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Localization of a macroscopic object induced by the factorization of internal adiabatic motion

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Abstract. To account for the phenomenon of quantum decoherence of a macroscopic object, such as the localization and disappearance of interference, we invoke the adiabatic quantum entanglement between its collective states (such as that of the center-of-mass (CM)) and its inner states based on our recent investigation. Under the adiabatic limit where motion of the CM does not excite the transition of inner states, it is shown that the wave function of the macroscopic object can be written as an entangled state with correlation between adiabatic inner states and quasi-classical motion configurations of the CM. Since the adiabatic inner states are factorized with respect to each component of the macroscopic object, this adiabatic separation can induce the quantum decoherence. This observation thus provides us with a possible solution to the Schrödinger cat paradox.

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1 Introduction

It is common sense that a macroscopic object should be localized in a certain spatial domain. However, a problem will arise if one directly uses quantum mechanics to describe the motion of a free macroscopic object with spatial localization.This issue originated from the correspondence between Einstein and Born [1]. They observed that, in a spatially-localized state, generally a macroscopic object can only be described by a time-dependent localized wave packet, which is a coherent superposition of the eigenstates of the center-of-mass Hamiltonian $H_0 = p^2/2M$. If the macroscopic object is regarded as a heavy particle of a large mass M, its initial state $|\varphi\rangle$ should be a very narrow wave packet of width a. Since the wave packet spreads in evolution by the law

$$w(t) = a\sqrt{1 + \frac{t^2}{4M^2a^4}}, \tag{1}$$

where $w\,(t)$ stands for the width of the wave packet, the spreading of an initially well localized wave packet can be reasonably ignored for very large mass. This seems to give a solution to the localization problem of the macroscopic object. But Einstein argued that the superposition of two narrow wave packets is no longer narrow with respect to the macro-coordinate, and on the other hand, it is still a possible state of the macroscopic object. So a contradiction to the superposition principle arises because of the requirement that the wave packet of a macroscopic object should be narrow [1].

To cope with this problem, Wigner [2], Joos and Zeh [3] propose the so-called scattering-induced-decoherence mechanism (or WJZ mechanism) [4]: scattering of photons and atoms off a macroscopic object records the information of its position to form a quantum measurement for the position. Indeed, the most recent experiments [5,6] show that Schrödinger's concept of entangled state, rather than the unavoidable measurement distribution, is crucial for the wave-particle duality in this "which-way" detection. Actually, similar gedanken experiments using photons and neutrons have been considered before [7,8]. From these experiments and theory, it seems reasonable to conclude that there does not exist coherent superposition of states of a macroscopic object due to the quantum decoherence resulting from its coupling to an external environment as a generalized detector.

Here a natural question arises: if a macroscopic object, such as the famous Schrödinger cat, is completely isolated from any external environment, can its quantum coherence be maintained to realize macroscopic superposition state? Such a question leads people to consider the influence of the inner particles of a macroscopic as the so-called "internal environment" [9]. Most recently, a novel experiment was presented to observe the matter wave interference of C_{60} molecules by diffraction at an absorbing grating [10]. Though decoherence phenomena have not been observed in this experiment, it might be possible to set up a decoherence experiment if one can find a new

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way to effectively record the "which-way" information of C_{60}. In fact, there does exist coherent superposition of macroscopic states in certain extreme cases, for example, in the case that superconductivity or Bose-Einstein condensation [11]. In such cases macroscopically-quantum phenomenon requires that each part of the macroscopic object has the same phase in the process of evolution. For this reason, they will not be considered in this paper.

In this paper, we will show that when quantum entanglement occurs between the states of the-center-of-mass (CM) of the macroscopic object and its inner states, the conception of adiabatic quantum entanglement (most recently proposed in reference [12] based on the Born-Oppenhemeir (BO) approximation) is suitable for the study of the decoherence phenomenon. In fact, when the motion of the CM does not excite the transition of inner states, the wave function of the macroscopic object can be adiabatically factorized with correlation between the adiabatic inner states and the quasi-classical motion configuration of the CM. By this correlation or entanglement, the spatial localization of a macroscopic object can be explained and the dilemma of the Schrödinger cat can be resolved in a natural way.

2 Adiabatic entanglement and the WJZ mechanism

In the following, based on the idea of adiabatic entanglement, we incorporate the WJZ mechanism and the relevant study in the dynamic theory of quantum measurement [13–16] developed by many people, including one (CPS) of the authors.

In the WJZ mechanism, the "which-way" information of the macroscopic object is recorded through the quantum entanglement caused by scattering of atoms or photons regarded as the so-called environment. Let x be the collective position (CM) of a macroscopic object. To study how different positions of the macroscopic object entangle with the environment (scattering atoms, photons, etc.), we suppose that the total system is initially in a product state $|\Psi_x(t=0)\rangle = |x\rangle \otimes |\phi\rangle$,where the first component $|x\rangle$ is the eigen-state of the collective position operator x while $|\phi\rangle$ is an arbitrary pure state of the environment. According to an argument by Joos and Zeh [4], only when the back-action is negligibly small, can the interaction between the collective and environment states realize a "measurement-like process":

$$|x\rangle \otimes |\phi\rangle \rightarrow |x(t)\rangle \otimes |\phi(x,t)\rangle \cdot \tag{2}$$

Here, $|x(t)\rangle$ represents the free evolution in the absence of the coupling to the environment and $|\phi(x,t)\rangle$ represents the environment state parameterized by the collective position x of the macroscopic object. If the collective motion is initially described by a wave packet $|\varphi\rangle = \int \varphi(x)|x\rangle \mathrm{d}x$, then equation (2) defines a reduced density matrix

$$\rho(x,x',t) = \varphi(x,t)\varphi^*(x',t)\langle\phi(x';t)|\phi(x;t)\rangle \tag{3}$$

of the macroscopic object. Considering the translational invariance of the scattering process, Joos and Zeh showed that, the off-diagonal terms $F(x,x') = \langle\phi(x';t)|\phi(x;t)\rangle$ take a damping form depending on the total cross-section.

Now we consider a macroscopic object with collective and internal variables, say x and q. By the above discussion one easily sees that the interaction between these two kinds of variables may lead to an ideal quantum entanglement between the collective and internal states, when the collective states are free of the back-action. But the question is whether the negligibility of the back-action is the unique cause of the appearance of the above mentioned "measurement-like process". If not, what are the other causes besides it? To resolve this problem, we use the BO approximation to adiabatically separate the collective and internal variables. Assume that the total Hamiltonian is $H = p^2/2M + h(q,x)$, where the Hamiltonian $h(q,x)$ describes the motion of the internal variables q coupling to the collective variable x. For a fixed value of the slow variable x, the eigen-state $|n[x]\rangle$ and the corresponding eigenvalues $V_n[x]$ are determined by the eigen-equation

$$h(q,x)|n[x]\rangle = V_n(x)|n[x]\rangle \cdot \tag{4}$$

Regarding x and q as the slow and fast variables respectively in the BO adiabatic approach, we approximately obtain the complete set $\{\langle x|n,\alpha\rangle \equiv \phi_{n,\alpha}(x)|n[x]\rangle\}$ of eigenstates of the total system, where $\phi_{n,\alpha}(x)$ comes from the eigen-equation

$$H_n\phi_{n,\alpha}(x) = E_{n,\alpha}\phi_{n,\alpha}(x) \tag{5}$$

and

$$H_n = p^2/M + V_n[x] \tag{6}$$

is the effective Hamiltonian associated with the internal state $|n[x]\rangle$. Here, we do not consider the induced gauge potential connected with Berry phase factor through the quantum adiabatic method [17,18]. Then, we can see how the "measurement-like process" naturally appears as a result of the adiabatic dynamic evolution.

In fact, under the BO approximation, we can expand the factorized initial state $|\Psi(0)\rangle = |x\rangle \otimes |\phi\rangle$ in terms of the adiabatic basis $\{|n,\alpha\rangle\}$ and then we obtain the total wave function [12]

$$|\Psi(t)\rangle = \sum_n \langle n[x]|\phi\rangle \int \mathrm{d}x'\, K(x',x,t)|x'\rangle \otimes |n[x']\rangle \tag{7}$$

where we have used the completeness relations for the full eigen-functions expressed in *x-representation* and $K(x',x,t) = \langle x'|\mathrm{e}^{-\mathrm{i}H_nt}|x\rangle$. Generally, the propagator $K(x',x,t)$ is not diagonal for $|x\rangle$, is not an eigen-state of H_n and then $|\Psi(t)\rangle$ cannot define an ideal entanglement state. However, for large mass M, we can prove that, to the first order approximation, $K(x',x,t)$ takes a diagonal form proportional to a *δ-function*. Actually, in the large limit, the kinetic term $p^2/2M$ can be regarded as a perturbation in comparison with the effective potential $V_n(x)$.

Using Dyson expansion to the first order of $1/M$, we have

$$\begin{aligned}\mathrm{e}^{-\mathrm{i}H_n t} &= \mathrm{e}^{-\mathrm{i}V_n t}\left(1-\mathrm{i}\int_0^t \mathrm{e}^{\mathrm{i}V_n t'}\frac{p^2}{2M}\mathrm{e}^{-\mathrm{i}V_n t'}\mathrm{d}t' + ...\right)\\ &= \mathrm{e}^{-\mathrm{i}V_n t}\left(1-\mathrm{i}\frac{p^2 t}{2M} - \frac{t^2}{4M}(p\partial_x V_n + [\partial_x V_n]p)\right.\\ &\qquad\left. -\frac{\mathrm{i}t^3(\partial_x V_n)^2}{6M} + ...\right)\cdot \quad (8)\end{aligned}$$

Since

$$\int \langle x'|p^n|x\rangle f(x')\,\mathrm{d}x = 0$$

for $n = 1, 2, ...$, we conclude that

$$\begin{aligned}K(x', x, t) &= \mathrm{e}^{-\mathrm{i}V_n[x]t}\left[\delta(x-x')\right.\\ &\left.+\frac{\mathrm{i}}{2M}\int_0^t \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}V_n[x']\tau}\frac{\partial^2}{\partial x'^2}\delta(x-x')\mathrm{e}^{-\mathrm{i}V_n(x)\tau}\right]\cdot \quad (9)\end{aligned}$$

Then, we observe that it is approximately diagonalized: $K(x', x, t) = \mathrm{e}^{-\mathrm{i}H_n(x)t}\delta(x-x')$, and the adiabatic wave-function leads to an ideal entanglement

$$|\Psi(t)\rangle = \sum_n \langle n[x]|\phi\rangle \mathrm{e}^{-\mathrm{i}H_n(x)t}|x\rangle \otimes |n[x]\rangle \quad (10)$$

We call this entanglement adiabatic entanglement.

In conclusion, up to the first order approximation of $1/M$, the quantum entanglement appears in the adiabatic evolution. The Born-Oppenheimer adiabatic approximation has provided us with a novel mechanism to produce a quantum entanglement between the macroscopic object and its internal variables.

3 Localization induced by factorized internal motion

We notice that the above simple result has the following physical explanation: the evolution state of a heavy particle for very large M, which is almost steady, is approximately an eigenstate of the position operator if it is initially in a state with a fixed position. Then, it follows that, in the large-mass limit, the wave function $|\Psi(t)\rangle$ can be factorized approximately: $|\Psi(t)\rangle = |x\rangle \otimes S(x, t)|\phi\rangle$ where the entangling *S-matrix*

$$S(x, t) = \sum_n \mathrm{e}^{-\mathrm{i}V_n t}\,|n[x]\rangle\langle n[x]| \quad (11)$$

is defined in terms of the adiabatic projection $|n[x]\rangle\langle n[x]|$.

According to our previous argument about the factorized structure of *S-matrix* in [11,12] developed based on the Hepp-Coleman model [13], if the internal degree of freedom has many components, *e.g.*, if $q = (q_1, q_2, ...q_N)$, then in their normal non-interaction modes, $S(x;t)$ can be factorized as:

$$S(x;t) = \prod_{j=1}^N S_j(x;t) \quad (12)$$

with

$$S_j(x;t) = \mathrm{e}^{-\mathrm{i}h_j(q_j,x)t} \quad (13)$$

where $h_j(q_j, x)$ is the single particle Hamiltonian of the macroscopic object. Of course, in the derivation of the above factorized structure for the *S-matrix*, we have made some simplifications. Roughly speaking, we have assumed that the adiabatic effective potential takes the form of a direct sum $V_n = \sum_j V_{nj}(q_j)$, and that the eigenstate takes the form of a direct product

$$|n[x]\rangle = \prod_{j=1}^N \otimes |n_j[x]\rangle \quad (14)$$

the higher order terms $\approx O(1/M)$ being neglected.

For the initial state $|\phi\rangle = \prod_{j=1}^N \otimes |\phi_j\rangle$ factorized with respect to internal components, the reduced density matrix

$$\rho(x, x', t) = \varphi(x)\varphi^*(x')F_N(x', x, t) \quad (15)$$

can be re-written in terms of the so called decoherence factor

$$\begin{aligned}F_N(x', x, t) &= \prod_{j=1}^N F^{[j]}(x', x, t)\\ &\equiv \prod_{j=1}^N \langle\phi_j|S^\dagger_{q_j}(x';t)S_{q_j}(x;t)|\phi_j\rangle\cdot \quad (16)\end{aligned}$$

This factor is expressed as an N-multiple product of the single decohering factors

$$F^j(x, x') = \langle\phi_j|S^\dagger_{q_j}(x';t)S_{q_j}(x;t)|\phi_j\rangle \quad (17)$$

with norms not larger than unity. Thus in the macroscopic limit $N \to \infty$, it is possible that $F_N(x', x, t) \to 0$, for $x' \neq x$. In fact, this factor reflects almost all of the dynamic features of the influence of the fast part on the slow part. Physically, an infinite N means that the object is macroscopic since it is made of infinite number of particles in that case. On the other hand, the occurrence of decoherence at infinite N manifests a transition of the object from the quantum realm to the classical realm. Here, as expected, the physical picture is consistent.

As to the localization problem raised by Einstein and Born [1], we, based on the above argument, comment that one can formally write down the wave function of a macroscopic object as a narrow pure state wave packet, but it is not the whole of the story. Actually, the statement that an object is macroscopic should physically imply that it contains many particles. So a physically correct description of its state must concern its internal motion coupling to the collective coordinates (*e.g.*, its center-of-mass). Usually, one observes this collective coordinate to determine whether two spatially-localized wave packets can interfere with each other. If there does not exist such interference, one may say that, the superposition of two narrow wave packets for the macro-coordinate is no longer a possible pure state of the macroscopic object. Indeed, because the "which-way" information of the macro-coordinate is recorded by the internal motion of particles making up the macroscopic object, the induced decoherence must destroy the coherence in the original superposition so that the state of the macroscopic object is no longer pure.

The present argument also provides a possible solution for the Schrödinger cat paradox. If we consider the Schrödinger cat as a macroscopic object consisting of many internal particles, then we can never observe anything corresponding to the interference between the dead and the living cats. This is because the macroscopically-dead and the macroscopically-living states, $|D\rangle$ and $|L\rangle$, of the cat are correlated to the corresponding internal states, $|d_j\rangle$ and $|l_j\rangle$. From the argument in this section the cat state can be written as

$$|\mathrm{Cat}\rangle = |L\rangle \otimes \prod_{j=1}^{N} |l_j\rangle + |D\rangle \otimes \prod_{j=1}^{N} |d_j\rangle \tag{18}$$

where $|D\rangle$ and $|L\rangle$ represent the collective states while $\prod_{j=1}^{N} |l_j\rangle$ and $\prod_{j=1}^{N} |d_j\rangle$ describe the corresponding internal motion. It leads to a reduced density matrix with the off-diagonal elements proportional to $\prod_{j=1}^{N} \langle d_j | l_j \rangle$. Thus if there is only a pair of inner states that are orthogonal, the off-diagonal elements will vanish and decoherence will happen. Even if there does not exist any pair of inner states orthogonal to each other, it is also highly possible that $\prod_{j=1}^{N} \langle d_j | l_j \rangle \to 0$ in the macroscopic limit $N \to \infty$ since the norm of each $\langle d_j | l_j \rangle$ is less than or equal to unity. In this sense, we conclude that the Schrödinger cat paradox is not a paradox at all in practice. Rather, it essentially arises from overlooking the internal motion of a macroscopic cat or the multi-particle scattering off it.

It is a little bit provocative that in practice the Schrödinger cat wavefucntion should take the above specific form to realize decoherence. Actually, as a macroscopic object, the cat may have a very large Hilbert space and a very dense energy spectrum. Thus it is imaginable that, the slightly different actions, as perturbations, exerted by different collective (living and dead) states will force the inner states with many variables to evolve into very different perturbed wave functions $\prod_{j=1}^{N} |l_j\rangle$ and $\prod_{j=1}^{N} |d_j\rangle$ in normal modes. For the case concerning the external environment, this point has been mentioned by Omnes. Another point we wish to make is that it must be difficult to distinguish the collective variables of a real cat from its internal ones. Therefore, strictly speaking, the above discussion about the Schrödinger cat paradox is only appropriate for an ideal Schrödinger cat, or in other words, a toy model of a Schrödinger cat. In fact, how to distinguish decoherence of a large system due to external and internal environments is an open question in the general case.

We have shown that the analysis of the localization phenomenon of a macroscopic object can be reduced to the study of entanglement between its collective position (or CM) and internal variables in the adiabatic evolution with the above mentioned factorization structure. Closely related to the Schrödinger cat phenomenon, this entanglement results from the adiabatic separation of collective and internal variables. To our surprise, in the C_{60} molecule interference experiment an elegant interference pattern appears. But there is no contradiction here. Firstly, at high temperature C_{60} would emit two or three infrared photons during its passage through the apparatus. Usually the emission would entangle the position of the C_{60} molecule with an ordinary environment, the background electromagnetic field as an external continuum. But as the wavelength of the emitted wave from the internal motion of C_{60} is much greater than the distance between the neighboring slits, the photons carry no information about the route the molecule takes. Secondly, though there exists interaction with the external air particles, the scattering rates on the macroscopic object are far too small to induce quantum decoherence. Finally, C_{60} can not well be considered as a usual macroscopic object since its internal variables are almost frizzed so that they are endowed with a single phase or the matched phases. The entanglement with the internal degrees of freedom could have the characteristics discussed above if the molecule C_{60} experiences inhomogeneous fields, but it does not seem to be the case in the present C_{60} experiment. These arguments explain the persistence of the interference pattern in the experiment [10].

However, we can imagine that in such experiments, the internal motion (such as radiation of photons of various frequencies) produces an effective coupling with the collective motion of the CM. Then, the configuration of internal motion can record the "which-way" information even through a single thermal photon so that the interference contrast should thus be completely destroyed. In that case, because a C_{60} molecule has only a finite number of internal degrees of freedom, the decoherence dynamics determined by the coupling to the inner states should be rather different from that happening in the ideal case concerning infinitely many internal degrees of freedom. For instance, coherence revival could occur since the factorized decoherence factor is a finite product of N parts and thus is an oscillating function of time t. Moreover, if one can realize the quantum decoherence of a large system (such as C_{60} fullerenes) of many degrees of freedom, the parameters (such as the internal temperature of the fullerenes, the temperature of the environment, the intensity and frequency of external laser radiation) can be controlled continuously so that quantitative natures such as those described in this paper could be tested. Unfortunately, the study in the present paper is not directly applicable to such "which-way" experiments because it is based on the assumption that the macroscopic object is composed of two-level subsystems and does not concern the concrete structure of C_{60} fullerenes. Nevertheless, for the quantitative investigation of the dynamic details of decoherence process in such experiments, it can serve as a starting point.

4 Simple model for macroscopic localization

To make a deeper elucidation of the above general arguments about the localization of a macroscopic object of mass M, we model the macroscopic object as consisting of N two level particles, which are fixed at certain positions to form a whole without internal spatial motion. The collective position x is taken to be its mass-center or any reference position in it while the internal variables are taken to be the quasi-spins associated with two level particles. Generally, if we assume that the back-action of the internal variables on the collective position is relatively small, the model Hamiltonian can be written as

$$\begin{aligned} H =& \frac{P^2}{2M} + h(x) \\ h(x) =& \sum_{j=1}^{N} \left(f_j(x)|e_j\rangle\langle g_j| + f_j^*(x)|g_j\rangle\langle e_j|\right) \\ &+ \sum_{j=1}^{N} \omega_j \left(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|\right) \end{aligned} \tag{19}$$

where $|g_j\rangle$ and $|e_j\rangle$ are the ground and the excited states of the jth particle and $f_j(x)$ denotes the position-dependent couplings of the collective variable to the internal variables. Let l_j be the relative distance between the jth particle and the reference position x. Further we assume $f_j(x) = f(x + l_j)$. Physically, we may think that these couplings are induced by an inhomogeneous external field, *e.g.*, they may be the electric dipole couplings of two-level atoms in an inhomogeneous electric field.

We remark that the above model enjoys some universality under certain conditions, compared with various environment models inducing both dissipation and decoherence of quantum processes. In fact, Caldeira and Leggett [19] have pointed out that any environment weakly coupling to a system may be approximated by a bath of oscillators under the condition that "each environmental degree of freedom is only weakly perturbed by its interaction with the system". We observe that any linear coupling only involves transitions between the lowest two levels (ground state and the first excitation state) of each harmonic oscillator in the perturbation approach though it has many energy levels. Therefore in such a case we can also describe the environment as a combination of many two level subsystems without losing generality [20]. To some extent, these arguments justify our choosing the two level subsystems to model the internal motion of the macroscopic object. We will soon see its advantage: the character of localization can be manifested naturally and clearly.

Now let us calculate the $S_j(x;t)$ for this concrete model. The single-particle Hamiltonian $h_j(x) = \omega_j(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|) + (f_j(x)|e_j\rangle\langle g_j| + \text{h.c.})$ has the x-dependent single-particle *S-matrix*

$$S_j(x;t) = \begin{pmatrix} \cos(\Omega_j t) - \mathrm{i}\sin(\Omega_j t)\cos\theta_j, & \mathrm{i}\sin(\Omega_j t)\sin\theta_j \\ \mathrm{i}\sin(\Omega_j t)\sin\theta_j, & \cos(\Omega_j t) + \mathrm{i}\sin(\Omega_j t)\cos\theta_j \end{pmatrix} \tag{20}$$

in the BO adiabatic approximation. Here $\Omega_j(x) \equiv \pm\sqrt{|f_j(x)|^2 + \omega_j^2}$. Explicitly, having obtained the above analytic results about *S-matrix*, we can further calculate the single-particle decoherence factors $F^{[j]}(x', x, t) \equiv \langle g_j|S_j^\dagger(x';t)S_j(x;t)|g_j\rangle$ for a given initial state $|\phi\rangle = \prod_{j=1}^{N} \otimes |g_j\rangle$. For simplicity we use the notation $f(x') = f'$. We have

$$\begin{aligned} F^{[j]}(x', x, t) =& \left\{\sin\left(\Omega_j' t\right)\sin\theta_j' \sin\left(\Omega_j t\right)\sin\theta_j \right. \\ &+\cos\left(\Omega_j' t\right)\cos\left(\Omega_j t\right) + \sin\left(\Omega_j' t\right)\cos\theta_j' \sin\left(\Omega_j t\right)\cos\theta_j' \cos\theta_j \\ &\left.+\mathrm{i}\left\{\cos\left(\Omega_j' t\right)\sin\left(\Omega_j t\right)\cos\theta_j - \sin\left(\Omega_j' t\right)\cos\theta_j' \cos\left(\Omega_j t\right)\right\}\right\} \end{aligned} \tag{21}$$

where $\tan\theta_j = f_j(x)/\omega_j$. In the weakly coupling limit, $g_j \ll \omega_j$ and the coupling $f_j \simeq g_j x$, thus we have $\sin\theta_j \simeq \theta_j \simeq f_j/\omega_j, \cos\theta_j \simeq 1 - \theta_j^2/2$ and $\Omega_j \simeq \omega_j$. Then, the decohering factors can be simplified

$$F(x', x, t) \simeq |F(x', x, t)| \times \exp\left(\frac{\mathrm{i}|g_j|^2}{4\omega_j^2}(x^2 - x'^2)\sin(2\omega_j t)\right) \tag{22}$$

where

$$|F(x', x, t)| = \exp\left(-(x - x')^2 \frac{|g_j|^2}{2\omega_j^2}\sin^2\left(\omega_j t\right)\right). \tag{23}$$

In the case of continuous spectrum, the sum

$$R(t) = \sum_{j=1}^{N} \frac{g_j^2}{2\omega_j^2}\sin^2\left(\omega_j t\right) \tag{24}$$

can be re-expressed in terms of a spectrum distribution $\rho(\omega_k)$ as

$$R(t) = \int_0^\infty \frac{\rho(\omega_k) g_k^2}{2\omega_k^2}\sin^2\omega_k\,\mathrm{d}\omega_k.$$

From some concrete spectrum distributions, interesting circumstances may arise. For instance, when $\rho(\omega_k) = (4/\pi)(\gamma/g_k^2)$ the integral converges to a negative number proportional to time t, precisely, $R(t) = \gamma t$. Therefore, our analysis recovers the result

$$\rho(x, x', t) = \varphi(x)\varphi^*(x')\mathrm{e}^{-\gamma t(x-x')^2} \times \exp\left[\mathrm{i}\pi(x^2 - x'^2)s(t)\right] \tag{25}$$

for the reduced density matrix of the macroscopic object, which was obtained by Joos and Zeh [3] through the multi particle external scattering mechanism and by Zurek separately through the Markov master equation. Here,

$$s(t) = \sum_{j=1}^{N} \frac{\sin(2\omega_j t)}{4\pi\omega_j^2} \tag{26}$$

is a time-dependent periodic function. This shows that the norm of the decoherence factor is exponentially decaying and as $t \to \infty$, the off-diagonal elements of the density matrix vanish simultaneously!

We will show that for a quite general distribution $\rho(\omega)$ the off-diagonal elements of the reduced density matrix decline rather sharply with time t if the particle number N is large. Assume that all $g_j's$ are equal: $g_j = g$. If the frequencies lie within an interval $[\omega_1, \omega_2]$ and the distribution is homogeneous, we have $\rho(\omega) = N/(\omega_2 - \omega_1)$. Then

$$\begin{aligned} R(t) &= \int_{\omega_1}^{\omega_2} \frac{g^2}{2\omega^2} \sin^2 \omega t \; \rho(\omega)\,\mathrm{d}\omega \\ &= \frac{N}{(\omega_2-\omega_1)} \frac{g^2}{2} \int_{\omega_1}^{\omega_2} \frac{1}{\omega^2} \sin^2 \omega t \; \mathrm{d}\omega \\ &\geqslant \frac{N}{(\omega_2-\omega_1)} \frac{g^2}{2\omega_2^2} \int_{\omega_1}^{\omega_2} \sin^2 \omega t \; \mathrm{d}\omega \\ &= \frac{N}{4} \frac{g^2}{\omega_2^2} \left(1 - \cos(\omega_2+\omega_1)t \frac{\sin(\omega_2-\omega_1)t}{(\omega_2-\omega_1)t} \right) \cdot \end{aligned} \tag{27}$$

For a general $\rho(\omega)$ in the interval $[\omega_1, \omega_2]$, we have

$$\int_{\omega_1}^{\omega_2} \rho(\omega)\,\mathrm{d}\omega = N.$$

Then there exists some $\overline{\omega}$ in $[\omega_1, \omega_2]$ such that

$$\rho(\overline{\omega}) = \frac{N}{\omega_2 - \omega_1} \cdot$$

If the frequency spectrum of the system is such that there exist ω_3 and ω_4 in the interval $[\omega_1, \omega_2]$ satisfying

$$\rho(\omega) \geqslant \frac{N}{\omega_2-\omega_1} \quad \text{for} \quad \omega_3 \leqslant \omega \leqslant \omega_4. \tag{28}$$

From the derivation of (44) it then follows that

$$R(t) \geqslant \frac{N}{4} \frac{g^2}{\omega_4^2} \frac{\omega_4-\omega_3}{\omega_2-\omega_1} \left(1 - \cos(\omega_4+\omega_3)t \times \frac{\sin(\omega_4-\omega_3)t}{(\omega_4-\omega_3)t} \right) \cdot \tag{29}$$

After a moment's thought, one can easily convince oneself that the condition (36) is rather easy to satisfy. From the inequality (37) we observe that although in the weakly coupling limit, we should have $g^2/\omega_4^2 \ll 1$, $R(t)$ can increase sharply with time t if the particle number is large enough. This just means that the off-diagonal elements of the reduced density matrix will decline sharply with time t. In conclusion, despite the complexity of $\rho(x, x', t)$ due to the presence of the oscillating factor $s(t)$, in many cases it can well describe the decoherence of macroscopic objects thanks to its simple decaying norm.

5 Decoherence of wave packets

Let us now turn to consider an example similar to that studied by Joos and Zeh. We take a coherent superposition of two Gaussian wave packets of width d

$$\varphi(x) = \frac{1}{\sqrt[4]{8\pi d^2}} \left\{ \exp\left(-\frac{(x-a)^2}{4d^2} \right) + \exp\left(-\frac{(x+a)^2}{4d^2} \right) \right\} \cdot \tag{30}$$

The norm of the corresponding reduced density matrix

$$|\rho(x, x', t)| = \sum_{k,l=0}^{1} P_{kl}(x, x', t) \tag{31}$$

contains 4 peaks:

$$\begin{aligned} P_{11}(x,x',t) &= \frac{1}{\sqrt{8\pi d^2}} \mathrm{e}^{-\gamma t (x-x')^2} \times \exp\left[-\frac{(x-a)^2}{4d^2} - \frac{(x'-a)^2}{4d^2} \right] \\ P_{10}(x,x',t) &= \frac{1}{\sqrt{8\pi d^2}} \mathrm{e}^{-\gamma t (x-x')^2} \times \exp\left[-\frac{(x-a)^2}{4d^2} - \frac{(x'+a)^2}{4d^2} \right] \\ P_{01}(x,x',t) &= \frac{1}{\sqrt{8\pi d^2}} \mathrm{e}^{-\gamma t (x-x')^2} \times \exp\left[-\frac{(x+a)^2}{4d^2} - \frac{(x'-a)^2}{4d^2} \right] \\ P_{00}(x,x',t) &= \frac{1}{\sqrt{8\pi d^2}} \mathrm{e}^{-\gamma t (x-x')^2} \times \exp\left[-\frac{(x+a)^2}{4d^2} - \frac{(x'+a)^2}{4d^2} \right] \end{aligned} \tag{32}$$

centering respectively around the points (a, a), $(a, -a)$, $(-a, a)$ and $(-a, -a)$ in $x - x'$-plane. The heights are respectively $1/\sqrt{8\pi d^2}$, $\mathrm{e}^{-4\gamma t a^2}/\sqrt{8\pi d^2}$, $\mathrm{e}^{-4\gamma t a^2}/\sqrt{8\pi d^2}$ and $1/\sqrt{8\pi d^2}$). Obviously, two peaks with centers at $(a, -a)$ and $(a, -a)$ decay with time while the other two keep their heights constant. Figure 1 shows this time-dependent configuration at $t = 0$, and a finite t. As $t \to \infty$, two off-diagonal terms P_{10} and P_{01} decay to zero so that the interference of the two Gaussian wave packets is destroyed. In this sense, we say that the pure state $\rho(x, x', t = 0) = \int \mathrm{d}x\, \varphi(x)\varphi^*(x')|x\rangle\langle x'|$ becomes a mixed state

$$\rho(t) = \int \mathrm{d}x\, \varphi(x)\varphi^*(x)|x\rangle\langle x| \tag{33}$$

in *x-representation.*

Interference of two plane waves of wave vector k_1, k_2 provides us with another simple example. Without decoherence induced by its internal motion or the external scattering, their coherent superposition

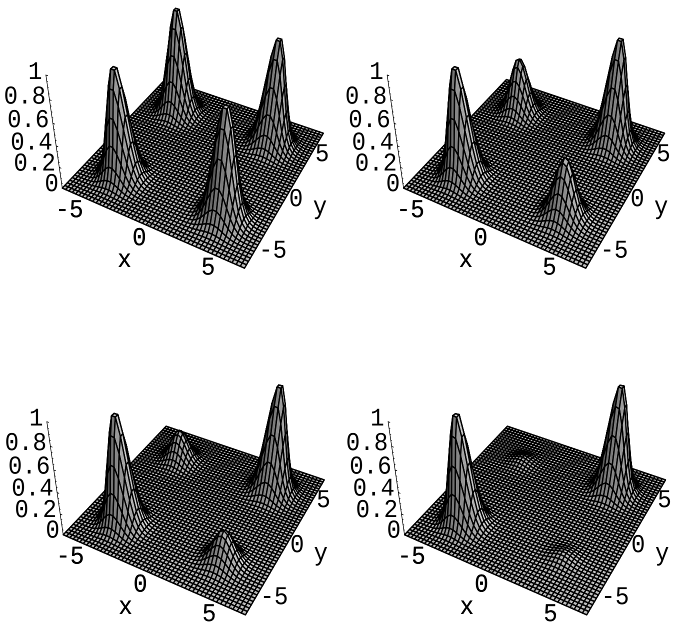

Fig. 1. Disappearance of the nondiagonal elements of the density matrix.

$\varphi(x) = \sqrt{(1/4\pi)}[\mathrm{e}^{\mathrm{i}k_1 x} + \mathrm{e}^{\mathrm{i}k_2 x}]$ yields a spatial interference described by the reduced density matrix

$$\rho_0(x, x', t) = \frac{1}{4\pi}\Bigg\{\mathrm{e}^{\mathrm{i}k_1(x-x')} + \mathrm{e}^{\mathrm{i}k_2(x-x')} + \exp\left[\mathrm{i}\left(\frac{k_1^2 t - k_2^2 t}{2m} + k_2 x - k_1 x'\right)\right] + \exp\left[\mathrm{i}\left(\frac{k_2^2 t - k_1^2 t}{2m} + k_1 x - k_2 x'\right)\right]\Bigg\}. \tag{34}$$

Under the influence of internal motion, it becomes

$$\rho(x, x', t) \approx \rho_0(x, x', t)\mathrm{e}^{-\gamma t(x-x')^2}$$

for large mass. We see that the difference created by decoherence is only reflected in the off-diagonal elements,and the pure decoherence (without dissipation) does not destroy the interference pattern described by the diagonal term $\rho(x, x, t) = \rho_0(x, x, t)$. This simple illustration tells us that the present quantum decoherence mechanism may not have to do with the interference pattern of the first order coherence, but it does destroy the higher order quantum coherence: $\rho(x, x', t) \to 0$ as $t \to \infty$. In fact, due to the induced loss of energy, quantum dissipation is responsible for the disappearance of the interference pattern of the first order coherence. The influence of internal motion or external scattering on the decoherence of a macroscopic object may be very complicated. Intuitively, these dynamic effects should depend on the details of interaction between the collective variables and the internal and external degrees of freedom. Practically,we can classify these influences into two categories, namely, quantum dissipation and quantum decoherence, and then study them separately by different models.

6 Concluding remarks

It is noticed that, so long as the "which-way" information of the collective motion of a macroscopic object already stored in the internal motion can be read out, the phenomenon caused by interference will be destroyed without any data being read out in practice [5,13–16]. In this sense the internal degrees of freedom interacting with the macroscopic object behave as a detector to realize a "measurement-like" process. Thus, the internal motion configuration is imagined as an objective detector detecting the collective states. Provided that the internal motion configuration couples with the collective motion and produces an ideal entanglement, the collective motion must lose its coherence. It is worth pointing out that this simple entanglement conserves the energy of the collective motion while it destroys the quantum coherence.

In the case without energy conservation, quantum dissipation can also induce the localization of macroscopic object. Based on the studies of quantum dissipation stimulated by Caldeira and Leggett [18], Yu and one (CPS) of the authors found a novel mechanism which sheds new light on the localization problem of macroscopic objects [21]. They studied the quantum dynamics of a simplest dissipative system: one particle moving in a constant external field and interacting with a bath of harmonic oscillators with Ohmic spectral density. It was found that the wave function of the total system can be factorized as a product of those of the system part and the bath part. When one ignores the effect of Brownian motion or the quantum fluctuation in the system caused by the bath, the product wave function becomes a direct product and the dissipative evolution of the system is governed by Caldirora-Kani (CK) Hamiltonian. Using this effective Hamiltonian, they discovered the following interesting result: the dissipation suppresses the wave packet spreading and causes the localization of the wave packet. Actually, it was shown that the breadth of the wave packet changes with time t in the following way: $w(t) = a\sqrt{1 + t_\eta^2/4M^2a^4}$. Here a is the initial breadth of the wave packet and $t_\eta = M(1 - \mathrm{e}^{-\eta t/M})/\eta$, where η is the damping rate. Comparing this formula with the equation (1), we find that the effect of the influence of the bath is the replacement of t by t_η in (1). We have $t_\eta \to t$ when $\eta/M \to 0$. So one can regard t_η as a deformation of time t caused by dissipation. Notice that t_η approaches the limit M/η as $t \to \infty$. This means localization of the wave packet in the presence of dissipation. Indeed, we have the limit breadth: $a_{\mathrm{limit}} = a\sqrt{1 + (1/2\eta a^2)^2}$. This suppression of the wave packet spreading by dissipation possibly provides a useful mechanism for the localization of quantum particles. It is a little bit surprising that as $t \to \infty$ the limit width of the damped particle wave packet is exactly the same as the "uncertainty product" of the damped particle, established by Schuch *et al.* through the nonlinear Schrödinger equation [22].

In summary, the environment induced dissipation as well as decoherence can provide an important mechanism for the localization of a macroscopic object. Mentioning macroscopicness implies the requirement that the macroscopic object must contain a large number of internal blocks. Then the macroscopic object, coupling to the internal variables, should be described by collective variables subject to an interaction similar to that concerning the external scattering in WJZ mechanism and the quantum dissipation of a particle in a bath.

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