Eur. Phys. J. D **61**, 215–220 (2011)
DOI: 10.1140/epjd/e2010-10353-2

THE EUROPEAN
PHYSICAL JOURNAL D

Regular Article

# Optically-driven cooling for collective atomic excitations

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Received 14 June 2010 / Received in final form 23 August 2010
Published online 5 November 2010 – 

**Abstract.** We explore how to cool collective atomic excitations in an optically-driven three-level atomic ensemble, which may be described by a model of two coupled harmonic oscillators (HOs) with a time-dependent coupling. Moreover, the model of two coupled HOs is further generalized to address the resolved sideband cooling issues, where the lower-frequency HO can be cooled whenever the cooling process dominates over the heating one during the sideband transitions. Unusually, due to the absence of the heating process, the optimal result for cooling collective excitations in an atomic ensemble could break the standard resolved sideband cooling limit for general models of two coupled HOs.

## 1 Introduction

Recently, quantum information processing based on collective excitations in atomic ensembles has attracted more and more attention. Atomic ensembles [1,2], usually consisting of three-level $\Lambda$-type atoms, have been considered as potential quantum memories for photons, because they could couple with quantized optical fields to form a stable dark-state polariton (DSP) via the electromagnetically induced transparency (EIT) mechanism [3–6]. In the limit of low excitations, DSP can be described as a hybrid bosonic mode [7,8] of quantized optical field and collective atomic excitations. By controlling the mixing angle between light and matter components of DSP, quantum state of photon can be mapped onto meta-stable collective-excitation state of matter, which realizes quantum information storage of photonic state [1,2,7–9]. More importantly, it was demonstrated that the long-distance quantum communications with quantum repeaters [10] can be implemented based on such collective atomic excitations [11–17].

As a requisite of quantum information processing for atomic ensembles, the preparation of initial ground state of collective atomic excitations is normally needed, that is, all the modes of collective excitations stay in their vacuum states, or equivalently all the atoms stay in the ground states. In a realistic atomic ensemble, a given collective-excitation mode may have a finite mean thermal excitations due to the interaction with its corresponding thermal bath at finite temperatures. This means it is necessary to cool the thermal excitations for the relevant quantum information processing based on atomic ensembles. Experimentally, the optical pumping technique [18,19] is used to achieve the above cooling by preparing all the atoms into certain states (e.g., the ground states). Usually, the corresponding theoretical explanation for the above cooling resorts to the single-atom picture for optical pumping. In this paper, based on the picture of collective atomic excitations instead of the single-atom picture, the cooling for atomic ensemble is proposed. We consider an optically-driven three-level atomic ensemble that can be modeled by two coupled harmonic oscillators (HOs), and then elaborate how to cool the collective-excitation modes (mainly the lower-frequency mode) near their vacuum states via the sideband-cooling-like scheme in this kind of systems, which means most of the atoms stay in the ground states according to the single-atom picture for optical pumping.

On the other hand, various nano- (or submicron-) mechanical resonators, modeled as single-mode HOs, have been investigated [20,21] extensively in recent years. To reveal quantum effect in nano-mechanical devices, a number of cooling schemes [22–36] were proposed to drive them to approach the standard quantum limit [37] or ground states of HOs. A famous one among them is the optical radiation-pressure cooling scheme [25–27] by coupling the resonator with a driven single-mode optical cavity. It is attributed to the (resolved) sideband cooling [28–36], which was previously well-developed to laser-cool the spatial motion of trapped ions [38,39] or neutral atoms [40].

Actually, the model of optical radiation-pressure cooling of mechanical resonator can be described by a model of two coupled HOs where the lower-frequency mode can be cooled when the cooling process dominates over the

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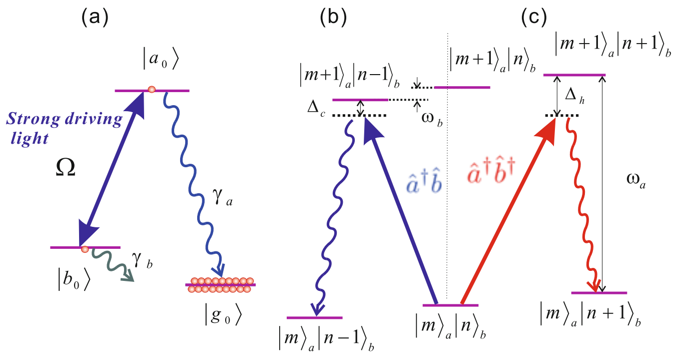


**Fig. 1.** (Color online) (a) Three-level atomic ensemble with most atoms staying in the ground states $|g_0\rangle$. The strong driving light couples to the transition from the meta-stable state $|b_0\rangle$ to the excited one $|a_0\rangle$ for each atom. The electric-dipole transition $|g_0\rangle \leftrightarrow |a_0\rangle$ is permitted, but $|g_0\rangle \leftrightarrow |b_0\rangle$ is forbidden. The wavy lines denote the decay processes with $\gamma_{a,b}$ the corresponding decay rates. (b) The cooling process ($|n\rangle_b \to |n-1\rangle_b$) and (c) the heating process ($|n\rangle_b \to |n+1\rangle_b$) for mode $b$ starting form $|m\rangle_a|n\rangle_b$ in the sideband structure forming by splitting $a$-mode with the lower-frequency $b$-mode. $\Delta_c$ ($\equiv \omega_a - \omega_b - \omega_d$) and $\Delta_h$ ($\equiv \omega_a + \omega_b - \omega_d$) are the detunings for the anti-Stokes (cooling) and Stokes (heating) transitions, respectively.

heating one during the sideband transitions. Moreover, we generalize the above optical radiation-pressure cooling model to other two types of cooling models of two coupled HOs addressing the standard resolved sideband cooling schemes. Notably, as a special model of two HOs, our cooling proposal for atomic ensembles can be understood as sideband-cooling-like scheme, that is the lower-frequency mode of collective atomic excitations losses its energy based on the induced sideband structure when it is time-dependently coupled with the higher-frequency one. However, compared with general cooling model of two coupled HOs which addresses the resolved sideband cooling limit, it is remarkable that our protocol for cooling atomic ensemble could break the limit of standard resolved sideband cooling due to the absence of counter-rotating heating terms in our special model of two coupled HOs for cooling atomic ensembles.

## 2 Three-level atomic ensemble modeled as two coupled harmonic oscillators

Let us consider an ensemble of $N$ identical three-level atoms as seen in Figure 1a. A strong classical driving optical field is homogeneously coupled to each atomic transition from the metastable state $|b_0\rangle$ to the excited one $|a_0\rangle$. Then the Hamiltonian reads ($\hbar = 1$ hereafter)

$$H = \omega_a \sum_{i=1}^{N} \sigma^{(i)}_{a_0 a_0} + \omega_b \sum_{i=1}^{N} \sigma^{(i)}_{b_0 b_0} + \varOmega \left(e^{i\omega_d t} + e^{-i\omega_d t}\right) \times \left(\sum_{i=1}^{N} \sigma^{(i)}_{b_0 a_0} + h.c.\right), \quad (1)$$

where $\omega_{g,a,b}$ are the corresponding energies of the atomic states $|g_0\rangle$, $|a_0\rangle$ and $|b_0\rangle$ respectively, and the ground state energy $\omega_g = 0$. $\varOmega$ is the coupling strength of the driving optical field (with the frequency $\omega_d$), which has been assumed to be real. Here $\sigma^{(i)}_{r,s} = |r\rangle_{ii}\langle s|$ $(r, s = g_0, a_0, b_0)$ is the flip operator for the $i$th atom.

We now introduce the bosonic operators $\hat{a} = \sum_i \hat{\sigma}^{(i)}_{g_0 a_0}/\sqrt{N}$ and $\hat{b} = \sum_i \hat{\sigma}^{(i)}_{g_0 b_0}/\sqrt{N}$ for low collective excitations [7,8,41][1], which satisfy $[\hat{a}, \hat{a}^\dagger] = 1$, $[\hat{b}, \hat{b}^\dagger] = 1$ and $[\hat{a}, \hat{b}^\dagger] = 0 = [\hat{a}, \hat{b}]$ in the limit of $N \to \infty$. Then, Hamiltonian (1) is modeled by a model of two coupled HOs, and can be further rewritten in the rotating framework as

$$\begin{aligned} H_I &= \varDelta \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \varOmega \left[\left(1 + e^{i2\omega_d t}\right) \hat{a}^\dagger \hat{b} + h.c.\right] \\ &\simeq \varDelta \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \varOmega \left(\hat{a}^\dagger \hat{b} + h.c.\right) \end{aligned} \quad (2)$$

with the detuning $\varDelta \equiv \omega_a - \omega_d$. In the derivation of last equation in Hamiltonian (2), we have used the rotating wave approximation (RWA), that is, neglecting the higher-oscillating terms ($e^{i2\omega_d t}\hat{a}^\dagger\hat{b} + h.c.$), since the RWA conditions $\{|\omega_{ab} - \omega_d|, |\varOmega|\} \ll (\omega_{ab} + \omega_d)$ (where $\omega_{ab} \equiv \omega_a - \omega_b$) are always fulfilled for most realistic atoms.

Normally, a weak quantized probe optical field would couple to the atomic transition $|g_0\rangle \leftrightarrow |a_0\rangle$ in the form of $\sim \sum_i \hat{\sigma}^{(i)}_{g_0 a_0}\hat{c}^\dagger/\sqrt{N} + h.c. = \hat{a}\hat{c}^\dagger + h.c.$ (with $\hat{c}^\dagger$ the creation operator of the quantized optical field). Thus, a so-called $\varLambda$-type three-level atomic ensemble configuration can be constructed associated with the well-known EIT and group-velocity slowdown phenomena. In such an ensemble, the DSP can also be obtained as the superposition of the optical mode and the atomic collective-excitation mode [1,2,7–9]. Based on the notations of EIT and DSP, the atomic ensemble can be a unit of quantum memory and be used to store the quantum information of, e.g., the photons. Here, we focus only on the cooling of the collective atomic excitations in the absence of the quantized probe optical field[2].

## 3 Cooling for collective atomic excitations

Generally, the atomic collective-excitation modes have non-vanishing mean thermal excitations due to their couplings to the bath at finite temperatures. In experiments, the frequency of the higher-frequency atomic collective-excitation, i.e., mode $a$, is of the order of $2\pi \times 10^{14}$ Hz,

[1] Strictly speaking, there are $N$ orthogonal degenerated collective modes $\{\hat{a}_k\}$ and $N$ orthogonal degenerated collective modes $\{\hat{b}_k\}$ for our present atomic ensemble consisted of $N$ three-level atoms. Here, we only consider the simplest case of $\hat{a}$ and $\hat{b}$. In principle, it is easy to generalize the present case to a general one including other collective modes since the present system can be constructed by $N$ uncoupled subsystems.

[2] Our above treatment based on collective atomic excitations may still work even in the presence of weak quantized optical field.

which implies that its mean thermal excitation number can be considered as zero even at room temperature. Normally, the atomic ground state $|g_0\rangle$ and meta-stable one $|b_0\rangle$ are selected as the atomic two hyperfine levels with the frequency difference being of the order of $2\pi \times 10^9$ Hz (that is, $\omega_b \sim 2\pi \times 10^9$ Hz). Although there is no optical dipole transition between $|b_0\rangle$ and $|g_0\rangle$ because of the electric dipole transition rule, the decay from $|b_0\rangle$ to $|g_0\rangle$ still exists due to some other cases such as magnetic dipole transition, with a very low decay rate. Such a very-low decay rate means that the lower-energy mode $b$ possesses a long coherence time, which is just a distinct advantage of using the collective atomic excitations as quantum memory units. However, in consideration of the high initial mean thermal excitation number $\bar{n}_b = [\exp(\omega_b/k_B T) - 1]^{-1} \sim 10^4 \gg 1$ at room temperature $T \sim 300$ K (with $k_B$ the Boltzmann constant), it is necessary to cool the modes of collective atomic excitations to their ground states (vacuum states) before quantum information processing based on atomic ensembles.

In the presence of noises, we may have the following Langevin equation from Hamiltonian (2)

$$\dot{\hat{a}} = -\Gamma_a \hat{a} - i\Omega \hat{b} + \hat{F}_a(t),$$
$$\dot{\hat{b}} = -\Gamma_b \hat{b} - i\Omega \hat{a} + \hat{F}_b(t), \tag{3}$$

where $\Gamma_a = \gamma_a/2 + i\Delta$ and $\Gamma_b = \gamma_b/2 + i\omega_b$. The noise operators are described by the correlations

$$\left\langle \hat{F}_C^\dagger(t)\hat{F}_C(t')\right\rangle = \gamma_C \bar{n}_C \delta(t - t'), \qquad (C = a, b). \tag{4}$$

Here, $\gamma_{a,b}$ are the decay rates of collective-excitation modes $a$ and $b$, respectively (for simplicity, we adopt the same symbols as those of the atomic levels $|a_0\rangle$ and $|b_0\rangle$), and $\bar{n}_{a,b} = [\exp(\omega_{a,b}/k_B T) - 1]^{-1}$ are the corresponding initial thermal excitation numbers with $T$ the initial temperature of thermal bath. Although the above quantum Langevin equation has vanishing steady state solutions $\langle \hat{a}\rangle = \langle \hat{b}\rangle = 0$, the corresponding quantum rate equations for the excitation numbers $\hat{n}_a = \hat{a}^\dagger \hat{a}$ and $\hat{n}_b = \hat{b}^\dagger \hat{b}$ read

$$\frac{d}{dt}\langle \hat{n}_a\rangle = \gamma_a(\bar{n}_a - \langle \hat{n}_a\rangle) - \left(i\Omega\left\langle \hat{\Sigma}\right\rangle + c.c.\right), \tag{5}$$
$$\frac{d}{dt}\langle \hat{n}_b\rangle = \gamma_b(\bar{n}_b - \langle \hat{n}_b\rangle) + \left(i\Omega\left\langle \hat{\Sigma}\right\rangle + c.c.\right), \tag{6}$$
$$\frac{d}{dt}\langle \hat{\Sigma}\rangle = -\zeta\left\langle \hat{\Sigma}\right\rangle + ig(\langle \hat{n}_b\rangle - \langle \hat{n}_a\rangle), \tag{7}$$

where $\hat{\Sigma} = \hat{a}^\dagger \hat{b}$ and $\zeta = (\gamma_a + \gamma_b)/2 + i(\omega_b - \Delta)$. Here we have used the non-vanishing noise-based relations [42]

$$\left\langle F_C^\dagger(t)\hat{C}(t)\right\rangle = \gamma_C \bar{n}_C/2, \qquad (C = a, b). \tag{8}$$

The steady state solutions of the quantum rate equations give the final mean thermal excitation number

$$\bar{n}_b^{\mathrm{f}} = \left\langle \left(\hat{b}^\dagger - \left\langle \hat{b}^\dagger\right\rangle\right)\left(\hat{b} - \left\langle \hat{b}\right\rangle\right)\right\rangle_{\mathrm{ss}}$$
$$\equiv \bar{n}_b - \xi(\bar{n}_b - \bar{n}_a) \tag{9}$$

with

$$\xi = \frac{\Omega^2\gamma_a(\gamma_a + \gamma_b)}{(\gamma_a + \gamma_b)^2\left(\Omega^2 + \frac{\gamma_a\gamma_b}{4}\right) + \gamma_a\gamma_b(\Delta - \omega_b)^2}. \tag{10}$$

Then, from the Bose-Einstein distribution, the effective temperature $T_{eff}$ of mode $b$ is expressed as

$$T_{eff} = \frac{\omega_b}{k_B \ln\left(1/\bar{n}_b^{\mathrm{f}} + 1\right)}. \tag{11}$$

It is obvious that $\bar{n}_b^{\mathrm{f}}$ depends on the coupling strength $\Omega$ and the detuning $\Delta$. The optimal cooling occurs in the strong driving strength limit ($\Omega \gg \gamma_a, \gamma_b$) and the case of $\Delta = \omega_b$ (namely, the driving light is exactly resonant to the atomic transition $|b_0\rangle \leftrightarrow |a_0\rangle : \omega_d = \omega_{ab}$) with the optimal final mean thermal excitation number of mode $b$

$$\bar{n}_b^{\mathrm{f}} = \frac{\gamma_b\bar{n}_b + \gamma_a\bar{n}_a}{\gamma_a + \gamma_b} \approx \frac{\gamma_b}{\gamma_a}\bar{n}_b + \bar{n}_a. \tag{12}$$

For a realistic atomic system, one has $\gamma_a \gg \gamma_b$ and $\bar{n}_b \gg \bar{n}_a$ ($\omega_a \gg \omega_b$). Especially, when $\gamma_b$ is sufficiently small such that $\gamma_b\bar{n}_b \ll \gamma_a\bar{n}_a$[3], the final mean thermal excitation number of mode $b$ reaches its limit: $\bar{n}_b^{\mathrm{f}} \to \bar{n}_b^{\mathrm{lim}} = \bar{n}_a$.

As mentioned above, the mean thermal excitation number of mode $a$ is usually tiny, which means that the atomic collective-excitation mode $b$ can be cooled close to its ground/vacuum state with the limit of final mean thermal excitation number

$$\bar{n}_b^{\mathrm{f}} \to \bar{n}_a \ll 1 \tag{13}$$

and the corresponding significantly-reduced effective temperature

$$T_{eff} \to \frac{\omega_a}{\omega_b} T \ll T. \tag{14}$$

A physical explanation of the above results can resort to the sideband-cooling-like mechanism (see Fig. 1b). The Jaynes-Cummings (JC) term ($\hat{a}^\dagger\hat{b}$) causes the anti-Stokes transition from $|m\rangle_a|n\rangle_b$ to $|m+1\rangle_a|n-1\rangle_b$, which will decay fast to the state $|m\rangle_a|n-1\rangle_b$. Thus, such a process makes the lower-frequency mode $b$ to lose one quantum and then results in its cooling. When the anti-Stokes transition is resonantly coupled, namely, $\Delta = \omega_b$, or $\Delta_c \equiv \omega_{ab} - \omega_d = 0$, the best cooling happens with the corresponding optimal final mean excitation number ($\bar{n}_b^{\mathrm{f}}$) given by the initial mean thermal excitation number $\bar{n}_a$ of higher-frequency mode $a$[4]. All in all, in order to reach the optimal cooling limit of lower-energy collective-excitation mode $b$, the following conditions should be satisfied: (i) strong enough pumping light $\Omega \gg \gamma_a, \gamma_b$;

[3] For typical alkali-(like)-metal $\Lambda$-type atoms, $\omega_a/2\pi \sim 10^{14}$ Hz, $\omega_b/2\pi \sim 10^9$ Hz, $\gamma_a/2\pi \sim 10^7$ Hz, $\gamma_b/2\pi \sim 10^{0-3}$ Hz (see Refs. [3–6,16,43] and references therein). The condition $\gamma_b\bar{n}_b \ll \gamma_a\bar{n}_a$ may be fulfilled for $\gamma_b/2\pi \sim 10^{0-1}$ Hz.

[4] Of course here the optimal case happens when one assumes that such a cooling process occurs much faster than the thermalization process of the mode $b$. In other words, one assumes $\gamma_b\bar{n}_b \ll \gamma_a\bar{n}_a$.

(ii) the resonant driving condition: $\Delta_c \equiv \omega_{ab} - \omega_d = 0$; (iii) $\gamma_b \ll \gamma_a$ and $\bar{n}_a \ll \bar{n}_b$ (that is, $\omega_b \ll \omega_a$); (iv) $\gamma_b \bar{n}_b \ll \gamma_a \bar{n}_a$. It is notable that the above four conditions may be met for experimentally accessible parameters of realistic atomic systems[3].

We would like to remark that extensive studies have been made in the framework of optically-pumping an individual atom into its internal lowest-energy ground state [18,19,44,45]. The present cooling scheme (for three-level atomic ensemble) based on collective atomic excitations is another description of the optical pumping technique based on a single-atom picture. Actually, both methods will lead to almost the same cooling results: both the limits of effective temperatures are the same if we define them respectively according to the effective thermal excitations of the lower-frequency collective mode in the case of collective atomic excitations and the effective stationary populations of the two lower levels of the atom in the case of single-atom optical pumping. Also note that, in processing the atomic-ensemble-based quantum information storage for photonic states, which is a main motivation of our work, an additional quantized optical field is needed to couple to the atomic transition, forming a $\Lambda$-type three-level atomic ensemble (for simplicity but without loss of generality, we have not considered this quantized optical field in the present cooling model). In such a case that involves collective atomic behaviors and quantum information storage for photonic states in atomic medium, a simple single-atom picture does not work any more, but our method of collective atomic excitations does. That means that our method is general and may be promising for cooling more general systems of atomic ensembles.

It is seen clearly from the above analysis that the time-dependent coupling between two large-detuned HOs could cool down the lower-frequency one. This cooling model is different from the existing mechanical cooling model based on the optical radiation pressure [25–36], with an external laser-driving. Nevertheless, we will show in the next section that these two cooling models may be generalized to a more universal effective one.

## 4 Generalized sideband cooling model of two coupled HOs

A naive cooling process could be realized when a hotter object contacts directly with a cold one. If there exists no external driving or no (equivalent) external control for two objects at the same initial temperature, it is obviously impossible that the temperature of any one can change via their direct interaction. But the situation changes dramatically when we add an additional time-dependent driving or manipulate the coupling between them to be time-dependent in largely-detuned two coupled HOs. This kind of setup leads to a more general sideband cooling framework, which is also motivated by previous heuristic explorations [46].

Let us consider the first type of two coupled HOs with largely-detuned frequencies ($\omega_a \gg \omega_b$) as seen in Figure 2a. The free Hamiltonian reads $\hat{H}_0 = \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b}$. A time-dependent coupling is generally expressed as

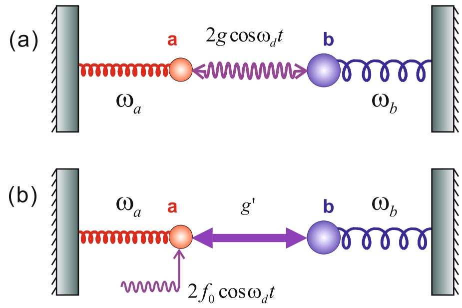


**Fig. 2.** (Color online) General model of two coupled HOs ($a$ and $b$). Mode $b$ is the desired lower-frequency HO to be cooled. (a) The interaction between two HOs is time-dependent modulated ($\propto 2g\cos(\omega_d t)$); (b) the coupling strength of the interaction between two HOs is time-independent but there is an external time-dependent linear driving ($\propto 2f_0\cos(\omega_d t)$) on the higher-frequency mode $a$.

$$\hat{V}_1(t) = 2g\cos(\omega_d t)\, F_1\left(\hat{a}^\dagger, \hat{a}\right)\left(\hat{b}^\dagger + \hat{b}\right), \tag{15}$$

where $\hat{a}^\dagger$ ($\hat{a}$) and $\hat{b}^\dagger$ ($\hat{b}$) are the creation (annihilation) operators of the oscillators $a$ and $b$ with $g$ the coupling coefficient between them and $\omega_d$ the modulating frequency. Here, $F_1(\hat{a}^\dagger, \hat{a})$ is a function of operators $\hat{a}^\dagger$ and $\hat{a}$. For simplicity, in what follows we consider only the simplest case of $F_1(\hat{a}^\dagger, \hat{a}) = \hat{a}^\dagger + \hat{a}$, though a more general function (i.e., $F_1(\hat{a}^\dagger, \hat{a}) = \sum_n c_n \hat{a}^{\dagger n}(\hat{a}^\dagger + \hat{a})\hat{a}^n$ with $c_n$ dimensionless coefficients) would lead to a similar result. In the time-varying reference frame defined by $\hat{R}(t) = \exp(-i\omega_d \hat{a}^\dagger \hat{a} t)$, the effective Hamiltonian of the coupled system reads

$$\hat{H}^{(1)}_{eff} = \Delta \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + g\left(\hat{a}^\dagger + \hat{a}\right)\left(\hat{b}^\dagger + \hat{b}\right), \tag{16}$$

where the high-oscillating terms have been neglected and the detuning $\Delta = \omega_a - \omega_d$ could be negative when $\omega_a < \omega_d$.

Next we consider another type of two coupled HOs (see Fig. 2b): a general time-independent interaction is assumed as

$$\hat{V}_2 = g' F_2'\left(\hat{a}'^\dagger, \hat{a}'\right)\left(\hat{b}'^\dagger + \hat{b}'\right) \tag{17}$$

with $g'$ the coupling strength and $F_2'(\hat{a}'^\dagger, \hat{a}')$ being Hermitian, and a periodically linear driving field on the higher-frequency HO reads

$$\hat{H}_d(t) = 2f_0\cos(\omega_d t)\left(\hat{a}'^\dagger + \hat{a}'\right). \tag{18}$$

In the time-varying reference frame defined by $\hat{R}'(t) = \exp(-i\omega_d \hat{a}'^\dagger \hat{a}' t)$, the total Hamiltonian reads

$$\hat{H}^{(2)} = \Delta_0 \hat{a}'^\dagger \hat{a}' + \omega_b \hat{b}'^\dagger \hat{b}' + g' F_2\left(\hat{a}'^\dagger, \hat{a}'\right)\left(\hat{b}'^\dagger + \hat{b}'\right) + f_0\left(\hat{a}'^\dagger + \hat{a}'\right) \tag{19}$$

(with $\Delta_0 = \omega_a - \omega_d$) after neglecting the high-oscillating terms, where $F_2(\hat{a}'^\dagger, \hat{a}')$ keeps the time-independent terms in $F_2'(\hat{a}'^\dagger e^{i\omega_d t}, \hat{a}' e^{-i\omega_d t})$. Around some quasi-classical state $|Q\rangle$ such that $\langle Q|\hat{a}'|Q\rangle = \alpha$ and $\langle Q|\hat{b}'|Q\rangle = \beta$, the quantum dynamics is determined by an effective Hamiltonian $\hat{H}_{eff}^{(2)} = \hat{H}_{eff}^{(2)}(\hat{a}^\dagger, \hat{b}^\dagger, \hat{a}, \hat{b})$ with the displacement operators $\hat{a} = \hat{a}' - \alpha$ and $\hat{b} = \hat{b}' - \beta$ for quantum fluctuations. Then, when the displacements $\beta$ and $\alpha$ take the equilibrium values $\beta = -F_2(\alpha,\alpha)/\omega_b$ and $\alpha = -[f_0 + 2\beta\partial_\alpha F_2(\alpha,y)|_{y=\alpha}]/\Delta_0$, the effective Hamiltonian $\hat{H}_{eff}^{(2)}$ has the same form as $\hat{H}_{eff}^{(1)}$ in equation (16) with the parameters $\Delta = \Delta_0 + 2\beta[\partial^2 F_2(x,y)/\partial_x\partial_y]|_{x,y=\alpha}$ and $g = g'\partial_\alpha F_2(\alpha,y)|_{y=\alpha}$. Therefore, these types of two coupled HOs should have the same cooling mechanism to cool the lower-frequency HO mode.

Let us recall the well-known laser-cooling for the spatial motion of atoms or trapped-ions, wherein the Hamiltonian has the similar form as that of the above first type actually. Recently, Tian [47] as well as Jacobs et al. [48] discussed the cooling of a nanomechanical resonator coupled with a LC oscillator, where the coupling is exactly the case of the above first type of two coupled HOs. We also remark that the optical radiation-pressure cooling of mechanical resonator [28–36], is just a simplest case of the second type with $F_2'(\hat{a}'^\dagger, \hat{a}') = g'\hat{a}'^\dagger\hat{a}'$. A similar linearization [28,29,35,36] of the effective Hamiltonian as given in equation (16) was also mentioned in the optical radiation-pressure cooling of mechanical resonator. Here we present only the cooling limit (so-called resolved sideband cooling limit)[5] of the general model of two coupled HOs:

$$\bar{n}_b^{\rm f} \rightarrow \bar{n}_b^{\rm lim,sid} = \bar{n}_a + \frac{\gamma_a^2}{4\omega_b^2} \approx \frac{\gamma_a^2}{4\omega_b^2} \tag{20}$$

in the resolved sideband case $\gamma_a^2 \ll \omega_b^2$ when $\Delta = \sqrt{\omega_b^2 + \gamma_a^2} \approx \omega_b$. Here the usual relation $\bar{n}_a \ll \gamma_a^2/4\omega_b^2$ has been used. Also note that the effective coupling strength has been taken as very large ($g \gg \gamma_b$) in order to reach the optimal cooling result.

Although the above Hamiltonian (16) describes a simple model of two coupled HOs, it may capture the essence of almost all sideband cooling schemes. We wish to emphasize the necessity of the time-dependence of modulating coupling or external driving. It lies in a fact that, when $\omega_a \gg \omega_b$, there still exists the effective interaction for $|\Delta| \sim \omega_b$ (or $\omega_a \pm \omega_b \sim \omega_d$). It is the effective resonance $|\Delta| \sim \omega_b$ that results in the sideband transitions to cool down (or heat up) the oscillator $b$ (see Figs. 1b and 1c): the JC term ($\hat{a}^\dagger\hat{b}$) (associated with the fast decay of mode $a$) denotes the cooling process of lower-frequency oscillator $b$ ($|n\rangle_b \rightarrow |n-1\rangle_b$); on the contrary, the anti-JC term (that is, the anti-rotating term) ($\hat{a}^\dagger\hat{b}^\dagger$) denotes the heating process of mode $b$ ($|n\rangle_b \rightarrow |n+1\rangle_b$). When the cooling process dominates (e.g., when $\Delta_c \equiv \omega_a - \omega_b - \omega_d \sim 0$), the cooling of mode $b$ happens with the optimal cooling subject to the usual standard sideband cooling limit ($\bar{n}_b^{\rm lim,sid} \approx \gamma_a^2/4\omega_b^2$).

We wish to point out that our current cooling scheme for atomic ensemble is one special example of the first type in the absence of anti-JC term $\hat{a}^\dagger\hat{b}^\dagger$ (and its conjugate) by comparing the cooling models described by the Hamiltonians (2) and (16). In equation (16), the anti-JC term $\hat{a}^\dagger\hat{b}^\dagger$ which is usually neglected due to the RWA, is kept here since the RWA may not be fulfilled in the present case of strong coupling strength $g$. Actually $\hat{a}^\dagger\hat{b}^\dagger$ would bring the heating contribution and play a role for the resolved sideband cooling limit. However, in equation (2), the non-RWA terms $e^{i2\omega_d t}\hat{a}^\dagger\hat{b}$ would bring a small cooling contribution, which can actually be neglected safely by using RWA. Thus, due to the absence of the heating process induced by the anti-JC term during the resolved sideband cooling, the optimal cooling for lower-frequency mode of collective atomic excitations happens at the exact resonant ($\Delta_c = 0$) of (first) anti-Stokes transitions, with the resolved sideband condition ($\gamma_a^2 \ll \omega_b^2$) being relaxed, and the corresponding $\bar{n}_b^{\rm lim}$ ($= \bar{n}_a$) for present cooling limit is certainly much less than that for the standard resolved sideband cooling limit ($\bar{n}_b^{\rm lim,sid} \approx \gamma_a^2/4\omega_b^2$).

We also note that the anti-JC heating terms are usually present in the general models of real two coupled harmonic oscillators (as the two general types of two coupled HOs mentioned above). This is different from the case of our scheme for cooling atomic ensemble, where the optical transitions can be chosen so that only the cooling terms appear with the heating terms disappearing. However, the present sideband-cooling-like scheme of atomic ensemble can still motivate us to construct new protocols for real two coupled HOs by suppressing the heating process in the sideband cooling models to break the standard resolved sideband cooling limit. For example, the suppressing of heating process can be achieved by inducing an extra process which cancels destructively the heating process by an EIT-like scheme, as like the EIT cooling for mechanical resonators by coupling with (artificial) atoms [49–51]. That is also our further consideration in future.

## 5 Summary

We have established a theory to cool collective atomic excitations in an optically-driven three-level atomic ensemble. Such a cooling protocol is quite useful and promising in quantum information processing based on collective atomic excitations, which breaks the standard sideband cooling limit. Moreover, motivated by the optical radiation-pressure cooling scheme of mechanical oscillator, we have also proposed two generalized cooling types of two coupled HOs: the first one possesses a time-dependent modulating coupling coefficient between the HOs without the external driving; while for the second one, an additional external time-dependent driving on the higher-frequency HO is involved, with the coupling coefficient between the HOs being time-independent. In fact, the second type is a generalized model of the optical radiation-pressure cooling of mechanical resonator. For both types, the lower-frequency HO can be cooled in the resolved sideband cooling case with the usual standard sideband cooling limit.

[5] For detailed calculations, one may refer to those in [28–34].

This work was supported by the RGC of Hong Kong under Grant Nos. HKU7051/06P & HKU7044/08P, and partially supported by the NFRP of China under Grant Nos. 10874091 and 2006CB921205 and NSFC Grants through ITP, CAS.

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