Decoherence of collective atomic spin states due to inhomogeneous coupling

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We investigate the decoherence of a superposition of symmetric collective internal states of an atomic ensemble due to inhomogeneous coupling to external control fields. For asymptotically large system, we find $\sqrt{N}$ as the characteristic decoherence rate scale with N being the total number of atoms. Our results shed new light on attempts for realizing quantum information processing and storage with atomic ensembles.

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Coherent quantum information encoding and processing has recently emerged as a major goal for the physics community. Despite the seemingly insurmountable difficulties, a rich variety of implementations are being pursued in laboratories across the globe. Among the early success is the apparent ability in simulating quantum operations with liquid based nuclear magnetic resonance (NMR) [1,2]. Recent theoretical efforts indicate, however, that in the pseudopure state approach using NMR, *quantum entanglement*, a key element for powerful quantum information processing, was, in fact, not present [3,4]. Room-temperature NMR technique, is therefore limited and cannot be explored fully to benefit from an exponentially large Hilbert space of only polynominally scaled resources and controls [5]. Nevertheless, early NMR based experiments have provided useful insight into the operations of genuine quantum computers [6]. Recently, seven effective qubits were used for a successful simulation of the Shor's algorithm with NMR technique [7].

Over the last few years, using symmetric collective internal states of an atomic ensemble has attracted much attention [8–12]. The pros and cons of such an approach assisted with cavity photon-atom interaction was recently discussed by Fleischhauer *et al.* [10,11]. In normal cavity QED based quantum computing implementations, atomic qubits are entangled and logic operations performed through their interaction with the common cavity photon quantum field. To maintain quantum coherence, it is important to reach the so-called-strong coupling regime, when the single-photon coherent coupling $g_0 \gg \gamma, \kappa$, the atomic and cavity dissipation or decoherence rates, respectively [13]. The symmetric collective internal states can reach the strong-coupling regime without requiring a high finesse cavity as $g_0 \propto \sqrt{N}$, with N the total number of atoms [14–16]. When implemented with protocols insensitive to individual atomic dissipation-decoherence rate as in the dark state based adiabatic transfer protocols [8,9,17], one can apparently gain an upper hand over systems based on single atoms inside a cavity [10,11]. This is, in fact, not quite surprising, earlier cavity QED experiments have relied on the enhanced dipole interaction of a collection of many atoms [14–16]. In free space, the phenomena of superfluorescence or super-radiance [14,18] constitutes another example of collective state dynamics. Recent experimental success clearly demonstrates the power of such an atomic ensemble based system for entangling macroscopic objects [19,20]. Several new ideas have raised further expectation of exciting developments to come [21,22]. Nevertheless, all ensemble based systems suffer from the reduced size of computational Hilbert space. In this case of symmetric collective internal states, the space used for quantum information is much less than the $\mathcal{V}^T = 2^N$ as for N two-level atoms [23]. In view of the recent experimental success in storage and recovery of light coherence in atomic gases [24,25], a related question to address is the sensitivity to errors when collective spin states are used as quantum memories.

In this paper, we investigate the decoherence for a superposition of symmetric internal states of an atomic gas due to its inhomogeneous coupling with external control fields. Our study is motivated by the simple observation that the symmetric states of an atomic ensemble spans the computation space only if atoms can be manipulated cooperatively. Namely, the coupling of both the external manipulating field and the environment surrounding the atomic ensemble should be homogeneous such that the collective motion of the atomic ensemble can be described by the collective quasispin operators. In essence, the effect of different spatial positions for atoms 1,2, . . . , and N, is ignored or absorbed into each single spin operators. In reality, an optically thick atomic ensemble suffers from inhomogeneous coupling to both classical and quantum light fields, i.e., the coupling strength is position dependent. Such a situation arises naturally for trapped ions due to its center-of-mass motion. In this case, it is well known that the loss of quantum coherence for a superposition of internal state occurs. In this study, we refer to such decoherence effect as inhomogeneous decoherence. We focus on introducing our technique and study the simplest example of superpositions of collective atomic Dicke states in this paper. The consideration of dark state, polariton approach based proposals [11] will be given in the future, as significant complications arise when the quantum cavity field is included.

Our model is comprised of an ensemble of two-level atoms described by the Hamiltonian

$$H=\sum_{k=1}^{N}\left[\frac{1}{2}\omega_a^{(k)}\sigma_z^{(k)}+\frac{1}{2}(g_0^{(k)}\sigma_+^{(k)}+\mathrm{H.c.})\right], \qquad (1)$$

where the σ's are the standard Pauli matrices $(\hbar=1)$, and the different local coupling $g_0^{(k)}$ (for the kth atom) may be due to a cavity mode profile as common in tightly focused cavities or when atomic motional wave packet is insufficiently localized [26]. $\omega_a^{(k)} = \varepsilon_a^{(k)} - \omega_L$ is the difference between atomic energy $\varepsilon_a^{(j)}$ and the near resonant laser frequency ω_L. For convenience, we further abstract Eq. (1) into the compact form $H = \Sigma_{k=1}^N \vec{B}^{(k)} \cdot \vec{\sigma}^{(k)}$ with real parameters $B_\mu^{(k)}$. This Hamiltonian represents the most general form for manipulating collective spin based quantum states using classical control fields.

The symmetric collective spin space $\mathcal{V}^S$ of dimension $2J+1 \ll 2^N$ ($J = N/2$) is spanned by the collective angular momentum states $\{|J,M\rangle, M = -J, \ldots, J-1, J\}$ of $J_\mu = \Sigma_{i=1}^N \sigma_\mu^{(i)}/2$ satisfying $[J_\mu, J_\nu] = i\epsilon_{\mu\nu\zeta} J_\zeta$ and $J_x^2 + J_y^2 + J_z^2 = \hat{J}^2 = J(J+1)$. $\epsilon_{\mu\nu\zeta}$ is the symmetric permutation tensor. The $|J,M\rangle$ space can be generated from the ladder operator $J_\pm = J_x \pm i J_y$ according to Ref. [27],

$$|J,M\rangle = \sqrt{\frac{(J-M)!}{(J+M)!(2J)!}} J_+^{J+M} |J,-J\rangle, \tag{2}$$

except we note that an arbitrary unimodular phasor can be self-consistently included with $J_\pm = \Sigma_{k=1}^N e^{\pm i\theta_k} \sigma_\pm^{(k)}/2$ and $|J,-J\rangle = |\downarrow, \downarrow, \ldots, \downarrow\rangle$.

For any realistic system, an inhomogeneous distribution of the parameter $\vec{B}^{(j)}$ makes it impossible to constrain the system dynamics within the subspace $\mathcal{V}^S$. To facilitate further discussion, denote $H = H_0 + H_1$ with $H_0 = \Sigma_{k=1}^N \vec{B} \cdot \vec{\sigma}^{(k)}$ and $H_1 = \Sigma_{k=1}^N \vec{b}^{(k)} \cdot \vec{\sigma}^{(k)}$, where $\vec{B}^{(k)} = \vec{B} + \vec{b}^{(k)}$ with $\vec{B} = \Sigma_k \vec{B}^{(k)}/N$. H_0 constitutes the intended coupling between the symmetric collective spin states, while H_1 represents a source of inhomogeneous decoherence. It causes decoherence as it provides a direct coupling from the subspace $\mathcal{V}^S$ to its complement $\mathcal{V}^O$ in $\mathcal{V}^T$. A quantitative measure for the unwanted coupling H_1 is in terms of the leakage parameter. Suppose initially the system is prepared in a superposition of collective spin states $|\phi(0)\rangle \in \mathcal{V}^S$. The intended dynamics governed by $U_0(t) = e^{-itH_0} = \Pi_{k=1}^N e^{-it\vec{B}\cdot\sigma^{(k)}}$ leads to the resultant state $|\phi(t)\rangle_0 = U_0(t)|\phi(0)\rangle$, still within the same subspace. The actual final state is $|\phi(t)\rangle = U(t)|\phi(0)\rangle$ with $U(t) = \Pi_{k=1}^N e^{-it\vec{B}^{(k)}\cdot\sigma^{(k)}}$, which will generally span more than $\mathcal{V}^S$. The leakage can therefore be defined as

$$\xi = 1 - |\langle \phi_0(t)|\phi(t)\rangle|^2. \tag{3}$$

$\xi = 0$ corresponds to no leakage, while $\xi \to 1$ indicates a complete loss of the system coherence and population. We note $|\langle \phi_0(t)|\phi(t)\rangle|^2$ is closely related to the usual definition of fidelity for a given quantum state operation. Our investigation of the model system decoherence behavior with this definition is motivated by recent interests in quantum information applications of using such collective states [8–12]. Benedict and Czirják have considered previously the decoherence of the so-called Schrodinger cat state of a similar system [28].

Denote $|\phi(0)\rangle \in \mathcal{V}^S$ as a normalized state expanded in terms of $|J,M\rangle$,

$$|\phi(0)\rangle = \sum_{M \ll N, \text{ or } M \sim N} c_M |J,M\rangle, \tag{4}$$

the overlap

$$\begin{aligned} |\langle \phi_0(t)|\phi(t)\rangle|^2 &= |\langle \phi(0)|U_0^\dagger(t)U(t)|\phi(0)\rangle|^2 \\ &= \sum_M \sum_{M'} c_{M'}^* c_M O_{M'M}(t) \leqslant 1, \end{aligned} \tag{5}$$

becomes the focus of our study with

$$\begin{aligned} O_{M'M}(t) &\equiv \langle J,M'|U_0^\dagger(t)U(t)|J,M\rangle \\ &= \langle J,M'|\prod_{k=1}^N O^{(k)}|J,M\rangle, \end{aligned} \tag{6}$$

$O^{(k)} = R^{(k)} + i\vec{I}^{(k)} \cdot \vec{\sigma}^{(k)}$, and

$$\begin{aligned} R^{(k)} &= \cos Bt \cos B^{(k)}t + (\hat{n}\cdot\hat{n}^{(k)}) \sin Bt \sin B^{(k)}t, \\ \vec{I}^{(k)} &= \hat{n} \sin Bt \cos B^{(k)}t + \hat{n}^{(k)} \cos Bt \sin B^{(k)}t + (\hat{n} \\ &\quad \times \hat{n}^{(k)}) \sin B \sin B^{(k)}t. \end{aligned} \tag{7}$$

We have defined $\hat{n} = \vec{B}/B$ and $\hat{n}^{(i)} = \vec{B}^{(i)}/B^{(i)}$.

When time t is small, a perturbative analysis of Eq. (6) can be carried out analytically. In this limit, we find that $O_{M'M}(t)$ decays with a time constant $\propto \sqrt{N}$. In general, however, the evaluation of Eq. (6) is difficult as state $|J,M\rangle$ involves a symmetric permutation of all atoms so that the Π_k factor cannot be pulled outside the inner product. Furthermore, $\Pi_{k=1}^N O^{(k)}$ expands into 2^N separate terms, involving asymmetric products of $\vec{\sigma}^{(k)}$ of up to powers of N. A similar product structure was found to be responsible for decoherence in quantum measurement models [29], where the decoherence factor (the overlaps of the final states of detector or a environment) suppresses the off-diagonal element of its reduced density matrix. In mathematical terms, for a factorized state $|f\rangle = \Pi_{k=1}^N |f^{(k)}\rangle$, the overlap integral $\langle f|\Pi_{k=1}^N W_{M'M}^{(k)}|f\rangle$ becomes $\Pi_{k=1}^N \langle f^{(k)}|W_{M'M}^{(k)}|f^{(k)}\rangle$, which approaches zero in the limit of macroscopic N as each factor $\langle f^{(k)}|W_{M'M}^{(k)}|f^{(k)}\rangle$ has a norm less than unity. To make a similar argument for the present problem, we need to find an expression such that the collective state $|J,M\rangle$ becomes factorized. Since we are interested in obtaining the asymptotically valid results in the limit of large N, a short-time approximation (small t) cannot be simply adopted. Following early discussions on atomic coherent states [30,31], we introduce

$$|\theta\rangle = \prod_{k=1}^N \frac{1}{\sqrt{2}} \left(1 + \frac{e^{i\theta}}{2} \sigma_+^{(k)}\right) |\downarrow\rangle = \frac{1}{2^{N/2}} e^{J_+ e^{i\theta}} |J,-J\rangle, \tag{8}$$

a phase coherent state that can be expanded according to the number of excitations

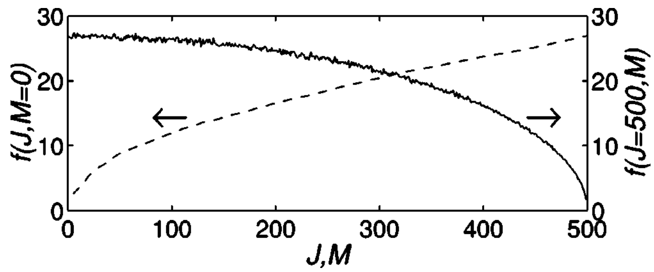

FIG. 1. M and J dependence of f for $J=500$ and $B_z=1$.

$$|\theta\rangle=\frac{1}{2^{N/2}}\left[1+e^{i\theta}J_{+}+\cdots+\frac{e^{in\theta}}{n!}J_{+}^{n}+\cdots\right]|J,-J\rangle$$
$$=\sum_{M=-J}^{J}\frac{e^{i(J+M)\theta}}{\mathcal{N}_{JM}}|J,M\rangle, \tag{9}$$

where $\mathcal{N}_{JM}=\sqrt{(J+M)!(J-M)!2^N/(2J)!}$. The inverse transformation gives

$$|J,M\rangle=\frac{\mathcal{N}_{JM}}{2\pi}\int_0^{2\pi}e^{-i(J+M)\theta}|\theta\rangle d\theta, \tag{10}$$

which helps to evaluate Eq. (6) as $O_{M'M}=\mathcal{N}_{JM}\mathcal{N}_{JM'}o_{M'M}$ with the reduced overlap

$$o_{M'M}=\frac{1}{4\pi^2}\int_0^{2\pi}d\theta\int_0^{2\pi}d\theta' e^{-i(J+M)\theta}e^{i(J+M')\theta'}$$
$$\times\prod_{k=1}^{N}G^{(k)}(\theta,\theta') \tag{11}$$

in a simple factorized form and

$$G^{(k)}=\frac{1}{2}{}_k\langle\downarrow|\left(1+e^{-i\theta'}\frac{\sigma_{-}^{(k)}}{2}\right)O^{(k)}\left(1+e^{i\theta}\frac{\sigma_{+}^{(k)}}{2}\right)|\downarrow\rangle_k .$$

$|G^{(k)}|\leqslant 1$ as both $(1+e^{i\theta}\sigma_{+}^{(k)}/2)|\downarrow\rangle_k/\sqrt{2}$ and $(1+e^{i\theta}\sigma_{+}^{(k)}/2)|\downarrow\rangle_k/\sqrt{2}$ are normalized. This points to a strong physical argument against rapid decoherence of collective spin state qubits. The question to answer is now clear: how does O_{MM} approach 0 due to inhomogeneous coupling. If the coupling coefficients $g_0^{(k)}$ and $\omega_a^{(k)}$ were constants (independent of atom label k), $O_{MM'}\equiv\delta_{MM'}$.

We now investigate the above question for several model cases of interest. First, we look at inhomogeneous broadening when $B_x^{(k)}=B_y^{(k)}=0$ and $B_z^{(k)}$ satisfies a normal distribution (with respect to k) with mean $\vec{B}=B_z\hat{z}=\langle B_z^{(k)}\rangle\hat{z}$ and variance σ_z^2. We find $O_{MM'}(t)\propto\delta_{MM'}$ with the coefficient being a constant unity for $|M|=J$ but decays with a time constant $T_{1/2}\propto 1/(\sqrt{N}\sigma_z)$ for $|M|<J$. Define $T_{1/2}\equiv 1/(f\sigma_z)$, we find f is essentially independent of σ_z for $\sigma_z\in[10^{-7},10^{-1}]B_z$. It contains an apparent dependence on J^2-M^2 as shown in Fig. 1 for a given J and B_z. The J dependence (for $M=0$) is also shown in the same figure. Based on our extensive numerical study, we find to a high level of accuracy

$$T_{1/2}(J,M,\sigma_z)=\frac{1}{\kappa\sigma_z\sqrt{J}\sqrt{1-M^2/J^2}}, \tag{12}$$

with κ (≈ 1.2), essentially independent of B_z for $B_z\in[10^{-2},10^2]$.

This result is to be expected based on the collapse and revival of a quantum wave packet [32], since each individual atom collapses with a time constant $\propto 1/\sigma_z$, the collective states of a Guassian ensemble should collapse with a time constant $\propto 1/(\sqrt{N}\sigma_z)$ as the net variance simply adds. This is indeed what we find for $M=0$ or, in general, for $|M|\ll J$. Equation (12) also indicates that significantly reduced decoherence does occur in this case for $|M|\sim J$, a regime where collective spin states are mostly useful [8–11,21,22]. In fact, for a single qubit quantum memory involving the two-state superposition of $M=-J$ and $-J+1$, one can easily check that the decoherence rate is just that of a single atom [10]. We caution, however, this corresponds to the special case of a coupling with $B_x=B_y=0$, which is of limited use as it does not allow for a general processing of collective spin superposition states. For small values of N, when ratios of different coupling strength $B_z^{(k)}$ match ratios of integers, we indeed were able to find the expected revival as shown in Fig. 2. This, of course, will not happen for an ensemble with a macroscopic N.

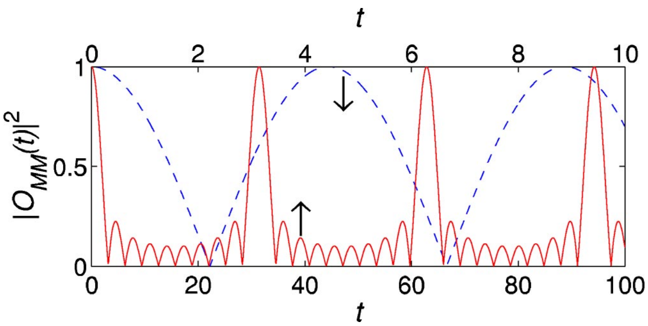

FIG. 2. Periodic behavior for $|O_{MM}(t)|^2$. The solid line denotes $M=4$ and $N=10$ with respect to the lower time axis, while the dashed line denotes $M=2$ and $N=2$; $B_z^{(k)}=k$ is taken for $N=10$ to assure the appearance of revival, (t is dimensionless). For $N=2$, revival, occurs for arbitrary random values of $B_z^{(k)}$.

Next we consider the case of inhomogeneous Rabi coupling with $B_{x/y}^{(k)}$ being Gaussian distributions with mean $B_x=B_y=B_r$ and variance $\sigma_x^2=\sigma_y^2=\sigma_r^2$, and $B_z^{(k)}=0$. Similar to the previous case, we find the diagonal term $O_{MM}(t)$ (including $M=\pm J$) decays with a time constant $T_{1/2}=1/f\sigma_r$. The J dependence of $T_{1/2}$ is in, fact, almost identical, i.e., $f_{M=0}=\kappa_1 J^{1/2}$, with $\kappa_1\approx 0.76$ when $B_r=10$. The M dependence, on the other hand, is more complicated as shown in Fig. 3. Obviously, f does not depend on M linearly as now $|O_{MM}(t)|$ seems to decay faster for larger values of $|M|$.

The off-diagonal element $O_{M\neq M'}(t)$ grows to significant nonzero values, as shown by the typical sampling of

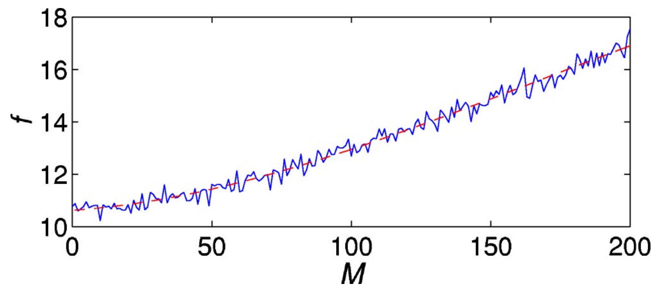

FIG. 3. The M dependence of f for $J=200$ and $B_r=10$. The smooth curve is a fit given by $-4.064\,83\times10^{-7}|M|^3+2.033\,93\times10^{-4}M^2+0.006\,97|M|+10.621\,88$.

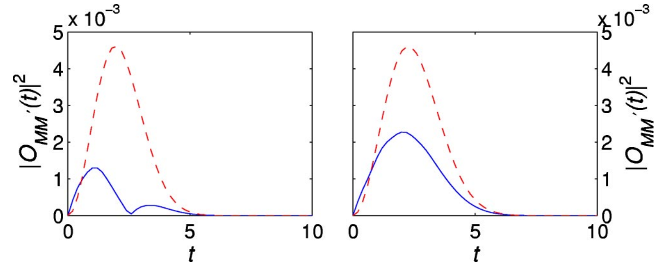

FIG. 4. Typical sampling of $O_{MM'}(t)$ for $J=100$, $M=J-1$, and $M'=M-1$ (solid line) and $M-2$ (dashed line). Different figures correspond to different random number sets.

$|O_{MM'}(t)|^2$ in Fig. 4 when N is not too large. Overall, we find the dependence on the random number sampling is strong only when $M-M'=\pm1$, so we focus on $M-M'=\pm2$ here. Define $T_{\max}$ as the time for $|O_{MM'}(t)|$ to reach its first maximum and $O_{\max}$ the value of the maximum. We find that similar to the diagonals, $T_{\max}=1/f\sigma_r$, with f a function of J,M,M', and B_r, although $O_{\max}$ seems to be largely independent of σ_r. To study the J dependence of f and $O_{\max}$, we consider the limiting case of $|M|\sim J$ when collective states are usually proposed to work. The result $f\propto J^{1/2}$ is once again as expected. In this case, we also find quite accurately $O_{\max}\propto J^{-1}$.

A naive conclusion from our investigation would be that when quantum state processing is attempted on a general coherent superposition state of collective spin ($|J,M_J\rangle$), the process is subject to enhance decoherence due to the inhomogeneous couplings with each individual atoms. In recent years, a special type of collective spin state, the so-called spin squeezed state (SSS), has attracted considerable attention [31]. Being a particular superposition of collective spin state $|J,M_J\rangle$, we expect much of our analysis also applies to quantum information processing with SSS. Another related topic of considerable recent interest is the quantum continuous variable representation of the collective spin for an atomic ensemble [19–21]. The decoherence of such systems constitutes an entirely different class of problem and is thus best addressed elsewhere.

Before concluding, let us briefly compare our studies with those of Ref. [28]. With the use of the Wigner function representation, Ref. [28] elucidated the decoherence and dissipation effects on maximum entangled Schrödinger cat states. The effect of decoherence was assumed to be due to a weak coupling with an external bath that consists of harmonic oscillators [28]. The use of the usual Born-Markov approximation then allowed for a compact analytic expression for decoherence and dissipation in terms of a superoperator on the system density matrix. In this study, however, we have investigated the decoherence effect of collective states involving a small number of excitations (as opposed to N excitations in Ref. [28]). Furthermore, our model of decoherence is due to inhomogeneous coupling rather than that of an external bath. Our results are obtained from rigorous numerical studies since we cannot use the Born-Markov approximation.

It is also interesting to address the question whether the powerful technique of "photon echo" [33,34] can be adopted in our model to slow down the rapid decoherence. According to the celebrated physical picture of echo experiments [34], an inhomogeneous broadened two-level ensemble can recover its coherence if its dynamics can be reversed. The possibility of echo, or recoherence, is due to the fact that an individual atom retains its own resonance frequency, with the ensemble distribution of which being the inhomogeneous broadening. In the model considered here, a different type of inhomogeneous distribution is involved. The coherent coupling for each atom now becomes different. Since any intended quantum operation (due to the coupling field) can be viewed as a tipping of an atomic dipole to a different direction on the Bloch sphere, an inhomogeneous distribution of control field strength tips individual atomic dipoles to different directions on the Bloch sphere. In contrast, individual dipoles are guaranteed to fall on the same equator of the Bloch sphere [33,34] in typical photon-echo arrangements. We thus conclude no simple schemes for recoherence in terms of a echolike mechanism is apparent.

To summarize, we find within our model, the apparent decoherence or dissipation rate for superpositions of collective spin states scales as $\sqrt{N}$. This evidence clearly demonstrates that asymptotically there is no advantage of using collective spin states for quantum information processing. The $\sqrt{N}$ enhanced coherent dynamics is simply being compensated by the $\sqrt{N}$ enhanced decoherence when inhomogeneous coupling arises. In the future, we will focus on applying the techniques as developed here to concrete quantum information processing protocols for superposition states of collective spins. To this end, we are unaware of any active experimental plan for quantum information processing based on collective spin states of an atomic ensemble.

Finally, we note that our result also applies to the case of entangled states between the collective spins of two separate ensembles. For instance, for two ensembles A and B, a state $\Sigma_{M_A,M_B}c_{M_A,M_B}|J_A,M_A\rangle_A|J_B,M_B\rangle_B$ can always be expressed as coherent superposition of the total angular momentum basis $\vec{J}=\vec{J}_A+\vec{J}_B$, i.e., into collective basis $|J_A,J_B;J,M=M_A+M_B\rangle$ with $J=J_A+J_B=(N_A+N_B)/2$.

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[1] D.G. Cory, A.F. Fahmy, and T.F. Havel, Proc. Natl. Acad. Sci. U.S.A. **94**, 1634 (1997).
[2] N.A. Gershenfeld and I.L. Chuang, Science **275**, 350 (1997).
[3] S.L. Braunstein, C.M. Caves, R. Jozsa, N. Linden, S. Popescu, and R. Schack, Phys. Rev. Lett. **83**, 1054 (1999).
[4] K. Zyczkowski, P. Horodecki, A. Sanpera, and M. Lewenstein, Phys. Rev. A **58**, 883 (1998).
[5] D.G. Cory *et al.*, Fortschr. Phys. **48**, 875 (2000).
[6] R. Laflamme, D.G. Cory, C. Negrevergne, and L. Viola, Quantum Inf. Comput. **2**, 166 (2002).
[7] L.M.K. Vandersypen, M. Steffen, G. Breyta, C.S. Yannoni, M.H. Sherwood, and I.L. Chuang, Nature (London) **414**, 883 (2001).
[8] M.D. Lukin, S.F. Yelin, and M. Fleischhauer, Phys. Rev. Lett. **84**, 4232 (2000).
[9] M. Fleischhauer and M.D. Lukin, Phys. Rev. Lett. **84**, 5094 (2000).
[10] M. Fleischhauer and C. Mewes, in *Proceedings of the International School "Enrico Fermi," Course CXLVIII*, edited by F. De. Martini and C. Monroe (IOS Press, Amsterdam, 2002), p. 511.
[11] M. Fleischhauer and M.D. Lukin, Phys. Rev. A **65**, 022314 (2002).
[12] M.D. Lukin, M. Fleischhauer, R. Cote, L.M. Duan, D. Jaksch, J.I. Cirac, and P. Zoller, Phys. Rev. Lett. **87**, 037901 (2001).
[13] O.G. Rempe *et al.*, Phys. Rev. Lett. **64**, 2783 (1990); G. Rempe *et al.*, *ibid.* **23**, 1727 (1991); M. Brune *et al.*, *ibid.* **76**, 1800 (1996); J.J. Childs *et al.*, *ibid.* **77**, 2901 (1996).
[14] Y. Kaluzny, P. Goy, M. Gross, J.M. Raimond, and S. Haroche, Phys. Rev. Lett. **51**, 1175 (1983).
[15] M.G. Raizen, R.J. Thompson, R.J. Brecha, H.J. Kimble, and H.J. Carmichael, Phys. Rev. Lett. **63**, 240 (1989).
[16] Yifu Zhu, Daniel J. Gauthier, S.E. Morin, Qilin Wu, H.J. Carmichael, and T.W. Mossberg, Phys. Rev. Lett. **64**, 2499 (1990).
[17] T. Pellizzari, S.A. Gardiner, J.I. Cirac, and P. Zoller, Phys. Rev. Lett. **75**, 3788 (1995).
[18] R. Bonifacio and A.L. Lugiato, Phys. Rev. A **12**, 587 (1975).
[19] A. Kuzmich, L. Mandel, and N.P. Bigelow, Phys. Rev. Lett. **85**, 1594 (2000).
[20] B. Julsgaard, A. Kozhekin, and E.S. Polzik, Nature (London) **413**, 400 (2001).
[21] Lu-Ming Duan, M. Lukin, I. Cirac, and P. Zoller, Nature (London) **414**, 413 (2001).
[22] M. Lukin and A. Imamoglu, Nature (London) **413**, 273 (2001).
[23] Julio Gea-Banacloche, Phys. Rev. A **62**, 062313 (2000).
[24] C. Liu, Z. Dutton, C.H. Behroozi, and L.V. Hau, Nature (London) **409**, 490 (2001).
[25] D.F. Phillips, A. Fleischhauer, A. Mair, R.L. Walsworth, and M.D. Lukin, Phys. Rev. Lett. **86**, 783 (2001).
[26] H.J. Carmichael and B.C. Sanders, Phys. Rev. A **60**, 2497 (1999).
[27] R.H. Dicke, Phys. Rev. **93**, 99 (1954).
[28] M.G. Benedict and A. Czirják, Phys. Rev. A **60**, 4034 (1999).
[29] C.P. Sun, Phys. Rev. A **48**, 878 (1993); C.P. Sun, X.F. Liu, D.L. Zhou, and S.X. Yu, Eur. Phys. J. Phys. D **17**, 85 (2001).
[30] G.S. Agarwal, D.H. Feng, L.M. Narducci, R. Gilmore, and R.A. Tuft, Phys. Rev. A **20**, 2040 (1979); J.M. Radcliffe, J. Phys. A **4**, 313 (1971); F.T. Arecchi, E. Courtens, R. Gilmore, and H. Thomas, Phys. Rev. A **6**, 2211 (1972).
[31] M. Kitagawa and M. Ueda, Phys. Rev. A **47**, 5138 (1993).
[32] J.H. Eberly, N.B. Narozhny, and J.J. Sanchez-Mondragon, Phys. Rev. Lett. **44**, 1323 (1980).
[33] E.L. Hahn, Phys. Rev. **80**, 580 (1950).
[34] *Optical Resonance and Two-level Atoms*, L. Allen and J.H. Eberley (Wiley, New York, 1975), p. 198, Chap. 9.