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Quantum Light Excitation of Atoms in Ideal Bose–Einstein Condensate[1]

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Abstract

The spectrum shape of atomic radiation in the case of ideal Bose–Einstein condensate (BEC) excited by quantized light is derived through the Wigner–Weisskopf approximation. Besides the split and gap in the excitation spectrum, a doublet structure appears in the emission spectrum with different widths for the non-resonant case and the same widths for the resonant case. The survival probability is found to decrease exponentially for the non-resonant case and with oscillation for the resonant case. A suppressed radiation also appears in the non-resonant case. All these results can be used to detect the existence of BE condensation.

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I. Introduction

In last months it was reported that the Bose–Einstein condensates (BEC) of the cooled ^{87}Rb atoms,[1] ^{7}Li atoms[2] and Na atoms[3] were observed at very low temperatures, e.g., 0.17 μK for Rb, with very high phase space density. The achievement of atomic BEC is very important for both fundamental aspect of modern quantum theory and development of high technology. Therefore the experimental achievement of BEC will urge people to investigate various properties and manifestations of BEC in details from various aspects. Among the explicit manifestations of BEC are its optical properties. In fact, optical features of atomic gas of BEC have been theoretically studied for various cases, such as, the coherent scattering of weak light on BEC,[4] the existence of the gap in the excitation spectrum of polaritons in BEC,[5] the temperature-dependence of scattering,[6] the line shapes of scattered light from BEC,[7,8] and the nonlinear optical response of BEC atoms.[9]

It was shown in Ref. [5] by solving an effective field equation that a split and a gap are found in the excitation spectrum for a system of BEC atoms interacting with light field. This result derived by Politzer is based on the basic field theory and the assumption that the light field is classical. Other theoretical studies[6–9] also deal with the cases with classical probing lights. This assumption is correct for the strong light field, but, for a weak light such as in a cavity, the light must be quantized. The aim of this paper is to consider the influences of quantized light on the line shape of the emitted spectrum from BEC. It is very interesting that there exits asymmetric doublet structure in the emission spectrum, which has different widths for the non-resonant case and the same widths for the resonant case. It is also found that the survival probability decreases exponentially and the emission is suppressed for the former and oscillates with decay for the later. All of these optical features can be used to detect the existence of BE condensation.

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II. Effective Hamiltonian for BEC Atoms and Light Field

Let us first consider a system of two-level bosonic atoms without trap interacting with a light field of vacuum. By using creation and annihilation operators b_{gk}, $b_{gk}^{\dagger}$, b_{ek} and $b_{ek}^{\dagger}$ for the ground and exited atoms respectively, and a_q and $a_q^{\dagger}$ for the light field, the Hamiltonian can be written as

$$H = \sum_q \hbar\omega_q a_q^{\dagger} a_q + \sum_k \hbar\varepsilon_k b_{gk}^{\dagger} b_{gk} + \sum_k \hbar(\omega_a + \varepsilon_k) b_{ek}^{\dagger} b_{ek} + \hbar \sum_{kq} g(q)(b_{gk-q}^{\dagger} b_{e,k} a_{\sigma q}^{\dagger} + b_{gk-q} b_{e,k}^{\dagger} a_{\sigma q}) , \tag{1}$$

where $\varepsilon_k = \hbar k^2/2m$, $g(q) = \boldsymbol{e_k} \cdot \boldsymbol{d}\sqrt{\omega_q/2\epsilon_0\hbar^3 V}$, V is the volume of the system, q denotes the state of a photon with certain momentum, $\boldsymbol{q}$, and polarization, $\boldsymbol{\sigma}$, and $g\boldsymbol{k}$ and $e\boldsymbol{k}$ correspond to the atoms with momentum $\boldsymbol{k}$ in ground and excited states respectively. For the case with very low temperature, a macroscopic number of the bosonic atoms denoted by N_0 will be condensed on the ground state with momentum $\boldsymbol{k} = 0$. Then the operators b_0 and $b_0^{\dagger}$ for the ground state with zero momentum are usually replaced by a c-number

$$b_{g0}^{\dagger} = b_{g0} = \sqrt{N_0} \ . \tag{2}$$

After it is substituted into the Hamiltonian, the conservation of atomic number in original Hamiltonian will be destroyed. In terms of the above quasi-particle operators as well as the relations (2), an effective Hamiltonian is obtained as follows:

$$H = \sum_q \hbar\omega_q a_q^{\dagger} a_q + \sum_k{}' \hbar\varepsilon_k b_{gk}^{\dagger} b_{gk} + \sum_k \hbar(\omega_a + \varepsilon_k) b_{ek}^{\dagger} b_{ek} + \hbar\sqrt{N_0} \sum_k g(k)(b_{e,k} a_{\sigma k}^{\dagger} + b_{e,k}^{\dagger} a_{\sigma k}) + \hbar \sum_{kq}{}' g(q)[b_{gk-q}^{\dagger} b_{e,k} a_{\sigma q}^{\dagger} + b_{gk-q} b_{e,k}^{\dagger} a_{\sigma q}] , \tag{3}$$

where the prime is used to exclude the terms with $k = 0$ from the summation and the constant terms representing the ground-state energy of the BEC have been neglected. In the derivation of the above Hamiltonian, we have assumed that the light field is so weak, e.g., only a single-photon excitation is considered in the following, that it will not influence the condensation of the atomic system. The above Hamiltonian will be applied to studying the excitation spectrum and the dynamic behavior for the coupled system of the BEC atoms and the light field by solving the Schrödinger equation. The Wigner–Weisskopf approximation (WWA) will be invoked in our treatment, from which the spectrum shape and the dynamic behavior of the BEC system interacting with light can be obtained.[10,11]

III. Time-Dependent Solution with WWA

We now consider an excitation of the BEC system by a photon with momentum $\boldsymbol{k}$ (in following discussions, the index, σ, is omitted for definite polarization), then the interaction between the atoms and the light will stimulate the following sequence of excitations

$$|\boldsymbol{k},0,0\rangle \to |0,\boldsymbol{k},0\rangle \to |\boldsymbol{q},0,\boldsymbol{k}-\boldsymbol{q}\rangle \to |0,\boldsymbol{q},\mathbf{k}-\mathbf{q}\rangle \to \cdots , \tag{4}$$

where

$$|\boldsymbol{k},0,0\rangle = a_{\boldsymbol{k}}^{\dagger}|0\rangle , \qquad |0,\boldsymbol{k},0\rangle = b_{e,\boldsymbol{k}}^{\dagger}|0\rangle , \tag{5}$$

$$|\boldsymbol{k},0,\boldsymbol{k}'\rangle = a_{\boldsymbol{k}}^{\dagger} b_{g\boldsymbol{k}'}^{\dagger}|0\rangle , \qquad |0,\boldsymbol{k},\boldsymbol{k}'\rangle = b_{e,\boldsymbol{k}}^{\dagger} b_{g\boldsymbol{k}'}^{\dagger}|0\rangle \tag{6}$$

represent the light and atomic excitations respectively and $|0\rangle$ is the vacuum state. The last term in the sequence (4) represents two-atom excitation and the dots stands for more than two atom excitations. Generally, the exact expression can be expanded in terms of them and the coefficients of these terms will be proportional to g, g^2, g^3, $\cdots$, respectively. Since general solutions cannot be obtained exactly, a truncation has to be made. Since the coupling between the atoms and the light is very small in comparison with the energies of photon and atoms, we will neglect the fourth as well as the higher order terms and only the first three terms are preserved. Then, for one-photon excitation the solution of the Schrödinger equation can approximately be written as

$$|\Psi(t)\rangle = \exp(-\mathrm{i}\omega_k t)\Big[A(t)|\boldsymbol{k},0,0\rangle + B(t)|0,\boldsymbol{k},0\rangle + \sum_{\boldsymbol{q}}{}' C_{\boldsymbol{q}}(t)|\boldsymbol{q},0,\boldsymbol{k}-\boldsymbol{q}\rangle + \cdots\Big]. \tag{7}$$

The corresponding equations for the coefficients satisfy

$$\mathrm{i}\frac{\mathrm{d}A(t)}{\mathrm{d}t} = g(k)\sqrt{N_0}B(t)\,, \quad \mathrm{i}\frac{\mathrm{d}B(t)}{\mathrm{d}t} + (\omega_k - \omega_a - \varepsilon_k)B(t) = g(k)\sqrt{N_0}A(t) + \sum_{\boldsymbol{q}} g(q)C_{\boldsymbol{q}}(t) \tag{8}$$

and

$$\mathrm{i}\frac{\mathrm{d}C_{\boldsymbol{q}}(t)}{\mathrm{d}t} + (\omega_k - \omega_q - \varepsilon_{k-q})C_{\boldsymbol{q}}(t) = g(q)B(t)\,. \tag{9}$$

The above system of equations just represents a typical problem in quantum dissipation theory[11,12] and can be solved approximately by the standard method with WWA and the solution for an initial condition with one photon and no atomic excitation is

$$A(t) = \frac{p_+ + \gamma_k/2 - \mathrm{i}\,\Delta}{p_+ - p_-}\,\mathrm{e}^{p_+ t} - \frac{p_- + \gamma_k/2 - \mathrm{i}\,\Delta}{p_+ - p_-}\,\mathrm{e}^{p_- t}\,, \tag{10}$$

$$B(t) = \frac{-\mathrm{i}\,g(k)\sqrt{N_0}}{p_+ - p_-}\,(\mathrm{e}^{p_+ t} - \mathrm{e}^{p_- t})\,, \tag{11}$$

$$C_{\boldsymbol{q}}(t) = -g(q)g(k)\sqrt{N_0}\Big[\frac{1}{p_+ - p_-}\Big(\frac{\mathrm{e}^{p_+ t}}{p_+ - p_3} - \frac{\mathrm{e}^{p_- t}}{p_- - p_3}\Big) + \frac{\mathrm{e}^{p_3 t}}{(p_3 - p_+)(p_3 - p_-)}\Big], \tag{12}$$

where

$$p_\pm = \frac{1}{2}\Big(-\frac{\gamma_k}{2} + \mathrm{i}\,\Delta\Big) \pm \frac{1}{2}\Big[\Big(\frac{\gamma_k}{2} - \mathrm{i}\,\Delta\Big)^2 - 4g(k)^2 N_0\Big]^{1/2}\,, \tag{13}$$

$$p_3 = \mathrm{i}(\omega_k - \omega_q - \varepsilon_{k-q}) \tag{14}$$

with $\Delta = \omega_k - \omega_a' - \varepsilon_k$ ($\omega_a' = \omega_a - \delta_k$ is the renormalized atomic frequency) where the width γ_k and the energy shift δ_k are given by

$$\gamma_k = 2\pi\rho(\omega_k - \varepsilon_{k-q})|g(\omega_k - \varepsilon_{k-q})|^2, \qquad \delta_k = P\int \frac{|g(\omega_q)|^2\rho(\omega_q)\mathrm{d}\omega_q}{\omega_q - \omega_k + \varepsilon_{k-q}}\,, \tag{15}$$

where P means principal value integration. Then the final spectrum distribution is given by

$$P(\omega_q) = \int |C_{\boldsymbol{q}}(\infty)|^2\rho(\omega_q)\Omega_q \propto \rho(\omega_q)\frac{g(k)^2\cdot N_0\cdot g(q)^2}{|p_3 - p_+|^2|p_3 - p_-|^2} = \frac{\rho(\omega_q)g(k)^2\cdot N_0\cdot g(q)^2}{|\omega_q - \omega_q^+|^2|\omega_q - \omega_q^-|^2}\,, \tag{16}$$

which gives a doublet structure and the positions of the peaks are defined by the real part of the corresponding effective photon frequencies

$$\omega_q^\pm = \omega_k - \varepsilon_{k-q} + \mathrm{i}p_\pm\,, \tag{17}$$

and the widths of the peaks are their imagine parts. Besides these stationary properties, the dynamic behavior can be derived from the solution, for example, the survival probability is given by $|\langle k,0,0|\Psi(t)\rangle|^2 = |A(t)|^2$.

IV. Line Shapes in Non-Resonance Cases

In the following discussions we consider two limit cases.

The first case is far from resonance. Since the frequencies of the photon and the atoms are of the order of $10^{12} \sim 10^{14}$ s^{-1}, the number of the BEC atoms, N_0, is now less than 10^8 and the interaction between the atom and the light with dipole approximation is $g \approx 10^3$ s^{-1}, we have the limit of effective weak interaction $g(k) \cdot \sqrt{N_0} \ll |\Delta i + \gamma_k/2|$, which leads to

$$p_\pm \approx \begin{cases} -\dfrac{\gamma_k}{2} \cdot \dfrac{N_0 g(k)^2}{\gamma_k^2/4 + \Delta^2}\,, \\ -\dfrac{\gamma_k}{2} + \mathrm{i}\Delta\,, \end{cases} \tag{18}$$

respectively. Then the spectral distribution is

$$P(\omega_q) = \frac{\rho(\omega_q) g(k)^2 \cdot N_0 \cdot g(q)^2}{[(\omega_q - \omega_k + \varepsilon_{k-q})^2 + (\gamma_k')^2/4] \cdot [(\omega_q - \omega_a' + \varepsilon_q - v_k q)^2 + \gamma_k^2/4]} \tag{19}$$

with

$$\gamma_k' = \gamma_k \frac{N_0 \cdot g(k)^2}{\gamma_k^2/4 + \Delta^2}, \qquad v_k = \frac{\hbar k}{m}\,. \tag{20}$$

The two peaks in the spectral distribution represent two atomic dressed states, one of which is approximately light excitation with width, γ_k, centered in $\omega_k - \varepsilon_{k-q}$ nearby the light frequency ω_k, and the other is the atomic excitation with decay width, γ_k', in the position $\omega_a' + \varepsilon_q - v_k q$, i.e., the renormalized atomic transition frequency shifted by Doppler effect and the photon recoil.

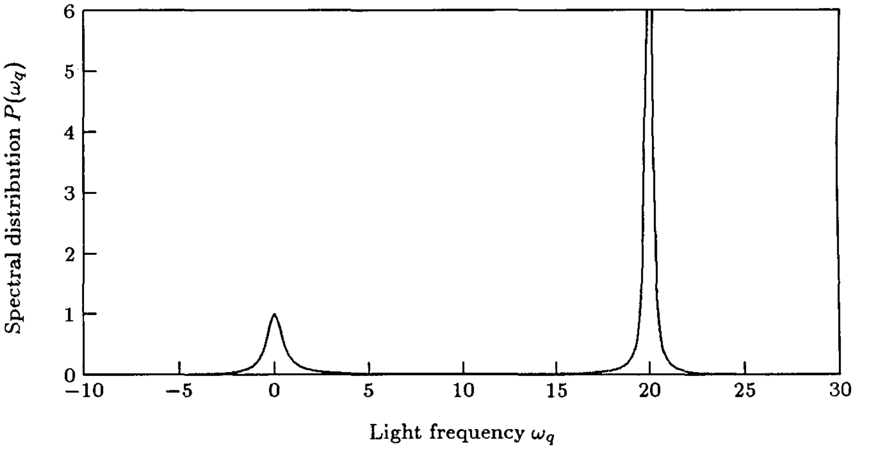

Fig. 1. The asymmetric doublet spectrum shape of the BEC atoms in the non-resonance case. The parameters and the units are given in the following: the atomic energy ω_a is selected as the origin, $g\sqrt{N_0} = 10$ and $\omega_k = 20$ with a unit of $\gamma_k = 1$ and an arbitrary unit for spectral strength.

Let us estimate numerically some of the above results. The natural width of the atom is about 10^6 s^{-1}, i.e., $\gamma_k \sim 10^6$ s^{-1}. With dipole approximation, $g\sqrt{N_0}$ is about 10^3 to 10^8 s^{-1} for the condensed atoms from one to 10^8. Therefore, even for the resonant case with small number of the condensed atoms, we may also have $g\sqrt{N_0} \ll \gamma_k$, then the above approximation is also correct. In these cases, which means that $\gamma'_k \ll \gamma_k$, namely, the width of the photon is larger suppressed by the atoms in BEC in comparison with the width of atomic decay in free space. From the above results, we find that an asymmetric doublet structure will appear on the emitted spectrum for the non-resonance case, which is shown in Fig. 1. Now we consider the dynamical behavior of the photon excitation. Substituting the approximation (18) into solution (10), the first term of $A(t)$ is approximately equal to one and the second term will be zero, then the survival probability of the incident photon will be obtained

$$|\langle \boldsymbol{k}, 0, 0|\Psi(t)\rangle|^2 = |A(t)|^2 \approx \exp(-\gamma'_k t)\,. \tag{21}$$

If the BEC happens in a superconducting cavity (high Q cavity) or the density of the condensed atoms is very high, the interaction between the atoms and the photon will be far greater than the width of the atoms, $g\sqrt{N_0} \ll \gamma_k$, which leads to another limit and is discussed in the following section.

V. Line Shapes in Resonance Cases

Next we consider the second case, i.e., the case of resonance, $\Delta = \omega_k - \omega'_a - \varepsilon_k = 0$. Since the number of the condensed atoms $N_0 \gg 1$, it is possible that $g(k)\sqrt{N_0} \gg \gamma_k$ according to the above estimation of orders and then we have

$$p_\pm \approx \frac{1}{2}\Big(-\frac{\gamma_k}{2} \pm 2\mathrm{i}g(k)\sqrt{N_0}\Big), \tag{22}$$

that is

$$\omega^\pm \approx \omega_k - \varepsilon_{k-q} \pm g(k)\sqrt{N_0} - \mathrm{i}\frac{\gamma_k}{4} = E_\pm - \mathrm{i}\frac{\gamma_k}{4}\,. \tag{23}$$

In this case, the spectral distribution becomes

$$P(\omega_q) = \frac{\rho(\omega_q)g(k)^2 \cdot N_0 \cdot g(q)^2}{[(\omega_q - E_+)^2 + \gamma_k^2/4] \cdot [(\omega_q - E_-)^2 + \gamma_k^2/4]}\,. \tag{24}$$

From this result we see that the distance of split in the spectral line is about $2g(k)\sqrt{N_0}$ proportional to the square root of the number of the condensed atoms. However, observing such split experimentally requires that $g(k)\sqrt{N_0} \gg \gamma_k$, namely, a high resolution spectroscopic instrument is needed for the detection of atomic BEC through this kind of spectral property. This fact can be used to detect the appearance of the atomic BEC. This symmetric spectral structure is illustrated in Fig. 2.

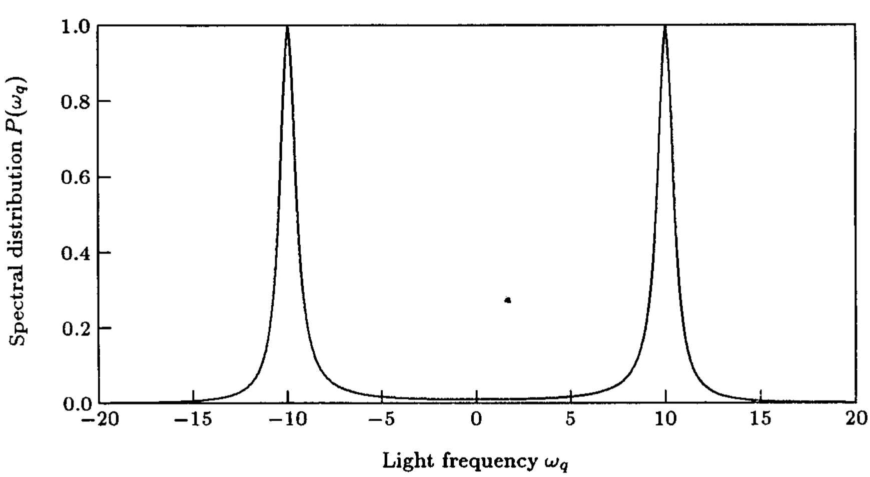

Fig. 2. The symmetric doublet spectrum shape of the BEC atoms in the resonance case. The parameters are the same as in Fig. 1 except $\omega_k = 0$.

The dynamic behavior can be obtained from the evolution solutions of $A(t)$ and $B(t)$ by substituting Eq. (22) into it, which exhibits a decay process with the oscillation structure

$$|\langle k,0,0|\Psi(t)\rangle|^2 = |A(t)|^2 = e^{-\gamma_k t/2}\cos^2(g(k)\sqrt{N_0}t)\,, \tag{25}$$

$$|\langle 0,k,0|\Psi(t)\rangle|^2 = |B(t)|^2 = e^{-\gamma_k t/2}\sin^2(g(k)\sqrt{N_0}t)\,. \tag{26}$$

These denote that the survival probability decays with oscillation and the width is equal to a half of that of atomic transition, but the oscillation frequency is proportional to square root of the number of the condensed atoms.

For the spontaneous emission of an excited atom on BEC, a similar doublet structure of the spectrum can be derived in the same procedure as the above by considering an initial condition of the atom in the excited state and no photon in BEC. The conclusion here is similar to that of the spontaneous emission of a single atom in a single-mode quantized cavity,[12] in which the vacuum Rabi splitting of the energy level of an atom into two closely spaced sublevels proportional to $g(k)$ was obtained by Kimble and his colleagues.[13] But in the present case, for atomic BEC the split is much easier to be observed since the split spacing is proportional to $g(k)\sqrt{N_0}$.

Finally let us discuss little bit about the case with interatomic interactions. The strength of the effective interaction between the atoms is about $g'N_0/\hbar < 10^{-2}\ \mathrm{s}^{-1}$ (here $g' = 4\pi l/m$ is proportional to the scattering length, l, of the S-wave for the low-energy ground-ground state scattering) and then the energy for the quasi-particle (resulting from the interatomic interaction) $\leq \hbar k^2/2m \ll g(k)\sqrt{N_0}$.[1,4,5] Therefore the energies of quasi-particle can be neglected apparently. However, because the situations in the recent experiments of atomic BEC[1-3] enjoy the weak interatomic interaction, our discussions in the present paper is only an ideal approach in comparison with these real experiments. A more detailed discussion about the case with interatomic interaction will be given in a separate paper.

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