Commun. Theor. Phys. 26 (1996) pp. 385–388
© International Academic Publishers Vol. 26, No. 4, December 15, 1996

Effect of Both Adiabatic and Non-Adiabatic Atomic Motion in Generalized JC Model

Cheng WANG, Li ZHANG and Chang-Pu SUN
Physics Department, Northeast Normal University, Changchun 130024, China
(Received July 3, 1995)

Abstract

This paper deals with the dynamic behavior of atom-evolution affected by motion of atomic mass-center in a high Q cavity. A generalized JC model is treated on this problem. The adiabatic condition is presented also in an explicit form. The time-evolution of the wavefunction is obtained, via quantum high-order adiabatic approximation (HOAA). Then the transition probability and average energetic distributions of each part at any time are discussed.

I. Introduction

Along with the development of experiment technique, the description of interaction between a two-level ideal atom and a single quantified mode of electromagnetic field, Jaynes–Cummngs (JC) models,[1] explained many quantum behaviors. Most of them were already verified by experiments. The quantum collapse and rehabilitate of inverted atom population were observed,[1] for instance. This model also is combined with cavity quantum electrodynamics to form a new area and show more physical phenomena. For example, there are experiments of atom cooling and trapping,[2] inverse Stern–Gerlach effect and atom-interference.[3] The key to problems mentioned above is the momentum exchange between the ultra-short electromagnetic field and the mass-center motion of a slower atom. Therefore the coupling principle between the motion of mass-center and atomic inter-energy is important. In this paper, we focus on the atom non-adiabatic dynamic process in the JC model due to atomic mass-center motion. The adiabatic condition is also discussed. The non-adiabatic modification of the wavefunction evolution is explicated, relating the atom mass-center movement, during the time evolving in the JC model with the method of high-order adiabatic approximation (HOAA). Then we search the transition probability and energy distribution.

II. The Non-Adiabatic Wavefunction of the JC Model

In JC model, $\hat{H}_{\rm JC}$ describes an interaction between a two-level atom and a single mode of electromagnetic field

$$\hat{H}_{\rm JC} = \hbar\omega_0 a^\dagger a + \frac{1}{2}\hbar\omega_a(|e\rangle\langle e| - |g\rangle\langle g|) + \hbar g(Z)(a|e\rangle\langle g| + a^\dagger|g\rangle\langle e|)\,, \tag{1}$$

where $|g\rangle$ and $|e\rangle$ represent lower and upper atomic states, respectively, ω_0 is the field frequency, a and $a^\dagger$ present photon annihilation and creation operators, respectively; ω_a is atomic transition frequency; we also have the coupling constant

$$g(Z) = g_0 \sin kZ\,, \tag{2}$$

which is the electric dipole matrix at the location Z of the atom.

In original JC model, $g(Z)$ is considered as a constant. Now, we have to regard Z as a function depending on the time in the coordinates of the mass-center motion, as the cavity field is inhomogeneous. If the initial velocity is ν, we could suppose $Z = \nu t$, under the zero-order approximation. The eigenvalue of $\hat{H}_{\rm JC}$

$$E_{+,-}(n) = \Big(n + \frac{1}{2}\Big)\hbar\omega_0 \pm \hbar\sqrt{\frac{\Delta^2}{4} + g^2(Z)(n+1)} \tag{3}$$

and the corresponding eigenfunctions

$$|\varphi_+\rangle = \cos\frac{\vartheta_n}{2}|g,n+1\rangle + \sin\frac{\vartheta_n}{2}|e,n\rangle\,, \qquad |\varphi_-\rangle = -\sin\frac{\vartheta_n}{2}|g,n+1\rangle + \cos\frac{\vartheta_n}{2}|e,n\rangle\,, \tag{4}$$

depend on time through $g = g(Z)$. Where $|n\rangle$ are number states of the field mode with $a^\dagger a|n\rangle = n|n\rangle$, the mixing angle is defined as

$$\tan\,\vartheta_n = \frac{2g\sqrt{n+1}}{\Delta}\,, \tag{5}$$

and $\Delta = \omega_0 - \omega_a$ is the field-atom frequency detuning.

Let the solution of the time-dependent Schrödinger equation be

$$|\psi(t)\rangle = \sum_{k=\pm} C_k(t)\,\mathrm{e}^{(1/\mathrm{i}\hbar)\int_0^t E_k\,\mathrm{d}t'}|\varphi_k\rangle\,. \tag{6}$$

Substituted it into Schrödinger equation

$$\mathrm{i}\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}_{\mathrm{JC}}(t)|\psi(t)\rangle\,. \tag{7}$$

The result is

$$\begin{aligned} C_k(t) - C_k(0) + \int_0^t \langle\varphi_k(t')|\dot{\varphi}_k(t')\rangle C_k(t')\,\mathrm{d}t' \;=\; \\ -\sum_{1=\pm}\int_0^t \langle\varphi_k(t')|\dot{\varphi}_l(t')\rangle \exp\Big[\mathrm{i}\int_0^{t'}\omega_{kl}(t'')\,\mathrm{d}t''\cdot C_1(t')\,\mathrm{d}t'\Big]\,, \end{aligned} \tag{8}$$

where $\omega_{kl} = (E_k - E_l)/\hbar$.

If we consider the following relationship

$$\begin{aligned} &\langle\varphi_+(t)|\dot{\varphi}_+(t)\rangle = 0\,, \qquad \langle\varphi_-(t)|\dot{\varphi}_-(t)\rangle = 0\,, \\ &\langle\varphi_-(t)|\dot{\varphi}_+(t)\rangle = \frac{\dot{\vartheta}}{2}\,, \qquad \langle\varphi_-(t)|\dot{\varphi}_+(t)\rangle = -\frac{\dot{\vartheta}}{2}\,, \end{aligned} \tag{9}$$

equation (8) can be integrated by part as

$$\begin{aligned} C_+(t) - C_+(0) &= \frac{\dot{\vartheta}_n}{2}\frac{\exp\left[\mathrm{i}\int_0^t\omega_{+-}(t')\,\mathrm{d}t'\right]}{\mathrm{i}\omega_{+-}}C_-(t) + O\Big(\frac{\dot{\vartheta}_n}{\omega_{+-}}\Big)\,, \\ C_-(t) - C_-(0) &= \frac{\dot{\vartheta}_n}{2}\frac{\exp\left[-\mathrm{i}\int_0^t\omega_{+-}(t')\,\mathrm{d}t'\right]}{\mathrm{i}\omega_{+-}}C_+(t) + O\Big(\frac{\dot{\vartheta}_n}{\omega_{+-}}\Big)\,. \end{aligned} \tag{10}$$

If the adiabatic condition $|\dot{\vartheta}_n/\omega_{+-}| \ll 1$ is satisfied, the right terms of Eq. (10) could be neglected. Then

$$C_+(t) = C_+(0)\,, \qquad C_-(t) = C_-(0)\,. \tag{11}$$

The wavefunction under the adiabatic condition can be gotten as

$$|\psi^0(t)\rangle = C_+(0)\exp\Big[-\mathrm{i}\int_0^t\omega_+(t')\,\mathrm{d}t'|\varphi_+(t)\rangle + C_-(0)\exp\Big[-\mathrm{i}\int_0^t\omega_-(t')\,\mathrm{d}t'\Big]|\varphi_-(t)\rangle\,, \tag{12}$$

where $\omega_k = E_k/\hbar$, $k = \pm$.

III. The Discussion About the Adiabatic Condition

Now let us discuss the adiabatic condition

$$\Big|\frac{\dot{\vartheta}}{\omega_{+-}}\Big| = \Big|\frac{2\Delta\sqrt{n+1}\,g_0k\nu\cos(k\nu t)}{(\Delta^2 + 4g_0^2(n+1)^2\sin^2(k\nu t))^{3/2}}\Big|, \tag{13}$$

which is enjoyed by the value of the term in Eq. (10). When $|\dot{\vartheta}/\omega_{+-}|$ is much smaller than unity, we say the adiabatic condition holds. Thus we deal with this factor to obtain the cases whenever this condition is satisfied.

The parameters for typical Redberg atom are[2] $n = 50$, $g_0 = 4.2 \times 10^5$ Hz, $k = \pi/\sqrt[3]{V} \approx 10^{-3}$ 1/m ($V = 95$ mm^3 is volume of the cavity).

In this case, the various situations about adiabatic conditions are listed in table 1.

Table 1. The adiabatic condition.

Mass center velocity ν	Relation of Δ and g_0	$\left|\dot\vartheta/\omega_{+-}\right|$ ($\sin kZ \approx 1$)	$\left|\dot\vartheta/\omega_{+-}\right|$ ($\sin kZ \approx 0.01$)	Adiabatic condition
$\nu = 0.25$ m/s	$\lvert\Delta\rvert \gg 2g_0\sqrt{n+1} \approx 1$	$\ll 1$	$\ll 1$	hold
	$\lvert\Delta\rvert \approx 10^3 \sim 10^4$	$\ll 1$	10^{-1}	not hold
($T = 3.2\ \mu k$)[2]	$\lvert\Delta \ll 2g_0\sqrt{n+1}$	$\ll 1$	$\ll 1$	hold
	$\lvert\Delta\rvert \approx 10^5$	10^{-2}	10^{-2}	not hold
$\nu = 2.5$ m/s ($T = 3.2 \times 10^{-4} k$)	$\lvert\Delta\rvert \approx 10^5$	10^{-1}	10^{-1}	not hold
$\nu = 25$ m/s ($T = 3.2 \times 10^{-2} k$)	$\lvert\Delta\rvert \approx 10^5$	1	1	not hold

For example, in column 2, when $\nu = 2.5$ m/s, or $T = 3.2 \times 10^{-4}$ k, at area $\sin kZ \approx 1$, non-adiabatic term is 0.1, thus the adiabatic condition does not hold.

If the above adiabatic condition could not be satisfied, it is necessary to discuss the case under the non-adiabatic condition. We calculated the wavefunction of the non-adiabatic case by means of the HOAA.[4,5]

Let initial state be $|\psi(0)\rangle = |\varphi_-\rangle$. From Eq. (10), we can get

$$C_-(t) = 1\,, \qquad C_+(t) = -\frac{\mathrm{i}\exp\mathrm{i}\,[\Omega_+(t) - \Omega_-(t)]}{\omega_{+-}}\frac{\dot\vartheta_n}{2}\,, \tag{14}$$

where $\Omega_\pm(t) = \int_0^t \omega_\pm(t')\,\mathrm{d}t'$. Substituting it into Eq. (5), we get the time-evolution wavefunction under the first-order approximation

$$\begin{aligned}|\psi^1(t)\rangle = {} & \mathrm{e}^{-\mathrm{i}\Omega_-(t)}\Big(-\sin\frac{\vartheta_n}{2}|g,n+1\rangle + \cos\frac{\vartheta_n}{2}|e,n\rangle\Big) - \\ & \mathrm{i}\,\mathrm{e}^{-\mathrm{i}\Omega_-(t)}\frac{1}{\omega_{+-}}\frac{\dot\vartheta_n}{2}\Big(\cos\frac{\vartheta_n}{2}|g,n+1\rangle + \sin\frac{\vartheta_n}{2}|e.n\rangle\Big)\,.\end{aligned} \tag{15}$$

IV. The Transition of Atom in the Cavity

When the adiabatic condition is satisfied, the adiabatic approximate wavefunction is

$$|\psi^0(t)\rangle = \mathrm{e}^{-\mathrm{i}\Omega_-(t)}\Big(-\sin\frac{\vartheta_n}{2}|g,n+1\rangle + \cos\frac{\vartheta_n}{2}|e,n\rangle\Big). \tag{16}$$

The transition probability from initial state to state $|g,n+1\rangle$ is

$$p(|\varphi_-\rangle \to |\varphi_+\rangle) = |\langle\varphi_+|\psi^0(t)\rangle|^2 = 0\,, \tag{17}$$

that shows no transition between the two states under the adiabatic condition. If we consider the influence of the non-adiabatic terms, the corresponding transition probability is

$$\begin{aligned}& P_{\mathrm{non}}(|\varphi_-\rangle \to |\varphi_+\rangle) = |\langle\varphi_+|\psi^1(t)\rangle|^2 = \\ & \Big|\Big(\langle n+1,g|\cos\frac{\vartheta_n}{2} + \langle n,e|\sin\frac{\vartheta_n}{2}\Big)\Big[\mathrm{e}^{-\mathrm{i}\Omega_-(t)}\Big(-\sin\frac{\vartheta_n}{2}|g,n+1\rangle + \cos\frac{\vartheta_n}{2}|e,n\rangle\Big) - \\ & \mathrm{i}\,\mathrm{e}^{-\mathrm{i}\Omega_-(t)}\frac{1}{\omega_{+-}}\frac{\dot\vartheta_n}{2}\Big(\cos\frac{\vartheta_n}{2}|g,n+1\rangle + \sin\frac{\vartheta_n}{2}|e,n\rangle\Big)\Big]\Big|^2 = \frac{1}{4}\frac{\dot\vartheta_n^2}{(\omega_{+-})^2}\,.\end{aligned} \tag{18}$$

Now the probability is no longer zero. It is directly proportional to the term $\dot{\vartheta}_n$. And it is also shown that the transition probability depends on the square of the atomic incident velocity. The larger incident energy is, the higher transition probability is. Additionally, when the atoms is near to the wall of the cavity, the transition probability is larger.

V. The Energy Motion of the System

Finally, let us discuss the average energy of each part: atom, field and interaction of both. The atom average energy at time t is

$$\langle\psi^1(t)|\frac{1}{2}\hbar\omega_a(|e\rangle\langle e|-|g\rangle\langle g|)|\psi^1(t)\rangle=\frac{1}{2}\hbar\omega_a\cos\vartheta\Big[1-\frac{1}{(\omega_{+-})^2}\Big(\frac{\dot{\vartheta}_n}{2}\Big)^2\Big], \tag{19}$$

where, the first term is the average energy at $t=0$, which is the energy when the atom at the initial state. The second term is contributed by the non-adiabatic term. The average energy is larger when the transition probability is higher.

Here, the average energy of the electromagnetic field at time t is

$$\begin{aligned}&\langle\psi^1(t)|\hbar\omega_0 a^\dagger a|\psi^1(t)\rangle=\\&\hbar\omega_0\Big\{\Big[n+\sin^2\frac{\vartheta_n}{2}\Big]+\Big[\frac{n}{(\omega_{+-})^2}\Big(\frac{\dot{\vartheta}_n}{2}\Big)^2+\frac{1}{(\omega_{+-})^2}\Big(\frac{\dot{\vartheta}_n}{2}\Big)^2\cos^2\frac{\vartheta_n}{2}\Big]\Big\},\end{aligned} \tag{20}$$

where, the first term is the average energy at $t=0$, which is the energy when the atom at the initial state. The second term is contributed by the non-adiabatic term.

The average interaction energy at time t is

$$\begin{aligned}&\langle\psi^1(t)|\hbar g(Z)(a^\dagger|g\rangle\langle e|+a|e\rangle\langle g|)|\psi^1(t)\rangle=\\&-\hbar g(Z)\sqrt{n+1}\,\sin\vartheta_n+\hbar g(Z)\frac{1}{(\omega_{+-})^2}\Big(\frac{\dot{\vartheta}_n}{2}\Big)^2\sqrt{n+1}\sin\vartheta_n\,,\end{aligned} \tag{21}$$

where the first term is the average energy at $t=0$, which is the energy when the atom is at the initial state; the second term is contributed by the non-adiabatic term. When the transition probability increases, the interaction energy increases, too. The coupling between the photon and atom is weakened.

VI. Conclusion

Bases on the above discussion, we can say that in JC model, the adiabatic condition may be unsatisfied. When the condition is not satisfied, the non-adiabatic effect is an important factor in the transition probability and the distribution of the energies.

References

[1] C.H. Hong and L. Mandel, Phys. Rev. Lett. **54(4)** (1985) 323.
[2] S. Haroche, M. Brane and J.M. Raimond, Europhys. Lett. **14** (1991) 19.
[3] E. Arimondo, W.D. Phillips and F. Strumia, *Laser Manipulation of Atoms and Ions*, North Holland, Amsterdam (1992).
[4] C.P. SUN, J. Phys. **A21** (1988) 1588.
[5] X.G. WANG and C.P. SUN, J. Mod. Optics **42** (1995) 515.