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Quantum memory based on Λ-atoms ensemble with two-photon resonance EIT

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Abstract. We study the Λ-atoms ensemble based quantum memory for the quantum information carried by a probe light field. Two atomic Rabi transitions of the ensemble are coupled to the quantum probe field and classical control field respectively with a same detuning. Our analysis shows that the dark states and dark-state polaritons can still exist for the present case of two-photon resonance EIT. Starting from these dark states we can construct a complete class of eigen-states of the total system. A explicit form of the adiabatic condition is also given in order to achieve the memory and retrieve of quantum information.

PACS. 03.67.-a Quantum information – 42.50.Gy Effects of atomic coherence on propagation, absorption, and amplification of light; electromagnetically induced transparency and absorption – 03.65.Fd Algebraic methods

With great interest and quick development in quantum information science [1], the implementation of quantum memory becomes one of the particular challenges to quest a realistic system transporting or communicating quantum states between different nodes of quantum networks. Quantum optical systems with atoms appear to be very attractive since photons are ideal carriers of quantum information with very fast velocity; and the atoms represent long-lived storage and reliable processing units.

A well known quantum optical system is of the electromagnetically induced transparency (EIT) [2], which can be used to make a resonant, opaque medium transparent. The essential property of EIT is induced by atomic coherence and quantum interference. The discovery of EIT has led to the occurrence of new effects and new techniques including ultraslow light pulse propagation [3,4] and the light signal storage [5,6] in atomic vapor. A following idea is then how to use the EIT system to transport and communicate the quantum state between photons and atoms.

Conventionally the EIT system consists of a vapor of 3-level atoms with two classic optical fields (the probe and control fields) being one-photon on-resonance with the relevant atomic transitions [2,3,7]. Later, it is noticed that such an on-resonance EIT is not a prerequisite for achieving significant group velocity reduction. The EIT phenomenon can also occur when the frequency difference between the probe and control fields matches the two-photon transition between the two lower states of the Λ-type atoms [8–10] (for this case, people call it the case of *two-photon resonance EIT*).

Recently, the method of atomic collective excitation [11,12] is used to implement the quantum memory and to transport the quantum state between photons and atomic ensemble by some people [5,6,13–16]. They have replaced the classical probe laser field by a weak quantum light field in the on-resonance EIT system with atomic collective excitations. Then, by adiabatically changing the coupling strength of the classic control field, they have demonstrated the possibility of coherently controlling the propagation of the quantum light pulses via the dark sates and dark-state polaritons. Most recently, to avoid the spatial-motion induced decoherence, we have considered an on-resonance EIT system of "atomic crystal" with each atom fixed on a lattice site [17]. With discovery of the hidden dynamic symmetry, we have shown that such a system is a robust quantum memory to transport the quantum states between photons and atomic ensemble.

Presently, most works about the memory of quantum probe light field within an atomic ensemble with collective excitations base on the on-resonance EIT [13,14,17], here we want to study a system under the case of two-photon resonance EIT with the method of atomic collective excitations. It is remarked that some experimental works of the light storage under the case of two-photon resonance EIT are studied [9,18]. The advantages about the case of two-photon resonance are advocated in references [8,9]. Some novel features are also founded in experiment [18]

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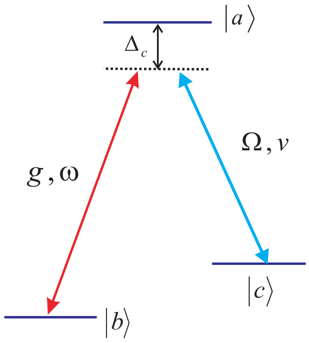

Fig. 1. The probe and control optical fields are respectively coupled to two atomic transitions with the same detuning Δ_c. That is, such a system consisted of 3-level Λ-atoms ensemble and two optical fields satisfies the two-photon resonance EIT condition.

in an atomic vapor under the case of two-photon resonance. In the former paper [19], we have calculated the susceptibility and group velocity of the probe field under two-photon resonance. Our results show that the EIT phenomenon exists indeed and an ultraslow group velocity can be obtained. In this work, we study how the excitonic system under two-photon resonance EIT serves as a robust quantum memory by means of the method of atomic collective excitations.

We consider an atomic ensemble consisted of N 3-level Λ-type atoms with each atom being coupled to two single-mode optical fields as shown in Figure 1. The atomic levels are labelled as the ground state $|b\rangle$, the excited state $|a\rangle$ and the meta-stable state $|c\rangle$. The atomic transition $|a\rangle \leftrightarrow |b\rangle$ with energy level difference $\omega_{ab} = \omega_a - \omega_b$ is coupled to a quantum probe light field of frequency ω with the coupling coefficient g and the detuning $\Delta_p = \omega_{ab} - \omega$, while the atomic transition $|a\rangle \leftrightarrow |c\rangle$ with energy level difference ω_{ac} is driven by a classical control field of frequency v with the Rabi-frequency $\Omega(t)$ and the detuning $\Delta_c = \omega_{ac} - \nu$. For simplicity, the coupling coefficients g and Ω are real and assumed to be identical for all the atoms in the ensemble. Under the two-photon resonance condition, that is, $\Delta_p \equiv \Delta_c$, the interaction Hamiltonian of total system can be written in the interaction picture as ($\hbar = 1$)

$$H_I = \Delta_c S + (g\sqrt{N} a A^\dagger + \Omega T_+ + \text{h.c.}) \tag{1}$$

in terms of the collective quasi-spin operators

$$S = \sum_{j=1}^{N} \sigma_{aa}^{(j)}, \quad A^\dagger = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \sigma_{ab}^{(j)}, \quad T_+ = \sum_{j=1}^{N} \sigma_{ac}^{(j)}. \tag{2}$$

Here $\sigma_{\mu\nu}^{(j)} = |\mu\rangle_{jj}\langle\nu|$ is the flip operator of the jth atom from state $|\mu\rangle_j$ to $|\nu\rangle_j$ ($\mu, \nu = a, b, c$); and $a^\dagger$ (a) is the creation (annihilation) operator of the probe field. In the large N and low atomic excitation limit with only a few atoms occupying states $|a\rangle$ or $|c\rangle$ [20], the atomic collective excitations A behaves as boson since it satisfies the bosonic commutation relation $[A, A^\dagger] = 1$.

We note that the above Hamiltonian is expressed in terms of the collective dynamic variables S, A, $A^\dagger$, T_+, and $T_- = (T_+)^\dagger$. To properly describe the cooperative motion of the atomic ensemble stimulated by the probe and control fields, it is convenient to introduce a new pair of collective excitation operators

$$C = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \sigma_{bc}^{(j)}, \quad C^\dagger = (C)^\dagger. \tag{3}$$

In the large N and low excitation limit, the corresponding collective excitation of C also behaves as boson since it satisfies the bosonic commutation relation $[C, C^\dagger] = 1$. These two quasi-spin collective excitations (A and C) are independent of each other in the same limit because of the vanishing commutation relations

$$[A, C] = 0, \quad [A, C^\dagger] = -T_-/N \longrightarrow 0 \tag{4}$$

by a straightforward calculation. It's noted that S is a Hermitian operator and has the commutation relations

$$\begin{aligned} &[S, A] = -A, && [S, A^\dagger] = A^\dagger, \\ &[S, C] = [S, C^\dagger] = 0, && [S, T_\pm] = \pm T_\pm. \end{aligned} \tag{5}$$

Moreover, it's easy to prove the following basic commutation relations

$$\begin{aligned} &[T_-, C] = -A, && [T_-, C^\dagger] = 0, \\ &[T_-, A] = 0, && [T_-, A^\dagger] = C^\dagger. \end{aligned} \tag{6}$$

The above relations (6) remind us of the similar relations appeared in our former work [17]. In reference [17], a semi-direct product algebra $SU(2)\overline{\otimes}h_2$ has been found in an "atomic crystal" under the case of the one-photon resonance EIT. Moreover, by means of this algebra and the spectral generating algebra method [21], we have obtained a complete class of eigen-states of the total system consisted of the atoms and optical fields. And a explicit form of adiabatic passage has been given to implement the quantum information memory and retrieve between the type of photons and the type of atomic collective excitations. In the following of this paper, we will study how the present system can serve as a robust quantum memory.

We can define a polariton operator

$$D = a\cos\theta - C\sin\theta, \tag{7}$$

where $\theta(t)$ satisfies $\tan\theta(t) = g\sqrt{N}/\Omega(t)$. This polariton that mixes the optical field and the atomic collective excitations behaves a boson in the large N and low excitation limit since $[D, D^\dagger] = 1$.

Let us introduce the "vacuum" state $|\mathbf{0}\rangle = |0\rangle_p \otimes |b^N\rangle$ where $|0\rangle_p$ shows the vacuum of the electromagnetic field and $|b^N\rangle = |b_1 b_2 ... b_N\rangle$ denotes all the N atoms staying in the ground states. The relations

$$H_I|\mathbf{0}\rangle = 0, \quad [D, H_I] = 0 \tag{8}$$

(by means of the Eqs. (5) and (6)) show that a degenerate class of eigen-states of H_I with zero eigen-value can be constructed naturally as follows:

$$|d_n\rangle = [n!]^{-1/2} D^{\dagger n}|\mathbf{0}\rangle, \quad n = 0, 1, 2, \cdots \tag{9}$$

Using the equation (7), we can expand $|d_n\rangle$ as

$$|d_n\rangle = \sum_{m=0}^{n} (-1)^m \sqrt{\frac{n!}{m!(n-m)!}} \cos^{n-m}\theta \sin^m\theta \times |\mathbf{c}^m\rangle \otimes |n-m\rangle_p, \quad n = 0, 1, 2, \cdots \tag{10}$$

where $|\mathbf{c}^m\rangle = [m!]^{-1/2} C^{\dagger m}|b^N\rangle$ represents there are m C-mode excitations in the atomic ensemble. Let us draw the energy level graphic as Figure 2. It can be observed from Figure 2 that the dark state $|d_n\rangle$ is a linear combination consisted of the terms $|b^N\rangle \otimes |n\rangle_p$, $|\mathbf{c}\rangle \otimes |n-1\rangle_p$, ..., $|\mathbf{c}^n\rangle \otimes |0\rangle_p$. It is remarked that the equation (10) and Figure 2 are in the similar forms as that in reference [16]. However, compared with the previous work [16], the present results are obtained by different method. Under the large N and low excitation limit, the collective excitation operators A and C can be simplified as independent Bosonic operators, so the present results can be obtained easily and it is convenient to study the present system further. More importantly, the present system is the case of two-photon resonance EIT, which is different from the previous one-photon resonance EIT [16]. It is also remarked that all the atomic decays are ignored in this work. Generally the decay related to the atomic excited state $|a\rangle$ is much larger than the decay related to the meta-stable state $|c\rangle$. However, the atomic parts of all the terms in the dark state only contain the single-atom lower states $|b\rangle$ and $|c\rangle$ so that the dark state is robust even if the physical decays are considered.

Physically, the above dressed state $|d_n\rangle$ is cancelled by the interaction Hamiltonian, thus it is called a dark state and D is called a dark-state polariton (DSP). The DSP traps the electromagnetic radiation from the excited state due to the quantum interference cancelling. For the case with an ensemble of free atoms, the similar DSP was obtained in references [6,13,14,17] to clarify the physics of the state-preserving slow light propagation in EIT associated with the existence of collective atomic excitations.

Now starting from these dark states $|d_n\rangle$, we can use the spectrum generating algebra method [21] to build other eigenstates for the total system. To this end we introduce the bright-state polariton operator

$$B = a\sin\theta + C\cos\theta. \tag{11}$$

It is obvious that

$$[B, B^\dagger] = 1, \quad [D, B^\dagger] = [D, B] = 0. \tag{12}$$

Evidently $[A, B] = [A, B^\dagger] = 0$ using the fact that A commutes with C and $C^\dagger$ in the large N with low excitation limit.

It is straightforward to obtain the commutation relations

$$[H_I, B^\dagger] = \varepsilon A^\dagger, \quad [H_I, A^\dagger] = \Delta_c A^\dagger + \varepsilon B^\dagger, \tag{13}$$

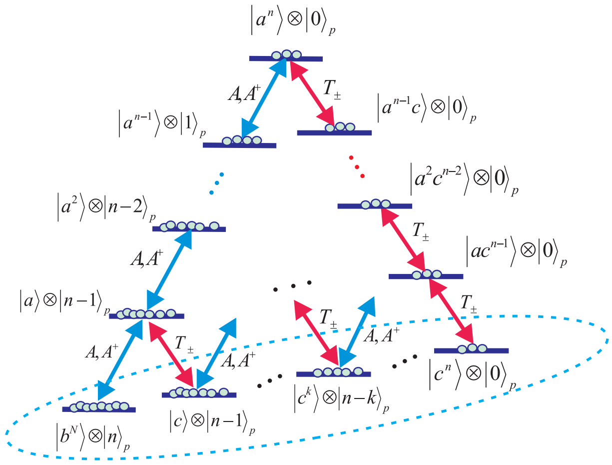

Fig. 2. The schematics of the states in the two-photon resonance system with atomic collective excitations, where $|\mathbf{a}^l\mathbf{c}^m\rangle = [l!m!]^{-1/2} A^{\dagger l} C^{\dagger m}|b^N\rangle$ denotes there are l A-mode and m C-mode excitations in the atomic ensemble. These states in the figure are connected by the interaction of the operators A, $A^\dagger$, and $T_\pm$ (also the action of the related optical fields). The dark state $|d_n\rangle$ is a superposition of the states $|b^N\rangle \otimes |n\rangle_p$, $|\mathbf{c}\rangle \otimes |n-1\rangle_p$, ..., $|\mathbf{c}^n\rangle \otimes |0\rangle_p$ in the dashing envelop.

where $\varepsilon = \sqrt{g^2 N + \Omega^2}$. Notice that the system of dynamical equations (13) are not simply as the same as that of the case of on-resonance [17]. However, we can introduce the norm mode variables $Q_\pm$:

$$Q_\pm = \sqrt{\frac{\Theta \pm \Delta_c}{2\Theta}} A \pm \sqrt{\frac{\Theta \mp \Delta_c}{2\Theta}} B, \tag{14}$$

where $\Theta = \sqrt{\Delta_c^2 + 4\varepsilon^2}$. What is crucial for our purpose is the commutation relations

$$[H_I, Q_\pm^\dagger] = e_\pm Q_\pm^\dagger, \tag{15}$$

where $e_\pm = \pm\sqrt{(\Theta \pm \Delta_c)/(\Theta \mp \Delta_c)}\varepsilon$. By introducing the norm mode transformation, it is almost the same as the case of on-resonance EIT. Based on these commutation relations we can construct the eigen-states

$$|e(m, k; n)\rangle = [m!k!]^{-1/2} Q_+^{\dagger m} Q_-^{\dagger k}|d_n\rangle, \tag{16}$$

as the dressed states of the total system. The corresponding eigen-values are

$$E(m, k) = me_+ - ke_-, \quad m, k = 0, 1, 2, \cdots \tag{17}$$

In the following discussion, we consider whether the dark states can work well as a quantum memory under the adiabatic manipulation. This means that we should consider how the adiabatic condition [22,23]

$$\left|\frac{\langle e(m, k; n)| \partial_t |d_l\rangle}{E(m, k) - 0}\right| \ll 1, \tag{18}$$

is satisfied for any $m, k, n, l = 0, 1, 2, \cdots$ The eigen-values of these instantaneous collective eigen-states are complicated and the corresponding energy levels can cross each other when adiabatically varying the Rabi frequency $\Omega(t)$. Fortunately, among all the terms $\langle e(m,k;n)|\,\partial_t|d_l\rangle$, only the terms $\langle e(0,1;l-1)|\,\partial_t|d_l\rangle$ and $\langle e(1,0;l-1)|\,\partial_t|d_l\rangle$ do not vanish:

$$\langle e(m,k;n)|\,\partial_t|d_l\rangle = \sqrt{l}\dot{\theta}\delta_{n,l-1} \times \left[\delta_{m,0}\delta_{k,1}\sqrt{\frac{\Theta+\Delta_c}{2\Theta}} - \delta_{m,1}\delta_{k,0}\sqrt{\frac{\Theta-\Delta_c}{2\Theta}}\right]. \quad (19)$$

From the above results, we can readily obtain the adiabatic condition:

$$\frac{g\sqrt{N}(\Theta+|\Delta_c|)}{\sqrt{\Theta(\Theta-|\Delta_c|)\varepsilon^3}}\left|\dot{\Omega}\right| \ll 1 \quad (20)$$

by means of $\dot{\theta} = -g\sqrt{N}\dot{\Omega}/\varepsilon^2$. When $\Delta_c = 0$, the above equation (20) will reduce to

$$g\sqrt{N}|\dot{\Omega}|/\varepsilon^3 \ll 1, \quad (21)$$

which is just as the same as the adiabatic condition under on-resonance EIT as shown in reference [17]. According to equation (19), we know that: the dark state $|d_l\rangle$ will not mix the states $|e(m,k;n)\rangle$ under the adiabatic condition (20); and the dark state $|d_l\rangle$ also can not mix the other degenerate state $|d_{l'}\rangle$ according to the adiabatic condition of degenerate states [17,23]. This means when the initial state is $|\Phi(0)\rangle = \sum_n c_n(0)|d_n(0)\rangle$, the total system will follow the superposition of dark-state

$$|\Phi(t)\rangle = \sum_n c_n(0)|d_n(t)\rangle \quad (22)$$

under the adiabatic evolution.

Then we can implement the quantum information memory and retrieve between the type of photons and the type of atomic collective excitations. As shown in Figure 3, we denote the dark state polariton $D(t)$ as a vector in the a-C plane. If $\Omega(t)$ is changed adiabatically to make $\theta(t)$: $0 \to \pi/2$, one have $D(t)$: $a \to -C$ and $|d_n(t)\rangle$ will change from $|b^N\rangle \otimes |n\rangle_p$ to $(-1)^n|\mathbf{c}^n\rangle \otimes |0\rangle_p$. Generally, the initial quantum state of the single-mode optical field is described by a density matrix

$$\rho_p = \sum_{n,m} \rho_{nm}|n\rangle_{pp}\langle m|, \quad (23)$$

the transfer process generates a quantum state of collective excitations according to

$$|b^N\rangle\langle b^N| \otimes \sum_{n,m}\rho_{nm}|n\rangle_{pp}\langle m| \to \sum_{n,m}(-1)^{n+m}\rho_{nm}|\mathbf{c}^n\rangle\langle\mathbf{c}^m| \otimes |0\rangle_{pp}\langle 0|. \quad (24)$$

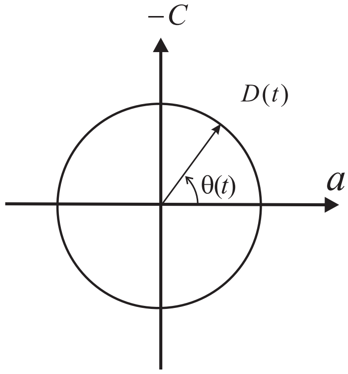

Fig. 3. Dark state polariton $D = a\cos\theta - C\sin\theta$ is dependent on the parameter $\theta(t) = \arctan g\sqrt{N}/\Omega(t)$. If initially $\theta(t_0) = 0$, $D(t_0) = a$, and the state of the total system is described as $|\psi(t_0)\rangle = |b^N\rangle \otimes |n\rangle_p$, then when θ is adiabatically changed to $\theta(t_1) = \pi/2$, one has $D(t_1) = -C$ and the state is described as $|\psi(t_1)\rangle = (-1)^n|\mathbf{c}^n\rangle \otimes |0\rangle_p$.

After an inverse adiabatic control to make θ: $\pi/2 \to 0$, the total quantum information will change from the type of the atomic ensemble to the type of photons:

$$\sum_{n,m}(-1)^{n+m}\rho_{nm}|\mathbf{c}^n\rangle\langle\mathbf{c}^m| \otimes |0\rangle_{pp}\langle 0| \to |b^N\rangle\langle b^N| \otimes \sum_{n,m}\rho_{nm}|n\rangle_{pp}\langle m|. \quad (25)$$

The quantum information can be adiabatically transferred from the optical field to the atomic ensemble, and vice versa. Such two adiabatic passages complement the "write" and "read" manipulation of quantum information. That is to say, the quantum information can be memorized in such an atomic ensemble.

In summary, we study the structure of the eigen-states and eigen-values of the collective exciton-photon system under the two-photon resonance EIT. Our results show that the dark states and dark-state polaritons can still exist under the case of two-photon resonance. We analyze in detail the possibility of dark states being staying the initial form under the adiabatic evolution of systemic parameter $\Omega(t)$. A precise adiabatic condition is presented in order to make sure that a dark state can not mix any other eigen-states. Then, with the help of the dark states under the two-photon resonance EIT system, we have described how one can transfer or communicate the quantum states between the type of photons and the type of atomic collective excitations by adiabatically changing the Rabi frequency $\Omega(t)$ of the classical control laser field. Our results show that the two-photon resonance EIT system can be used as the same robust quantum memory as that under the case of one-photon resonance EIT.

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