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Dynamics of Bose-Einstein Condensed and Excited Atoms in a Quantized Cavity*

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Abstract *The dynamics of the Bose–Einstein condensed and excited atoms in a cavity interacting with quantized electromagnetic field of single mode is studied for both the cases with and without dissipation. It is shown that the frequencies of oscillation between the condensed atomic state and an excited state are related to the number of Bose–Einstein condensed atom in a certain form. This relation may be used for the reliable detection of the appearance of the Bose–Einstein condensate.*

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Most recently a Bose–Einstein condensate (BEC) was produced in a vapor of rubidium atoms that confined by magnetic field and evaporatively cooled.[1] The achievement of atomic BEC is quite important for both the foundational aspect of modern quantum theory and the development of high technology. It is an urgent task to investigate the details of property of BEC atom gas so that the BEC of atoms can be reliably diagnosed and confirmed. In fact, the optical properties of a Bose–Einstein condensed atomic gas have been theoretically studied for various situations, such as the coherent scattering of weak light on BEC,[2,3] the temperature dependence of the scattering,[4] the line shapes of scattered light from BEC[5,6] and the nonlinear optical response of BEC atoms.[7]

It should be pointed out that all of the above-mentioned studies only concerned the classical probing light and hence relied on the semiclassical theory. The goal of this paper is trying to consider the fully quantum effect of the light on the BEC atoms. Suppose the BEC atom is injected in a ring-cavity with a single-mode quantized field. The dynamic behaviors of the dressed bosonic atoms derived by the cavity field is expected to manifest the appearance of the BEC. In present studies, the processes of many-photon excitation of BEC and excited atoms are analyzed in detail for the two cases with and without dissipation respectively. It is shown that, the frequency of oscillation between the dressed bosonic states is proportional to the square root of number of the BEC. The probability of transition between the BEC atoms and excited atoms is a damping function of time when the system behaves as a dissipation one.

From the theoretical point of review, it is very convenient to used the second quantization framework for describing the system of cooled atoms. In terms of the creation and annihilation operators $b_{gn}^\dagger, b_{en}^\dagger$, b_{gn} and b_{en} corresponding to the jointed states $|g_n\rangle$ and $|e_n\rangle$ respectively, the Hamiltonian describing many identical bosonic atoms interacting with a single mode ω quantized cavity field is[8]

$$H = \sum_n (E_n b_{gn}^\dagger b_{gn} + (E_n + \hbar\omega_a) b_{en}^\dagger b_{en}) + \hbar\omega a^\dagger a + \sum_{mn} g(W_{mn} b_{gm}^\dagger b_{en} a^\dagger + W_{mn}^* b_{gm}^\dagger b_{em} a)\,, \quad (1)$$

where $W_{mn} = \langle\psi_m| \mathrm{e}^{ikx} |\psi_n\rangle$ and $|\psi_n\rangle$ is eigenstate of trapped atom. It is emphasized that the above Hamiltonian describes transitions from $|g\rangle$ to $|e\rangle$ accompanied by certain changes of motion of atomic mass center represented by the matrix elements W_{mn}. When BEC happens in trapped atoms, most atoms are forced to the lowest level $|g_n\rangle|_{n=1}$. A customary treatment for condensate in quantum field theory is enjoyed by the broken-symmetry. Namely, if there are N_c condensed atoms, one can let the annihilation and creation operators of $|g_{n=1}\rangle$ as c-numbers, $b_{g1} = \sqrt{N_c} = b_{e1}^\dagger$ which is the square root of the number of BEC atom, and hence

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the Hamiltonian can be written by

$$H = \hbar\omega a^\dagger a + \sum_n (E_n b^\dagger_{gn} b_{gn} + (E_n + \hbar\omega_a) b^\dagger_{en} b_{en}) + g\sqrt{N_c}\left[(\sum_n W_{1n} b_{en}) a^\dagger + (\sum_n W^*_{1n} b^\dagger_{en}) a\right].$$

Since $\sqrt{N_c} \gg 1$, one can neglect the contribution of the interaction term of $m > 1$. According to Ref. [5] one can also introduce the collective excited operator $b_1 = \sum_n W_{1n} b_{en}$ and its conjugate $b_1^\dagger$. With the above approximation, one has $H = \hbar\omega a^\dagger a + \sum_n (E_n + \hbar\omega_a) b^\dagger_{en} b_{en} + g\sqrt{N_c}(b_1 a^\dagger + b_1 a^\dagger)$, where we also ignore the term $\sum_n E_n b^\dagger_{gn} b_{gn} = E_1 N_c + \sum_{n)1} E_n b^\dagger_{gn} b_{gn} \sim E_1 N_c =$ constant. Finally, we obtain an effective Hamiltonian for a system of condensed atoms

$$H = \hbar\omega a^\dagger a + \hbar\omega_a b^\dagger b + g\sqrt{N_c}(b a^\dagger + a b^\dagger), \tag{2}$$

where $b^\dagger = b_1^\dagger$. This Hamiltonian just describes a familiar problem of coupling of two harmonic oscillators with larger coupling coefficient proportional to the number of condensed atoms.

Before the discussion for the dynamics with N-photon process based on the model Hamiltonian (2), we must clarify the physical meaning of the collective operators $b^\dagger$ and b, from the definition of $b^\dagger$ and b, we see the $b^\dagger$ and b describe independent wave packets, i.e. $[b, b^\dagger] = 1$, and $b^\dagger$ denotes an excitation with momentum translation $\hbar k$ form the lowest level $|g_{n=1}\rangle$.

The Heisenberg equations that follow from the Hamiltonian (2) are

$$\mathrm{i}\hbar\dot{b} = \hbar\omega_a b + \hbar g\sqrt{N_c} a, \qquad \mathrm{i}\hbar\dot{a} = \hbar\omega a + \hbar g\sqrt{N_c} b \tag{3}$$

after applying a Laplace and an inverse Laplace transformation to Eq. (3) yields an exact equation of motion for the operators $a(t)$ and $b(t)$

$$a(t) = g(t)a(0) + v(t)b(0), \qquad b(t) = f(t)a(0) + w(t)b(0), \tag{4}$$

where

$$g(t) = \frac{p_1 + \mathrm{i}\omega_a}{p_1 - p_2}\mathrm{e}^{p_1 t} + \frac{p_2 + \mathrm{i}\omega_a}{p_2 - p_1}\mathrm{e}^{p_2 t}, \qquad v(t) = \frac{-\mathrm{i}g\sqrt{N_c}}{p_1 - p_2}\mathrm{e}^{p_1 t} + \frac{-\mathrm{i}g\sqrt{N_c}}{p_2 - p_1}\mathrm{e}^{p_2 t},$$

$$w(t) = \frac{p_1 + \mathrm{i}\omega}{p_1 - p_2}\mathrm{e}^{p_1 t} + \frac{p_2 + \mathrm{i}\omega}{p_2 - p_1}\mathrm{e}^{p_2 t}, \qquad f(t) = \frac{-\mathrm{i}g\sqrt{N_c}}{p_1 - p_2}\mathrm{e}^{p_1 t} + \frac{-\mathrm{i}g\sqrt{N_c}}{p_2 - p_1}\mathrm{e}^{p_2 t},$$

and

$$p_{1,2} = \frac{-\mathrm{i}(\omega_a + \omega) \pm \sqrt{-(\omega_a + \omega)^2 - 4g^2 N_c + 4\omega\omega_a}}{2}.$$

Further, we use the above solution (Eq. (4)) to study the dynamics of BEC atoms with many photon process, for this end, we must select one of initial conditions, since different initial conditions would cause different dynamics. Firstly, consider the case that the cavity field is initially in Fock state $|N\rangle$ and there are M atoms in excited state i.e., $|\psi(0)\rangle = |M\rangle \otimes |N\rangle \equiv |M, N\rangle$, at t, the transition probability to $|n, m\rangle = |n\rangle \otimes |m\rangle$ is

$$P = |\langle m, n|U(t)|M, N\rangle|^2 = \left|\frac{1}{\sqrt{m!n!}}\langle 0, 0|a^m(0) b^n(0) U(t)|M, N\rangle\right|^2 =$$

$$\left|\sqrt{m!n!M!N!}\sum_{k=0}^{m}\frac{g^k(t) v^{m-k}(t) f^{N-k-M}(t) w^{M+k-m}(t)}{k!(m-k)!(n+m-k-M)!(M+k-m)!}\right|^2. \tag{5}$$

For the special case of the atoms being initially in BEC and the field having exact m photons, the probability of the atoms remaining in BEC is given by

$$P_m = \left(\frac{v}{w}\right)^{2m} = \left(\frac{\mathrm{i}g\sqrt{N_c}(\mathrm{e}^{p_1 t} - \mathrm{e}^{p_2 t})}{p_2 - p_1 + \mathrm{i}\omega(\mathrm{e}^{p_1 t} - \mathrm{e}^{p_2 t})}\right)^{2m} \tag{6}$$

The result of numerical calculations using Eq. (5) is represented in Fig. 1. Figure 1 shows the effect of detuning $\delta = \omega_a - \omega$ on the occupation probability of state $|M, N\rangle$ with $M = 10, N = 10, N_c = 100$. The figures correspond to different δ. From Fig. 1 we see that the detuning affects the period and amplitude of the occupation probability, the frequency of oscillation also relates to the number of BEC atoms. Figure 2 shows the occupation probability of BEC atoms in the case that the field is initially in the Fock state $|N\rangle$ with $N = 20$ (namely, transition from $|020\rangle$ to final state $|020\rangle$). From Fig. 2 we see, the occupation probability of the BEC atoms is oscillatory function, frequency of the oscillation depends on the number of

BEC atoms, the large the number of BEC atoms, the shorter the period of oscillations. This property can be used to probe the appearance of BEC. However, it is difficult to prepare an exact number (Fock) state, but it is practical to use the coherent state as an initial state of the cavity field. Because $b^+b + a^+a = N$ is invariant under action of H, then the subspace $\{|m,n\rangle, m+n = N\}$ spanned by eigenstate of N is an invariant subspace which is closes under the action of the interaction term $H_I = g\sqrt{N_c}(ba^\dagger + ab^\dagger)$, therefore, in the case that field is initially in coherent state $|\alpha\rangle$, the occupation probability of BEC atoms is given by $P = \sum_n |c_n|^2 |(1/\sqrt{n!})\langle 0,0|a^n(t)|0,n\rangle|^2 = \sum_n |c_n|^2 P_n$, where $|\alpha\rangle = \sum_n \exp(-|\alpha|^2/2)\alpha^n/\sqrt{n!}$.

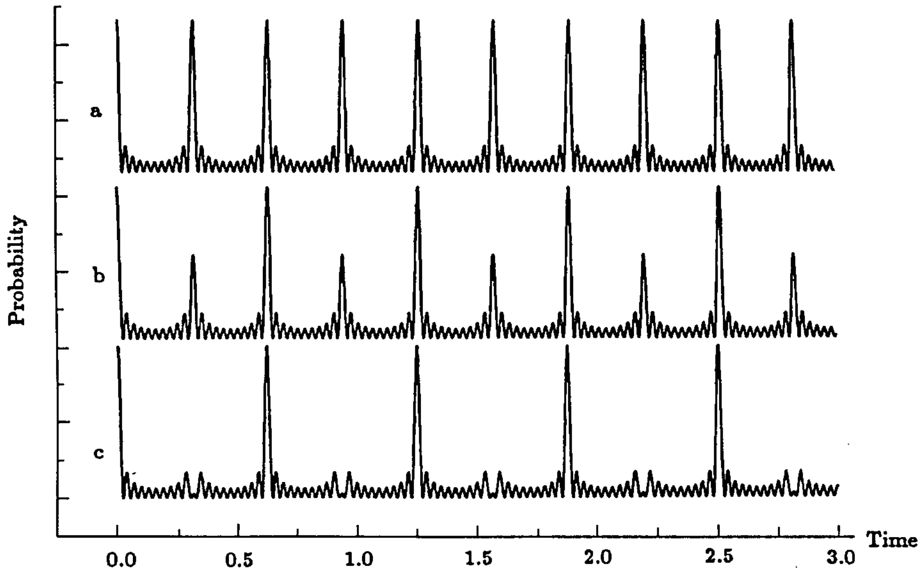

Fig. 1. Effect of detuning on the occupation probability of state $|M,N\rangle$ with $M = N = 10, N_c = 100$. Figures 1a, 1b and 1c correspond to different detuning δ, (a) $\delta = 0$, (b) $\delta = 0.5$, (c) $\delta = 1$.

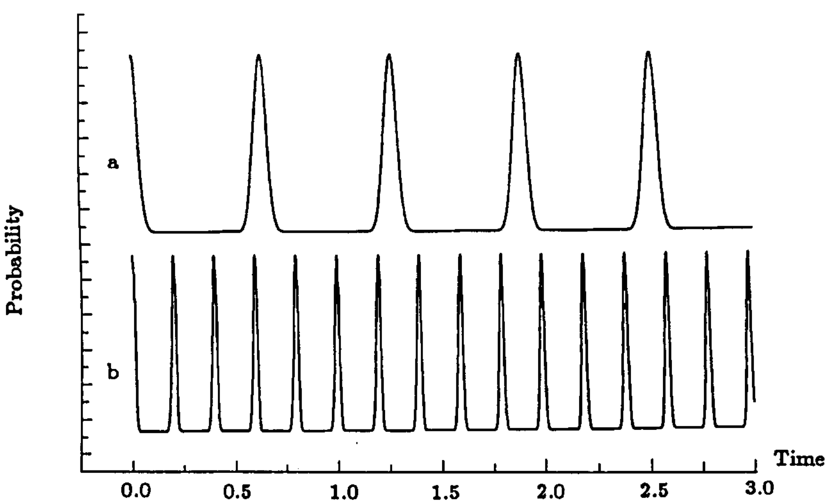

Fig. 2. The occupation probability of BEC atoms with the field having exact $N = 20$ photons. Figures 2a and 2b correspond to different N_c. (a) $N_c = 100$, (b) $N_c = 1000$.

Further we study the dynamics of BEC atoms with dissipation. The two-level atom excited to its upper-state $|e\rangle$ must undergoes a transition to the ground state $|g\rangle$ due to the interaction with the background of the electromagnetic vacuum. This process of spontaneous emission can be treated with the Wigner–Weisskopff or the Born–Markov approximation base on the "first" laws in quantum mechanics microscopically, the dissipation of the cavity field must be taken into account since the field might interact with wall of the cavity as well. However, only for the need of application, we can directly introduce a phenomenological losing term $H' = -i\hbar\Gamma|e\rangle\langle e| - i\hbar\gamma a^{\dagger}a$ to be added to the original Hamiltonian (1). It is emphasized that 2Γ is the rate of spontaneous emission of atoms into the free-space background and the total Hamiltonian $\tilde{H} = H + H'$ is non-Hermitian. A similar procedure to the case without dissipation is used to deduce the Hamiltonian for BEC

$$\tilde{H} = \hbar(\omega - i\gamma)a^{\dagger}a + \hbar(\omega_a - i\Gamma)b^{\dagger}b + g\sqrt{N_c}(ba^{\dagger} + ab^{\dagger}). \tag{7}$$

The above equation is very similar to the Hamiltonian without dissipation, however, the system described by Eq. (7) is a open one different from Eq. (2), by using the same procedure of the case without dissipation, we obtain the $a(t)$ and the $b(t)$ as same as Eq. (4) beside replacing $p_{1,2}$ by $p_{\pm}$, where

$$p_{\pm} = \frac{-i(\omega_a - i\Gamma + \omega - i\gamma) \pm \sqrt{-(\omega_a - i\Gamma + \omega - i\gamma)^2 - 4g^2N_c + 4(\omega - i\gamma)(\omega_a - i\Gamma)}}{2},$$

therefore, the probability of transition from initial state $|M, N\rangle$ to final state $|n, m\rangle$ is

$$P = \sqrt{m!n!M!N!}\sum_{k=0}^{m}\frac{g^k(t)v^{m-k}(t)f^{N-k-M}(t)w^{M+k-m}(t)}{k!(m-k)!(n+m-k-M)!(M+k-m)!}. \tag{8}$$

Figure 3 shows the numerical results of Eq. (8) for the field is initially in the Fock state. From Fig. 3 we see that the occupation probability is a damping function of time, the damping rate depends on the number of BEC atoms.

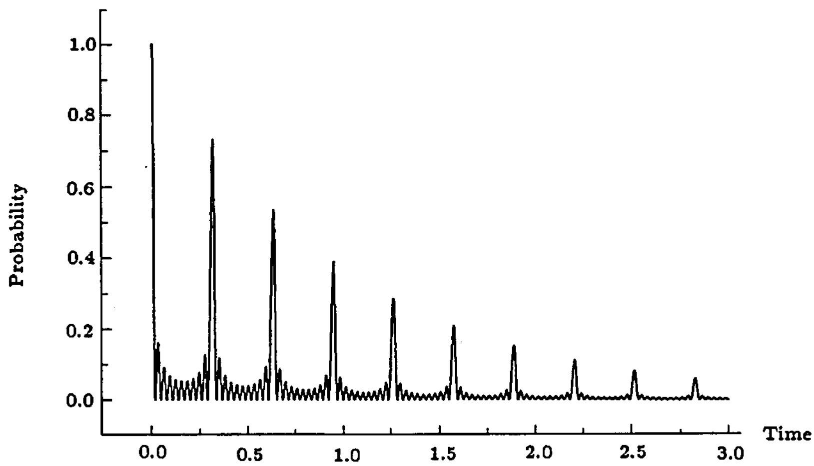

Fig. 3. The occupation probability of state $|10, 10\rangle$ with dissipation rate $\gamma = \Gamma = 0.025$, and there are $N_c = 100$ atoms in BEC.

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