Scattering of electrons by a Bose-Einstein condensate of alkali-metal atoms

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The scattering of an incident electron by a pure Bose-Einstein condensate of alkali-metal atoms confined to a three-dimensional harmonic-oscillator potential is investigated to give the corresponding differential cross section. It is shown that for the elastic scattering process the differential cross section is proportional to the square of the number of condensed atoms, and hence can be called the ''coherent scattering.'' For the inelastic scattering process, the stopping power is obtained by considering the state of the center of mass and the internal electronic state of the condensed atom being excited separately and simultaneously, which is found to be proportional to the $\frac{7}{5}$ power of the number of condensed atoms when the binary collision interaction between atoms is taken into account.

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I. INTRODUCTION

The phase transition of the Bose-Einstein condensations (BECs) is one of the fascinating predictions of quantum-statistical mechanics, which states that below the critical temperature, a macroscopic population would condense in the ground state of the system consisting of identical bosons. Following the outstanding experimental discovery of liquid ^{4}He, the recent experimental realizations of BEC in several systems of ultracold alkali-metal atoms again verify this prediction and have become a vigorous topic in physics [1–5]. This impressive progress not only opened the possibility of investigating macroscopic quantum phenomena but also offered the chance of applying it in various fields. Since the experimental demonstration of atomic BEC in 1995 [1–3], a large number of problems related to BEC have attracted many experimental and theoretical efforts for the purpose of understanding and utilizing this phenomenon. So far much research on the optical properties [6–9], statistical properties [10], interference and phase properties [9,11–13], tunneling effect [14,15], elementary excitations [16–18], atomic scattering [19–21], and other properties [22,23] of atomic BEC has been undertaken. In particular, it has been pointed out that the scattering of photons [8] and atoms [19–21] by atomic BEC could provide an effective method to investigate various properties of atomic BEC. In fact, as an important method, the scattering techniques have been frequently utilized to probe the structure and obtain other related information of the target particle or object. In the pioneering works on the scattering of photons and atoms by atomic BEC [8,21], the dependence of the differential and total cross sections on the number of condensed atoms has been predicted. For instance, Lewenstein and co-workers [8] have shown that the number of scattered photons can increase dramatically as the BEC phase transition occurs and the part of coherent scattering spectrum is proportional to the square of the number of the condensed atoms. While for the scattering of atoms by atomic BEC, Idziaszek and co-workers [21] have shown that the total elastic cross section σ_{el} scaled with the number of condensed atoms N_0 as $\sigma_{el} \sim N_0^2$ when the energy of the scattered particle is small, and scaled as $\sigma_{el} \sim N_0^{8/5}$ when the energy of the probe atom is of the order of a single excitation in the trapped potential.

It is well known that an incident electron can excite an atom from its ground state to an excited state under the interaction of the incident electron with those bound electrons. Similarly, it is possible that the incident electron excites the ultracold atom being in the BEC state and therefore, the scattering of electron also provides a way of probing atomic BEC.

In this paper we give a theoretical study of the scattering of electrons by a BEC of alkali-metal atoms trapped in a three-dimensional (3D) harmonic potential. For the case that an incident electron is scattered by a single atom, the origin is usually situated at the position of the nucleus, and then the state of center of mass (c.m.) may be considered to be unchanged in this process. However, for the present system consisting of the incident electron and atomic BEC, the c.m. wave function of atomic BEC should be taken into account because the interaction of the incident electron with a large number of atoms is involved. Because the state of the condensate is symmetric under all atom pair permutations, the origin of the system should not coincide with the position of a given condensed atom.

As a matter of fact, because the incident electron is identical to the atomic electrons, there are exchange effects or a rearrangement of the atomic electrons under the interactions among them. Hence, it is a nontrivial task to treat this problem rigorously. For simplicity, in the present paper the velocity of the incident electron is considered to be large compared to that of valence electrons of the condensed alkali-metal atoms, that is, we consider the scattering of a fast electron by an atomic BEC. Under this condition, the exchange effect and rearrangement of atomic electrons can be neglected due to the little overlap between the relatively fast incident electron and ultracold atoms' bound electrons in momentum space. Moreover, the spin interaction between the incident electron and atomic electrons and spin-orbit coupling are also neglected as an approximation.

This paper is organized as follows. In Sec. II the problem

of scattering of an incident electron by the atomic BEC consisting of N_0 alkali-metal atoms is outlined. In Sec. III the differential cross section for the elastic and inelastic scattering of the electron by atomic BEC is given. In Sec IV the energy loss of the incident electron in the inelastic scattering process, i.e., the stopping power, is discussed by considering the c.m. state and the internal electronic state of an atom in the Bose-Einstein condensed state being excited separately and simultaneously. Furthermore, the stopping power is found to scale as $N_0^{7/5}$ as the binary collision interaction between the condensed atoms is taken into account and scale as N_0^2 for an ideal BEC with the *s*-wave scattering length being zero. In Sec. V the main results are summarized.

II. SCATTERING OF AN INCIDENT ELECTRON BY AN ATOMIC BEC

In this section we study the scattering of an incident electron by a pure BEC sample consisting of N_0 identical alkali-metal atoms with nucleus charge Ze and mass M. In second quantization notation, the ground canonical Hamiltonian describing atomic BEC reads

$$H_{\mathrm{BEC}}=\int d^3\vec{R}\,\hat{\Psi}^+(\vec{R})\Big[-\frac{\hbar^2}{2M}\nabla^2+V_{\mathrm{trap}}(\vec{R}) +\frac{1}{2}U_0\hat{\Psi}^+(\vec{R})\hat{\Psi}(\vec{R})\Big]\hat{\Psi}(\vec{R}), \tag{2.1}$$

where $V_{\mathrm{trap}}(\vec{R})=\frac{1}{2}M(\omega_1^2R_1^2+\omega_2^2R_2^2+\omega_3^2R_3^2)$, with angular frequencies ω_1, ω_2, and ω_3, is the external trapping potential for a single particle whose c.m. displacement from the center of the trap is located at $\vec{R}(R_1,R_2,R_3)$. Here the subscripts 1, 2, and 3 are used to denote *x*, *y*, and *z* components of the corresponding quantities, respectively. Whereas $U_0=4\pi\hbar^2a_s/M$ is the contact interaction induced by the binary collision characterized by the *s*-wave scattering length a_s, $\hat{\Psi}^+(\vec{R})$ and $\hat{\Psi}(\vec{R})$ are the boson field operators which create or annihilate an atom at the position $\vec{R}$, respectively, and fulfill the standard bosonic commutation relation $[\hat{\Psi}(\vec{R}'),\hat{\Psi}^+(\vec{R})]=\delta(\vec{R}-\vec{R}')$.

Under the Hartree-Fock approximation [24], for the condensed atoms confined to the 3D harmonic-oscillator potential, we can suppose that $\psi_{n,m_l}(\vec{R})\equiv\langle\vec{R}|n,m_l\rangle$ is the single-particle wave function of an atom in the c.m. state $|n\rangle\equiv|n_1,n_2,n_3\rangle$ and internal state $|m_l\rangle$, where $|n,m_l\rangle=|n\rangle\otimes|m_l\rangle$, $|m_l\rangle$ with $l=0,1,2,\ldots$ denote the configurations of the atomic ground state, the first excited state, the second excited state, ..., respectively. Therefore the creation operator B^+_{n,m_l} which creates an atom in the c.m. state $|n\rangle$ and internal state $|m_l\rangle$ can be defined as

$$B^+_{n,m_l}=\int d^3\vec{R}\,\psi_{n,m_l}(\vec{R})\Psi^+(\vec{R}). \tag{2.2}$$

In this way, applying the atomic creation operator, the ground state $|\Psi_g\rangle$ occupied by N_0 alkali-metal atoms can be expressed as

$$|\Psi_g\rangle=\frac{1}{\sqrt{N_0!}}(B^+_{0,m_0})^{N_0}|0,0\rangle, \tag{2.3}$$

where the state $|0,0\rangle$ denotes the c.m. and internal vacuum states of the atomic BEC sample and should be considered as the direct product form $|0,0\rangle=|0\rangle_{\mathrm{c.m.}}\otimes|0\rangle_{\mathrm{inter}}$.

Now let us turn our attention to the discussion on the scattering of an incident electron by the atomic BEC sample. Suppose that the incident electron with mass m_e and charge $(-e)$ has momentum $\hbar\vec{k}_i$ and is scattered by the interaction with alkali-metal atomic BEC into momentum $\hbar\vec{k}_f$. The interaction of the incident electron with the atomic BEC reads $H_I=\Sigma_{i=1}^{N_0}V_i$, in which V_i is the interaction of the incident electron with the *i*th atom and has the following form:

$$V_i=-\frac{Ze^2}{|\vec{r}-\vec{X}_i|}+\sum_{j=1}^{Z}\frac{e^2}{|\vec{r}-\vec{x}_{ij}|}, \tag{2.4}$$

where $\vec{r}$ is the position of the incident electron, $\vec{X}_i$ and $\vec{x}_{ij}$ are the positions of the *i*th nucleus and the *j*th electron of the atom, and where the spin-spin and spin-orbit interactions have been ignored.

Under these interactions, the incident electron will be scattered by the condensed atoms and, any atom in the BEC can be excited from the ground state to one of its excited states. Hence the BEC sample either remains in its ground state $|\Psi_g\rangle$ or is excited to one of its excited states. The former is called the elastic scattering process because the incident electron energy remains constant and the later is known as the inelastic scattering process since the incident electron will transfer part of its kinetic energy to the atomic BEC.

The initial state of the incident electron and atomic BEC system can be written as the following direct product form:

$$|\phi_i\rangle=|\vec{k}_i\rangle\otimes|\Psi_g\rangle, \tag{2.5}$$

where $|\vec{k}_i\rangle$ refers to the initial state of the incident electron with momentum $\hbar\vec{k}_i$, which is usually expressed as $|\vec{k}_i\rangle=L^{-3/2}\exp[i\vec{k}_i\cdot\vec{r}]$ using the box normalization.

It is of great interest to consider the final state $|\phi_f\rangle=|\vec{k}_f\rangle\otimes|\Psi_f\rangle$ of the system with $|\vec{k}_f\rangle=L^{-3/2}\exp[i\vec{k}_f\cdot\vec{r}]$ the final state of the incident electron and $|\Psi_f\rangle$ the atomic BEC state after the scattering process. In the present paper the cases in which the incident electron excites more than one atom simultaneously are neglected and for simplicity we only consider the single-particle excitation process. Thus, using the atomic creation operators, the excited state $|\Psi_f\rangle$ of the atomic BEC can be chosen as

$$|\Psi_f\rangle = \frac{1}{\sqrt{(N_0 - 1 + \delta_{n,0}\delta_{m_1,m_0})!}} B^+_{n,m_l}(B^+_{0,m_0})^{N_0-1}|0,0\rangle. \tag{2.6}$$

Since we only discuss the case of fast electron scattering, in which the velocity of the incident electron is supposed larger than that of the bound electrons (typically, of the order $c/137$, where c is the speed of light), we can treat the present problem using the time-dependent perturbation theory. It should be noted that the main requirement for the validity of the Born approximation is $Ze^2/\hbar\nu_i \ll 1$, where $\nu_i = \hbar k_i/m_e$ is the initial velocity of the incident electron. In the opposite limit, where $Ze^2/\hbar\nu_i \gg 1$, the Born approximation is no longer valid, and the system may make many successive transitions. Then higher approximations must be considered, or one applies other methods such as the method of the Green function, the method of partial waves, and classical approximation, etc. In general, the scattering of a slow electron from an atom may lead to the exchange effects or the rearrangement of atomic electrons, which is not considered in this paper. For the fast electron scattering under investigation, in the first Born approximation we can immediately write the differential cross section $d\sigma/d\Omega$ in the following form:

$$\frac{d\sigma}{d\Omega} = \left(\frac{m_e L^3}{2\pi\hbar^2}\right)^2 \frac{k_f}{k_i}|(H_I)_{if}|^2, \tag{2.7}$$

where $k_i \equiv |\vec{k}_i|$, $k_f \equiv |\vec{k}_f|$, and $(H_I)_{if} = \langle \phi_f|H_I|\phi_i\rangle = \langle \phi_f|\Sigma_{i=1}^{N_0} V_i|\phi_i\rangle$ is the matrix element between the initial and final states of the system under the interaction H_I.

In order to evaluate the differential cross section, it is necessary to calculate the matrix element $(H_I)_{if}$

$$(H_I)_{if} = \frac{1}{L^3}\int d^3\vec{r}\exp[i\vec{q}\cdot\vec{r}]\langle\Psi_f|\sum_{i=1}^{N_0}\left[-\left(\frac{Ze^2}{|\vec{r}-\vec{X}_i|}\right) + \sum_{j=1}^{Z}\frac{e^2}{|\vec{r}-\vec{x}_{ij}|}\right]|\Psi_g\rangle, \tag{2.8}$$

in which $\vec{q} = \vec{k}_i - \vec{k}_f$ denotes the change of the wave vector of the incident electron in the scattering process. It is directly integrated over $\vec{r}$ to give the equation

$$(H_I)_{if} = \frac{4\pi e^2}{q^2 L^3}\sum_{i=1}^{N_0}\langle\Psi_f|\left[\sum_{j=1}^{Z}\exp(i\vec{q}\cdot\vec{x}_{ij}) - Z\exp(i\vec{q}\cdot\vec{X}_i)\right]|\Psi_g\rangle. \tag{2.9}$$

In obtaining this, we have used the integral $\int d^3\vec{r}\exp(i\vec{q}\cdot\vec{r})/r = 4\pi/q^2$.

It is useful to rewrite Eq. (2.9) as

$$(H_I)_{if} = \frac{4\pi e^2}{q^2 L^3}\sum_{i=1}^{N_0}\langle\Psi_f|\exp(i\vec{q}\cdot\vec{X}_i) \times\left[\sum_{j=1}^{Z}\exp(i\vec{q}\cdot\vec{r}_{ij}) - Z\right]|\Psi_g\rangle, \tag{2.10}$$

where $\vec{r}_{ij} = \vec{x}_{ij} - \vec{X}_i$ is the relative coordinate of the jth electron bound by the ith atom. For the ith atom we make use of the coordinate transformation between the relative motion and atomic c.m. motion for Z electrons and nucleus,

$$\vec{R}_i = \vec{X}_i + \sum_{j=1}^{Z}\frac{m_e}{M}\vec{r}_{ij}, \tag{2.11}$$

where $\vec{R}_i$ is the c.m. coordinate of the ith atom. Furthermore, we can write the equation $\exp(i\vec{q}\cdot\vec{X}_i) = \exp(i\vec{q}\cdot\vec{R}_i)$ under the approximation $\exp[i\Sigma_{j=1}^{Z} m_e/M\vec{q}\cdot\vec{r}_{ij}] \cong 1$, since the sum is approximately zero. The reason is twofold. On the one hand, it is known that the alkali-metal atoms are all hydrogenlike atoms, that is, they all have only one valence electron outside the closed shell, and their inner rare-gas configurations are spherically symmetric. Thus, Eq. (2.11) can be approximately expressed as $\vec{R}_i = \vec{X}_i + (m_e/M)\vec{\tilde{r}}_i$ with $\vec{\tilde{r}}_i$ being the relative position of the valence electron of the ith atom. On the other hand, because the mass of electron is much smaller than the mass of the alkali-metal atoms [e.g., m_e/M is of the order of 10^{-4}, 10^{-5}, and 10^{-6} for lithium (Li), sodium (Na), and rubidium (Rb), respectively], the quantity $\Sigma_{j=1}^{Z} m_e/M\vec{q}\cdot\vec{r}_{ij}$ can be neglected in comparison with $\vec{q}\cdot\vec{r}_{ij}$, so that $\exp[\Sigma_{j=1}^{Z}(m_e/M)\vec{q}\cdot\vec{r}_{ij}] \cong 1$. This leads to the result

$$\exp(i\vec{q}\cdot\vec{X}_i)\left[\sum_{j=1}^{Z}\exp(i\vec{q}\cdot\vec{r}_{ij}) - Z\right] \cong \exp(i\vec{q}\cdot\vec{R}_i)\left[\sum_{j=1}^{Z}\exp(i\vec{q}\cdot\vec{r}_{ij}) - Z\right]. \tag{2.12}$$

In doing so, Eq. (2.10) can be further rewritten as

$$(H_I)_{if} = \frac{4\pi e^2}{q^2 L^3}\sum_{i=1}^{N_0}\langle\Psi_f|\exp(i\vec{q}\cdot\vec{R}_i) \times\left[\sum_{j=1}^{Z}\exp(i\vec{q}\cdot\vec{r}_{ij}) - Z\right]|\Psi_g\rangle, \tag{2.13}$$

which is the starting point of the following discussions on the differential cross sections.

III. THE DIFFERENTIAL CROSS SECTION

In this section we will calculate the differential cross section for the elastic and inelastic scattering processes. We introduce the form factor of a single atom defined by

$$F_l(\vec{q}) = \frac{1}{Z}\langle m_l|\sum_{j=1}^{Z}\exp(i\vec{q}\cdot\vec{r}_j)|m_0\rangle. \tag{3.1}$$

For the elastic scattering process no energy is transferred from the incident electron to the atomic BEC, which means that the final state of atomic BEC is the same as its initial state with $k_i = k_f$. Then, from Eq. (2.13), we have

$$(H_I)_{if} = \frac{4\pi e^2 N_0 Z}{q^2 L^3} \langle 0|\exp(i\vec{q}\cdot\vec{R})|0\rangle [F_0(\vec{q}) - 1], \qquad (3.2)$$

where the subscript i, which denotes different atoms in the state of BEC, has been dropped because only the single-particle excitation process is considered.

For the inelastic process, the c.m. and internal states of an atom can be excited separately or simultaneously [see Eq. (2.13)]. Thus using the state given by Eq. (2.6), one gets

$$(H_I)_{if} = \frac{4\pi e^2 \sqrt{N_0} Z}{q^2 L^3} \langle n|\exp(i\vec{q}\cdot\vec{R})|0\rangle F_l(\vec{q})$$

$$(l \neq 0 \quad \text{or} \quad n \neq 0). \qquad (3.3)$$

Substituting Eqs. (3.2) and (3.3) into Eq. (2.7), the expressions for the elastic and inelastic differential cross sections become

$$\frac{d\sigma_{\text{el}}^{(0,0\to 0,0)}}{d\Omega} = \frac{4N_0^2 Z^2 a_0^2}{(qa_0)^4} |\langle 0|\exp(i\vec{q}\cdot\vec{R})|0\rangle|^2 |[F_0(\vec{q}) - 1]|^2, \qquad (3.4)$$

$$\frac{d\sigma_{\text{inel}}^{(0,0\to n,l)}}{d\Omega} = \frac{4N_0 Z^2 a_0^2 k_f}{(qa_0)^4 k_i} |\langle n|\exp(i\vec{q}\cdot\vec{R})|0\rangle|^2 |F_l(\vec{q})|^2$$

$$(l \neq 0 \quad \text{or} \quad n \neq 0). \qquad (3.5)$$

In Eqs. (3.4) and (3.5), $a_0 = \hbar^2/m_e e^2$ is the Bohr radius.

From Eqs. (3.4) and (3.5) one finds that for the elastic scattering of electrons by the atomic BEC, the differential cross section is proportional to the square of the number of condensed atoms; in other words, a dramatic enhancement of the differential cross section appears, hence this process could be called the ''coherent scattering'' of electron by the atomic BEC in analogy with the scattering of photons and atoms by atomic BEC [8,21]. For the inelastic scattering processes, this kind of ''coherence'' is absent. Moreover, it should be noted that for both elastic and inelastic scattering process, the c.m. states of the condensed atoms play an important role because of the influence of the squared modulus of the matrix elements $\langle n|\exp(i\vec{q}\cdot\vec{R})|0\rangle$ on the corresponding differential cross section.

Now let us turn our attention to calculating the quantity $|\langle n|\exp(i\vec{q}\cdot\vec{R})|0\rangle|^2$ by means of the theory of a displaced number state [25]. To this end we first outline the properties of the displaced number states for a one-dimensional (1D) harmonic oscillator described by the Hamiltonian $H = \hbar\omega(a^\dagger a + \frac{1}{2})$, in which $a^\dagger$ and a are the creation and annihilation operators, respectively. The displaced number state $|\alpha,m\rangle$ associating with the harmonic oscillator is defined as

$$|\alpha,m\rangle = \hat{D}(\alpha)|m\rangle, \qquad (3.6)$$

where $|m\rangle$ is the eigenstate of the Hamiltonian of the 1D harmonic oscillator under consideration, $\hat{D}(\alpha)$ is the displaced operator and takes the form

$$\hat{D}(\alpha) = \exp(\alpha a^\dagger - \alpha^* a), \qquad (3.7)$$

that satisfies the equations

$$\hat{D}(\alpha) a \hat{D}^\dagger(\alpha) = a - \alpha, \quad \hat{D}(\alpha) a^\dagger \hat{D}^\dagger(\alpha) = a^\dagger - \alpha^*, \qquad (3.8)$$

in which the quantity α denotes the translational parameter, which may be real or complex. It can be easily found that for the case of $m = 0$, the displaced number state $|\alpha,0\rangle$ is the well-known coherent state.

Furthermore, as an interesting application of the displaced number state, the following equation has been obtained [25]

$$|\langle l|\hat{D}(\alpha)|m\rangle|^2 = \frac{\exp(-|\alpha|^2)|\alpha|^{2(l-m)} m!}{l!} L_m^{l-m}(|\alpha|^2)$$

$$(\text{for} \quad l \geqslant m), \qquad (3.9)$$

where $L_m^{l-m}(x)$ is the associated Laguerre polynomial with argument x.

To evaluate the result of $|\langle n|\exp(i\vec{q}\cdot\vec{R})|0\rangle|^2$, we rewrite the matrix element $\langle n|\exp(i\vec{q}\cdot\vec{R})|0\rangle$ as

$$\langle n|\exp(i\vec{q}\cdot\vec{R})|0\rangle = \prod_j \langle n_j|\exp(iq_j R_j)|0\rangle \quad (j = 1,2,3), \qquad (3.10)$$

in which the subscript $j = 1$, 2, and 3 corresponds to the x, y, and z components of the corresponding physical quantities. Notice that in the 3D external trapping potential, the following relations:

$$R_j = \sqrt{\frac{1}{2}}(a_j^\dagger + a_j) L_j, \quad L_j = \sqrt{\frac{\hbar}{M\omega_j}} \quad (j = 1,2,3), \qquad (3.11)$$

$$\hat{D}_j(\alpha_j) = \exp(iq_j R_j), \quad \alpha_j = iq_j L_j \quad (j = 1,2,3), \qquad (3.12)$$

can be obtained immediately. Here $L_j = \sqrt{\hbar/M\omega_j}$ with $j = 1$, 2, and 3 are the geometric mean size of the c.m. ground state, and also the characteristic length of the trap. As a consequence of the theory of the displaced number state, we have

$$|\langle n|\exp(i\vec{q}\cdot\vec{R})|0\rangle|^2 = \prod_j \frac{1}{(n_j)!} |\alpha_j|^{2n_j} \exp(-|\alpha_j|^2)$$

$$= \prod_j \frac{1}{(n_j)!} \left(\frac{q_j^2 L_j^2}{2}\right)^{n_j} \exp\left(-\frac{q_j^2 L_j^2}{2}\right)$$

$$(j = 1,2,3). \qquad (3.13)$$

Based on this equation, the differential cross section for the elastic and inelastic scattering can be explicitly rewritten as

$$\frac{d\sigma_{\rm el}^{(0,0\to 0,0)}}{d\Omega}=\frac{4N_0^2Z^2a_0^2}{(qa_0)^4}|[F_0(\vec{q})-1]|^2\prod_j \exp\left(-\frac{q_j^2L_j^2}{2}\right), \tag{3.14}$$

$$\frac{d\sigma_{\rm inel}^{(0,0\to n,l)}}{d\Omega}=\frac{4N_0Z^2a_0^2k_f}{(qa_0)^4k_i}\prod_j \frac{1}{(n_j)!}\left(\frac{q_j^2L_j^2}{2}\right)^{n_j} \times\exp\left(-\frac{q_j^2L_j^2}{2}\right)|F_l(\vec{q})|^2. \tag{3.15}$$

IV. THE STOPPING POWER IN THE SCATTERING PROCESS

In the inelastic scattering process, the kinetic energy of an incident electron passing through the atomic BEC would be transferred to the condensate by exciting the c.m. state or an internal state of atomic BEC. Therefore the stopping power, i.e., the energy loss of the incident electron, play a key role and can be obtained by means of the differential cross section for the inelastic scattering process.

In this paper, as is usually the case in scattering problems, we are interested in the energy loss of the incident electron along the path. We write $-dE/dx$, which takes the form

$$-\frac{dE}{dx}=\rho_0\sum_{n,l}(E_{n,l}-E_{0,0})\int_{q_{\rm min}}^{q_{\rm max}}\frac{d\sigma_{\rm inel}^{(0,0\to n,l)}}{dq}dq, \tag{4.1}$$

where ρ_0 is the density of atomic BEC trapped in the external potential, $E_{n,l}$ denotes the energy of an atom being the c.m. state $|n\rangle$ and internal state $|m_l\rangle$, and the quantities $q_{\rm min}$ and $q_{\rm max}$ are the integral limits.

Note that $q^2=|\vec{q}|^2=|(\vec{k}_i-\vec{k}_f)|^2=k_i^2+k_f^2-2k_ik_f\cos\theta$, where θ is the angle between $\vec{k}_i$ and $\vec{k}_f$. From this relation we can obtain that

$$\frac{d\sigma_{\rm inel}^{(0,0\to n,l)}}{dq}=\frac{2\pi q}{k_ik_f}\frac{d\sigma_{\rm inel}^{(0,0\to n,l)}}{d\Omega}. \tag{4.2}$$

Using this equation together with Eq. (3.5), Eq. (4.1) can be expressed as

$$-\frac{dE}{dx}=\frac{8\pi\rho_0N_0Z^2}{a_0^2k_i^2}\sum_{n,l}(E_{n,l}-E_{0,0}) \times\int_{q_{\rm min}}^{q_{\rm max}}|\langle n|\exp(i\vec{q}\cdot\vec{R})|0\rangle|^2|F_l(\vec{q})|^2\frac{dq}{q^3}. \tag{4.3}$$

In order to evaluate the stopping power, it is useful to rewrite the energy difference $(E_{n,l}-E_{0,0})$ in Eq. (4.3) as $(E_{n,l}-E_{0,0})=(\varepsilon_l-\varepsilon_0)+(\tilde{\varepsilon}_n-\tilde{\varepsilon}_0)$, where ε_l is the energy corresponding to the lth internal state $|m_l\rangle$ and $\tilde{\varepsilon}_n$ is the energy corresponding to the nth c.m. state $|n\rangle$. Then for the case where only the internal state is excited, i.e., $n=0$ in Eq. (4.3), we find

$$-\left.\frac{dE}{dx}\right|_{n=0}=\frac{4\pi\rho_0N_0Ze^4}{m_e\nu_i^2}\int_{q_{\rm min}}^{q_{\rm max}}|\langle 0|\exp(i\vec{q}\cdot\vec{R})|0\rangle|^2\frac{dq}{q}, \tag{4.4}$$

where $\nu_i=\hbar k_i/m_e$ is the initial velocity of the incident electron. In this calculation, the famous result

$$\sum_l(\varepsilon_l-\varepsilon_0)|F_l(\vec{q})|^2=\frac{1}{Z}\frac{\hbar^2q^2}{2m_e}, \tag{4.5}$$

presented by Gottfried and Bethe [26], has been used.

Making use of the result given by Eq. (3.13) and arranging $n_1=n_2=n_3=0$, Eq. (4.4) becomes

$$-\left.\frac{dE}{dx}\right|_{n=0}=\frac{4\pi\rho_0N_0Ze^4}{m_e\nu_i^2}\left[\int_{q_{\rm min}}^{q_{\rm max}}\exp\left[-\frac{1}{2}(q_1^2L_1^2+q_2^2L_2^2+q_3^2L_3^2)\frac{dq}{q}\right]\right]. \tag{4.6}$$

The final result of this equation depends on the detailed form of potential function.

For the case with $l=0$ in Eq. (4.3), which implies that only the c.m. state is excited, we get

$$-\left.\frac{dE}{dx}\right|_{l=0}=\frac{4\pi\rho_0N_0Z^2e^4}{m_e\nu_i^2}\frac{m_e}{M}\int_{q_{\rm min}}^{q_{\rm max}}|F_0(\vec{q})|^2\frac{dq}{q}, \tag{4.7}$$

where we have used the result

$$\sum_n(\tilde{\varepsilon}_n-\tilde{\varepsilon}_0)|\langle n|\exp(i\vec{q}\cdot\vec{R})|0\rangle|^2=\frac{\hbar^2q^2}{2M}, \tag{4.8}$$

which can be explicitly obtained in the spirit of Gottfried and Bethe [26].

For the case that the c.m. state and internal state are excited simultaneously, using the decomposition of the equation $(E_{n,l}-E_{0,0})=(\varepsilon_l-\varepsilon_0)+(\tilde{\varepsilon}_n-\tilde{\varepsilon}_0)$ together with Eqs. (4.5) and (4.8), the quantity $-dE/dx$ can be expressed as

$$-\frac{dE}{dx}=\frac{4\pi\rho_0N_0Z^2e^4}{m_e\nu_i^2}\left[\int_{q_{\rm min}}^{q_{\rm max}}\left(\frac{m_e}{M}\sum_l|F_l(\vec{q})|^2+\frac{1}{Z}\sum_n|\langle n|\exp(i\vec{q}\cdot\vec{R})|0\rangle|^2\right)\frac{dq}{q}\right]. \tag{4.9}$$

Note that $\Sigma_n|\langle n|\exp(i\vec{q}\cdot\vec{R})|0\rangle|^2=1$, which may be directly derived from Eq. (3.13). The physical interpretation of this result is rather obvious because the term $\Sigma_n|\langle n|\exp(i\vec{q}\cdot\vec{R})|0\rangle|^2$ stands for the sum of the probabilities that the c.m. ground state is excited to all excited states, thus it must equal to 1. Based on the same consideration, we know that $\Sigma_l|F_l(\vec{q})|^2=1$ as well. This allows Eq. (4.9) to be further simplified to

$$-\frac{dE}{dx}=\frac{4\pi\rho_0 N_0 Ze^4}{m_e\nu_i^2}\left(1+\frac{m_eZ}{M}\right)\left[\int_{l_{\min}}^{q_{\max}}\frac{dq}{q}\right]. \quad (4.10)$$

Following the Bethe's treatment with respect to the limits $q_{\min}$ and $q_{\max}$, that is, $q_{\min}=k_i-k_f$ and $q_{\max}=2k_i$, we obtain

$$-\frac{dE}{dx}=\frac{4\pi\rho_0 N_0 Ze^4}{m_e\nu_i^2}\left(1+\frac{m_eZ}{M}\right)\ln\left(\frac{2m_e\nu_i^2}{I}\right), \quad (4.11)$$

where I is a semiempirical parameter, which is related to the average excitation energy. This is just the stopping power in the scattering process.

Equations (4.6), (4.7), and (4.11) correspond to the stopping power for the case where the internal and c.m. states are excited separately and simultaneously. From these expressions, it is evident that both the form factor and the quantity $|\langle n|\exp(i\vec{q}\cdot\vec{R})|0\rangle|^2$ determine the value of the stopping power.

Having obtained the expression of the stopping power in the inelastic scattering process, we show below the dependence of the stopping power on the number of the condensed atoms. To our knowledge, this is an interesting feature distinguishing the atomic BEC from the normal target object. For an ideal atomic BEC (where $a_s=0$) trapped in a 3D harmonic-oscillator potential under consideration, the density of the atomic BEC can be written as $\rho_0=(2\pi)^{3/2}(N_0/L_1L_2L_3)$. Thus the stopping power is proportional to the square of the number of the condensed atoms, i.e., $-(dE/dx)\propto N_0^2$. However, when the binary collision interaction $U_0=4\pi\hbar^2a_s/M$ with a nonzero s-wave scattering length a_s is taken into account, the density of the condensed atoms becomes $\rho_0=\mu/U_0$ as the temperature approaches zero [27], where μ is the chemical potential. For the 3D trapping potential used in this paper, the chemical potential μ takes the following form [28]:

$$\mu=\frac{\hbar}{2}\left(\sqrt{\frac{M}{\hbar}}15a_sN_0\omega_1\omega_2\omega_3\right)^{2/5}, \quad (4.12)$$

which can reduce to the results given by Baym and Pethick and Timmermans *et al.* [27] for the spherically and azimuthally symmetric potentials.

From Eq. (4.12), one further gets

$$\rho_0=\frac{\mu}{U_0}=\frac{1}{8\pi a_s}\left(\frac{15a_sN_0}{L_1^2L_2^2L_3^2}\right)^{2/5}. \quad (4.13)$$

Substituting Eq. (4.13) into Eq. (4.11), we obtain the desired result

$$-\frac{dE}{dx}=\frac{1}{a_s}\left(\frac{15a_s}{L_1^2L_2^2L_3^2}\right)^{2/5}\frac{N_0^{7/5}Ze^4}{2m_e\nu_i^2}\left(1+\frac{m_eZ}{M}\right)\ln\left(\frac{2m_e\nu_i^2}{I}\right). \quad (4.14)$$

From this expression we conclude that the stopping power scales as the $\frac{7}{5}$ power of N_0, i.e., $-dE/dx\propto N_0^{7/5}$. This conclusion also holds true for the case of the c.m. and internal states being excited separately.

Now, as an example, we calculate the stopping power by assuming that the atomic BEC is trapped in a 3D spherically symmetric harmonic-oscillator potential, i.e., $\omega_1=\omega_2=\omega_3=\omega$ and $L_1=L_2=L_3=L=\sqrt{\hbar/M\omega}$. In this case, Eq. (4.6) reduces to

$$-\left.\frac{dE}{dx}\right|_{n=0}=\frac{4\pi\rho_0N_0Ze^4}{m_e\nu_i^2}\int_{q_{\min}}^{\max}\exp\left[-\frac{1}{2}q^2L^2\right]\frac{dq}{q}. \quad (4.15)$$

Let $y=q^2$, then

$$\int_{q_{\min}}^{q_{\max}}\exp\left[-\frac{1}{2}q^2L^2\right]\frac{dq}{q}=\int_{q_{\min}^2}^{q_{\max}^2}\frac{1}{2}\exp\left[-\frac{yL^2}{2}\right]\frac{dy}{y},$$

and we have

$$-\left.\frac{dE}{dx}\right|_{n=0}=\frac{4\pi\rho_0N_0Ze^4}{m_e\nu_i^2}\left[\ln\frac{2m_e\nu_i^2}{I}+\sum_{k=1}^{\infty}\left(-\frac{L^2}{2}\right)^k\frac{(q_{\max}^{2k}-q_{\min}^{2k})}{k!k}\right]. \quad (4.16)$$

If q satisfies the condition $\hbar^2q^2/2M\ll\hbar\omega$, the leading term in the right-hand side of Eq. (4.16) becomes $-dE/dx|_{n=0}=4\pi\rho_0N_0Ze^4/m_e\nu_i^2\ln 2m_e\nu_i^2/I$. Moreover, it can be shown that $|F_l(\vec{q})|^2$ will approach unity as $q\to0$. Then under this condition, Eq. (4.7) can be approximately expressed as

$$-\left.\frac{dE}{dx}\right|_{l=0}=\frac{4\pi\rho_0N_0Z^2e^4}{m_e\nu_i^2}\frac{m_e}{M}\ln\frac{2m_e\nu_i^2}{I}. \quad (4.17)$$

Obviously, the Z dependence of the stopping power in Eqs. (4.16) and (4.17) is different, since the total-energy loss of the incident electron is $(1/Z)(\hbar^2q^2/2m_e)$ when only the internal state is excited and $\hbar^2q^2/2M$ when only the c.m. state is excited.

As shown above, we find that the scattering of electrons by an atomic BEC may be used to determine the number of condensed atoms and the density of the atomic BEC by measuring elastic scattering cross section and the stopping power.

V. SUMMARY

In conclusion, we have studied the scattering of an incident electron by a Bose-Einstein condensate of alkali-metal atoms and obtained the differential cross section for the elastic and inelastic scattering processes. For the former, it is shown that the elastic scattering differential cross section is proportional to the square of the number of the condensed atoms, which indicates the intrinsic coherent property of the atomic BEC, and we may therefore call this kind of elastic scattering ''coherent scattering'' because of the dramatic enhancement of the scattering cross section. For the later, we calculated the stopping power by considering the internal and c.m. states being excited separately and simultaneously, and the dependence on the number of the condensed atoms

N_0 is found to be two power of N_0 for an ideal atomic BEC ($a_s=0$) and $\frac{7}{5}$ power of N_0 for a nonideal BEC ($a_s\neq 0$), respectively. This is an interesting feature of atomic BEC and can be used to distinguish the atomic Bose-Einstein condensate from normal targets.

The limitations of the present paper are as follows. The exchange effects of the incident electron and atomic electrons, the rearrangement of atomic electrons, and the spin-spin and spin-orbit interactions in the scattering process have been neglected. However, in practice, these interactions may give rise to different influences on the scattering result with the different range of velocity of the incident electron. Some further discussions will be performed under the condition of relaxing these limitations.

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