

Finite-temperature excitations of an inhomogeneous trapped Bose gas with Feshbach resonances

X X Yi^{†‡}, D L Zhou[‡], C P Sun[‡] and W M Zheng[‡]

[†] Institute of Theoretical Physics, Northeast Normal University, Changchun 130024, People's Republic of China

[‡] Institute of Theoretical Physics, Academia Sinica, PO Box 2735, Beijing 100080, People's Republic of China[§]

Received 22 October 1999, in final form 15 December 1999

Abstract. We derive and discuss the temperature dependence of the condensate and noncondensate density profile of a Bose–Einstein condensate gas with Feshbach resonance in a parabolic trap. These quantities are calculated self-consistently using the generalized Hartree–Fock–Bogoliubov (HFB) equations within the Bogoliubov approximation. At zero temperature, the HFB equation can be solved by means of a variation method that gives the low-excitation spectrum. Moreover, within the two-body collision theory, we estimate the relationship between the number of atoms in the condensate and the external magnetic field B . The result is in good agreement with data from recent experiments.

Bose–Einstein condensates (BECs) [1–4] of atomic gases offer new opportunities for studying quantum-degenerate fluids. Almost all the essential properties of BEC systems such as the formation and shape of the condensate and noncondensate, the nature of its collective excitations and statistical fluctuations, and the formation and dynamics of vortices are determined by the strength of atom–atom interactions. In contrast with the situation in traditional superfluids, the strength of inter-particle interactions in the atomic condensate can vary over a wide range of values [5–12]. In particular, the scattering length that characterizes the atom–atom interactions can be negative, corresponding to an effective inter-atom attraction.

Most recently, in trapped atomic Bose–Einstein condensation, Ketterle's group reported evidence for modifying the scattering length by magnetic-field-induced Feshbach resonance [13]. Feshbach resonance was studied 21 years ago [14] at much higher energies, but the Feshbach resonance energy observed in ultracold atoms can be tuned to near zero. Theoretical studies of the ultracold atoms with Feshbach resonance [15, 16] show that the two-body interactions responsible for the Feshbach resonance produce an additional condensate of molecules [17], which differs qualitatively from the properties of a single condensate. In this paper, we discuss the temperature dependence of the hybrid atomic/molecular condensate and noncondensate density profile as well as the excitation spectrum. Especially, for an inhomogeneous system with negative scattering length, the excitation spectrum shows an upper constraint on the atom number, which, together with the conventional estimation of the condensate atom number, gives the relationship between the atom number and the external magnetic field B . The theoretical results are in good agreement with the experiment.

[§] Mailing address.

The binary atom Feshbach resonances studied by varying a strong external magnetic field in an alkali-atom trap are hyperfine-induced spin-flip processes that bring the colliding atoms to a bound molecular state of different electron spin. This process can be described by the Hamiltonian

$$H_{FR} = \alpha \int d^3r \Psi_m^+(r) \Psi_a(r) \Psi_a(r) + \text{h.c.}, \quad (1)$$

where $\Psi_m(r)$, $\Psi_m^+(r)$ ($\Psi_a(r)$, $\Psi_a^+(r)$) are the annihilation and creation field operators of the molecules (atoms), α stands for the transition matrix element proportional to the overlap of the molecular continuum and bound state wavefunctions. Usually, the transition matrix element depends on the magnetic field as $\alpha \sim \sqrt{\lambda_a \Delta^2 / 2|B - B_0|}$, where B_0 is the resonant magnetic field and Δ characterizes the width of the resonance as a function of B . The Hamiltonian H_{FR} together with the atomic Hamiltonian

$$H_a = \int d^3r \Psi_a^+(r) \left[-\frac{\nabla^2}{2M} + V_a(r) - \mu_a \right] \Psi_a(r) + \frac{\lambda_a}{2} \int d^3r \Psi_a^+(r) \Psi_a^+(r) \Psi_a(r) \Psi_a(r), \quad (2)$$

the molecular Hamiltonian

$$H_m = \int d^3r \Psi_m^+(r) \left[-\frac{\nabla^2}{4M} + V_m(r) - \mu_m + \epsilon \right] \Psi_m(r) + \frac{\lambda_m}{2} \int d^3r \Psi_m^+(r) \Psi_m^+(r) \Psi_m(r) \Psi_m(r), \quad (3)$$

and the atom–molecule interaction Hamiltonian

$$H_{am} = \lambda \int d^3r \Psi_a^+(r) \Psi_m^+(r) \Psi_m(r) \Psi_a(r) \quad (4)$$

forms a total Hamiltonian $H = H_a + H_m + H_{am} + H_{FR}$, which governs the dynamics of the system under investigation. Here, $V_{a(m)}(r)$ represents the trapped potential for the atom (molecule), $\mu_{a(m)}$ is the chemical potential of the atoms (molecules), $\lambda_{a(m)} = \frac{4\pi a_{a(m)}}{(2)M}$, with M being the atomic mass and $a_{a(m)}$ the s-wave scattering length of the atom–atom interaction, λ denotes the coupling constant of the atom–molecule interaction, and ϵ is the energy of the intermediate molecular state relative to the continuum of the incident atoms.

We separate out the condensate part in the usual fashion (Bogoliubov approximation), i.e.

$$\Psi_{a(m)}(r) = \phi_{a(m)}(r) + \hat{\phi}_{a(m)}(r), \quad (5)$$

where $\phi_{a(m)}(r) = \langle \Psi_{a(m)}(r) \rangle$ plays the role of a spatially varying macroscopic Bose field of the atoms (molecules). The possibility that the resonantly formed quasibound atom pairs form a molecular condensate was previously suggested by Timmermans *et al* [15]. Using a Raman photon-association process, the quasibound pairs start to be formed from the atomic condensate and form a molecular condensate [17]. Here, we assume that there is a large number of atoms and molecules in the condensate. It is easy to show that the operators $\hat{\phi}_{a(m)}(r)$ and $\hat{\phi}_{a(m)}^+(r)$ obey the Bose commutation relations

$$[\hat{\phi}_{a(m)}(r), \hat{\phi}_{a(m)}^+(r')] = \delta(r - r'). \quad (6)$$

In terms of $\hat{\phi}_{a(m)}$ and $\phi_{a(m)}$, the Hamiltonian can be expanded as

$$H = H_0 + H',$$

$$H_0 = \int d^3r \left\{ \phi_a^* \left(-\frac{\nabla^2}{2M} - \mu_a + V_a(r) \right) \phi_a + \frac{\lambda_a}{2} \phi_a^* \phi_a^* \phi_a \phi_a \right.$$

$$\begin{aligned}
 & +\phi_m^* \left(-\frac{\nabla^2}{4M} - \mu_m + V_m(r) + \epsilon \right) \phi_m + \frac{\lambda_m}{2} \phi_m^* \phi_m^* \phi_m \phi_m \\
 & + \lambda \phi_a^* \phi_a \phi_m^* \phi_m + \alpha \phi_m^* \phi_a \phi_a + \alpha \phi_m \phi_a^* \phi_a^* \left. \right\} \quad (7)
 \end{aligned}$$

$$\begin{aligned}
 H' = \int d^3r \left\{ \hat{\phi}_a^+ \left(-\frac{\nabla^2}{2M} - \mu_a + V_a(r) \right) \hat{\phi}_a + 2\lambda_a \hat{\phi}_a^+ \hat{\phi}_a \phi_a^* \phi_a \right. \\
 + \frac{\lambda_a}{2} (\hat{\phi}_a^+ \hat{\phi}_a^+ \phi_a \phi_a + \hat{\phi}_a \hat{\phi}_a \phi_a^* \phi_a^*) \\
 + \hat{\phi}_m^+ \left(-\frac{\nabla^2}{4M} + \epsilon - \mu_m + V_m(r) \right) \hat{\phi}_m + 2\lambda_m \hat{\phi}_m^+ \hat{\phi}_m \phi_m^* \phi_m \\
 + \frac{\lambda_m}{2} (\hat{\phi}_m^+ \hat{\phi}_m^+ \phi_m \phi_m + \hat{\phi}_m \hat{\phi}_m \phi_m^* \phi_m^*) \\
 \left. + \lambda \hat{\phi}_a^+ \hat{\phi}_a \phi_m \phi_m^* + \lambda \hat{\phi}_m^+ \hat{\phi}_m \phi_a^* \phi_a + \alpha (\hat{\phi}_a \hat{\phi}_a \phi_m^* + \hat{\phi}_a^+ \hat{\phi}_a^+ \phi_m) \right\}. \quad (8)
 \end{aligned}$$

In the derivation of (7), (8), the following coupling equations are used [15]:

$$\begin{cases} -\frac{\nabla^2}{2M} + \lambda_a |\phi_a|^2 + V_a(r) + \lambda |\phi_m|^2 \} \phi_a + 2\alpha \phi_m \phi_a^* = \mu_a \phi_a, \\ -\frac{\nabla^2}{4M} + \lambda_m |\phi_m|^2 + V_m(r) + \epsilon + \lambda |\phi_a|^2 \} \phi_m + \alpha \phi_a \phi_a = \mu_m \phi_m. \end{cases} \quad (9)$$

These coupling equations may be yielded by the expectation value of the Heisenberg equations

$$i\hbar \dot{\Psi}_a = [\Psi_a, H], \quad i\hbar \dot{\Psi}_m = [\Psi_m, H], \quad (10)$$

and replacing the time derivatives by the chemical potentials

$$i\hbar \dot{\phi}_a \rightarrow \mu_a \phi_a, \quad i\hbar \dot{\phi}_m \rightarrow \mu_m \phi_m.$$

The chemical potential of the molecules is twice the chemical potential of the atoms, in accordance with the condition for chemical equilibrium. The α -terms that couple the equations describe the tunnelling of pairs of atoms between ϕ_m and ϕ_a fields, which leads to the formation of a second condensate—a molecular condensate in an atomic BEC [15–17]. Using the coupling equations (9), Timmermans *et al* [15] investigate the behaviours of the hybrid atomic/molecular condensates near- and off-resonance. The Hamiltonian (8) can be diagonalized by using the Bogoliubov transformation

$$\begin{aligned}
 \hat{\phi}_a(r) &= \sum_j [u_j(r) \alpha_j - v_j^*(r) \alpha_j^+], \\
 \hat{\phi}_a^+(r) &= \sum_j [u_j^*(r) \alpha_j^+ - v_j(r) \alpha_j], \\
 \hat{\phi}_m(r) &= \sum_j [x_j(r) \beta_j - y_j^*(r) \beta_j^+], \\
 \hat{\phi}_m^+(r) &= \sum_j [x_j^*(r) \beta_j^+ - y_j(r) \beta_j], \quad (11)
 \end{aligned}$$

where the quasiparticle operators $\alpha_j, \alpha_j^+, \beta_j, \beta_j^+$ obey boson commutation relations

$$\begin{aligned}
 [\alpha_i, \alpha_j^+] &= \delta_{ij}, [\alpha_i, \alpha_j] = [\alpha_i^+, \alpha_j^+] = 0, \\
 [\beta_i, \beta_j^+] &= \delta_{ij}, [\beta_i, \beta_j] = [\beta_i^+, \beta_j^+] = 0, \\
 [\alpha_i^+, \beta_j^+] &= [\alpha_i, \beta_j] = [\alpha_i, \beta_j^+] = 0,
 \end{aligned}$$

and $u_j(r)$, $v_j(r)$, $x_j(r)$, $y_j(r)$ are c -number functions. Substituting (11) into (8), one obtains

$$H' = \sum_j E_j \alpha_j^+ \alpha_j + \sum_i e_i \beta_i^+ \beta_i - \sum_j E_j \int d^3r |v_j(r)|^2 - \sum_i e_i \int d^3r |y_i(r)|^2 \quad (12)$$

with

$$\begin{aligned} & \left(-\frac{\nabla^2}{2M} + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2 - \mu_a + V_a(r) \right) u_j - (\lambda_a\phi_a^*\phi_a^* + 2\alpha\phi_m^*)v_j = E_j u_j, \\ & \left(-\frac{\nabla^2}{2M} + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2 - \mu_a + V_a(r) \right) v_j - (\lambda_a\phi_a^*\phi_a^* + 2\alpha\phi_m^*)u_j = -E_j v_j, \\ & \left(-\frac{\nabla^2}{4M} + \epsilon + \lambda|\phi_a|^2 + 2\lambda_m|\phi_m|^2 - \mu_m + V_m(r) \right) x_j - \lambda_m\phi_m^*\phi_m^*y_j = e_j x_j, \\ & \left(-\frac{\nabla^2}{4M} + \epsilon + \lambda|\phi_a|^2 + 2\lambda_m|\phi_m|^2 - \mu_m + V_m(r) \right) y_j - \lambda_m\phi_m^*\phi_m^*x_j = -e_j y_j. \end{aligned} \quad (13)$$

In order to study the temperature dependence of the excitation spectrum as well as the spatial distribution of the hybrid atom/molecular condensate and noncondensate, we need to solve the coupled mean-field Bogoliubov equations (13), and the condensate equation (9) self-consistently. The calculation procedure can be summarized for an arbitrary confining potential as follows: first, we solve (9) self-consistently and, once ϕ_a and ϕ_m are known, the solution of u_j , v_j , x_j and y_j can be generated. To illustrate this procedure, we present its first calculation step analytically. The trapped potential considered here is taken to be an isotropic harmonic potential $V_{a(m)}(r) = \frac{1}{2}M\omega_{a(m)}^2 r^2$, for which ϕ_a and ϕ_m are spherically symmetric functions,

$$\phi_{a(m)}(r) = R_{00}(r)Y_{00}(\theta, \psi), \quad (14)$$

with

$$R_{00}(r) = \alpha^{3/2} \sqrt{\frac{4}{\pi}} \exp\left[-\frac{1}{2}\alpha^2 r^2\right], \quad Y_{00}(\theta, \psi) = \frac{1}{\sqrt{4\pi}}, \quad \alpha = ((2)M\omega)^{1/2}.$$

Rather than solving the coupled equations (13) directly, we introduce a new method based on the auxiliary functions

$$\begin{aligned} u_j &= A_j \langle r|j \rangle_a, & v_j &= B_j \langle r|j \rangle_a, \\ x_j &= C_j \langle r|j \rangle_m, & y_j &= D_j \langle r|j \rangle_m, \end{aligned} \quad (15)$$

where $|j \rangle_{a(m)}$ is defined by

$$\left[-\frac{\nabla^2}{2(4)M} + V_{a(m)}(r) \right] |j \rangle_{a(m)} = \hbar\omega_{a(m)}(j + \frac{1}{2})|j \rangle_{a(m)}.$$

The reason for such a selection is that the level shifts caused by atom–atom interactions weakly depend on the shape of the wavefunction. A combination of (13)–(15) gives

$$\begin{aligned} & (\hbar\omega_a(j + \frac{1}{2}) + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2 - \mu_a)A_j - (\lambda_a\phi_a^*\phi_a^* + 2\alpha\phi_m^*)B_j = E_j A_j, \\ & (\hbar\omega_a(j + \frac{1}{2}) + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2 - \mu_a)B_j - (\lambda_a\phi_a^*\phi_a^* + 2\alpha\phi_m^*)A_j = -E_j B_j, \\ & (\hbar\omega_m(j + \frac{1}{2}) + \epsilon + \lambda|\phi_a|^2 + 2\lambda_m|\phi_m|^2 - \mu_m)C_j - \lambda_m\phi_m^*\phi_m^*D_j = e_j C_j, \\ & (\hbar\omega_m(j + \frac{1}{2}) + \epsilon + \lambda|\phi_a|^2 + 2\lambda_m|\phi_m|^2 - \mu_m)D_j - \lambda_m\phi_m^*\phi_m^*C_j = -e_j D_j, \end{aligned} \quad (16)$$

where the eigenfunctions and the corresponding eigenvalues are given by

$$\begin{aligned} B_j^\pm &= \left[\frac{1}{f^\pm(r, j) - 1} \right]^{\frac{1}{2}}, & A_j^\pm(r) &= f^\pm(r, j)B_j^\pm, \\ D_j^\pm &= \left[\frac{1}{g^\pm(r, j) - 1} \right]^{\frac{1}{2}}, & B_j^\pm(r) &= g^\pm(r, j)D_j^\pm, \end{aligned} \quad (17)$$

and

$$\begin{aligned} E_j^\pm(r) &= \pm\{(\lambda_a\phi_a^*\phi_a^* + 2\alpha\phi_m^*) - [\hbar\omega_a(j + \frac{1}{2}) - \mu_a + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2]\}, \\ e_j^\pm(r) &= \pm\{\lambda_m\phi_m^*\phi_m^* - [\hbar\omega_m(j + \frac{1}{2}) - \mu_m + \epsilon + 2\lambda_m|\phi_m|^2]\}. \end{aligned} \quad (18)$$

Here,

$$f^\pm(r, j) = \frac{\lambda_a\phi_a^*\phi_a^* + 2\alpha\phi_m^*}{\hbar\omega_a(j + \frac{1}{2}) - \mu_a + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2 - (E_j^\pm)^2},$$

and

$$g^\pm(r, j) = \frac{\lambda_m\phi_m^*\phi_m^*}{\hbar\omega_m(j + \frac{1}{2}) - \mu_m + \epsilon + 2\lambda_m|\phi_m|^2 - (e_j^\pm)^2}.$$

These explicit solutions enable us to construct the one-body density matrix

$$\begin{aligned} \rho(r, r') &= \rho_a(r, r') + 2\rho_m(r, r'), \\ \rho_a(r, r') &= \phi_a^*(r)\phi_a(r') + \sum_{p=\pm, i=1}^{\infty} [u_i^{p*}(r)u_i^p(r')F_i^p + v_i^{p*}(r)v_i^p(r')(1 + F_i^p)], \\ \rho_m(r, r') &= \phi_m^*(r)\phi_m(r') + \sum_{p=\pm, i=1}^{\infty} [x_i^{p*}(r)x_i^p(r')f_i^p + y_i^{p*}(r)y_i^p(r')(1 + f_i^p)], \end{aligned} \quad (19)$$

where $F_i^p = \frac{1}{\exp(\beta E_i^p) - 1}$ and $f_i^p = \frac{1}{\exp(\beta e_i^p) - 1}$ are the Bose distributions for the quasiparticle excitations with energies E_i^p and e_i^p , respectively. Setting $r = r'$, (19) follows the resulting particle density.

We need to point out that (17) and (18) are results of the first step of the numerical calculations. To complete numerical calculations, we should repeat the above procedures until the eigenvalues E_j and e_j do not depend on position r . In what follows, we present a variation method to study the excitations at zero temperature. This method was first introduced in [18] to study the ground state of BEC in the harmonic trap of a boson system, and it was generalized in [19] to investigate the excited states in the BEC. Considering (13) as well as

$$\int [u_j(r)u_j^*(r) - v_j(r)v_j^*(r)] dr = 1,$$

and

$$\int [x_j(r)x_j^*(r) - y_j(r)y_j^*(r)] dr = 1,$$

which were derived from the Bose commutation relation (6), we arrive at

$$\begin{aligned} E_j &= \int u_j^*(r) \left(-\frac{\nabla^2}{2M} + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2 + V_a(r) \right) u_j dr \\ &\quad + \int v_j^*(r) \left(-\frac{\nabla^2}{2M} + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2 + V_a(r) \right) v_j dr \\ &\quad - \int u_j^*(r) (\lambda_a\phi_a^*\phi_a^* + 2\alpha\phi_m^*) v_j(r) dr \\ &\quad - \int v_j^*(r) (\lambda_a\phi_a^*\phi_a^* + 2\alpha\phi_m^*) u_j(r) dr, \end{aligned} \quad (20)$$

$$\begin{aligned} e_j &= \int x_j^*(r) \left(-\frac{\nabla^2}{4M} + \epsilon + \lambda|\phi_a|^2 + 2\lambda_m|\phi_m|^2 + V_m(r) \right) x_j dr \\ &\quad + \int y_j^*(r) \left(-\frac{\nabla^2}{4M} + \lambda|\phi_a|^2 + \epsilon + 2\lambda_m|\phi_m|^2 + V_m(r) \right) y_j dr \\ &\quad - \int x_j^*(r) \lambda_m\phi_m^*\phi_m^* y_j(r) dr - \int y_j^*(r) \lambda_m\phi_m^*\phi_m^* x_j(r) dr. \end{aligned} \quad (21)$$

For simplicity, we study only the case of the spherical harmonic trap. In this case, we may choose the trial wavefunctions of the excitation components $u_j(r)$, $v_j(r)$, $x_j(r)$ and $y_j(r)$ in the form of the spherical harmonic oscillator wavefunction $\xi_{n_r, l, m}$ with quantum numbers (n_r, l, m) :

$$\begin{pmatrix} u_j(r) \\ v_j^*(r) \end{pmatrix} = \begin{pmatrix} u \\ v^* \end{pmatrix} \xi_{n_r, l, m}(\omega_{n_r, l, m}, r), \quad (22)$$

$$\begin{pmatrix} x_j(r) \\ y_j^*(r) \end{pmatrix} = \begin{pmatrix} x \\ y^* \end{pmatrix} \xi_{n_r, l, m}(\omega_{n_r, l, m}, r),$$

where $\omega_{n_r, l, m}$ is an adjustable scaling factor of variation. Equations (20) and (21) show that E_j and e_j take a similar form, hence we here discuss branches E_j of the excitation spectra in detail. For $(n_r, l, m) = (0, 1, 0)$, we have

$$\xi_{0,1,0} = \alpha_{010}^{3/2} \left[\frac{8}{3\sqrt{\pi}} \right]^{1/2} \alpha_{010} r e^{-\alpha_{010}^2 r^2 / 2} Y_{1,0}(\theta, \psi).$$

The excitation spectrum in this case is reduced to

$$\begin{aligned} E = E[v, \omega_{010}] &= (1 + 2v^2) \left[\frac{5}{4} \hbar \omega_{010} + \frac{5}{4} \hbar \frac{\omega_a^2}{\omega_{010}} \right] \\ &+ \lambda(1 + 2v^2) N_m \omega_m^{3/2} \left[\frac{2M}{\pi \hbar} \right]^{3/2} \left[\frac{\omega_{010}}{\omega_{010} + 2\omega_m} \right]^{5/2} \\ &+ \left[2\lambda_a(1 + 2v^2) - 2\lambda_a v \sqrt{1 + v^2} \right] N_a \omega_a^{3/2} \left[\frac{M}{\pi \hbar} \right]^{3/2} \left[\frac{\omega_{010}}{\omega_{010} + \omega_a} \right]^{5/2} \\ &- 4\alpha v \sqrt{1 + v^2} \left[\frac{\omega_{010}}{\omega_{010} + \omega_m} \right]^{5/2} N_m^{1/2} \omega_m^{3/4} \left[\frac{2M}{\pi \hbar} \right]^{3/4}, \end{aligned} \quad (23)$$

where $\alpha_{010}^2 = \frac{M\omega_{010}}{\hbar}$. Similarly, for $(n_r, l, m) = (1, 0, 0)$, we have

$$\xi_{1,0,0} = \alpha_{100}^{3/2} \left[\frac{8}{3\sqrt{\pi}} \right]^{1/2} \left(\frac{3}{2} - \alpha_{100}^2 r^2 \right) r e^{-\alpha_{100}^2 r^2 / 2} Y_{0,0}(\theta, \psi)$$

and

$$\begin{aligned} E = E[v, \omega_{100}] &= (1 + 2v^2) \left[\frac{7}{4} \hbar \omega_{100} + \frac{7}{4} \hbar \frac{\omega_a^2}{\omega_{100}} \right] \\ &+ \lambda(1 + 2v^2) N_m \omega_m^{3/2} \left[\frac{2M}{\pi \hbar} \right]^{3/2} f(\omega_{100}, 2\omega_m) \\ &+ \left[2\lambda_a(1 + 2v^2) - 2\lambda_a v \sqrt{1 + v^2} \right] N_a \omega_a^{3/2} \left[\frac{M}{\pi \hbar} \right]^{3/2} f(\omega_{100}, \omega_a) \\ &- 4\alpha v \sqrt{1 + v^2} f(\omega_{100}, \omega_m) N_m^{1/2} \omega_m^{3/4} \left[\frac{2M}{\pi \hbar} \right]^{3/4}, \end{aligned} \quad (24)$$

where

$$f(x, y) = \frac{3}{2} \left(\frac{x}{x+y} \right)^{3/2} - 3 \left(\frac{x}{x+y} \right)^{5/2} + \frac{5}{2} \left(\frac{x}{x+y} \right)^{7/2}.$$

Minimizing the energies of (23) and (24) with respect to the variation parameters v , ω_{010} and ω_{100} , we can determine the excitation spectrum for the modes $(0, 1, 0)$ and $(1, 0, 0)$. The numerical results are illustrated in figures 1 and 2. The dashed curves show the excitation spectrum in atomic Bose–Einstein condensation, i.e. $\alpha = \lambda = 0$. In contrast, the solid curves

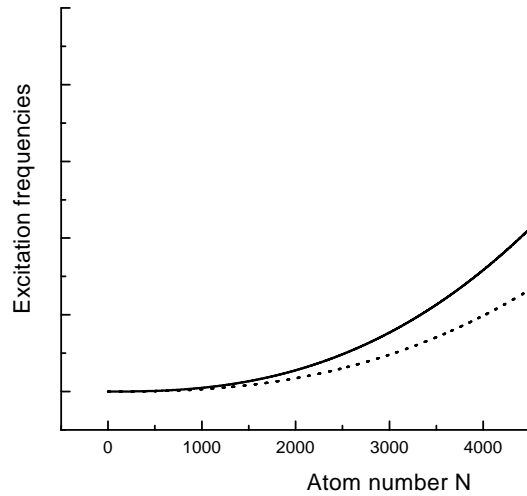


Figure 1. Excitation spectrum of mode $(0, 1, 0)$ versus the number of atoms. The dotted and dashed curves indicate those with and without Feshbach resonance, respectively.

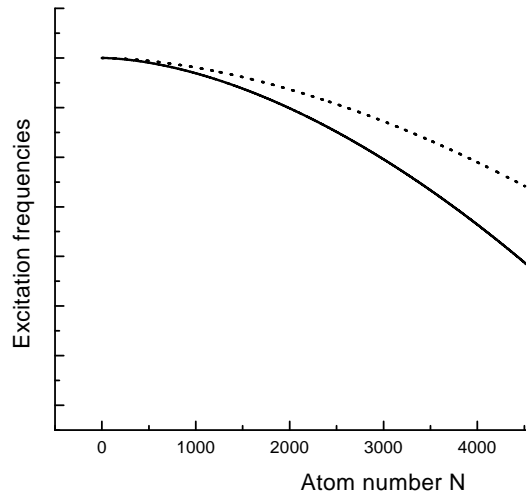


Figure 2. Same as figure 1 but for mode $(1, 0, 0)$.

are those of hybrid atomic/molecular condensates near the Feshbach resonance. From these figures we see that while the excitation frequency for mode $(0, 1, 0)$ increases due to the Feshbach resonance effect, the excitation frequency for mode $(1, 0, 0)$ decreases. We would like to point out that the numerical results presented here depend on the coupling constant as well as the parameter α . In figures 1 and 2, we let $\alpha = 5\lambda_a$, and $\lambda_a = 0.1$ (arbitrary units). The other parameters are $\omega_m = 1.4\omega_a = 7500$ Hz and $N_a = N_m = 10^6$, respectively.

For clarity, we illustrate the above somewhat formal discussion by considering the binary atom system for a uniform system ($V_{a(m)}(r) \rightarrow 0$), in this case H_{FR} gives a resonant contribution to the atom–atom interaction strength a_a : $a_{\text{eff}} = a_0(1 + \frac{\Delta}{B_0 - B})$, where a_0 is the off-resonant scattering length, and Δ characterizes the width of the resonance. For small

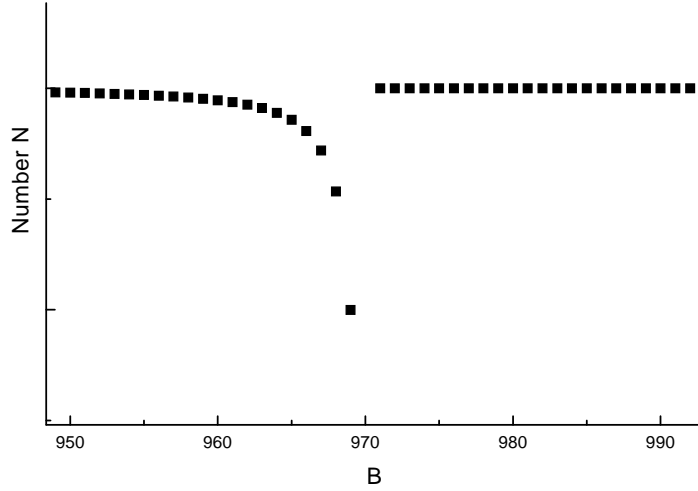


Figure 3. The number of atoms in the condensate versus magnetic field B .

p , these excitations are phonons, and their energy tends to zero with p . Hence,

$$\mu_a = \lambda|\phi_m|^2 + \lambda_a|\phi_a|^2 - 2\alpha\phi_m^*, \quad (25)$$

which leads to

$$E_j^2 = E^2(p) = \left(\frac{\hbar^2}{2m}\right)^2 p^2(p^2 + 16\pi n a_{\text{eff}}). \quad (26)$$

For a uniform dilute Bose gas with negative scattering length a_{eff} , (26) implies an instability of those modes with $p^2 \leq 16\pi n |a_{\text{eff}}|$. For a gas in a trap, however, the wavenumber cannot be arbitrarily small, and the minimum value is of order $p_{\text{min}} \simeq \pi/R_0$ (R_0 is the mean size of the ground state). Hence the system can remain stable if $\frac{\pi^2}{R_0^2} \geq 16\pi n |a_{\text{eff}}|$. Since the density is of order $n \simeq N/R_0^2$, this means that the critical number of the system is

$$N_0 \simeq \frac{\pi}{16} \frac{R_0}{|a_{\text{eff}}|}. \quad (27)$$

For a positive scattering length a_{eff} , however, there are no constraints in N . The Bogoliubov quasiparticle theory shows that the number of condensate atoms N_0 depends on the scattering length and satisfies (for $(a_{\text{eff}} \frac{N}{V})^{\frac{1}{3}} \ll 1$).

$$N_0 = N \left(1 - \frac{8}{3} \sqrt{\frac{N a_{\text{eff}}^3}{\pi V}} \right). \quad (28)$$

The numerical results of (27) and (28) are illustrated in figure 3, which shows the atom number N_0 versus external magnetic field B . The parameters in figure 3 are $N/V = N/R^2 = 10^{15} \text{ cm}^{-3}$, $\Delta = 0.01 \text{ mT}$.

To sum up, we have derived a set of four coupled equations of the atomic and molecular excitations within the standard Hartree–Fock–Bogoliubov approximation. As shown in (9) and (13), the α -terms describing the process that converts atoms into molecules play an important role in the atomic/molecular BEC. In particular, two low-excitation spectra have been given at zero temperature, which show that the interaction between the hybrid atomic/molecular BECs increases one excitation mode, while decreasing another excitation mode. The mode

(0, 1, 0) comes from the density fluctuation of the condensate-like vibrating oscillation, in this sense the fact that the mode (0, 1, 0) increases near the Feshbach resonance indicates that the presence of the Feshbach resonance enhances the density fluctuation like vibrating oscillations in an atomic/molecular condensation system, whereas the breathing mode (like the breathing oscillation) (1, 0, 0) decreases near the Feshbach resonance. Within the two-body collision regime, we show the number of atoms remaining in the BEC versus the external magnetic field B ; the result is in good agreement with the recent experimental data. This work removes from consideration the resonance case, since at resonance the Bogoliubov approximation is not available (at resonance, there are few atoms in the condensate). The contributions of the noncondensate atoms (molecules) to the excitation spectrum are also ignored (see equation (9)). These need further investigation.

Acknowledgment

This work was supported by the Chinese Postdoctoral Foundation.

References

- [1] Anderson M H, Ensher J R, Matthews M R, Wieman C E and Cornell E A 1995 *Science* **269** 198
- [2] Bradley C C, Sackett C A, Tollett J J and Hulet R G 1995 *Phys. Rev. Lett.* **75** 1687
- [3] Davis K B, Mewes M O, Andrews M R, Van Druten N J, Durfee D S, Kurn D M and Ketterle W 1995 *Phys. Rev. Lett.* **75** 3969
- [4] Fried D G, Killian T C, Willmann L, Landhuis D, Moss S C, Kleppner D and Greytak T J 1998 *Phys. Rev. Lett.* **81** 3811
- [5] Tiesinga E, Moerdijk A J, Verhaar B J and Stoof H T C 1992 *Phys. Rev. A* **46** R1167
- [6] Tiesinga E, Verhaar B J and Stoof H T C 1993 *Phys. Rev. A* **47** 4114
- [7] Moerdijk A J, Verhaar B J and Axelsson A 1995 *Phys. Rev. A* **51** 4852
- [8] Vogels J M *et al* 1997 *Phys. Rev. A* **56** R1067
- [9] Boesten H M J M, Bogels J M, Tempelaars J G C and Verhaar B J 1996 *Phys. Rev. A* **54** R3726
- [10] Fedichev P O, Kagan Yu, Shlyapnikov G B and Walraven J T M 1996 *Phys. Rev. Lett.* **77** 2913
- [11] Bohn J L and Julienne P S 1997 *Phys. Rev. A* **56** 1486
- [12] Moerdijk A J, Verhaar B J and Nagtegaal T M 1996 *Phys. Rev. A* **53** 4343
- [13] Inouye S, Andrews M R, Stenger J, Miesner H J, Stamper-kurn D M and Ketterle W 1998 *Nature* **392** 151
- [14] Bryant H C *et al* 1977 *Phys. Rev. Lett.* **38** 228
- [15] Timmermans E, Tommasini P, Cote R, Hussein M and Kerman A 1999 *Phys. Rev. Lett.* **83** 2691
- [16] Tommasini P *et al* Feshbach resonance and hybrid atomic/molecular BEC systems unpublished
- [17] Julienne P S, Burnett K, Band Y B and Stwalley W C 1998 *Phys. Rev. A* **58** R797
- [18] Baym G and Pethick C J 1996 *Phys. Rev. Lett.* **76** 6
- [19] Shi Hua-lin and Zheng Wei-mou 1997 *Phys. Rev. A* **55** 2930