Semiconductor-cavity QED in high-Q regimes with q-deformed bosons

Yu-Xi Liu, C. P. Sun,* S. X. Yu, and D. L. Zhou
Institute of Theoretical Physics, Academia Sinica, P.O. Box 2735, Beijing 100080, China
(Received 3 January 2000; published 4 January 2001)

The high-density Frenkel excitons, which interact with a single-mode cavity field, are investigated in the framework of the q-deformed boson. It is shown that the q-deformed bosonic commutation relations are satisfied naturally by the exciton operators in the high-density limit. An analytical expression of the physical spectra of the excitons is obtained by using the dressed states of the cavity field and the excitons. We also give the numerical study and compare the theoretical results with the experimental results.

DOI: 10.1103/PhysRevA.63.023802 PACS number(s): 42.50.Fx, 71.35.−y

I. INTRODUCTION

It's well known that the system of the excitons is a quasiparticle system. In case of the low density, the excitons are approximately treated as bosons that obey Bose statistics [1–3]. But when the density of the excitons becomes relative higher, the excitons, which somewhat deviate the bosonic model, are no longer ideal bosons. There are two ways of dealing with this problem: one way is to put these deviations into the effective interaction between the hypothetical ideal bosons and the exciton operators as still presented by the bosonic operators [4,5]. Another way is the implementation of the atomic operators [6–8], which naturally gives rise to the question of whether a system of excitons is equivalent to an atomic system.

The concept of the q-deformed boson was extensively applied in physics due to the q-deformed boson realization of the quantum group theory by different authors ten years ago. Since then, many physicists made great efforts to find its real physical implementation [9–13]. For example, they did some phenomenological investigations to fit the deformed spectra of the rotation and oscillation for molecules and nuclei [14–16]. In our opinion, those investigations can be regarded as merely phenomenological explanations, because these q-deformed structures were postulated *ad hoc* and no underlying microscopic mechanism was presented in advance.

In this paper, it will be shown that a physical and natural realization of the q-deformed boson is provided by the exciton operators, which were proposed recently by Gardiner [17] for the description of Bose-Einstein condensation (BEC). Here the deformation parameter q is well-defined by the total molecule number N rather than its phenomenological explanation given in the investigations in the past. In fact, the similar quasiparticles scheme for the particle-number conservation was already introduced by Girardeau and Arnowitt almost 40 years ago [18]. The relationship between the Gardiner's phonons and these quasiparticles has been discussed in a recent comment [19]. We find that when the density of the excitons (the particles excited in upper state) is low enough, it returns naturally to the ideal boson case. According to that theory, we could give a good explanation to the semiconductor-cavity QED in high-Q regimes. What will be investigated here is the case where the total molecule number N is very large but not infinite by using the method of the q-deformed boson algebra. That is, we shall consider the effects of order $O(1/N)$. As it turns out, the commutation relations for the exciton operators will no longer obey the commutation relations of the Heisenberg-Weyl algebra but the q-deformed bosonic commutation relations

$$[b_q, b_q^\dagger]_q \equiv b_q b_q^\dagger - q b_q^\dagger b_q = 1, \tag{1}$$

where the deformation constant q depends on the total atom number.

This paper is organized as follows. In Sec. II we first deduce the q-deformed commutation relations of the exciton operators in the high-Q cavity in the case of the large but finite lattice molecule number N. In Sec. III, by keeping only the first-order term of $1/N$, we model the Frenkel excitons in a microcavity as a dressed q-deformed boson system. In Sec. IV, the quantum approach of the angular momentum is used to obtain the eigenvalues and eigenfunction of the system under the first-order approximation. The stationary physical spectrum of the system is calculated in Sec. V. Finally we summarize our results with some comments.

II. Q-DEFORMED BOSONIC ALGEBRA FOR EXCITON

Gardiner's starting point [17] to introduce the exciton operators is to consider a system of the weakly interacting Bose gas. Without losing generality, we consider a thin molecular crystal film containing N identical two-level molecules interacting resonantly with a single-mode quantum field. The intermolecular interaction is neglected in the low-molecular excitation. We assume that all molecules have equivalent mode positions, so they interact with the cavity field by the same coupling constant κ. By using Dicke model [20], we could write the Hamiltonian in the rotating wave approximation as follows:

$$H = \hbar\Omega(S_z + a^\dagger a) + \hbar\kappa(aS_+ + a^\dagger S_-), \tag{2}$$

where a and $a^\dagger$ are the annihilation and creation operators of the quantum cavity field, respectively, and

*URL: http://www.itp.ac.cn/~suncp. Electronic address: suncp@itp.ac.cn

$$S_z=\sum_{n=1}^{N} s_z(n), \quad S_{\pm}=\sum_{n=1}^{N} s_{\pm}(n), \tag{3}$$

where $s_z(n)=(|e_n\rangle\langle e_n|-|g_n\rangle\langle g_n|)/2, s_+(n)=|e_n\rangle\langle g_n|$, and $s_-(n)=|g_n\rangle\langle e_n|$ are quasispin operators of the nth molecule. States $|e_n\rangle$ and $|g_n\rangle$ denote the excited state and the ground state of the nth molecule.

We consider the second quantization of the above model. Let $b_e^\dagger$ and b_e denote the creation and annihilation operators of the molecules in the excited state. $b_g^\dagger$ and b_g are the creation and annihilation operators of the molecules in the ground state. The simplified Hamiltonian after the second quantization is written

$$H=\hbar\Omega(b_e^\dagger b_e-b_g^\dagger b_g+a^\dagger a)+\hbar\kappa(ab_e^\dagger b_g+\text{H.c.}). \tag{4}$$

Note that the total molecular number $\mathbf{N}=b_e^\dagger b_e+b_g^\dagger b_g$ is conserved. For convenience we define $\eta=1/N$ with large particle number N.

In the thermodynamical limit $N\to\infty$, the Bogoliubov approximation [21,22] is usually applied, in which the ladder operators $b_g^\dagger, b_g$ of the ground state are replaced by a c number $\sqrt{N_c}$, where N_c is the average number of the initial condensated atoms. As a result Hamiltonian Eq. (4) describes a system of two coupled harmonic oscillators

$$H_b=\hbar\Omega(b_e^\dagger b_e+a^\dagger a)+\hbar\kappa\sqrt{N_c}(ab_e^\dagger+\text{H.c.}). \tag{5}$$

However, this approximation destroys a symmetry of the Hamiltonian Eq. (4), i.e., the conservation of the total particle number is violated because of $[\mathbf{N},H_b]\neq 0$.

To avoid this problem, the exciton operators are defined as:

$$b_q=\frac{1}{\sqrt{N}}b_g^\dagger b_e, \quad b_q^\dagger=\frac{1}{\sqrt{N}}b_g b_e^\dagger \tag{6}$$

according to Gardiner [17]. These operators act invariantly on the subspace V^N spanned by bases $|N;n\rangle\equiv|N-n,n\rangle (n=0,1,\ldots,N)$, where Fock states

$$|m,n\rangle=\frac{1}{\sqrt{m!n!}}b_e^{\dagger m}b_g^{\dagger n}|0\rangle \quad (m,n=0,1,2,\ldots)$$

span the Fock space H_{2b} of a two-mode boson.

A straightforward calculation leads to the following commutation relation between the exciton operator b_q and its Hermitian conjugate:

$$[b_q,b_q^\dagger]=1-\frac{2}{N}b_e^\dagger b_e=f(b_q^\dagger b_q;\eta), \tag{7}$$

with $f(x;\eta)=\sqrt{1+2(1-2x)\eta+\eta^2}-\eta$. Keeping only the lowest order of η for a very large total particle number, the commutator above becomes

$$[b_q,b_q^\dagger]=1-2\eta b_q^\dagger b_q \tag{8a}$$

or

$$[b_q,b_q^\dagger]_q=b_q b_q^\dagger-q b_q b_q^\dagger=1, \tag{8b}$$

with $q=1-2\eta$. This is exactly a typical q-deformed commutation relation. As $N\to\infty$ or $q\to 1$, the usual commutation relation of Heisenberg-Weyl algebra is regained.

In the above discussion about the phonon excitation, we have linearized the commutator $h\equiv f(b^\dagger b;\eta)$ so that a q-deformed commutation rule was obtained. Essentially this linearization establishes a physical realization of the q-deformed algebra. However, if the total particle number N is not large enough, then h cannot be approximated by a linear function. From the commutation relations between h and $b_q, b_q^\dagger$

$$[h,b_q^\dagger]=-\frac{2}{N}b_q^\dagger, \quad [h,b_q]=\frac{2}{N}b_q, \tag{9}$$

we see that the algebra of exciton operators is a rescaling of algebra SU(2) with factor N.

III. THEORETICAL MODEL

Based on the above analysis about the algebraic structure of exciton operators, we consider the case of the relatively high density of molecules in the excited state for the Hamiltonian (2).

Since the second quantization forms of S_+ and S_- are $S_+=b_e^\dagger b_g$ and $S_-=b_g^\dagger b_e$, respectively, it is straightforward to prove that the collective operators $S_+/\sqrt{N}$ and $S_-/\sqrt{N}$ are approximately considered as the simple bosonic operators as $N\to\infty$. These collective operators are called exciton operators. But in case of the relatively high density of molecules in the excited state with finite N, since many molecules are in the excited state the bosonic approximation can no longer work well. The Hamiltonian (2) can be rewritten in terms of q-deformed bosons as the following effective Hamiltonian

$$H=\hbar\Omega(a^\dagger a+b_q^\dagger b_q)+\hbar g(a^\dagger b_q+b_q^\dagger a), \tag{10}$$

where $g=\sqrt{N}k$ and q-deformed boson operators b_q, $b_q^\dagger$ satisfy the q-deformed commutation relation

$$[b_q,b_q^\dagger]_q=1, \tag{11}$$

where

$$q=1-\frac{2}{N}. \tag{12}$$

It should be emphasized here that the deformation parameter q is determined by the lattice molecule number and is no longer phenomenological.

Up to the first-order approximation, the q-deformed boson operators $b_q^\dagger$ and b_q could be expressed as

$$b_q^\dagger=b^\dagger+\frac{b^\dagger b^\dagger b}{2N}, \tag{13}$$

$$b_q=b+\frac{b^\dagger b b}{2N}. \tag{14}$$

in terms of the normal bosonic operators $b^\dagger$ and b. As a consequence the Hamiltonian H in Eq. (10) is rewritten in the form of perturbation

$$H = H_0 + H', \tag{15}$$

where

$$H_0 = \hbar\Omega(a^\dagger a + b^\dagger b) + \hbar g(a^\dagger b + b^\dagger a), \tag{16}$$

$$H' = \frac{\hbar}{2N}(2\Omega b^\dagger b^\dagger bb + gab^\dagger b^\dagger b + ga^\dagger b^\dagger bb). \tag{17}$$

It is clear that the first term of H' describes the attractive exciton-exciton collisions due to the bi-exciton effect and the last two terms of H' describe the decrease of the exciton-photon coupling constants due to the phase-space filling effect [24].

IV. APPROXIMATE ANALYTICAL SOLUTIONS

To solve the Schrödinger equation governed by Hamiltonian Eq. (15), we shall make use of the quantum angular momentum theory [23].

According to the Schwinger representation of the angular momentum by using two bosons, we can build the angular momentum operators

$$J_z = \frac{1}{2}(a^\dagger a - b^\dagger b), \quad J_+ = a^\dagger b, \quad J_- = ab^\dagger. \tag{18}$$

From the ladder operators a, $a^\dagger$ of the cavity field and the exciton operators b and $b^\dagger$ then

$$J_x = \frac{1}{2}(a^\dagger b + ab^\dagger), \quad J_y = \frac{1}{2i}(a^\dagger b - ab^\dagger). \tag{19}$$

We rewrite the Hamiltonian Eq. (16)

$$H_0 = \hbar\Omega\hat{N} + 2\hbar gJ_x = \hbar\Omega\hat{N} + 2\hbar ge^{-i(\pi/2)J_y}J_z e^{i(\pi/2)J_y} \tag{20}$$

in terms of an $SO(3)$ rotation $e^{i(\pi/2)J_y}$ of $\hbar(\Omega\hat{N} + 2gJ_x)$. Note that the excitation number operator $\hat{N} = a^\dagger a + b^\dagger b$ is a constant under any $SO(3)$ rotation and

$$J^2 = J_x^2 + J_y^2 + J_z^2 = \frac{\hat{N}}{2}\left(\frac{\hat{N}}{2} + 1\right) \tag{21}$$

is the total angular momentum operator. The common eigenstates of J^2 and J_z are

$$|jm\rangle = \frac{(a^\dagger)^{j+m}(b^\dagger)^{j-m}}{\sqrt{(j+m)!(j-m)!}}|0\rangle, \tag{22}$$

where the eigenvalues of the J^2 and J_z are, respectively,

$$j = \frac{\mathcal{N}}{2}, m = -\frac{\mathcal{N}}{2}, \cdots, \frac{\mathcal{N}}{2}. \tag{23}$$

The eigenfunctions ψ^0_{jm} and the eigenvalues $E^{(0)}_{jm}$ of H_0 can be easily constructed as

$$|\psi^0_{jm}\rangle = e^{-i\pi/2J_y}|jm\rangle, \quad E^{(0)}_{jm} = \hbar\Omega\mathcal{N} + 2\hbar gm. \tag{24}$$

Up to the first-order approximation, the eigenvalues of H are obtained as

$$E_{jm} = E^{(0)}_{jm} + \langle jm|e^{i(\pi/2)J_y}H'e^{-i(\pi/2)J_y}|jm\rangle, \tag{25}$$

with their corresponding eigenfunctions given by

$$|\psi_{jk}\rangle = |\psi^{(0)}_{jk}\rangle + \sum_{n\neq k}\frac{\langle jn|H'|jk\rangle}{E^{(0)}_{jk} - E^{(0)}_{jn}}|\psi^{(0)}_{jn}\rangle. \tag{26}$$

We calculate the matrix elements of the perturbation Hamiltonian H':

$$\begin{aligned}\langle jm'|e^{i\pi/2J_y}H'e^{-i\pi/2J_y}|jm\rangle = {} & \frac{\hbar}{4N}\Omega\sqrt{(j+m)(j+m-1)}\times\sqrt{(j-m+1)(j-m+2)}\,\delta_{m-2,m'} + \frac{\hbar}{4N}\Omega\sqrt{(j+m+1)(j+m+2)} \\ & \times\sqrt{(j-m)(j-m-1)}\,\delta_{m+2,m'} + \frac{\hbar}{4N}(2\Omega - g)\sqrt{(j-m)(j+m+1)}\times(j-m-1)\,\delta_{m+1,m'} \\ & + \frac{\hbar}{4N}(2\Omega + g)\sqrt{(j-m)(j+m+1)}\times(j+m)\,\delta_{m+1,m'} + \frac{\hbar}{4N}(2\Omega + g)\sqrt{(j+m)(j-m+1)} \\ & \times(j+m-1)\,\delta_{m-1,m'} + \frac{\hbar}{N}(2\Omega - g)\sqrt{(j+m)(j-m+1)}\times(j-m)\,\delta_{m-1,m'} + \frac{\hbar}{4N}(\Omega + g) \\ & \times(j+m)(j+m-1)\,\delta_{m,m'} + \frac{\hbar}{4N}(\Omega - g)(j-m)(j-m-1)\,\delta_{m,m'} + \frac{\hbar}{N}\Omega(j^2 - m^2)\,\delta_{m,m'}.\end{aligned} \tag{27}$$

Therefore the eigenvalues of H are

$$E_{jm}=\hbar\Omega\mathcal{N}+2m\hbar g+\frac{\hbar}{N}\Omega(j^2-m^2)+\frac{\hbar}{4N}(\Omega+g)(j+m)\times(j+m-1)+\frac{\hbar}{4N}(\Omega-g)(j-m)(j-m-1). \quad (28)$$

In general, we could obtain all the eigenfunctions of H under the first-order approximation by using Eqs. (24), (26), and (27). So the time evolution operator of the system is written as:

$$U(t)=e^{-itH/\hbar}=\sum_{2j=0}^{\infty}\sum_{m=-j}^{j}e^{-itE_{jm}/\hbar}|\psi_{jm}\rangle\langle\psi_{jm}|. \quad (29)$$

V. FLUORESCENCE SPECTRUM OF THE EXCITONS

We firstly give an analytic expression for the physical spectrum of the q-deformed excitons in terms of the Fock state of the quantum field and the excitons. The standard definition of the physical spectrum is [25]

$$S(\omega)=2\gamma\int_0^t dt_1\int_0^t dt_2 e^{-(\gamma-i\omega)(t-t_2)}e^{-(\gamma+i\omega)(t-t_1)}G(t_1,t_2), \quad (30)$$

where γ is the half-bandwidth of spectrometer that is being used to measure the spectrum, and t is time length of the excitation in the cavity. Provided an arbitrary initial state $|i\rangle$ of the system, the dipole correlation function is

$$G(t_1,t_2)\equiv\langle i|U^{\dagger}(t_2)b_q^{\dagger}U(t_2)U^{\dagger}(t_1)b_qU(t_1)|i\rangle=\sum_{j,k,l,m,n,}\langle\psi_{jl}|b_q^{\dagger}|\psi_{km}\rangle\langle\psi_{km}|b_q|\psi_{jn}\rangle\times\langle i|\psi_{jl}\rangle\langle\psi_{jn}|i\rangle e^{i\omega_{jl,km}t_2}e^{-i\omega_{jn,km}t_1} \quad (31)$$

with $\omega_{jl,km}=(E_{jl}-E_{km})/\hbar$ and $\omega_{jn,km}=(E_{jn}-E_{km})/\hbar$. Here the approximate evolution operator $U(t)$ in Eq. (29), keeping only the first order of $1/N$, has been used and in the following calculation the bosonic expansion of the q-deformed exciton operators, Eqs. (13)–(14), will also be used. It's evident that j is determined only by the initial state $|i\rangle$. So we have

$$S(\omega)=\sum_{j,l,k,m}\frac{2\gamma}{\gamma^2+(\omega-\omega_{jl,km})^2}\times|\langle i|\psi_{jl}\rangle|^2|\langle\psi_{jl}|b_q^{\dagger}|\psi_{km}\rangle|^2. \quad (32)$$

Noting that we have omitted the transient terms and slowly variation terms. This equation gives the stationary physical spectrum in terms of the eigenvalues and eigenstates of the system. If $\langle m'j'|b_q^{\dagger}|jm\rangle\neq 0$, then we have $j'=j+\frac{1}{2}$ and $m'=m-\frac{1}{2}$. So Eq. (33) can be rewritten as

$$S(\omega)=\sum_{j,l,m}\frac{2\gamma}{\gamma^2+(\omega-\omega_{jl,(j-1/2)m})^2}\times|\langle i|\psi_{jl}\rangle|^2|\langle\psi_{jl}|b_q^{\dagger}|\psi_{(j-1/2)m}\rangle|^2. \quad (33)$$

The eigenvalues determine the position of the spectral component and $|\langle i|\psi_{jl}\rangle|^2|\langle\psi_{jl}|b_q^{\dagger}|\psi_{(j-\frac{1}{2})m}\rangle|^2$ determine the intensity of the spectral lines.

Under the experimental condition of Ref. [26], the bare excitons could be prepared by resonant femtosecond pulse pumping. If we prepare the initial state $|i\rangle$ in the subspace of $\mathcal{N}=1$, then Eq. (34) shows that the emission spectrum of $\mathcal{N}=1$ to $\mathcal{N}=0$ transition has double peaks structure, which is exactly equal to that of the two-level atomic system. When the pumping power is increased, the emission spectrum is quite different from the case of the two-level atomic system. For example, if the system is initially prepared in the subspace $\mathcal{N}=2$, then we have:

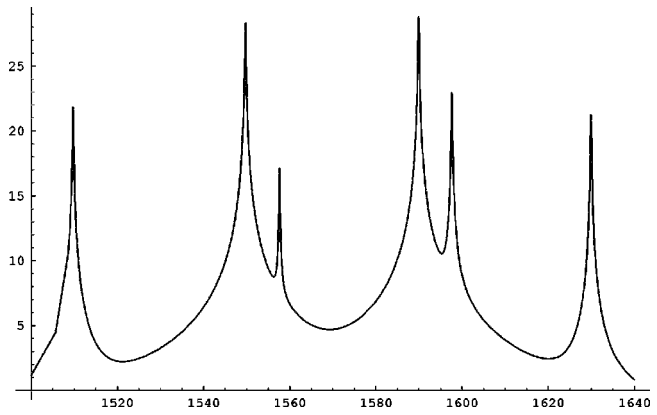

FIG. 1. The physical spectrum $S(\omega)$ is plotted as a function of ω in the case of $\Omega=1562$ MeV, N=100, g=20 MeV, and $\gamma=0.1$ MeV.

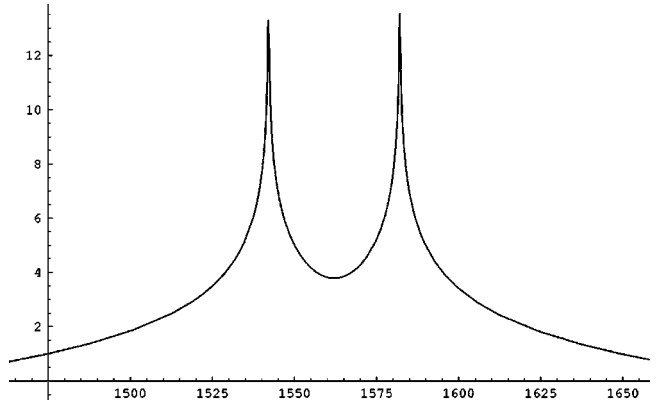

FIG. 2. The physical spectrum $S(\omega)$ is plotted as a function of ω when the molecular number $N=10\,000$ and other parameters are the same as those in Fig. 1 ($\Omega=1562$ MeV, g=20 MeV, γ=0.1 MeV).

$$S(\omega)=\sum_{l,m}\frac{2\gamma}{\gamma^2+(\omega-\omega_{lm})^2}\times|\langle i|\psi_{1l}\rangle|^2|\langle\psi_{1l}|b_q^{\dagger}|\psi_{\frac{1}{2}}m\rangle|^2 \tag{34}$$

with $\omega_{lm}=\omega_{1l,(1/2)m}$. As expected, from this equation we see immediately that there are six peaks in the emission spectrum when $\mathcal{N}=2$.

Although there are three different initial states, they will result in similar shapes of spectra. As an example, in Fig. 1 we plot the physical spectrum $S(\omega)$ as a function of the frequency ω when $|j=1,m=0\rangle$ is taken as the initial state. In contrast, the emission spectrum from a strong pumped two-level system possesses a triplet structure [27] instead of the sextet structure manifested here.

When the molecular number of the system is increased, e.g., there are 10 000 molecules in the system, the other conditions are the same as that in Fig. 1, the coupling between the molecules and the cavity field is weak. In this case there are only two peaks in the emission spectrum as shown in Fig. 2 and the Bose approximation is valid.

VI. CONCLUSION

It has been shown that Frenkel excitons with relative high density obey naturally the q-deformed commutation relation keeping the first order of $1/N$. Based on this observation the quantum theory of the angular momentum is employed here to obtain the eigenvalues and eigenfunctions, i.e., the dressed states of the cavity field and the q-deformed excitons of the exciton system under the same approximation. Compared with the usual approach to the Frenkel exciton dynamics our Hamiltonian is Hermitian and closed in form. And above all, the deformation parameter q is no longer phenomenological and is determined by the total molecular number. An analytical expression for the stationary physical spectrum for the excitons is obtained with the help of the dressed states of the cavity field and the excitons.

ACKNOWLEDGMENTS

This work was supported by the NSF of China and ''9-5 Pandeng'' project.

[1] H. Haken, *Quantum Field Theory of Solids* (North-Holland, Amsterdam, 1976).
[2] R.S. Knox, *Theory of Excitons* (Academic Press, New York, 1963).
[3] A.S. Davydov, *Theory of Molecular Excitons* (Plenum Press, New York, 1971).
[4] E. Hanamura, J. Phys. Soc. Jpn. **37**, 1545 (1974); **37**, 1553 (1974).
[5] M.L. Steyn-Ross and C.W. Gardiner, Phys. Rev. A **27**, 310 (1983).
[6] Y. Manabe, T. Tokihiro, and E. Hanamura, Phys. Rev. B **48**, 2773 (1993).
[7] T. Tokihiro, Y. Manabe, and E. Hanamura, Phys. Rev. B **47**, 2019 (1993).
[8] T. Tokihiro, Y. Manabe, and E. Hanamura, Phys. Rev. B **51**, 7655 (1995).
[9] M. Arik and D. Coon, J. Math. Phys. **17**, 524 (1976).
[10] L.C. Biedenharn, J. Phys. A **22**, L873 (1989).
[11] C.P. Sun and H.C. Fu, J. Phys. A **22**, L983 (1989).
[12] A.J. Macfarlan, J. Phys. A **22**, 4851 (1989).
[13] For a review article, see C.P. Sun and M.L. Ge, *The q Deformed Boson Realization of Quantum Algebra and its Application to Yang-Baxter Equation*, in *Quantum Group and Quantum Integrable Systems* edited by M.L. Ge (World Scientific, Singapore, 1992), p. 133.
[14] D. Bonatsos, E.N. Argyres, S.B. Prenska, P.P. Reychev, R.R. Roussev, and Y.F. Smirnov, Phys. Lett. B **251**, 477 (1990); J. Phys. G **16**, 137 (1990).
[15] Z. Chang, H.Y. Guo, and H. Yan, Phys. Lett. A **192**, 156 (1991); Z. Chang and H. Yan, *ibid.* **254**, 154 (1991).
[16] J.M. Li and C.P. Sun, Phys. Lett. A **180**, 50 (1993).
[17] C.W. Gardiner, Phys. Rev. A **56**, 1414 (1997).
[18] M. Girardeau and R. Arnowitt, Phys. Rev. **113**, 755 (1959).
[19] M.D. Girardeau, Phys. Rev. A **58**, 775 (1998).
[20] R. Dicke, Phys. Rev. **93**, 99 (1954).
[21] N.N. Bogoliubov, J. Phys. (Moscow) **11**, 23 (1947).
[22] A.L. Fetter and J.D. Walecka, *Quantum Theory of Many-Particle System* (McGraw-Hill, New York, 1971).
[23] L.C. Beidenharn and J.D. Louck, *Angular Momentum in Quantum Physics* (Addison-Wesley, London, 1981), pp. 205–264
[24] Jun-ichi Inoue, Tobias Brandes, and Akira Shimizu, e-print cond-mat/9905085.
[25] J.H. Eberly and K. Wodkiewicz, J. Opt. Soc. Am. **67**, 1252 (1977).
[26] Y. Yamamoto, J. Jacobson, S. Pau, H. Cao, and G. Bjork, in *Quantum Coherence and Decoherence*, edited by K. Fujikawa and Y.A. Ono (Elsevier Science, Amsterdam, 1996), pp. 331–334.
[27] B.R. Mollow, Phys. Rev. **188**, 1969 (1969).