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Group velocity of a probe light in an ensemble of Λ atoms under two-photon resonance

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We study the propagation of a probe light in an ensemble of Λ-type atoms, utilizing the dynamic symmetry as recently discovered when the atoms are coupled to a classical control field and a quantum probe field [Sun *et al.*, Phys. Rev. Lett. **91**, 147903 (2003)]. Under two-photon resonance, we calculate the group velocity of the probe light with collective atomic excitations. Our result gives the dependence of the group velocity on the common one-photon detuning, and can be compared with the recent experiment of E. E. Mikhailov, Y. V. Rostovtsev, and G. R. Welch, e-print quant-ph/0309173.

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Electromagnetically induced transparency (EIT) [1] has become an active area of theoretical and experimental research [2–4]. Since the discovery of EIT, a host of new effects and techniques for light–matter interaction has occurred; e.g., the propagation of ultraslow light pulses [5,6], the storage of light in atomic vapors [7,8] or in an "atomic crystal" [9], the cooling of ground-state atoms, and the giant cross-Kerr nonlinearity [10].

A conventional EIT system consists of a vapor cell with 3-level atoms near-resonantly coupled to two classical fields (from the control and probe lasers) [1,5,11]. To investigate its application as a quantum memory, or for transferring quantum information between light (photons) and atoms, several groups [12–16] replaced the classical probe laser field with a weak quantum field. By adiabatically changing the coupling strength of the classic control field, it was shown that the propagation of the quantum probe field can be coherently controlled via the *so-called* dark states and dark-state polaritons. The recent experiments [7,8] on light storage have further demonstrated the possibility of using this system for storage of quantum information.

In most studies of quantum memory based on EIT systems [9,13], both the probe and control fields are required to be on resonance with the relevant (one-photon) atomic transitions. We note, however, on-resonance EIT is in fact not a prerequisite for achieving significant group-velocity reduction [17]. More generally, the EIT phenomenon occurs when the probe and control fields are two-photon Raman resonant with the Λ-type atoms. References [18–21] reported theoretical and experimental results on significant group-velocity reduction when both fields are classical and two-photon resonant with the atoms. A more recent experiment [22] demonstrated the dependence of ultraslow group velocity on the probe light detuning under two-photon resonance, with or without a buffer gas. Some of their experimental results are, however, difficult to explain using the conventional EIT theory with a single atom.

In this article, we revisit the above two-photon resonant EIT system with the dynamic symmetry analysis as developed earlier [9]. In Ref. [9], we find the EIT system, which consists of Λ-type atoms *exactly* resonantly coupled by the quantum probe light and the classical control light, possesses a hidden dynamic symmetry described by the semidirect product of quasispin SU(2) and the boson algebra of the excitons. Here, we will further prove that the same hidden dynamic symmetry persists in the more general two-photon resonant case. This observation allows us to build a dynamic equation describing the propagation of the probe light in this atomic ensemble with atomic collective excitations. We calculate the group velocity of the quantum probe field, and investigate how it depends on the detuning of the control and probe fields. Putting aside the influence of atomic spatial motion, atomic collisions, and buffer gas atoms, our results are consistent with some of the recent experiment of Ref.[22].

We consider an ensemble of N three-level Λ-type atoms, coupled to a classical control field and a quantum probe field as shown in Fig. 1. The atomic levels are labeled as the ground state $|b\rangle$, the excited state $|a\rangle$, and the final state $|c\rangle$. The atomic transition $|a\rangle \leftrightarrow |b\rangle$ with energy level difference $\omega_{\mathrm{ab}}=\omega_{\mathrm{a}}-\omega_{\mathrm{b}}$ is coupled to the quantum probe field of frequency ω with the coupling coefficient g and the detuning $\Delta_{\mathrm{p}}=\omega-\omega_{\mathrm{ab}}$, while the atomic transition $|a\rangle \leftrightarrow |c\rangle$ with energy level difference ω_{ac} is driven by a classical control field of frequency ν with the Rabi frequency Ω and the detuning $\Delta_{\mathrm{c}}=\nu-\omega_{\mathrm{ac}}$.

In the interaction picture, the interaction part of our Hamiltonian reads ($\hbar=1$)

$$H_I = -\Delta_{\mathrm{p}} S + (g\sqrt{N} a A^{\dagger} + e^{i(\Delta_{\mathrm{p}}-\Delta_{\mathrm{c}})t}\Omega T_{+} + \mathrm{H.c.}), \tag{1}$$

in terms of the collective quasispin operators

$$S = \sum_{j=1}^{N} \sigma_{\mathrm{aa}}^{(j)}, \quad A^{\dagger} = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \sigma_{\mathrm{ab}}^{(j)}, \quad T_{+} = \sum_{j=1}^{N} \sigma_{\mathrm{ac}}^{(j)}. \tag{2}$$

Here, $\sigma_{\mu\nu}^{(j)}=|\mu\rangle_{jj}\langle\nu|$ is the flip operator of the jth atom from state $|\mu\rangle_j$ to $|\nu\rangle_j$ ($\mu,\nu=a,b,c$), and $a^{\dagger}$ (a) is the creation (annihilation) operator of the probe light. In the large-N limit with low atomic excitations, only a few atoms occupy states $|a\rangle$ or $|c\rangle$ [23], and the atomic collective excitations of the atoms behave as bosons since in this case they satisfy the bosonic commutation relation $[A,A^{\dagger}]=1$. When at two-photon resonance defined by $\Delta_{\mathrm{p}}=\Delta_{\mathrm{c}}$, the Hamiltonian (1) is

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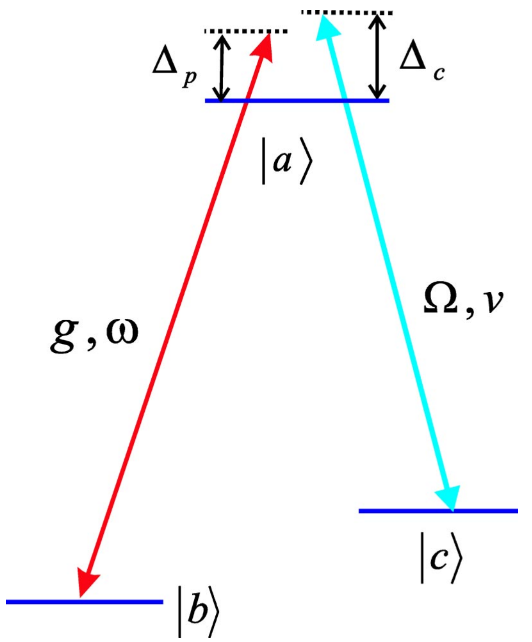

FIG. 1. (Color online) A three-level Λ atom coupled to classic control and quantum probe fields with respective detuning, Δ_c and Δ_p. When $\Delta_p = \Delta_c$, the system satisfies the two-photon resonance EIT condition.

time independent and thus there exists the same dark state and dark-state polariton as shown before for the one-photon resonant case [9,13].

We note that the above Hamiltonian is expressed in terms of the collective dynamic variables S, A, $A^\dagger$, $T_- = (T_+)^\dagger$, and T_+. To properly describe both the probe light propagation and the cooperative motion of the atomic ensemble stimulated by the two fields, we consider the closed Lie algebra generated by A, $A^\dagger$, T_-, and T_+. To this end, a new pair of atomic collective excitation operators,

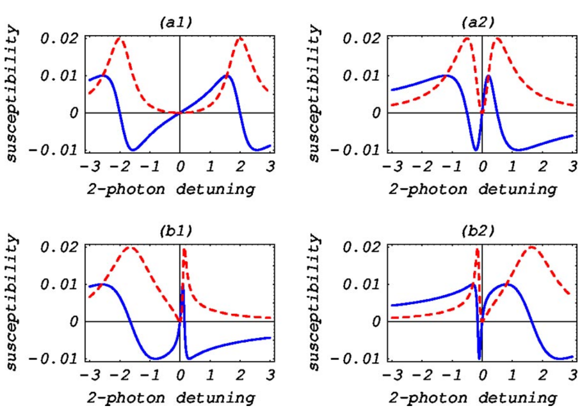

FIG. 2. (Color online) Real part χ_1 (solid) and imaginary part χ_2 (dashed) of the linear susceptibility vs two-photon detuning $\Delta(=\Delta_p - \Delta_c)$ in normalized units according to: (a1) $\Omega = 2$, $\Delta_c = 0$; (a2) $\Omega = 1/2$, $\Delta_c = 0$; (b1,b2) $\Omega = 1/2$, $\Delta_c = \pm 1.5$. Other parameters are given as $\Gamma_A = 1$, $\Gamma_C = 10^{-4}$, $g\sqrt{N} = 100$, and $\omega_{ab} = 10^6$.

$$C = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \sigma_{bc}^{(j)}, \quad C^\dagger = (C)^\dagger, \tag{3}$$

is introduced here to form a closed algebra. In the low excitation limit, when a few atoms occupy states $|a\rangle$ and $|c\rangle$, the corresponding atomic collective excitations also behave as bosons since they satisfy the bosonic commutation relation $[C, C^\dagger] = 1$. These atomic collective excitations are independent of each other in the same limit because of the vanishing commutation relations $[A, C] = 0, [A, C^\dagger] \to 0$ by a straightforward calculation. Together with the above commutators, the following basic commutation relations:

$$\begin{aligned} [S, C^\dagger] &= 0, \quad [A, S] = A, \\ [T_-, C^\dagger] &= 0, \quad [T_+, C^\dagger] = A^\dagger, \end{aligned} \tag{4}$$

define a dynamic symmetry hidden in our dressed atomic ensemble described by the semidirect-product algebra containing the algebra SU(2) generated by T_- and T_+.

We now calculate the probe field group velocity from the time-dependent Hamiltonian (1). With the help of the above dynamic algebra, we can write the Heisenberg equations of operators A and C as

$$\begin{aligned} \dot{A} &= -(\Gamma_A - i\Delta_p)A - ig\sqrt{N}a - ie^{i(\Delta_p - \Delta_c)t}\Omega C + f_A(t), \\ \dot{C} &= -\Gamma_C C - ie^{-i(\Delta_p - \Delta_c)t}\Omega A + f_C(t), \end{aligned} \tag{5}$$

where we have phenomenologically introduced the decay rates Γ_A and Γ_C of the states $|a\rangle$ and $|c\rangle$, and $f_A(t)$ and $f_C(t)$ are the quantum fluctuation of operators with $\langle f_\alpha(t) f_\alpha(t')\rangle \neq 0$, but $\langle f_\alpha(t)\rangle = 0$, $(\alpha = A, C)$.

To find the steady-state solution for the above motion equations of atomic coherent excitation, it is convenient to remove the fast-changing factors by making a transformation $C = \tilde{C} e^{-i(\Delta_p - \Delta_c)t}$. The steady-state solution can be achieved from the transformed equations

$$\begin{aligned} \dot{A} &= -(\Gamma_A - i\Delta_p)A - ig\sqrt{N}a - i\Omega\tilde{C} + f_A(t), \\ \dot{\tilde{C}} &= -\Gamma_C\tilde{C} + i(\Delta_p - \Delta_c)\tilde{C} - i\Omega A + f_C(t), \end{aligned} \tag{6}$$

by letting $\dot{A} = \dot{\tilde{C}} = 0$. The mean expression of A explicitly obtained is

$$\langle A\rangle = \frac{-ig\sqrt{N}[\Gamma_C - i(\Delta_p - \Delta_c)]\langle a\rangle}{(\Gamma_A - i\Delta_p)[\Gamma_C - i(\Delta_p - \Delta_c)] + \Omega^2}. \tag{7}$$

It is noted that the single-mode probe quantum light is described by

$$E(t) = \varepsilon e^{-i\omega t} + \text{H.c.} \equiv \sqrt{\frac{\omega}{2V\epsilon_0}} a e^{-i\omega t} + \text{H.c.}, \tag{8}$$

where V is the effective mode volume, which for simplicity is chosen to be equal to the interaction volume. Its corresponding polarization is

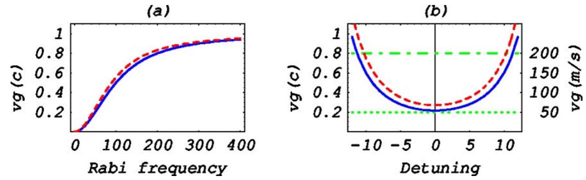

FIG. 3. (Color online) The probe light group velocity v_g vs (a) Rabi frequency Ω (in normalized units) for $\Delta_c=5$, and $g\sqrt{N}=100$ (blue, solid) or $g\sqrt{N}=80$ (red, dashed); (b) the detuning Δ_c (in normalized units) for $\Omega=2g\sqrt{N}=200$ (green, dotted-dashed) or $\Omega=g\sqrt{N}/2=50$ (green, dotted), or $\Omega=0.04$ and $g\sqrt{N}=100$ (blue, solid), or $\Omega=0.04\Gamma_A$ and $g\sqrt{N}=80$ (red, dashed), where the green, dotted-dashed, and dotted lines are related to the left axis, and the blue, solid and red, dashed lines are related to the right axis. Other parameters are given as $\Gamma_A=1$, $\Gamma_C=10^{-4}$ and $\omega_{ab}=10^6$.

$$\langle P\rangle = \langle p\rangle e^{-i\omega t} + \text{H.c.} \equiv \epsilon_0 \chi \langle \varepsilon\rangle e^{-i\omega t} + \text{H.c.}, \tag{9}$$

where $\chi=\langle p\rangle/(\langle\varepsilon\rangle\epsilon_0)$ is the susceptibility. Let μ denote the dipole moment between states $|a\rangle$ and $|b\rangle$. The average polarization

$$\langle p\rangle = \mu\left\langle \sum_{j=1}^{N} \sigma_{\text{ba}}^{(j)} \right\rangle \Big/ V = \frac{\mu\sqrt{N}}{V}\langle A\rangle \tag{10}$$

can be expressed here in terms of the average of the exciton operators A. Since the coupling coefficient $g=-\mu\sqrt{\omega/2V\epsilon_0}$, the susceptibility can be obtained as

$$\chi = \frac{2ig^2N(\Gamma_C - i(\Delta_p-\Delta_c))}{\omega[(\Gamma_A - i\Delta_p)(\Gamma_C - i(\Delta_p-\Delta_c)) + \Omega^2]}. \tag{11}$$

The real and imaginary parts χ_1 and χ_2 of this complex susceptibility $\chi=\chi_1+i\chi_2$ can be explicitly expressed as

$$\chi_1 = \frac{[(\Delta_p-\Delta_c)\Theta - \Gamma_C\Xi]F}{\Theta^2+\Xi^2}, \tag{12}$$

$$\chi_2 = \frac{[\Gamma_C\Theta + (\Delta_p-\Delta_c)\Xi]F}{\Theta^2+\Xi^2}, \tag{13}$$

where $F=2g^2N/\omega$ and

$$\begin{aligned}\Theta &= \Gamma_A\Gamma_C - \Delta_p(\Delta_p-\Delta_c) + \Omega^2,\\ \Xi &= \Delta_p\Gamma_A + \Gamma_A(\Delta_p-\Delta_c).\end{aligned} \tag{14}$$

It is well-known that χ_1 and χ_2 are related to dispersion and absorption, respectively. In Fig. 2, χ_1 and χ_2 are plotted versus the two-photon detuning $\Delta(=\Delta_p-\Delta_c)$. In Figs. 2(a2,b1,b2), $\Delta_c=0,\pm1.5$, respectively, and other parameters are fixed. When $\Delta\to 0$, both χ_1 and χ_2 are almost equal to zero. This result is consistent with that in the case of one-photon on-resonance EIT [11,14]. This fact shows that the medium indeed becomes transparent when driven by the classical control field as long as the system is prepared in the two-photon resonance ($\Delta=\Delta_p-\Delta_c=0$). We also notice that the width of the transparency window (which is determined by χ_2) also depends on the Rabi frequency Ω. It can be observed clearly from Fig. 2(a1) (where $\Delta_c=0$, $\Omega=2$) compared with Fig. 2(a2) (where $\Delta_c=0$, $\Omega=1/2$).

Next, we consider the properties of refraction and absorption of the single-mode probe light in the atomic ensemble medium in more detail. To this end, we analyze the complex refractive index

$$n(\omega) = \sqrt{\epsilon(\omega)} = \sqrt{1+\chi}, \tag{15}$$

and generally the real and imaginary parts, n_1 and n_2, of n are, respectively,

$$n_1 = \sqrt{\frac{[(1+\chi_1)^2+\chi_2^2]^{1/2} + (1+\chi_1)}{2}}, \tag{16}$$

$$n_2 = \sqrt{\frac{[(1+\chi_1)^2+\chi_2^2]^{1/2} - (1+\chi_1)}{2}}\,\text{sgn}(\chi_2), \tag{17}$$

where $\text{sgn}(\chi_2)=+1(-1)$ if $\chi_2>0(<0)$, n_1 represents the refractive index of the medium and n_2 is the associated absorption coefficient. Together with the formulas for the group velocity of the probe light

$$v_g(\Delta_p,\Delta_c) = \frac{c}{\text{Re}[n+\omega \mathrm{d}n/\mathrm{d}\omega]} = \frac{c}{n_1+\omega\dfrac{\mathrm{d}n_1}{\mathrm{d}\omega}} \tag{18}$$

(where c is the light velocity in vacuum) depending on the frequency dispersion, one can obtain the explicit expression for the group velocity v_g from Eqs. (12)–(16) for arbitrary reasonable values of Δ_p and Δ_c. Now, we consider the group velocity of the probe light v_g for the two-photon resonance, where χ_1 and χ_2 are almost zero. We find approximately

$$n_1 \simeq 1+\chi_1/2 \to 1, \quad n_2 \simeq \chi_2 \to 0,$$

and v_g is given briefly [25] as

$$\begin{aligned} v_g(\Delta_c) &= \frac{c}{n_1 + \omega\left.\dfrac{\mathrm{d}n_1}{\mathrm{d}\omega}\right|_{\Delta_p=\Delta_c}} \\ &= \frac{c}{1+\dfrac{\omega}{2}\left.\dfrac{\mathrm{d}\chi_1}{\mathrm{d}\omega}\right|_{\Delta_p=\Delta_c}}. \end{aligned} \tag{19}$$

It is worth pointing out that, in the calculation of the term $\mathrm{d}\chi_1/\mathrm{d}\omega$, $\Delta_p(=\omega-\omega_{ab})$ is a function of ω. In what follows we should perform a numerical calculation of $v_g(\Delta_c)$ by means of Eqs. (12) and (19) (and/or Eqs. (12)–(18)) since its analytical expression is redundant. According to Eq. (12), the group velocity $v_g(\Delta_c)$ of the weak probe field depends on Δ_c, Ω, and g^2N when given the other relevant parameters (typically, $\Gamma_A=1$, $\Gamma_C=10^{-4}$, $\omega_{ab}=10^6$).

Figure 3(a) shows the dependence of $v_g(\Delta_c)$ on the Rabi frequency Ω, where the blue, solid (red, dashed) line is drawn for $g\sqrt{N}=100$ ($g\sqrt{N}=80$). This provides one with a technique that can be used to accomplish the storage and retrieval of the probe pulse. Initially, when the probe field enters the atomic medium, the Rabi frequency Ω is very large (relative to $g\sqrt{N}$) and $v_g \to c$. When one reduces Ω adiabatically to zero, v_g reduces to zero accordingly and one can then store the pulse in the medium. Conversely, if one wants to retrieve the probe pulse, he needs only to increase Ω adiabatically so as to increase v_g. Figure 3(b) shows the dependence of $v_g(\Delta_c)$ versus the common detuning Δ_p $(=\Delta_c)$ under two-photon resonance EIT. When $\Omega \sim g\sqrt{N}$, v_g hardly depends on the detuning Δ_c and is close to the simplified result $c/(1+g^2N/\Omega^2)$ given in Ref. [14]. However, when $\Omega \ll g\sqrt{N}$, as denoted by the blue, solid and red, dashed curves in Fig. 3(b), v_g becomes very small and depends on Δ_c. In the symmetric spectral configuration we find that the group velocity v_g of the quantum probe light takes its minimum near the zero detuning, and v_g increases when $|\Delta_p|$ increases in the case of two-photon resonance. This theoretical result is consistent with the experimental phenomena as discovered in Ref. [22] when no buffer gases are used.

Finally, we notice that, in our model, the density of the medium is proportional to the atom number N. Figure 3(a) and 3(b) demonstrates how v_g depends on atomic density. In Fig. 3(a) and 3(b), the blue, solid curve is plotted for a denser medium ($g\sqrt{N}=100$ and g is given as constant) than that of the red, dashed curve ($g\sqrt{N}=80$). We also find that a denser medium leads to a slower v_g, consistent with our physical intuition.

In the present parameters given in this paper, the group velocity v_g is within the zone (0,c). It is remarked that v_g can be negative or superluminal in other Λ-atoms systems as in Refs. [24,25]. Different from our EIT system, a system consisting of Λ atoms coupled to three optical fields is studied in Ref. [24], and it is the issue of coherent-population trapping (not EIT) that is considered in Ref. [25]. A theoretical work [21] about the Λ-atoms EIT system shows that a negative group velocity can appear since the effect of atomic spatial motion (and/or the buffer gases) in the hot atoms is considered. Contrarily, the group velocity is always within $(0,c)$ in our EIT system. In our opinion, this difference is mainly due to our ignoring the effect of the atomic spatial motion and the buffer gases in this work.

In conclusion, based on the algebraic dynamics method, our theoretical studies on the light propagation in an atomic ensemble with two-photon resonance EIT show a similar phenomenon as discovered in the experiment in Ref. [22]. Our analysis ignores the generated Stokes field, which is also detected in the above experiment and described in Refs. [26,27]. We also neglect the influence of atomic spatial motion, atomic collisions, and the effects of buffer gases, since in principle these effects can be taken into account as the perturbations in our present study when the atomic ensemble is prepared under low enough temperature.

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