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Wei-Norman Algebraic Method Solving the Evolution of the Coherent States of Electron in Two Dimensions[1]

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Abstract

The Wei-Norman algebraic method is used to solve the evolution of electron in two-dimensional cases in this letter. We show that the coherent states of electron in two dimensions can be generated by the interaction of electron with a certain type of external electromagnetic field.

There is much interest in the behaviour of electron in two dimensions at present. A main reason for this is that investigating the motion of electron in two dimensions is essential to understanding the quantized Hall effect[1–2].

In this letter we consider an electron in a two-dimensional plane π pierced everywhere by a strong stationary magnetic field $\vec{B} = B\hat{e}_z$ normal to the plane π. If we apply an electric field $\vec{E} = E(t)\vec{e}_y$ perpendicular to $\vec{B}$ and let it change slowly so that the inducted magnetic field can be neglected in comparison with the strong magnetic field $\vec{B}$, the electrons in certain initial states will evolve into the coherent states of an equivalent harmonic oscillator[3]. These states are called the coherents of electrons.

The Hamiltonian of our problems is

$$\hat{H}(t) = \frac{1}{2m}\left[\hat{p}_x + \frac{yeB}{c}\right]^2 + \frac{1}{2m}\hat{p}_y^2 - \frac{eB\hat{S}_z}{mc} - eE(t)y\,, \tag{1}$$

where we use the Landau gauge $\vec{A} = (-By, 0, 0)$.

Now, assuming

$$\Psi(t) = \phi(y, t) \otimes \exp\left[\frac{i}{\hbar}p_x \cdot x\right] \otimes x_{\frac{1}{2}}(m_s) \tag{2}$$

and substituting it into the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(t) = \hat{H}(t)\Psi(t)\,, \tag{3}$$

we obtain

$$i\hbar\frac{\partial}{\partial t}\phi(y, t) = \hat{H}_{\text{eff}}(t) \cdot \phi(y, t)\,, \tag{3a}$$

where the effective Hamiltonian is

$$\hat{H}_{\text{eff}}(t) = \frac{1}{2m}\hat{p}_y^2 + \frac{1}{2}m\omega_B^2(y - y_0)^2 - E(t)\cdot ey + \mathcal{E}_0 - \frac{1}{2}\hbar\omega_B\,, \tag{4}$$

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where

$$\omega_B = \frac{|e|B}{mc}, \qquad y_0 = -\frac{cp_x}{eB}, \qquad \mathcal{E}_0 = \frac{\hbar\omega_B}{2} - \frac{eB\hbar m_s}{cm}.$$

Equation (4) is formally identified with the Hamiltonian for the forced oscillator along the axis y.

Using a canonical transformation

$$\begin{aligned} A &= \frac{[m\omega_B(y-y_0)+i\hat{p}_y]}{\sqrt{2m\hbar\omega_B}}, \\ A^+ &= \frac{[m\omega_B(y-y_0)-i\hat{p}_y]}{\sqrt{2m\hbar\omega_B}}, \end{aligned} \tag{5}$$

that satisfies $[A, A^+] = 1$, the above Hamiltonian can be expressed in a normal form

$$\hat{H}_{\text{eff}}(t) = \hbar\omega_B A^+A + f(t)[A^+ + A] + \lambda(t) + \mathcal{E}_0, \tag{6}$$

where

$$f(t) = -eE(t)\sqrt{\frac{\hbar}{2m\omega_B}}, \qquad \lambda(t) = -ey_0E(t).$$

For the case with $E(t) = 0$, the eigenvalues of this effective Hamiltonian (6) are

$$\tilde{E}_n = \hbar\omega_B n + \mathcal{E}_0, \qquad n = 0, 1, 2, \cdots, \tag{7}$$

which are called Landau levels[4]. The eigenfunctions corresponding to the energy levels $\tilde{E}_n$ are easily written as

$$\begin{aligned} \phi_n(y) = \langle y|n\rangle &= \frac{\langle y|A^{+n}|0\rangle}{\sqrt{n!}} \\ &= \exp\Big[-\frac{m\omega_B(y-y_0)^2}{2\hbar}\Big]H_n\Big[\sqrt{\frac{m\omega}{\hbar}}(y-y_0)\Big]. \end{aligned} \tag{8}$$

We define the coherent states of electron as

$$|z, p_x, m_s\rangle = g(t)\exp\Big[\frac{i}{\hbar}p_x x\Big]x_{\frac{1}{2}}(m_s)|z\rangle, \tag{9}$$

where $z \in$ the complex field $\mathbb{C}$, and

$$|z\rangle = \exp[zA^+ - z^*A]|0\rangle = e^{-|z|^2/2}\sum_{n=0}^{\infty}\frac{z^n}{n!}|n\rangle \tag{10}$$

is a coherent state of the equivalent harmonic oscillator

$$\hat{H}_0 = \hbar\omega A^+A + \mathcal{E}_0.$$

It will be shown that the solutions of the Schrödinger equation subjected to the time-dependent Hamiltonian $\hat{H}_{\text{eff}}$ are just the coherent states of electrons as defined above.

To this end, we use the Wei-Norman technique[5] to solve the Schrödinger equation in the interaction representation

$$i\hbar\frac{\partial}{\partial t}\hat{U}_I(t) = V_I(t)\hat{U}_I(t), \tag{11}$$

where

$$V_I(t) = \exp\Big[-\frac{i\hat{H}_0 t}{\hbar}\Big]\hat{V}_I \exp\Big[\frac{i\hat{H}_0 t}{\hbar}\Big]$$
$$= f(t)[A^+ e^{i\omega_B t} + A e^{-i\omega_B t}] + \lambda(t) , \qquad (12)$$
$$\hat{H}_0 = \hbar\omega_B A^+ A + \mathcal{E}_0 , \qquad \hat{V}_I = f(t)[A^+ + A] + \lambda(t) .$$

Supposing

$$\hat{U}_I(t) = \exp\Big[\frac{\tilde{x}(t)}{i\hbar}\Big]\exp\Big[\frac{\tilde{y}(t)A^+}{i\hbar}\Big]\exp\Big[\frac{\tilde{z}(t)A}{i\hbar}\Big] \qquad (13)$$

and substituting it into Eq. (11), we obtain

$$\begin{cases} \dot{\tilde{x}}_{(t)} - \dfrac{\dot{\tilde{z}}(t)\tilde{y}(t)}{i\hbar} = \lambda(t) , \\ \dot{\tilde{y}}_{(t)} = f(x)e^{i\omega_B t} , \\ \dot{\tilde{z}}_{(t)} = f(x)e^{-i\omega_B t} . \end{cases} \qquad (14)$$

It is easily solved as

$$\begin{cases} \tilde{x}_{(t)} = \displaystyle\int_0^t \lambda(\tau)d\tau - \frac{i}{\hbar}\int_0^t \Big[f(\tau)e^{-i\omega_B \tau}\int_0^\tau f(u)e^{i\omega_B u}du\Big]d\tau , \\ \tilde{y}_{(t)} = \displaystyle\int_0^t f(\tau)e^{i\omega_B \tau}d\tau , \\ \tilde{z}_{(t)} = \displaystyle\int_0^t f(\tau)e^{-i\omega_B \tau}d\tau . \end{cases} \qquad (15)$$

Then, we have the time evolution operator

$$\hat{U}(t) = \exp\Big[\frac{i\hat{H}_0 t}{\hbar}\Big]\hat{U}_I(t)\exp\Big[-\frac{i\hat{H}_0 t}{\hbar}\Big]$$
$$= \exp\Big[\frac{1}{i\hbar}\Big(\int_0^t \lambda(\tau)d\tau - \frac{i}{\hbar}\int_0^t\Big[f(\tau)e^{-i\omega_B\tau}\int_0^\tau f(u)e^{i\omega_B u}du\Big]d\tau\Big)\Big] \qquad (16)$$
$$\times\exp\Big[\frac{1}{i\hbar}\cdot e^{-i\omega_B t}\int_0^t f(\tau)e^{i\omega_B\tau}d\tau A^+\Big]\times\exp\Big[\frac{1}{i\hbar}e^{i\omega_B t}\int_0^t f(\tau)e^{-i\omega_B\tau}d\tau A\Big] .$$

Thus, if an electron in the ground state $|0(f)\rangle$ of $\hat{H}(0)$ at time $t = 0$

$$|\,0(f)\,\rangle = \exp\Big[-\frac{f(t)A^+}{\hbar\omega_B} - \frac{f^*(t)}{\hbar\omega_B}\cdot A\Big]|0\rangle \otimes \exp\Big[\frac{i}{\hbar}p_x x\Big] \otimes x_{\frac{1}{2}}(m_s) , \qquad (17)$$

then, this state has evolved into the state at time $t = T$

$$|\tilde{\phi}(T)\rangle = \hat{U}(T)|0(f)\rangle = \exp\Big(\frac{\tilde{x}(T)}{i\hbar}\Big)\exp\Big(\frac{\tilde{y}(T)A^+}{i\hbar}\Big)|0(f)\rangle \otimes \exp\Big[\frac{i}{\hbar}p_x x\Big] \otimes x_{\frac{1}{2}}(m_s) , \qquad (18)$$

which is just the coherent state of electron and satisfies

$$A|\tilde{\phi}(T)\rangle = \frac{1}{i\hbar}\tilde{y}(T)|\tilde{\phi}(T)\rangle . \qquad (19)$$

It is easy to prove that the state (18) results in the minimun uncertainty of measurements in the canonical variables, i.e.,

$$\langle(\Delta y)^2\rangle\langle(\Delta p_y)^2\rangle = \frac{1}{4}\hbar^2 . \qquad (20)$$

Thus, the coherent state of electron is a most classical state in two dimensions.

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References

[1] V. Klitzing, G. Dorda and M. Pepper, Phys. Rev. Lett. **45**(1980)479.

[2] H. Aoki, Rep. Prog. Phys. **50**(1987)655.

[3] R.J. Glauber, Phys. Rev. **131**(1963)2766; J.R. Klauder and B.S. Skagerstan, *Coherent State*, World Scientific (1985).

[4] L.D. Landau and E.M. Lifshitz, *Quantum Mechanics*, Pergamon Press (1977).

[5] J. Wei and E. Norman, J. Math. Phys. **4A**(1963)575.