Quantum measurement via Born-Oppenheimer adiabatic dynamics

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(Received 10 November 1999; published 13 December 2000)

The Born-Oppenheimer adiabatic approximation is used to describe the dynamic realization of wave-function collapse in quantum measurement. In the adiabatic limit, it is shown that the wave function of the total system formed by the measured quantum system plus the measuring apparatus can be factorized as an entangled state with correlation between adiabatic quantum states and quasiclassical motion configurations of the large system. When the apparatus effectively behaves as a classical object, this adiabatic entanglement leads to the wave-function collapse, which creates an ideal quantum measurement process.

DOI: 10.1103/PhysRevA.63.012111 PACS number(s): 03.65.Ta, 03.67.−a

In von Neumann's quantum measurement theory, the wave-packet collapse (WPC) of a measured system S can be described as a dynamic evolution process through an appropriate coupling with the measuring apparatus D (detector) [1]. But this approach brings with it a philosophical difficulty known as the von Neumann chain. A second detector should be introduced to monitor the first one so that the first one can be decohered classically, and for the same reason a third one, a fourth one, and so on should be introduced until we finally have a final detector, which is not described by quantum mechanics and thus gives definite outputs [2]. To overcome this difficulty that physicists have to confront, the boundary between the classical and quantum worlds should be physically clarified so that the sequence of detectors could be cut off reasonably [3].

A direct way to avoid the introduction of the sequence of detectors after the first one is to take the macroscopic character of the detector D into account. This idea was proposed in 1972 by Hepp and Coleman with an explicit illustration [4]. The crucial major point in this approach was then clarified by Bell in a reasonable criticism [5]. In the spirit of this approach, it was manifested via a simple exactly solvable model [the Hepp-Coleman (HC) model] that the WPC appears dynamically when the detector is a ''large system'' and the number of its constituting blocks approaches infinity. Later on, Namiki *et al.* generalized this work and put forward various new models for quantum measurement [6].

In 1992, after analyzing the original HC model and its various generalizations, one of the authors (C.P.S.) found that what underlies these models is a factorization structure in the off-diagonal elements of the reduced density matrix for the measured system [7,8]. First, starting from a pure state of S, an appropriate interaction between S and D will force the total system to evolve into a quantum entangled state [3,9] for S plus D. Then, by tracing out the variables of D, the reduced density matrix of S is obtained with the off-diagonal elements proportional to decoherence factors $F_{m,n}$. Finally, under the assumption that the detector is composed of N particles, a *factorization structure* implied in the WPC models is exposed: $F_{m,n}=\Pi_{j=1}^{N} f_{m,n}^{[j]}$. Here, each factor $f_{m,n}^{[j]}$ has a norm less than unity. So the product $F_{m,n}$ of an infinite number of such factors may approach zero as $N\to\infty$. Correspondingly, the WPC phenomenon or quantum decoherence is dynamically described through the vanishing of the off-diagonal elements of the reduced density matrix. Recently, this factorization theory has been applied to the analysis of the universality of the environmental influences on a quantum computation process [10].

The above discussion concerns only one of the macroscopic and classical behaviors of D. As regards the latter, a particular situation is that a certain dynamic variable of the detector takes a huge quantum number, e.g., the angular momentum approaches infinity [7,11,12]. Here the point is, in order to discuss the classical feature, the detector need not be composed of many particles. Actually one can assume it only possesses a single degree of freedom. In this case, a quantum system regarded as a measuring apparatus differs from other quantum systems in that it is a ''large object,'' which is effectively classical in a certain sense. In the subsequent discussion, we assume the detector to be a ''heavy system,'' which is prepared in a quantum state beforehand but has to approach the classical limit. In the classical limit, a particle approximately possesses a definite trajectory and the relevant mean-square deviation of the observable is zero. We will return to this point and further explain it later. For the moment, we only point out that a quantum system interacting with a ''heavy'' system can decohere to realize a quantum measurement. This paper is devoted to a study of the classical limit of a measuring apparatus only with reference to the quantum mechanics of the total system formed by S plus D.

In a very wide sense, any interaction between two quantum systems can cause an entanglement between them. This then creates a quantum measurement in a certain sense. This is due to the fact that one quantum system in different states can act on another quantum system with different effects correspondingly. But usually this entanglement and the related quantum measurement are not very ideal because the usual interaction cannot produce a clean one-to-one correspondence between the states of the two systems. Indeed, only a very particular interaction or its effective reduction directly leads to an ideal entanglement and realize an ideal quantum measurement. Nevertheless, we will prove in the following that if one of the two systems can be separated *adiabatically* and behaves *classically*, a general interaction can result in an ideal entanglement and create the WPC. In our discussion, quantum decoherence plays a crucial role in the definition of the classical limit of the detector as well as in the realization of the wave-packet collapse of the measured system. Using the Born-Oppenheimer adiabatic approximation [13,14], we will show that, in the adiabatic limit, the wave function of the total system formed by the measured quantum system plus the measuring apparatus can be factorized as an entangled state with a correlation between adiabatic quantum states and quasiclassical motion configurations of the large system. When the apparatus effectively behaves as a classical object (this is the case in the classical limit), one can draw information about the measured system from quasiclassical states (e.g., a moving and slowly spreading narrow wave packet which centers around a definite classical trajectory) of the detector. Thus this adiabatic entanglement means decoherence of the coherent superposition of quantum states of the measured system. It leads to the wave-function collapse and creates an ideal quantum measurement process.

In the spirit of the Born-Oppenheimer (BO) approximation, we consider a total quantum system (''molecular'') with two sets of variables, a fast (''electric'') one q and a slow (nuclear) one x. Resolving the dynamics of the fast part q for a given motion of the slow part, we obtain certain quantum states labeled by n for q. When the x part moves so slowly that the internal transition of the q part is not excited by the backaction of the x part, the left effective Hamiltonian governing x involves an external scalar potential $V_n(x)$. It is noticed that there may be a magnetic-potential-like vector potential $A_n(x)$ induced by the q part [15], but for our major purpose here we need only consider the generic case without $A_n(x)$. In fact, the following discussions do not concern the cyclic evolution giving prominence to the effects of the Berry phase [16]. If we assume that the motions of the slow subsystem are ''classical,'' we naturally observe that, due to the backaction of the fast part, there are different induced forces $F_n=-\nabla_x V_n(x)$ exerting on the slow part. Then the direct physical consequence is that the information of the ''fast'' states labeled by n is recorded in the different motion configurations of the slow part. An entanglement just stems from this correlation.

Let us now analyze the quantum dynamics governed by the interaction between a quantum system S with fast dynamic variable q and the detector D, a large system with slow variable x. Their Hamiltonians are $H_s=H_s(q)$ and $H_d=H_d(x)$, respectively. In general, the interaction Hamiltonian is written as $H_I=H_I(x,q)$. For a fixed value of the slow variable x of D, the dynamics of the quantum system is determined by the eigenequation of $H_q[x]=H_s(q)+H_I(x,q)$. The corresponding eigenvalues $V_n[x]$ and eigenstates $|n\rangle=|n[x]\rangle$ all depend on *the slow parameter x*.

When the variable x changes slowly enough, the transition from an energy level $V_n(x)$ to another $V_m(x)$ caused by the variation of the Hamiltonian $H_q[x]$ with x, can be physically neglected. Specifically, this requires that the adiabatic condition [14]

$$\frac{|\langle n|\{\partial_x H_I(x,q)\}|m\rangle\dot{x}|}{|V_m(x)-V_n(x)|^2}\ll 1 \tag{1}$$

hold for any two of the different energy levels $\{V_n(x)\}$ within the spatial domain R to which the slow variable x belongs. Let $|\Phi_{n,\alpha}\rangle$ be the full eigenfunction of the full Hamiltonian $H=H_d(x)+H_q[x]$ for the total system. The BO approximation treats it as a partially factorized function $|\Phi_{n,\alpha}\rangle=|n\rangle\otimes|\phi_{n,\alpha}\rangle$ with respect to the fast and slow variables q and x. Here, the set of slow components $\{|\phi_{n,\alpha}\rangle\}$ and the corresponding eigenvalues $\omega_{n,\alpha}$ can be obtained by solving the effective eigen equation of the BO effective Hamiltonian,

$$H_n(x)=H_d(x)+V_n(x). \tag{2}$$

After obtaining the complete set of eigenstates $\{|n\rangle\otimes|\phi_{n,\alpha}\rangle\}$ of the total system, we can consider how the entanglement appears in adiabatic dynamic evolution.

Let the total system be initially in a state $|\Psi(t=0)\rangle=|\varphi\rangle\otimes|\phi\rangle$, where the initial state of the system, $|\varphi\rangle=\Sigma_n c_n|n\rangle$, is a coherent superposition of the adiabatic eigenstates of S, and $|\phi\rangle$ is a single pure state of D. It will be proved that, starting from $|\varphi\rangle$, the adiabatic evolution of the system will lead to the WPC in the quantum measurement about an observable Q, which commutes with the Hamiltonian $H_q[x]$ and satisfies $Q|n\rangle=\lambda_n|n\rangle$. Expanding the total initial state $|\Psi(t=0)\rangle$ in terms of the complete set $\{|n\rangle\otimes|\phi_{n,\alpha}\rangle\}$, we obtain the evolution wave function at time t,

$$|\Psi(t)\rangle=\sum_n c_n|n\rangle\otimes|d_n(t)\rangle, \tag{3}$$

where $|d_n(t)\rangle=\exp[-iH_n t]|\phi\rangle$ are just those evolution states starting from the same initial state $|\phi\rangle$, but driven by different effective potentials $V_n(x)$. The full wave function $|\Psi(t)\rangle$ is obviously an entangled state, a superposition of $|n\rangle\otimes|d_n(t)\rangle$ with the correlations between different final states $|d_n(t)\rangle$ of D and different adiabatic eigenstates of S.

This intuitive argument shows us that, even in the case of a quite general interaction, there can exist an entanglement between the two quantum systems when one of them moves so slowly that their dynamic variables can be adiabatically factorized according to the BO approximation. This adiabatic formalism for quantum measurement is similar to the Stern-Gerlach (SG) experiment [17], which detects the spin states of an A_g atom on the ground state by observing the spatial distribution on a screen. In the SG experiment, the entanglement is reflected by two correlations: the spin-up state $|\uparrow\rangle$ corresponds to the spatial state $|d_\uparrow(t)\rangle$ of the upper spot; the spin-down state $|\downarrow\rangle$ corresponds to the spatial state $|d_\downarrow(t)\rangle$ of the lower spot. It is noticed that the adiabaticity is crucial to produce such an entangled state $|\Psi(t)\rangle$. To describe it in a quantitative way, an adiabatic parameter was precisely introduced recently [18]. Its dual version can be considered in certain perturbation series with different choices of the leading terms corresponding to the different perturbative Hamiltonians [18]. In this sense, the BO approximation can be interpreted in the following way: the kinetic term $P^2/2M$ acts as a perturbation for the ''large object'' with a large mass parameter M and a very small initial momentum. This argument shows that the interaction between a large and a small objects implies a reasonable extension of the concept of quantum measurement.

It will be argued that, when the detector behaves classically under the BO approximation, the entangled adiabatic $|d_n(t)\rangle$ $(n=1,2,\dots)$ may be orthogonal to one another, i.e., $|d_n(t)\rangle$ may be ''classically distinguishable.'' Then we can say the above-mentioned adiabatic measurement is ideal since the WPC happens as a result of quantum decoherence of the reduced density matrix, namely a transition from $\rho_s(0)=|\varphi\rangle\langle\varphi|$ characterizing the initial pure state to $\rho_d(t)=\Sigma_n|c_n|^2|n\rangle\langle n|$ describing a completely mixed state. To show it, we will need the following expression of the general reduced density matrix of the measured system:

$$\rho_s(t)=\mathrm{Tr}_D(|\Psi(t)\rangle\langle\Psi(t)|)$$
$$=\sum_n |c_n|^2|n\rangle\langle n|+\sum_{n\neq m} c_m c_n^* F_{nm}|m\rangle\langle n|\langle d_n(t)|d_m(t)\rangle. \tag{4}$$

Here the overlapping $F_{nm}=\langle d_n(t)|d_m(t)\rangle$ of the two detector states is called the decoherence factor. A complete decoherence is defined by $F_{n,m}=0$ while a complete coherence is defined by $F_{n,m}=1(m\neq n)$.

We are now in a position to study the dynamical realization of the WPC quantitatively. It boils down to considering in what case the decoherence factors become zero so that the off-diagonal elements of the reduced density matrix vanish simultaneously. We recall the following widely accepted viewpoint clearly stated by Landau and Lifshitz [19]: in the classical limit, the expectation value of an observable for a particular state (e.g., a coherent state or its squeezed versions) can recover its classical value form. Specifically, it takes Fejér's arithmetic mean of the partial sums of the Fourier series of its corresponding classical quantity [19]. In this view, these particular states can well describe definite classical trajectories of a particle in this limit and the relevant mean-square deviation of the observable is zero. The mean of the position operator defines a classical path in such a limit. Physically, the zero mean-square deviation of the position operator implies the zero width of each wave packet $\langle x|d_m(t)\rangle$, and the overlapping $F_{n,m}=\langle d_n(t)|d_m(t)\rangle$ of wave packets of almost-zero width must vanish. In such a semiclassical picture, the initial state $|\varphi\rangle$ can be regarded as a very narrow wave packet for a heavy particle. Then we describe the detector by a moving wave packet with the center along a classical path $x(t)$ on a manifold with local coordinates x. For a proper initial state $|\varphi\rangle$, we will see that the wave packet will split into several very narrow peaks with the centers moving along different paths without efficient overlaps. These quasiclassical paths are determined by the effective forces $F_n=-\nabla_x V_n(x)$ through different motion equations.

To support the above physically intuitive argument, we carry out an explicit calculation. We assume $H_d(x)=p^2/2M$ and the interaction $H_I(x,q)$ is a smooth function of x such that the effective potential $V_n(x)$ is also satisfactorily smooth. This assumption is reasonable since the adiabatic condition of the BO approximations requires that the velocity $\dot{x}\approx-[\nabla V_n(x)/Mt]$ (if the initial velocity of D is zero) should be small enough. So we can linearize the effective potentials: $V_n(x)\approx V_n(0)-F_n x$, where $F_n\approx-\nabla V_n(0)$ are the classical forces of backaction on the detector D, which correspond to different system states. Accordingly, we have the effective Hamiltonians

$$H_n=\frac{p^2}{2M}+V_n(0)-F_n x. \tag{5}$$

In the semiclassical picture, driven by different forces F_n, the detector will finally form some macroscopically distinguishable spots on the detecting screen, each of which is correlated to one of the adiabatic states.

To see the WPC in the measurement process clearly, we assume that the detector is initially in the state of a Gaussian wave packet of width a:

$$|\phi\rangle=\int\left(\frac{1}{2\pi a^2}\right)^{1/4}e^{-x^2/4a^2}|x\rangle dx \tag{6}$$

distributed along direction x and centered around the original point. Following the Wei-Norman method [20,21], we first factorize the evolution operator $U_n(t)=\exp[-iH_n t]$ as $U_n(t)\equiv e^{\alpha_n(t)p^2}e^{\beta_n(t)p}e^{\gamma_n(t)x}e^{\mu_n(t)}$ formally [21]. Then we can exactly obtain the effective wave functions $|d_n(t)\rangle$:

$$\langle x|d_n(t)\rangle=\left(\frac{a^2}{2\pi^3}\right)^{1/4}\left(\frac{\pi}{a^2+\frac{it}{2M}}\right)^{1/2}$$
$$\times\exp\left[-i\Omega_n(t)+ixF_n t-\frac{[x-x_n(t)]^2}{4\left(a^2+\frac{it}{2M}\right)}\right], \tag{7}$$

where $\Omega_n(t)=V_n(0)t+(F_n^2t^3/6M)$.

It is seen from Eq. (9) that the Gaussian wave packets $\langle x|d_n(t)\rangle$ are centered on the classical trajectories $x_n(t)=\frac{1}{2}(F_n/M)t^2$ and have different group speeds $v_n=(F_n/M)t$, respectively, but they have the same width $a(t)=a[1+(t^2/4M^2a^2)]^{1/2}$ spreading with time. It is obvious that the wave-packet centers move in the same way as a classical particle of mass M forced by F_n. Here the quantum character is reflected in the spreading of the wave packets. The macroscopic distinguishability of wave packets in quantum measurement requires that the distance between the centers of two wave packets should be larger than the width of each wave packet, i.e.,

$$\frac{|F_n-F_m|}{M}t^2\gg a\left(1+\frac{t^2}{4M^2a^2}\right)^{1/2}. \tag{8}$$

This condition is easily satisfied when t is sufficiently large.

To see how the decoherence happens quantitatively, we compute the norm of the decoherence factor: $F_{mn}(t)$, which completely determines the extent of the quantum coherence of the measured system. This is an overlapping integral, which we can explicitly integrate as follows:

$$|F_{mn}(t)|=\exp\left[-\frac{(F_n-F_m)^2t^4}{32M^2a^2}-\frac{1}{2}a^2(F_n-F_m)^2t^2\right]. \tag{9}$$

One easily sees from this formula that the decoherence process indeed appears as $t\to\infty$. The character time τ_{mn} of the WPC is determined by the equation $F_{mn}(\tau_{mn})=e^{-1}$, that is,

$$\tau_{mn}=2\sqrt{Ma[\sqrt{4M^2a^6+2(F_n-F_m)^{-2}}-Ma^3]}. \tag{10}$$

We have associated the WPC or quantum decoherence problem with the adiabatic separation of the total system formed by the measured system plus the detector. We have seen that under certain reasonable conditions, ideal entanglement does happen adiabatically to meet the requirement that the result of a measurement should be classically observable . At this point, it is worth pointing out that in the adiabatic limit, the nondemolition interaction [22] can also effectively realize a quantum measurement. In our discussion, besides the use of the Born-Oppenheimer approximation with adiabaticity, the other key point is the classical limit of the detector. On the other hand, in the theory of continuous quantum measurement there have been a lot of discussions about the attainment of the classical limit, but in a different sense [23]. It is apparent that any quantum system coupling to another ''large'' system cannot stay in a pure state and its dynamics must be described by a mixed state, or more specifically, by a reduced density matrix with rank larger than 1 (in the case of a pure state, the rank is exactly 1). Usually, dealing with the environment with many degrees of freedom, one can apply the Master equation to describe the evolution of the mixed state of a quantum system. But it should be noticed that the present ''heavy'' system may possess only one single dynamic variable. Thus the corresponding Master equation description should be essentially different from that used for an environment with many dynamic variables or many particles inside. Thus we think the question of how to use the Master equation to describe dissipation and decoherence of a quantum system coupling to a ''heavy'' system with one single effective variable might be an intricate problem, which is appropriate to be addressed in further works.

We also point out that the adiabatic entanglement presented here can be well understood in the picture of coupled channels [24]. In terms of certain ''internal'' states $\{|n\rangle\}$, the total eigenfunction $\Psi_E(x,q)$ can be expressed as $\Psi_E(x,q)=\Sigma\phi_n(x)|n\rangle$. The *channel wave function* $\phi_n(x)$ defined here obeys the *coupled channel equations* $H_n\phi_n(x)=E\phi_n(x)+\Sigma_{m\neq n}H_{nm}\phi_m(x)$. When the channel coupling $H_{m,n}$ can be ignored physically, each internal state $|n\rangle$ correlates to a channel state ϕ_n. The diagonal elements H_n play a dominant role in measuring the internal states by ϕ_n. The recent experiments [25,26] give support to the following argument: as long as the ''which-way'' information already stored in the detector *could be read out*, the interference pattern implied by the off-diagonal elements in the reduced density matrix would be destroyed *without any data being read out in practice*. According to this argument, we can compare the environment-induced decoherence with the quantum measurement dealt with above and come to the conclusion that actually the environment surrounding the quantum system behaves as a detector to realize a ''measurement-like'' process. This is because the environment *never needs to read out* the data. Thus, the argument in this paper is also valid for the analysis of decoherence problems in an interfering quantum system coupling to the environment.

Before concluding this paper, we notice that the actual calculation in this paper is only carried out in a general case without emphasizing its inherent experimental significance. To develop the present study so as to provide insight into possible experiments, we should probe various coupling systems that permit the Born-Oppenheimer approximation. There are concrete problems such as the dynamics of the small spin–large spin interacting system, microcavity-mirror coupling dynamics with a classical source [27], and the localization of a macroscopic object through adiabatic scattering, which can serve this purpose [2]. For instance, we consider the intracavity dynamics. In this case, there is a cavity with two end mirrors, one of which is fixed while the other is treated as a simple harmonic oscillator with a large mass. In the radiofrequency range, when the cavity field is driven by macroscopic currents, the cavity field can be treated as a forced harmonic oscillator. This cavity field–mirror coupling system can also be used to detect the photon number in the cavity. The detection can be realized by the motion of the mirror. Obviously, the motion of the mirror is slow with respect to the oscillation of the cavity field. Thus we can use the BO approximation to approach the quantum decoherence problem in the measurement of the cavity field. Most recently, the special case of this model without a classical source has been used to present a scheme for probing the decoherence of a macroscopic object [27]. In the presence of a classical source, this cavity-field–mirror coupling system can be analyzed using the general approach that this paper provides.

This work was supported by a direct grant (Project ID No. 2060150) from The Chinese University of Hong Kong. It was also partially supported by the NSF of China. One of the authors (C.P.S.) wishes to express his sincere thanks to P. T. Leung, C. K. Law, and K. Young for many useful discussions.

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