[18] M. Kozuma, D. Akamatsu, L. Deng, E. W. Hagley, and M. G. Payne, Phys. Rev. A **66**, 031801(R) (2002).
[19] M. G. Payne and L. Deng, Phys. Rev. Lett. **91**, 123602 (2003).
[20] G. R. Jin, P. Zhang, Yu-xi Liu, and C. P. Sun, Phys. Rev. B **68**, 134301 (2003).
[21] A. Zee, Phys. Rev. A **38**, 1 (1988).
[22] C. P. Sun, Phys. Rev. D **41**, 1318 (1990).
[23] See, e.g., D. P. DiVincenzo and C. Bennet, Nature (London) **404**, 247 (2000), and references therein.