Consistent approach for quantum measurement

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In a close form without referring the time-dependent Hamiltonian to the total system, a consistent approach for quantum measurement is proposed based on Zurek's triple model of quantum decoherence [W. Zurek, Phys. Rev. D **24**, 1516 (1981)]. An exactly solvable model based on the intracavity system is dealt with in detail to demonstrate the central idea in our approach: by peeling off one collective variable of the measuring apparatus from its many degrees of freedom, as the pointer of the apparatus, the collective variable decouples with the internal environment formed by the effective internal variables, but still interacts with the measured system to form a triple entanglement among the measured system, the pointer, and the internal environment. As another mechanism to cause decoherence, the uncertainty of relative phase and its many-particle amplification can be summed up to an ideal entanglement or a Shmidt decomposition with respect to the preferred basis.

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I. INTRODUCTION

von Neumann's quantum theory of measurement emphasizes that, after a measurement, the emergence of classicality of a quantum system S from quantum dynamics is due to a perfect correlation between this system and its measuring apparatus A described by quantum mechanics [1]. But Zurek argued that this theory does not thoroughly solve the core problem in quantum measurement [2]. His argument is that the interaction between the quantum system and the apparatus can only produce a quantum entanglement like the Einstein-Podolsky-Rosen (EPR) state with quantum uncertainty [3], rather than a classical correlation described by a statistical operator with classical probability distribution $|c_s|^2$—the density matrix $\rho_c=\Sigma_s|c_s|^2|s\rangle\langle s|\otimes|p_s\rangle\langle p_s|$, where $|s\rangle$ and $|p_s\rangle$ are orthonormal basis vectors of the system to be measured and the pointer state of the apparatus, respectively. To go beyond von Neumann's theory, Zurek proposed an elegant "triple model" for quantum measurement 20 years ago. In his theory, besides the quantum system and the apparatus, an environment E must be introduced as a necessary element to generate the triple entanglement

$$|\Phi_{tri}\rangle=\sum_s c_s|s\rangle\otimes|p_s\rangle\otimes|e_s\rangle \tag{1}$$

through the coupling of the apparatus to the environment. It is obvious that the classical mixture state ρ_c of correlation can be obtained by ignoring (mathematically "tracing over") the environment states.

Zurek's triple model, in principle, overcomes the key difficulty in quantum measurement theory, but it still needs microscopic refinement in terms of quantum dynamics and there remain details to be filled in. Actually, just as Zurek points out, to implement such triple entanglement as dynamic Schmidt decomposition, the interactions among the triple parts should be time dependent. To be more precise, two steps are required to finish the measurement: turning on the interaction H_{SA} between S and A at the instant $t=0$ and turning on the interaction H_{AE} between A and E at another instant $t=t_m$. The process can be represented as follows:

$$\sum_s c_s|s\rangle\otimes|p\rangle\otimes|e\rangle_{t=0}\overset{H_{SA}}{\rightarrow}\times\left(\sum_s c_s|s\rangle\otimes|p_s\rangle\right)\otimes|e\rangle_{t=t_m}\overset{H_{AE}}{\rightarrow}|\Phi_{tri}\rangle. \tag{2}$$

However, the time dependence of the Hamiltonian means that there exists another extra system governing the "universe" formed by the triple system. So the quantum dynamic theory describing the measurement is not in a close form. Moreover, to realize a real measurement process, one should switch the couplings at certain exact instants. In practice, it is difficult to exactly control the interaction between A and E so that it occurs only after the correlation between A and S has just been established. Another point we wish to mention is that according to Zurek's model, to produce an ideal triple entanglement such that $\{|s\rangle\},\{|p_s\rangle\}$, and $\{|e_s\rangle\}$ form three orthonormal sets, it is even required that there is no interaction between S and E as described in Eq. (1); otherwise the Schmidt decomposition structure of Eq. (1) would be destroyed.

Most recently, we studied the phenomenon of quantum decoherence of a macroscopic object along a different direction: we investigated the adiabatic quantum entanglement [4] between its collective states [such as that of the center-of-mass (c.m.)] and its inner states. It is shown that the adiabatic wave function of a macroscopic object can be written as an entangled state with correlation between adiabatic inner states and quasiclassical motion configurations of the c.m. Since the adiabatic inner states are factorized with respect to the composing parts of the macroscopic object [5], this adiabatic separation can induce quantum decoherence. This ob-

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servation thus provides us with a possible solution to the Schrödinger cat (macroscopic quantum interference) paradox. This approach to quantum decoherence only concerns a double system rather than a triple one, so it does not solve the quantum measurement problem completely. Rather, it provides an example for von Neumann's quantum measurement theory, which does not produce a classical correlation.

In this paper, integrating the above-mentioned results with Zurek's triple model, we present a consistent quantum-mechanical approach for measurement process in a close form with a time-independent total Hamiltonian. In this alternative, the measuring apparatus is taken as a macroscopic object with effective inner variables and a pointer variable. In our treatment, the complete separation of the pointer variable from the effective inner variables of the macroscopic apparatus A is carried out. With this separation there is no coupling between the pointer variable and the inner variables of A, but the effective interaction of the pointer with S is induced by that of the original variables of A in an adequate way. Just for this reason, the triple entanglement [2] can be dynamically generated without the time-dependent control. To sketch our basic idea, we start with an exactly solvable model in the intracavity dynamics. Using this example we also show that the back action of the inner environment on the system plus pointer implied by Heisenberg's position-momentum uncertainty relation will disturb the phases of the states between the system and pointer and then decoher the quantum entanglement system plus pointer, which is formed dynamically just before measurement.

II. OUTLINE OF OUR APPROACH FOR QUANTUM MEASUREMENT BASED ON ZUREK'S THEORY

The quantum theory of measurement based on von Neumann's theory usually treats the measuring process as a quantum-mechanical evolution by considering the measuring apparatus A as a proper quantum system. This is just in contrast to the Copenhagen interpretation with the hypothesis of classicality on the part of the apparatus. According to the theory of the Copenhagen school, the apparatus should behave classically so that the experimental outcome of measurement can be recorded in the classical way. Zurek's theory does not stress the classicality of apparatus directly since the meaning of classicality of apparatus is not clear without association with the measured system S. The important discovery by Zurek is the decoherence of the quantum entanglement between the measuring apparatus and the measured system induced by an external or inner environment E [6]. Led by Zurek's observation one may imagine that it is the direct interaction of the environment with the pointer of apparatus that leads to the classicality of apparatus. However, it is not true.

In the following, we can show that to decoher the quantum entanglement between A and S, only two proper couplings of the measured system to the pointer and to the environment are needed and the interaction between A and E is not necessary. The requirement of no interaction between A and E will result in a time-independent reformulation of Zurek's triple theory . In our quantum approach of measurement by generalizing Zurek's triple theory, we consider E as the collection of the internal relative degrees of freedom. Then, the pointer (also denoted by A) of the apparatus can be defined as the collective (or macroscopic) degree of freedom of the apparatus, e.g., the coordinate of the c.m.

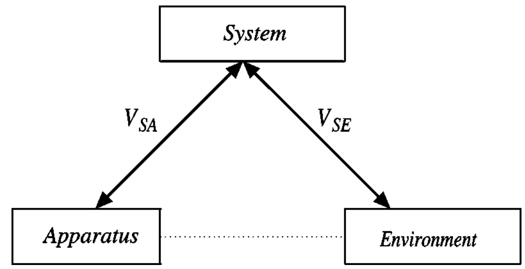

FIG. 1. The effective interactions among the system S, the pointer A, and the inner environment E.

In general, we write a time-independent total Hamiltonian as

$$H = H_S + H_A + H_E + V_{SA} + V_{SE}. \tag{3}$$

Here $H_S = H_S(q_S)$, $H_A = H_A(q_A)$, and $H_E = H_E(q_E)$ are, respectively, the free Hamiltonian for S, A, and E; q_S, q_A, and q_E roughly stand for the system variable, the pointer variable and, the environment variable, correspondingly; $V_{sa} = V_{SA}(q_A, q_S)$ describes the interaction between S and A while $V_{SE} = V_{SE}(q_E, q_S)$ describes that between S and E.

To gain a close form for quantum measurement based on Zurek's triple theory, it is important that no interaction exists between the pointer A and the ''inner'' environment E (in Fig. 1). Only by assuming that the system satisfies the following double nondemolition condition:

$$[H_S, V_{SA}] = 0, \quad [H_S, V_{SE}] = 0. \tag{4}$$

The evolution operator for the total system can be written as

$$U(t) = \sum_n e^{-iE_n t} |n\rangle_{SS}\langle n| U_{An}(t) \otimes U_{En}(t). \tag{5}$$

Here, $\{|n\rangle_S\}$ is an eigenvector of H_S corresponding to the eigenvalue E_n,

$$U_{An} = {}_S\langle n| \exp[H_A(q_A) + V_{SA}(q_A, q_S)] |n\rangle_S$$

and

$$U_{En} = {}_S\langle n| \exp[-i\{H_E(q_E) + V_{SE}(q_E, q_S)\}] |n\rangle_S$$

are the effective evolution operators. They describe the feedbacks of the measured system on the pointer of apparatus and the environment, respectively, when S is just in its eigenstate $|n\rangle_S$. It is worthy to point out that in the present approach for entanglement, the energy of the measured system is conserved while the quantum coherence is destroyed.

This kind of unitary evolution operator $U(t)$ can establish a nonseparable correlation among the system, the pointer, and the environment. Namely, if the initial state $|\Psi\rangle_{initial} = |S\rangle \otimes |A\rangle \otimes |E\rangle$ of the total system is of a factorized form with the system state $|S\rangle=\Sigma_n c_n|n\rangle_S$, the pointer state $|A\rangle$, and the environment state $|E\rangle$, then the final state of the total system will be

$$|\Psi\rangle_{final}=U(t)|\Psi\rangle_{initial}=\sum_n c_n e^{-iE_n t}|n\rangle_S\otimes|A_n\rangle\otimes|E_n\rangle. \tag{6}$$

Here, $|A_n\rangle=U_{An}|A\rangle$ and $|E_n\rangle=U_{En}|E\rangle$ are the final states of the pointer and the environment entangling with the system states $|n\rangle_S$. Thus, a triple correlation among the measured system, the pointer, and the environment is established. Obviously, the reduced density matrix for the composite subsystem formed by the system plus the pointer is

$$\rho=\sum_n |c_n|^2|n\rangle_{SS}\langle n|\otimes|A_n\rangle \times\left\langle A_n\right|+\sum_{m\neq n} c_m^* c_n\left|n\right\rangle_{SS}\langle m|\otimes|A_n\rangle\langle A_m|\langle E_m|E_n\rangle. \tag{7}$$

The off-diagonal terms on the right-hand side of this equation is responsible for the interference pattern. It is easy to see that the interference fringe completely vanishes when the states of the inner part E are orthogonal to one another. In this situation, an ideal Zurek's classical correlation

$$\rho=\sum_n |c_n|^2|n\rangle_{SS}\langle n|\otimes|A_n\rangle\langle A_n| \tag{8}$$

results from the ideal entanglement with the correlated components $|E_n\rangle$ orthogonal to one another.

The above Zurek's classical correlation [2] just describes the fact like the weather forecast impersonally predicting whether it rains or not tomorrow. Equation (7) deterministically tells us the classical correlation that the system is in $|n\rangle$ when the pointer is just in $|A_n\rangle$ with probability $|c_n|^2$. This is unlike the quantum entanglement $|S\rangle=\Sigma_n c_n|n\rangle\otimes|A_n\rangle$ that not only indicates the correlation between $|n\rangle$ and $|A_n\rangle$, but also simultaneously tells us the correlation with probability $p_n=\Sigma_{n'}|s^{-1}_{n'n}c_{n'}|^2$ between any superposition state $|S_n\rangle=\Sigma_{n'}s_{nn'}|n'\rangle$ of S and the corresponding one

$$|t_n\rangle=\sqrt{\frac{1}{p_n}}\sum_{n'} s^{-1}_{n'n}c_{n'}|A_{n'}\rangle$$

of A. This is because $|S\rangle$ can also be reexpressed as

$$|S\rangle=\sum_n p_n|S_n\rangle\otimes|t_n\rangle. \tag{9}$$

In fact, the classical correlation does not say anything about the correlation of different pairs $|S_n\rangle$ and $|t_n\rangle$ but for the original pair $|n\rangle$ and $|A_n\rangle$, and its prediction is independent of what to be measured. On the contrary, what the quantum entanglement tells us depends on what we measure according to the EPR argument [3]. With this understanding, it can be said that quantum measurement is implemented completely when quantum decoherence happens to result in the so-called classical correlation.

The above argument shows that the vanishing overlap of the final states of the inner environment is necessary to obtain the classical correlation after measurement. We define this overlap $F_{m,n}=\langle E_m|E_n\rangle$ as the decoherence factor. Now an immediately following question is in what case the decoherence factor becomes zero. Our previous works on quantum measurement theory [5] showed that an ideal entanglement appears in the macroscopic limit where the number N of particles making up the detector approaches infinity. In the present case, we assume that there are N degrees of freedom in the apparatus. We can peel off one (or more) collective variable as the pointer of the apparatus, and the inner environment is formed by N relative internal variables $q_k (k=1,2,\ldots,N)$. We can imagine that there are N blocks constituting the inner environment, so we may write $H_E(q_E)=\Sigma_k H_E^{(k)}(q_k)$ and $V_{SE}(q_E,q_S)=\Sigma_k V_{SE}^{(k)}(q_k,q_S)$ in sum forms. If all $V_{SE}^{(k)}(q_k,q_S)$ $(k=1,2,\ldots,N)$ commute with one another, we can factorize the effective evolution operator U_{En}:

$$U_{En}=\prod_{j=1}^{N} U_{En}^{[j]}.$$

When the measured system is initially prepared in $|n\rangle$ and the environment in a factorized state $|E\rangle=\Pi_{j=1}^N|E^{[j]}\rangle$, the environment will obey a factorized evolution

$$|E\rangle\rightarrow|E_n^{[j]}\rangle\equiv\prod_{j=1}^{N}|E_n^{[j]}\rangle=\prod_{j=1}^{N}U_{En}^{[j]}|E^{[j]}\rangle q \tag{10}$$

entangling with the system state $|n\rangle$. It results in the factorization structure [5] of the decoherence factor

$$F_{m,n}=\prod_{j=1}^{N}\langle E_m^{[j]}|E_n^{[j]}\rangle. \tag{11}$$

Since each factor $\langle E_m^{[j]}|E_n^{[j]}\rangle$ in $F_{m,n}$ has a norm less than unity, the product of infinite such factors may approach zero. This investigation was developed based on the Hepp-Coleman model and its generalizations [7,8] and was applied to analyzing the universality of the influence of the environment on the quantum computing process [9].

III. POINTER MODEL FOR QUANTUM MEASUREMENT IN INTRACAVITY SYSTEM

In this section and the subsequent sections, we will use an exactly solvable model based on the over simplified intracavity system to demonstrate our central ideas.

Consider a cavity with two end mirrors (as in Fig. 2), one of which is fixed while the other is treated as a macroscopic object consisting of N particles of mass m_i with position coordinate x_i and momentum coordinate p_i $(i=1,2,\ldots,N)$. The radiation pressure of the cavity field on the moving mirror is proportional to the intracavity photon

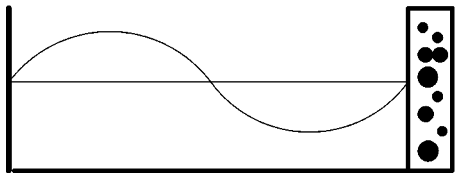

FIG. 2. A cavity with a moving end mirror B and a fixed one A. The moving one is treated as a macroscopic object consisting of N free particles.

density. Let $a^\dagger$ and a be the creation and annihilation operators of the cavity with a single mode of frequency ω. The cavity-mirror coupling is described by an interaction Hamiltonian $H_I = -\Sigma_i^N g_i x_i a^\dagger a$, where g_i is coupling constant depending on the electric dipole.

In this situation we describe the cavity-field dynamics with the free Hamiltonian $H_S = \omega_0 a^\dagger a$. This cavity-field-mirror coupling system can also be used to detect the photon number in the cavity by the motion of the mirror. Obviously, the total Hamiltonian governing the motion of the mirror is

$$H = \omega_0 a^\dagger a + \sum_{i=1}^{N} \frac{p_i^2}{2m_i} - a^\dagger a \sum_{i}^{N} g_i x_i . \tag{12}$$

By taking the moving mirror as a whole, this intracavity model is associated with the interferometric detection of the gravitational wave [10–12]. This system has already been studied quite extensively by many authors [12], under the assumption that the moving mirror is a macrocosm without internal structure. Like the discussions in these previous references [10–12], we also assume that the oscillation of the movable mirror is so small that the frequency of the single-cavity mode does not undergo a considerable change due to the displacement of the mirror.

The above-described physical system can be viewed as a quantum measurement system, in which the cavity field monitored by the macroscopic moving mirror is to be measured. Since $[H_S, H_I] = 0$ and $[H_S, \Sigma_i p_i^2/2m_i] = 0$, the moving mirror has no influence on the cavity field. On the other hand, different eigenstates $|n\rangle$ of H_s can imprint on the moving mirror differently. Indeed, the interaction term $-n\Sigma_i^N g_i x_i$ implies that different forces will act on the moving mirror when the cavity is prepared in different number states $|n\rangle$. We notice that this is a typical characteristic of nondemolition measurement [13].

We take the c.m. position $x = M^{-1}\Sigma_i^N m_i x_i$ to be the pointer of the apparatus—the moving mirror and the relative coordinates $\xi_j = x_j - x (j = 1,2,\ldots,N-1)$ to be the internal variables. Here, $M = \Sigma_{i=1}^N m_i$ is the total mass of the moving mirror. We denote the conjugate momenta of x and ξ_j by p_x and $p_{\xi j}$. Then, we obtain an interesting realization of Zurek' s triple model with the time-independent Hamiltonian

$$H = H_S + H_A + H_E + V_{SA} + V_{SE} , \tag{13}$$

where $H_A = p_x^2/2M$ and

$$H_E = \frac{1}{2} \sum_{i,j=1}^{N-1} \tau^{-1}{}_{ij} p_{\xi i} p_{\xi j} \tag{14}$$

are the free Hamiltonians for the pointer part and the internal environment, respectively,

$$V_{SA} = -G a^\dagger a x \tag{15}$$

is the effective interaction describing the coupling of the system to the pointer, and

$$V_{SE} = -\sum_{j=1}^{N-1} G_i a^\dagger a \xi_j \tag{16}$$

describes an interaction between the system and the internal environment. Here

$$G_i = \left(g_i - \frac{m_i}{m_N} g_N \right), \quad G = \sum_{i=1}^{N-1} g_i \tag{17}$$

are the effective coupling constants, and the mass matrix τ is defined by the matrix elements

$$\tau_{ij} = m_i \delta_{ij} + \frac{m_i m_j}{m_N}. \tag{18}$$

This expression of τ is obtained by substituting the individual laboratory coordinate

$$x_N = \frac{1}{m_N} \left[x - \frac{1}{M} \sum_{i}^{N-1} m_i (\xi_j + x) \right] \tag{19}$$

into the interaction term $-a^\dagger a \Sigma_i^N g_i x_i$ of the total system Hamiltonian.

In this model the couplings of the system to the pointer and to the environment can be turned on or turned off simultaneously, for V_{SA} and V_{SE} are proportional to the same coupling constant. Physically this is reasonable as the pointer variable is a reflection of the collective average effect of the internal degrees of freedom of the macroscopic apparatus. Another remarkable character of the above model is the absence of coupling between the pointer variable and the inner variables of A. Physically this is just what is required of an ideal measuring apparatus, whose effective inner motion should not directly affect the reading of the pointer. In fact this property guarantees that a triple entanglement will form dynamically from a factorized initial state. This just realizes our central ideas in Sec. II: the decoupling of pointer variable with the inner ones in the apparatus and the factorization of the inner environment with respect to each inner variable.

IV. EXACT SOLUTION WITH FACTORIZATION FOR IDEAL ENTANGLEMENT

Now let us consider the exact solution to the dynamic evolution problem of the intracavity model introduced in Sec. II. To this end we invoke a canonical transformation

$$\eta_i=\sum_{j=1}^{N-1} U_{ij}\xi_j, \quad i,j=1,\ldots,N-1 \tag{20}$$

from the inner variables ξ_j to a new set of canonical variables η_i. Here, U is an $(N-1)(N-1)$ matrix diagonalizing the mass matrix τ, i.e., $U\tau U^T$ is diagonal. Denote the conjugate momenta of x and η_i by p_x and p_i, respectively. The inner environment is described by the new internal variable $\{\eta_i\}$. Then, the total Hamiltonian can be reexpressed as

$$H=\frac{p_x^2}{2M}-Ga^\dagger ax+\sum_{i=1}^{N-1}\left(\frac{1}{2m_i'}p_i^2-f_ia^\dagger a\,\eta_i\right)+\omega_0 a^\dagger a, \tag{21}$$

where the eigenvalues m_i' of matrix τ are the effective masses with respect to the new coordinates $\{\eta_j\}$; $f_i=\Sigma_j G_j U_{ji}^T$ represent the strengths of forces on each inner coordinate η_i by one photon of the cavity field. Obviously, the inner motion of the moving mirror is factorizable since the effective Hamiltonian $H_{SE}=\Sigma H_{SE}^j$ is only a simple sum of the single component,

$$H_{SE}^j(a^\dagger a)=\frac{1}{2m_i'}p_i^2-f_ia^\dagger a\,\eta_i. \tag{22}$$

This direct sum structure results in the factorization of the effective evolution matrix defined by

$$U_{En}(t)=\langle n|U(t)|n\rangle=\prod_{i=1}^{N-1} U_{En}^i(t),$$

$$U_{En}^i(t)=\exp\left[-i\left(\frac{p_i^2}{2m_i'}-nf_i\eta_i\right)t\right] \tag{23}$$

$$=e^{-itp_i^2/2m_i'}e^{inf_it^2p_i/2m_i'}\exp\left[inf_it\,\eta_i-i\frac{n^2f_i^2t^3}{6m_i'}\right], \tag{24}$$

where $|n\rangle$ is the Fock state and $U(t)=\exp(-iHt)$ is the evolution matrix of the total system. Here, we have used the Wei-Norman algebraic method [14] to rewrite the single-particle evolution matrix.

Without loss of generality, we assume each inner component of the moving mirror to be described by the same Gaussian wave packet $|i\rangle$ of width a, i.e.,

$$\langle\eta_i|i\rangle=\left(\frac{1}{2\pi a^2}\right)^{1/4}e^{-\eta_i^2/4a^2}. \tag{25}$$

This is a physically reasonable preparation of initial state. This is because the initial state of the inner $\Phi(0,\{\eta_j\})=\Pi_{i=1}^{N-1}\langle\eta_i|i\rangle$ also defines the factorized Gaussian wave packet $\Phi(0,\{\xi_i\})=\Pi_{i=1}^{N-1}\langle\xi_i|i\rangle$ in ξ representation, since $\Sigma_i\xi_i^2$ is a canonical invariant, i.e., $\Sigma_i\xi_i^2=\Sigma_i\eta_i^2$. A Gaussian wave packet $\langle\xi_i|i\rangle\simeq\exp[-(x_i-x)^2/4a^2]$ implies basically that the particles composing the moving mirror are almost peaked on the position of c.m. We also assume that the c.m. of the moving mirror is just in the position eigenstate $|X\rangle$ and the cavity is initially in a coherent state $|\alpha\rangle=\Sigma_n c_n|n\rangle$, where $c_n=e^{-1/2|\alpha|^2}\alpha^n/n!$. In this case using the effective evolution matrix of the pointer,

$$U_{nx}=\exp\left[-i\left(\frac{p_x^2}{2M}-Gnx\right)t\right], \tag{26}$$

with respect to the cavity Fock state $|n\rangle$, we explicitly obtain the triple entanglement

$$|\Psi(t)\rangle=\sum_n c_n(t)|n\rangle\otimes|x_n\rangle\otimes|E_n\rangle \tag{27}$$

at any instance $t\neq0$. Here,

$$c_n(t)=c_ne^{-i(n+1/2)\omega_0t}, \quad |x_n\rangle=U_{nx}|x\rangle,$$

and

$$|E_n\rangle=U_{En}(t)|\Phi(0)\rangle=\prod_{j=1}^{N} U_{En}^j(t)|j\rangle.$$

We can also calculate the decoherence factor with factorization

$$F_{mn}(t)=\langle E_n|E_m\rangle=\prod_{j=1}^{N}\langle E_n^{[j]}|E_m^{[j]}\rangle$$

$$\equiv\prod_{j=1}^{N}\langle j|U_{En}^{j\dagger}(t)U_{Em}^j(t)j\rangle$$

to give the decaying norm

$$f_{mn}(t)=|F_{mn}(t)|$$
$$=\exp\left[-(n-m)^2\sum_{j=1}^{N-1}f_i^2\left(\frac{t^4}{32m_i'^2a^2}+\frac{a^2t^2}{2}\right)\right]. \tag{28}$$

In Fig. 2, the decaying behavior of the decoherence factor is demonstrated for different N. It is seen that a decoherence process indeed happens as $t\to\infty$, but it does not obey the simple exponential law $e^{-\gamma t}$. In a long-time scale, the temporal behavior of decoherence is described by

$$F(t)\approx\exp[-(m-n)^2\Gamma t^4]$$

with $\Gamma=\Sigma_{j=1}^{N-1}f_i^2/32m_i'^2a^2$. If we define the characteristic time τ_d of the decoherence process by $F(\tau_d)=e^{-1}$, then (see Fig. 3)

$$\tau_d=[(m-n)^2\Gamma]^{1/4}. \tag{29}$$

This shows that the long-time behavior of decoherence depends directly on interaction.

In the macroscopic limit $N\to\infty$ or for the long-time evolution $t\to\infty$, the vanishing decoherence factors $\langle E_n|E_m\rangle$

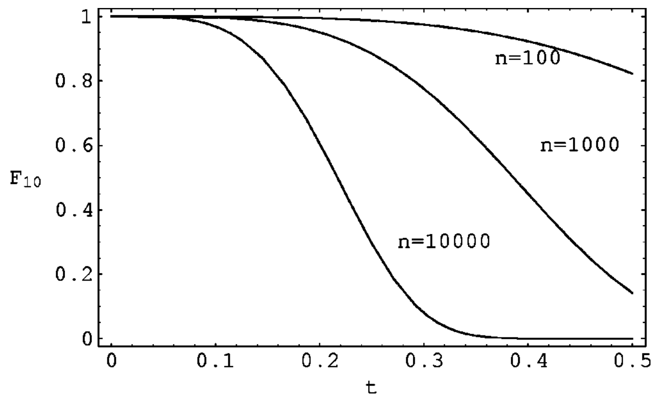

FIG. 3. The decoherence factor F_{10} plotted as a function of time and for various values of the particle number N. We have taken the mass of a particle, $m=10^{-6}$ kg; the wave width of the Gaussian wave packet as the length of the cavity, $a=10^{-5}$ m; and the force strength on each inner coordinate, $f=10^{-14}\ \mathrm{kg\,m\,s^{-2}}$. The time unit is second. The real particle number $N=10^6 n$.

leads to an ideal triple entanglement (1) with invariant probability distribution $p_n=|c_n(t)|^2=|c_n|^2$.

Finally we need to show that the measuring process we modeled above is ideal since the pointer states entangling with each system state are orthogonal to one another, namely, $\langle x_n|x_m\rangle\sim\delta_{n,m}$ for $m\neq n$. In fact, in the coordinate representation, the pointer state $|x_n\rangle$ can be calculated explicitly as

$$\langle x|x_n\rangle=\sqrt{\frac{-iM}{t}}\exp\left[i\left\{\frac{[-nGt^2+2M(x-X)]^2}{8Mt}+\left(nGtx-\frac{n^2G^2t^3}{6M}\right)\right\}\right]. \tag{30}$$

This means that the width of each wave packet $\langle x|x_n(t)\rangle$ is zero and then the overlaps

$$|O_{n,m}|=|\langle x_n(t)|x_m(t)\rangle|=\frac{8\pi M}{3Gt^2}\delta(m-n) \tag{31}$$

of wave packets vanish for $m\neq n$. This indicates an ideal classical correlation between the measured system and the pointer of the apparatus in our intracavity model. Therefore, our triple model for quantum measurement leads to an ideal quantum measurement in a purely quantum dynamical way with neither the introduction of the hypothesis of classicality for the apparatus, nor the artificial control of interactions.

As shown above, an ideal entanglement of the double system (formed by the measured system plus pointer of apparatus) with an (inner) environment is a necessary element to force the apparatus to behave so classically that a deentanglement process occurs in the double system. Another necessary element to implement quantum measurement is the ideal entanglement between the measured system and the pointer of apparatus. The work of this mechanism depends on the choice of the form of the initial state of the pointer. For a general initial state of the pointer, it is difficult to obtain an explicit condition under which $\langle x_n|x_m\rangle$ becomes zero. The solution to this problem concerns the consideration of the classical limit of the motion of the pointer as a large system. In general, for certain particular states, the classical limit of the expectation value of an observable should recover its classical value form. Such quasiclassical states can give definite classical trajectories of a particle in the classical case. In this sense the mean-square deviation of the observable is zero, and accordingly the expectation value of the position operator defines a classical path. In the following section we will discuss a similar problem in greater detail in association with the random phase induced by Heisenberg's momentum-coordinate uncertainty.

V. CLASSICALITY OF APPARATUS DUE TO POSITION-MOMENTUM UNCERTAINTY

In the above discussion the quantum realization of measuring process boils down to the appearance of decoherence in the entanglement between the measured system and the measuring apparatus only due to the coupling of the measured system with the inner environment rather than that between the pointer of the apparatus and the inner environment.

It is well known that quantum decoherence can be explained in two ways: the usual explanation for decoherence in a which-way experiment based on Heisenberg's position-momentum uncertainty relation [15], and the current explanation based on quantum entanglement, which is not related to this uncertainty relation directly. In the latter explanation, the quantum correlations between the environment and the considered system are responsible for the destruction of quantum coherence. In the present paper the considered system is a composition formed by the measured system plus the pointer. In fact, in the previous sections we have adopted the second viewpoint to deal with the quantum measurement problem. In this section we will see that the first explanation can also work well in our modeled quantum measurement problem. To this end, we first show that the back action of the inner environment on the system plus pointer implied by Heisenberg's position-momentum uncertainty relation will disturb the phases of states of the system plus pointer. This result is shown in Ref. [16].

Let us return to our model mentioned in the preceding section. We assume that the initial state $|j\rangle$ of each component of the inner environment is a real wave packet, which is symmetric with respect to both the canonical coordinate η_j and the corresponding canonical momentum p_j,

$$\langle\eta_j\rangle\equiv\langle j|\eta_j|j\rangle=0,\ \ \langle p_j\rangle=0. \tag{32}$$

We will not need its concrete form. Rather we assume it to be of the Gaussian type with the variance $a_j=\Delta\eta_j$ in η_j space. Here, we adopt the definition of the standard deviation

$$\Delta\phi\equiv\sqrt{\langle(\phi-\langle\phi\rangle)^2\rangle}=\sqrt{\langle\phi^2\rangle-\langle\phi\rangle^2} \tag{33}$$

for a given phase ϕ. Physically, once $\Delta\eta_j$ is given, the variance of p_j cannot be arbitrary since there is Heisenberg's position-momentum uncertainty relation $\Delta\eta_j\Delta p_j\geqslant\frac{1}{2}$. In the previous two sections, the conclusion drawn seems to depend on the choice of the concrete form of the initial state, but now we can argue that this is not the case with the above consideration.

After a measurement, the final state of j's environment component η_j entangling with the system-pointer state $|n\rangle \otimes |x_n\rangle$ is just

$$|E_n^{[j]}\rangle = U_{En}^j(t)|j\rangle. \tag{34}$$

The j'th factor in the decoherence factor $F_{mn} = \langle E_n|E_m\rangle = \Pi_{j=1}^N F_{mn}^j$ can be written as

$$F_{mn}^j \equiv \langle E_n^{[j]}|E_m^{[j]}\rangle = \langle j|\exp[i\hat{\phi}_{mn}^{[j]}]|j\rangle \tag{35}$$

by the Wei-Norman algebraic method. Here, the time-dependent global phase is neglected. We can understand the Hermitian operator

$$\hat{\phi}_{mn}^{[j]} = (n-m) f_i t \hat{D}_j \tag{36}$$

in terms of the generalized phase difference between $|E_m^{[j]}\rangle$ and $|E_n^{[j]}\rangle$ for

$$\hat{D}_j = \frac{t}{2m_j'}\hat{p}_j + \eta_j. \tag{37}$$

The standard deviation

$$\Delta\hat{\phi}_{mn}^{[j]} = (n-m) f_j t \Delta D_j \tag{38}$$

is proportional to time t and represents a random phase change of $|n\rangle \otimes |x_n\rangle$ by the j'th inner component.

The whole random phase change of $|n\rangle \otimes |x_n\rangle$, contributed by the inner variables, is determined by

$$\hat{\phi}_{mn} = \sum_{j=1}^{N-1} \hat{\phi}_{mn}^{[j]}. \tag{39}$$

Physically, each variable of the inner environment can exert a different impact independently on the different components of the entangling states of the measured system plus the pointer. If we consider each uncertain phase change by this perturbation as an independent stochastic variable, we have

$$\begin{aligned}\Delta\hat{\phi}_{mn} &= \sqrt{\sum_{j=1}^{N-1} (\Delta\hat{\phi}_{mn}^{[j]})^2} \\ &\geqslant \sqrt{N}\ \min\{(\Delta\hat{\phi}_{mn}^{[j]})^2 | j = 1,2,\ldots,N-1\}.\end{aligned} \tag{40}$$

We observe that the phase uncertainty $\Delta\hat{\phi}_{mn}$ caused by all inner variables can be amplified to a number much greater than 2π when $N\rightarrow\infty$. Then the system-pointer states acquire a very large random-phase factor. Therefore, the inner environment washes out the interference of any two components of the system plus the pointer.

In the above reasoning we make the connivance that there exists a finite minimum of $\Delta\hat{\phi}_{mn}^{[j]}$. This point is just guaranteed by Heisenberg's position-momentum uncertainty relation $\Delta\eta_j \Delta p_j \geqslant \frac{1}{2}$. In fact, because D_j is a linear combination of η_j and p_j and

$$\langle p_i \eta_i\rangle + \langle \eta_i p_i\rangle = 0 \tag{41}$$

for the real wave-function average, there should be a quantum limit for its variance ΔD_j:

$$\begin{aligned}\Delta D_{j\cdot} &= \sqrt{(\Delta\eta_{j\cdot})^2 + \left(\frac{t}{2M}\Delta\hat{p}_j\right)^2} \\ &\geqslant \sqrt{(\Delta\eta_{j\cdot})^2 + \frac{1}{(\Delta\eta_{j\cdot})^2}\left(\frac{t}{2M}\right)^2}.\end{aligned}$$

If one wishes the highest possible for D_j, one should not make $(\Delta\eta_{j\cdot})^2$ arbitrarily small, because this will make Δp_j arbitrarily large. So it is optimal to take $(\Delta\eta_{j\cdot})^2 = t/2M$, and we have the finite minimum $\sqrt{t/M}$. So we have a minimum of phase uncertainty

$$\Delta\hat{\phi}_{mn} = \sqrt{N}\ \frac{t}{M}.$$

This result qualitatively illustrates the many-particle amplification effect of uncertain phase change.

The direct relationship between the two explanations for decoherence can also be revealed explicitly in our present model. As a matter of fact, this problem has been tackled by Stern, Aharonov, and Imary, with the observation $\langle E_n|E_m\rangle = \langle e^{i\hat{\phi}_{mn}^{[j]}}\rangle$ [17]. Consider the specially chosen initial state of Gaussian type $\langle\eta_j|j\rangle = (1/2\pi a^2)^{1/4}\exp(-\eta_j^2/4a^2)$. Since the standard deviation $\Delta\eta_j$ is the width a of Gaussian wave packet and the uncertainty Δp_j of the momentum fluctuation is $1/2a$, we have

$$(\Delta D_i)^2 = \left(\Delta\frac{t}{2m_i}p_i\right)^2 + (\Delta\eta_i)^2 = (\Delta D_i)^2 = \frac{t^2}{16 m_i^2 a^2} + a^2.$$

Then we can probe the relationship between the two explanations by using the exact solution in the preceding section,

$$\begin{aligned}F_{mn} &= \prod_{j=1}^{N-1}\exp\left[-(n-m)^2 f_j^2 t^2 \times\frac{1}{8}\left(\frac{1}{(\Delta p_j)^2} + \frac{1}{\left(\Delta\frac{2m_i'}{t}\eta_j\right)^2}\right)\right] \\ &= \prod_{j=1}^{N-1}\exp[-(n-m)^2 f_j^2 t^2 \tfrac{1}{2}(\Delta D_j)^2] \\ &= \exp\left[-\frac{1}{2}\sum_{j=1}^{N-1}(\Delta\phi_{mn}^j)^2\right] = \exp\left[-\frac{1}{2}(\Delta\phi_{mn})^2\right].\end{aligned} \tag{42}$$

This result just shows that the condition $F_{mn}=0$, which is required by an ideal measurement, implies a large random-phase variance $(\Delta\phi_{mn})^2$. Furthermore, we can conclude from this exact solution that the large random-phase change just originates from Heisenberg's position-momentum uncertainty relation $\Delta\eta_j\Delta p_j=\frac{1}{2}$. In the terminology of classical stochastic process, η_j and p_j can be regarded as a pair of uncorrelated stochastic variables, but the uncertainty relation $\Delta\eta_j\Delta p_j=\frac{1}{2}$ exerts a constraint on them. This constraint just reflect the uncertainty of phase change in the measuring process.

VI. CONCLUDING REMARKS

The above arguments shed a new light on the understanding of the relationship between Bohr's complementarity principle and Heisenberg's uncertainty principle. It is well known that the principle of complementarity usually refers to the wave-particle duality in quantum mechanics. It says that the quantum-mechanical entity can behave as a particle or wave under different experimental conditions, but these two natures excluded each other in the experiment. For example, in the famous Yang's double-slit experiment, the matter wave of a single particle can apparently pass through both slits simultaneously. In this sense the experiment emphasizes the nature of the wave and so there forms an interference pattern. On the contrary, if a which-way detector is employed to determine the particle's path, the interference pattern is destroyed. This is because the which-way experiment focuses on the nature of particle with a classical location, "path." This can further be explained in terms of Heisenberg's uncertainty principle: the uncertainty in the particle's momentum will introduce a random phase difference between two paths and thus destroy the interference. At this point it is worth mentioning that Durt *et al.* [18] reported a which-way experiment in an atom interferometer in which the back action of path detection on the atom's momentum is too small to explain the disappearance of the interference pattern.

We would also like to emphasize that the arguments about the quantum description of the macroscopic object such as the moving mirror is closely related to the Schrödinger cat problem [19,20], a conventional topic of quantum decoherence concerning the state superposition for macroscopic object. Most recent progress has been made in demonstrating the Schrödinger cat state in various macroscopic quantum systems such as superconductors, laser-cooled trapped ions, photons in a microwave cavity, and C_{60} molecules [21]. According to the viewpoint in this paper, a system with many microscopic degrees of freedom can behave quantum mechanically only if it is sufficiently decoupled from the environment and the phases of its inner states match very well.

In summary, we have proposed an alternative Zurek's triple approach for quantum measurement with a time-independent Hamiltonian. The calculation for a specific model shows the possibility of implementing our approach. It should be pointed out that in the above discussed extremely idealized model, the interaction between the particles composing the moving mirror has not been considered. But we do not think this is a big problem. Starting with the main idea developed in this paper, we can discuss a more general situation with inter particle interaction in a similar way. Although the exactly solvable model in this paper might not exist in reality, the idea of quantum adiabatic separation or the master equation is physically meaningful.

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